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STUDY OF $\pi^+\pi^-$ SCATTERING IN $\pi^- p \longrightarrow \pi^+\pi^- X^o$ VIA A CHEW LOW EXTRAPOLATION*

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ABSTRACT

An unconstrained Chew Low extrapolation has been performed using 50,000 events from the reaction $\pi^- p \longrightarrow \pi^+\pi^- X^o$ at 15 GeV/c, with $-t<0.3$ $(\text{GeV/c})^2$. An energy independent phase shift analysis was performed. Results on the $\pi\pi$ cross section, p-wave and s-wave phase shifts in the mass range (525-975) MeV are presented. We find a unique solution for the s-wave phase shift below 750 MeV.

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We have studied π-π elastic scattering for dipion masses between 525 and 975 MeV using data from a wire spark chamber experiment performed at SLAC on the process $\pi^- p \rightarrow \pi^+ \pi^- X^0$ at 15 GeV. Information on real $\pi\pi$ scattering has been obtained from these data by extrapolating the observed $\pi^+\pi^-$ differential cross section as a function of the momentum transfer squared, t, given to the neutral baryonic system, to the pion pole $t = m_\pi^2$, as originally suggested by Goebel[1] and Chew and Low.[2] Two aspects of the present analysis should be emphasized. (i) For the first time a substantial amount of data, with good t resolution, exists for $-t < m_\pi^2$; this is the t region for which the extrapolation function varies most rapidly. (ii) The effects of absorption were taken into account by removing the conventional OPE constraint that the cross section for $\pi^- p \rightarrow \pi^+ \pi^- n$ vanishes at $t = 0$.[3] In this paper, we report the results for the $\pi\pi$ elastic cross section, the differential cross section, and the p-wave and the I = 0 s-wave $\pi\pi$ phase shifts.

The subject of $\pi\pi$ scattering has been intensively studied during the past few years.[4] For dipion masses below $K\bar{K}$ threshold, the scattering is elastic and dominated by the p-wave resonance, the ρ meson. The I = 0 s-wave phase shift, δ_0^0, has proven to be the most difficult component to determine. Early analyses[5] determined δ_0^0 largely from the s-p interference term (the $\cos\theta$ term) in the $\pi^+\pi^-$ angular distribution, which leads to two solutions for the s-wave: δ_0^0 and $\delta_0^{0'} = \pi/2 - (\delta_0^0 - \delta_1^1)$, where δ_1^1 is the p-wave phase shift. Inasmuch as the two solutions were found to coincide in the neighborhood of the ρ, one was left with the well-known up-down ambiguity for δ_0^0, both above and below the ρ meson.[6] In principle, this ambiguity can be resolved by evaluating the isotropic term in the π-π angular distribution at the pion pole. In practice, this imposes a requirement of large statistics and accurate small t data because of the smallness of the isotropic contribution to the cross section in the ρ region. The present analysis extrapolates the complete angular distribution to the pion pole. We find a unique solution for δ_0^0 below the ρ meson; however, the ambiguity persists above the ρ. Similar results have been reported recently by Baton et al.[7]

The Chew-Low extrapolation formulae used in this analysis are given below:

(a) for $\pi^- p \longrightarrow \pi^+ \pi^- n$

$$\frac{d\sigma_{\pi\pi}}{d\cos\theta} = \lim_{t \to \mu^2} \frac{2\pi k_{lab}^2 (t-\mu^2)^2}{f^2 M_{\pi\pi} \sqrt{0.25 M_{\pi\pi}^2 - \mu^2}} \cdot \frac{d^3\sigma_n}{dt\, dM_{\pi\pi}\, d\cos\theta} \qquad (1)$$

where k_{lab} is the incident pion momentum in the laboratory, μ is the pion mass, $M_{\pi\pi}$ is the mass of the outgoing π-π system, $f^2 = 0.081$ is the πN coupling constant, $d^2\sigma_n/dt\, dM_{\pi\pi}$ is the cross section for the reaction $\pi^- p \longrightarrow \pi^+\pi^- n$ in the intervals dt and $dM_{\pi\pi}$, $\sigma_{\pi\pi}$ is the cross section for $\pi^+\pi^- \longrightarrow \pi^+\pi^-$, and θ is the angle between the incident π^- and the outgoing π^- in the $\pi^+\pi^-$ rest frame.

(b) for $\pi^- p \longrightarrow \pi^+\pi^- X^o$ (where X^o represents all final states at the nucleon vertex),

$$\frac{d\sigma_{\pi\pi}}{d\cos\theta} = \lim_{t \to \mu^2} C(t-\mu^2)^2 \frac{d^3\sigma_{X^o}}{dt\, dM_{\pi\pi}\, d\cos\theta} \qquad (2)$$

where C is a numerical constant which cannot be determined as easily as in reaction (a). $\frac{d^2\sigma_{X^o}}{dt\, dM_{\pi\pi}}$ has the same definition as in (1) except that the cross section is integrated over all the missing mass available.

The data from reaction (a) are used to determine the absolute normalization at the pion pole. The larger sample comprising reaction (b) is used to determine the shape of the angular distribution at the pion pole, and when taken together with the normalization constraint from reaction (a) the decomposition into partial waves is obtained.

A detailed description of the geometry and properties of the wire spark chamber spectrometer has been given previously.[8,9] Briefly, the spectrometer consists of scintillation hodoscopes and wire spark chambers before and after an analysing magnet. The trigger required two charged particles to pass through the spectrometer. The data sample contains 17,121 events of reaction (a) and 48,516 events of reaction (b) with the X^0 mass less than 1.39 GeV[10] where only events with $|t| < 0.3$ $(GeV/c)^2$ and with $525 \leq M_{\pi^+\pi^-} \leq 975$ MeV are included. Approximately one half of these data have been used in our previous studies of the $\pi^+\pi^-n$ system.[3,8,11]

Possible sources of background in these two reactions have been investigated and only two sizable sources identified: (i) K^+K^- pairs reconstructed as $\pi^+\pi^-$ for that portion of the data where the Cerenkov counter did not allow π-K separation (~6% effect);[12] and (ii) $N\pi$ missing mass background being included within the neutron missing mass cut defining reaction (a), (~5% effect). Neither of these effects contributes significantly at the pion pole and the use of a high order unconstrained extrapolation function allows for their small contribution in the physical region.

The acceptance of the spectrometer was evaluated by generating fake events from those $\pi^+\pi^-$ events which were detected but averaging over the target production volume and the azimuthal angle about the incident π^- beam using standard Monte Carlo techniques. A weight inversely proportional to the detection probability was assigned to each event. Only data for which $|\cos\theta| \leq 0.7$ and with weights <50[13] have been used in this analysis; θ is the polar angle of the outgoing π^- in the Gottfried Jackson frame.[14] A correction must be made to this procedure since there are certain kinematic configurations which are never detected by the spectrometer, and the location of these regions varies with t.

From our studies of the angular distribution[8] and from previous analyses, it has been shown that no partial waves higher than $\ell = 1$ contribute significantly to the $\pi^+\pi^-$ final states for $M_{\pi\pi} < 950$ MeV. This implies that there is no term higher than $\cos 2\phi$ in the $\pi\pi$ angular distribution, where ϕ is the azimuthal angle. As a consequence, for fixed t, θ, and missing mass, the average value of $D(\phi) = 1 - 2\eta(\cos\theta \cos 3\phi)$ must be zero $[\eta(x)$ being the usual step function $\eta(x)=0$ for $x<0$, $\eta(x) = 1$ for $x>0]$. In a ϕ, $\cos\theta$ plot the regions for which acceptance losses occur are less than a half period of $\cos 3\phi$.[15] $D(\phi)$ has been chosen to equal -1 in those regions for which acceptance zeroes occur. Consequently, the average of $D(\phi)$ for all the real weighted events will measure precisely the missing fraction discussed above. The correction at the rho mass is of the order of 20% at $t = -0.08$ $(\mathrm{GeV}/c)^2$, 10% at $t = -0.02$ and less than 5% at $t = -0.01$. This method has nevertheless the disadvantage of increasing the statistical errors by $\sqrt{2}$.

In order to perform the extrapolations to the pion pole, the data were binned into a matrix consisting of nine 50 MeV wide mass intervals from 525 to 975 MeV and 13 t bins with $-t<0.3$ $(\mathrm{GeV}/c)^2$, including five t bins within the region $-t<m_\pi^2$. The $\pi^- p \longrightarrow \pi^+\pi^- n$ events comprising reaction (a) were integrated over the angular region, $|\cos\theta| < 0.7$, and this integrated cross section was then extrapolated to the pion pole for each mass interval using Eq. (1) to give the absolute normalization. The angular distribution at the pion pole was determined by dividing the events of reaction (b) into three or four bins in $\cos\theta$ between -0.7 and 0.7 according to the statistics available, and then extrapolating using Eq. (3) shown below. Equation (3) explicitly includes the normalization constant obtained from the separate integrated $\pi^+\pi^-n$ extrapolation.

$$\frac{d\sigma_{\pi\pi}}{d\cos\theta} = \left\{ \int_{-0.7}^{0.7} \frac{d\sigma_{\pi\pi}}{d\cos\theta} \cdot d\cos\theta \right\}_{\pi^+\pi^-n} \cdot \lim_{t\to\mu^2} \frac{\dfrac{d^3\sigma_{X^0}}{dt\,dM_{\pi\pi}\,d\cos\theta}}{\displaystyle\int_{-0.7}^{0.7} \frac{d^3\sigma_{X^0}}{dt\,dM_{\pi\pi}\,d\cos\theta} \cdot d\cos\theta} \tag{3}$$

Two methods of extrapolation were used to reach $t = \mu^2$: (1) a polynomial in t fit to the physical region data, which is then evaluated for $t = \mu^2$; and (2) conformal mapping[7,16] with a change of variable $x = \frac{at+b}{t+d}$, where a, b, d were determined such as to have our measured region from $x = -1$ to $x = 1$, and the cut for $9\mu^2 < t < \infty$, symmetrically disposed around the physical region; i.e., the t interval $(9\mu^2, \infty)$ will be transformed to the x interval $(\infty, -a)$, (a, ∞). The second technique proved to be more powerful. The results presented below were obtained from extrapolations using a cubic polynomial in the variable x with no constraints imposed on the cross sections at $t = 0$. The results for the $\pi\pi$ differential cross section are shown in Fig. 1. The black points are the results of the subsequent phase shift analysis.

A phase shift analysis was performed for each of the nine $\pi^+\pi^-$ mass intervals separately (i.e., an energy independent analysis). Only s and p-waves have been included, and the $\pi^+\pi^-$ scattering has been assumed to be purely elastic. Further, the I=2 part which is small was taken from the study of Baton et al.[7] The results are shown in Fig. 2. Note that there are two solutions for the elastic $\pi\pi$ cross section, $\sigma_{\pi\pi}$ (Fig. 2a), and for the p-wave phase shift, δ_1^1 (Fig. 2b), for those mass values for which we find two solutions in the s-wave; the up and down notation refers to the corresponding s-wave solution. This occurs because (i) important contributions come from outside our normalization interval ($|\cos\theta| < 0.7$) and therefore depend on the phase shift; (ii) the normalization coming from reaction (a) is used only as a constraint in the fit. The results of Baton et al.,[7] for $\sigma_{\pi\pi}$ and δ_0^0 (obtained from their analogous purely elastic phase shift analysis) are included in Fig. 2a and 2c for comparison.

The elastic $\pi\pi$ cross section does reach the unitary limit in the ρ region. Furthermore, although no particular mass dependence is imposed in this energy independent phase shift analysis, the p-wave result, δ_1^1, is well represented by a Breit Wigner shape including centrifugal barriers.[17] The parameters found are $M = 761 \pm 5$ MeV and $\Gamma = 108 \pm 20$ MeV.

We obtain a unique solution for the $I = 0$ s-wave phase shift for dipion masses below the ρ meson. A similar result for δ_0^0 in this region was found in the analysis of Baton et al.[7] Above the ρ, the well-known up-down ambiguity persists for the three mass bins 800 through 900 MeV, again in good agreement with the results of Baton et al., shown in Fig. 2c; however, for the 950 MeV bin we find just a single solution.

It is of interest to examine the nature of the ambiguity in more detail. Figure 3 displays the χ^2 obtained for each mass interval when δ_0^0 is fixed and the p-wave refitted. It should be noted that although each of the bins 800 through 900 MeV has a minimum solution, the difference in χ^2 is no longer sufficient to exclude the possibility of the other solution. Rather, in this region, one has a relatively broad valley in χ^2 space. The two branches for the δ_0^0 solutions would not appear to be as distinct as the traditional up-down notation implies and the possibility must be considered that the s-wave may cross 90^o at a mass substantially higher than the mass range 700-750 MeV which is customarily assumed. Indeed, the data on $\pi^- p \to \pi^o \pi^o n$ implies that the s-wave phase shift remains near 90^o through the ρ region,[18] which, taken with Fig. 2c, would then imply that δ_0^0 cross 90^o in the vicinity of 900-950 MeV, and be changing rapidly just below $K\overline{K}$ threshold. Such a behavior is in accord with recent data on $\pi^+ p \rightarrow \pi^+\pi^-\Delta^{++}$,[19] where rapid variations in the s-wave amplitude in this mass region were reported.

To conclude, we have performed an unconstrained Chew Low extrapolation of our data on $\pi^- p \rightarrow \pi^+\pi^- X^0$ and determined the differential cross section for $\pi\pi$ scattering. An energy independent phase shift analysis finds that the elastic $\pi\pi$ cross section does reach the unitary limit at the ρ and yields a unique solution for the $I=0$ s-wave phase shift below the ρ. While the precise behavior of δ_0^0 above the ρ remains an open question, arguments are presented for the phase shift crossing 90^o at higher masses than conventionally accepted, viz. ~950 MeV. It is important to investigate this possibility further by extending systematic $\pi\pi$ phase shift studies to higher masses. This requires that D-wave effects and inelasticity be included, and therefore requires very high statistic experiments.

REFERENCES

1. C. Goebel, Phys. Rev. Letters 1, 337 (1958).

2. G. F. Chew and F. W. Low, Phys. Rev. 113, 1640 (1959).

3. P. Baillon *et al.*, Phys. Letters 35B, 453 (1971).

4. For a comprehensive discussion of $\pi\pi$ scattering results, see Proceedings of the Conference on $\pi\pi$ and $K\pi$ Interactions, Argonne National Laboratory, 14-16 May 1969.

5. E. Malamud and P. E. Schlein, Phys. Rev. Letters 19, 1056 (1967); L. J. Gutay *et al.*, Phys. Rev. Letters 18, 142 (1967); W. D. Walker *et al.*, Phys. Rev. Letters 18, 630 (1967).

6. Several authors have selected a preferred solution. See, for example, S. Marateck *et al.*, Phys. Rev. Letters 21, 1613 (1968); J. H. Scharenguivel *et al.*, Phys. Rev. 186, 1387 (1969). Both of these papers choose the down-up solution which corresponds to an I=0 s-wave $\pi\pi$ resonance near the ρ.

7. J. P. Baton, G. Laurens, and J. Reignier, Phys. Rev. Letters 33B, 525 (1970); Phys. Letters 33B, 528 (1970).

8. F. Bulos *et al.*, Phys. Rev. Letters 26, 1453 (1971).

9. G. T. Armstrong *et al.*, Report No. SLAC-PUB-801 (1970) (unpublished); B. Ratcliff, Stanford Doctoral Dissertation, Report No. SLAC-141; H. H. Williams, Stanford Doctoral Dissertation, Report No. SLAC-142.

10. For $M(X^o)$ much larger, the minimum momentum transfer becomes so large that the events are not useful in the extrapolation to $t = \mu^2$.

11. F. Bulos *et al.*, Phys. Rev. Letters 26, 1957 (1971).

12. Approximately one half of the data were obtained with the Cerenkov counter hodoscope gas pressure set to distinguish pions from kaons and protons and half with pions and kaons separated from protons.

13. The average weight is 10.

14. K. Gottfried and J. D. Jackson, Nuovo Cimento 33, 309 (1964).

15. Acceptance losses occur (i) for $\cos\theta<0$ only for $\frac{5}{6}\pi<\phi<\frac{7}{6}\pi$, (ii) for $\cos\theta>0$ only for $-\pi/6<\phi<\pi/6$.

16. R. E. Cutkosky and B. B. Deo, Phys. Rev. 174, 1859 (1968); S. Ciulli, Nuovo Cimento 61A, 787 (1969); 62A, 307 (1969).

17. Blatt and Weiskopf, Theoretical Nuclear Physics (Wiley and Sons, 1962), p. 361.

18. P. Sonderegger and P. Bonomy, Preprint of contribution to Lund International Conference on Elementary Particles (Abstract 372) June 25-July 1, 1969; E. I. Shibata *et al.*, Phys Rev. Letters 25, 1227 (1970); and private communication; J. T. Carroll, Ph.D. Thesis, Univ. of Wisconsin (1971).

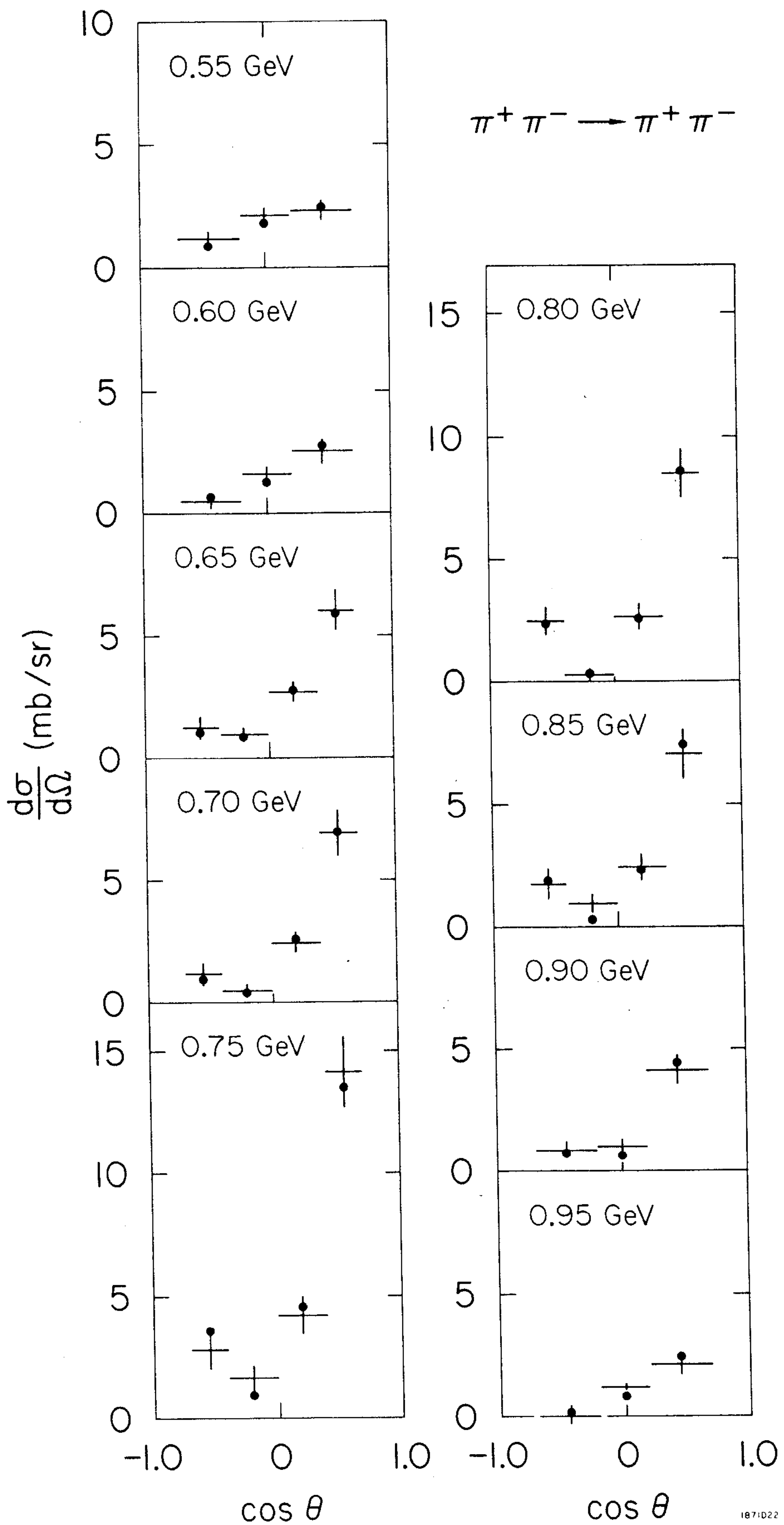
19. M. Alston-Garnjost *et al.*, Phys. Letters 36B, 152 (1971).

FIGURE CAPTIONS

1. Results for the $\pi^+\pi^-$ differential cross section for each 50 MeV wide $\pi^+\pi^-$ mass interval. The black points are the results of the phase shift analysis.

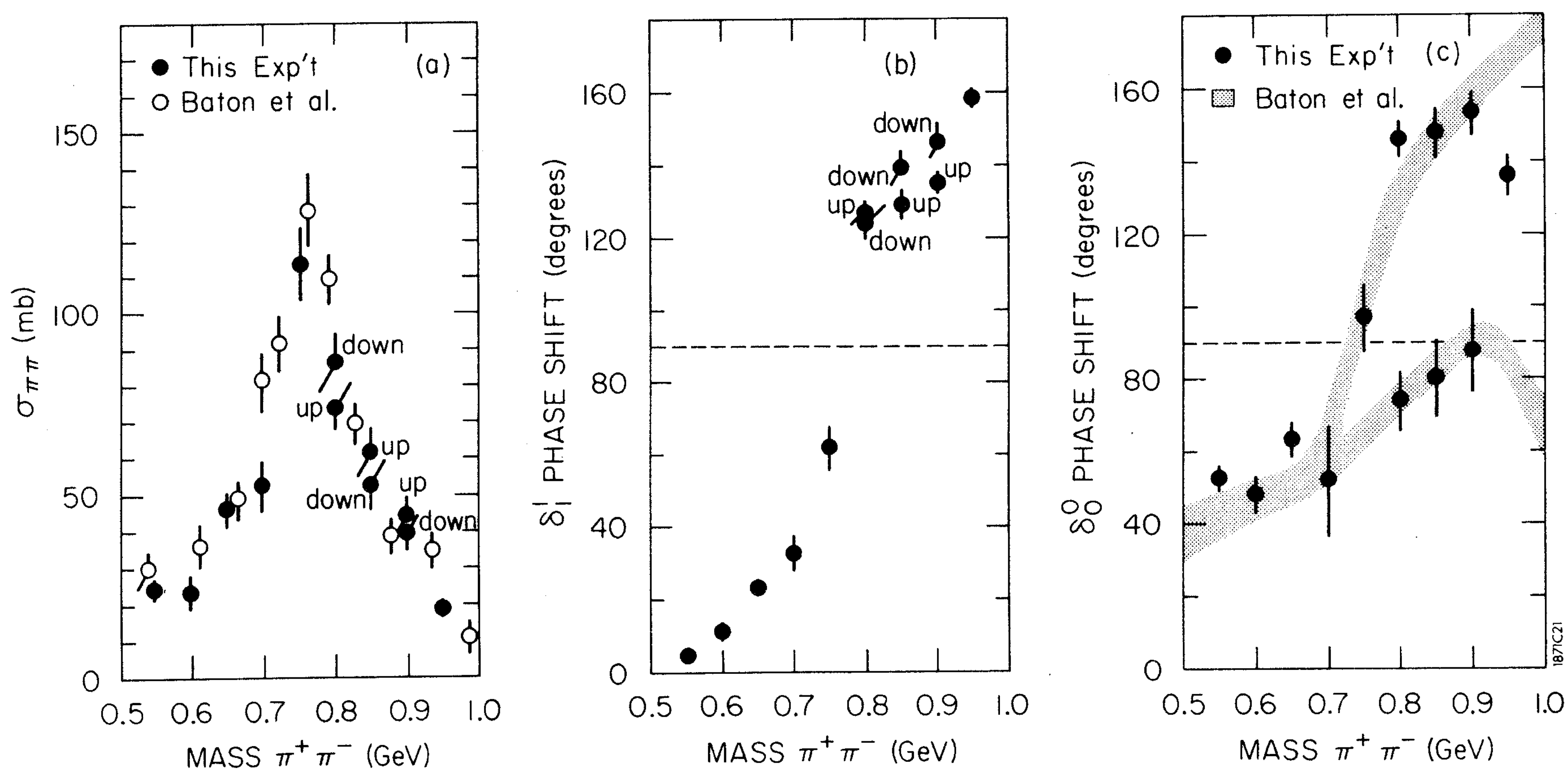
2. Results of the energy independent elastic $\pi\pi$ phase shift analysis: (a) the elastic $\pi\pi$ cross section, (b) the p-wave phase shift, (c) the I = 0 s-wave phase shift. In (a) and (b) up and down refers to the corresponding s-wave solution. For comparison, the results of Baton et al., are included in (a) and (c).

3. Variation of χ^2 as a function of the I = 0 s-wave phase shift for each of the nine mass intervals.



0.55 GeV
0.60 GeV
0.65 GeV
0.70 GeV
0.75 GeV
0.80 GeV
0.85 GeV
0.90 GeV
0.95 GeV
$\pi^+\pi^- \longrightarrow \pi^+\pi^-$
$\frac{d\sigma}{d\Omega}$ (mb/sr)
cos θ
cos θ
1871D22

Fig. 1



This Exp't
Baton et al.
(a)
$\sigma_{\pi\pi}$ (mb)
MASS $\pi^+\pi^-$ (GeV)
down
up
down
(b)
δ^1_1 PHASE SHIFT (degrees)
MASS $\pi^+\pi^-$ (GeV)
This Exp't
(c)
Baton et al.
δ^0_0 PHASE SHIFT (degrees)
MASS $\pi^+\pi^-$ (GeV)
1871C21

Fig. 2

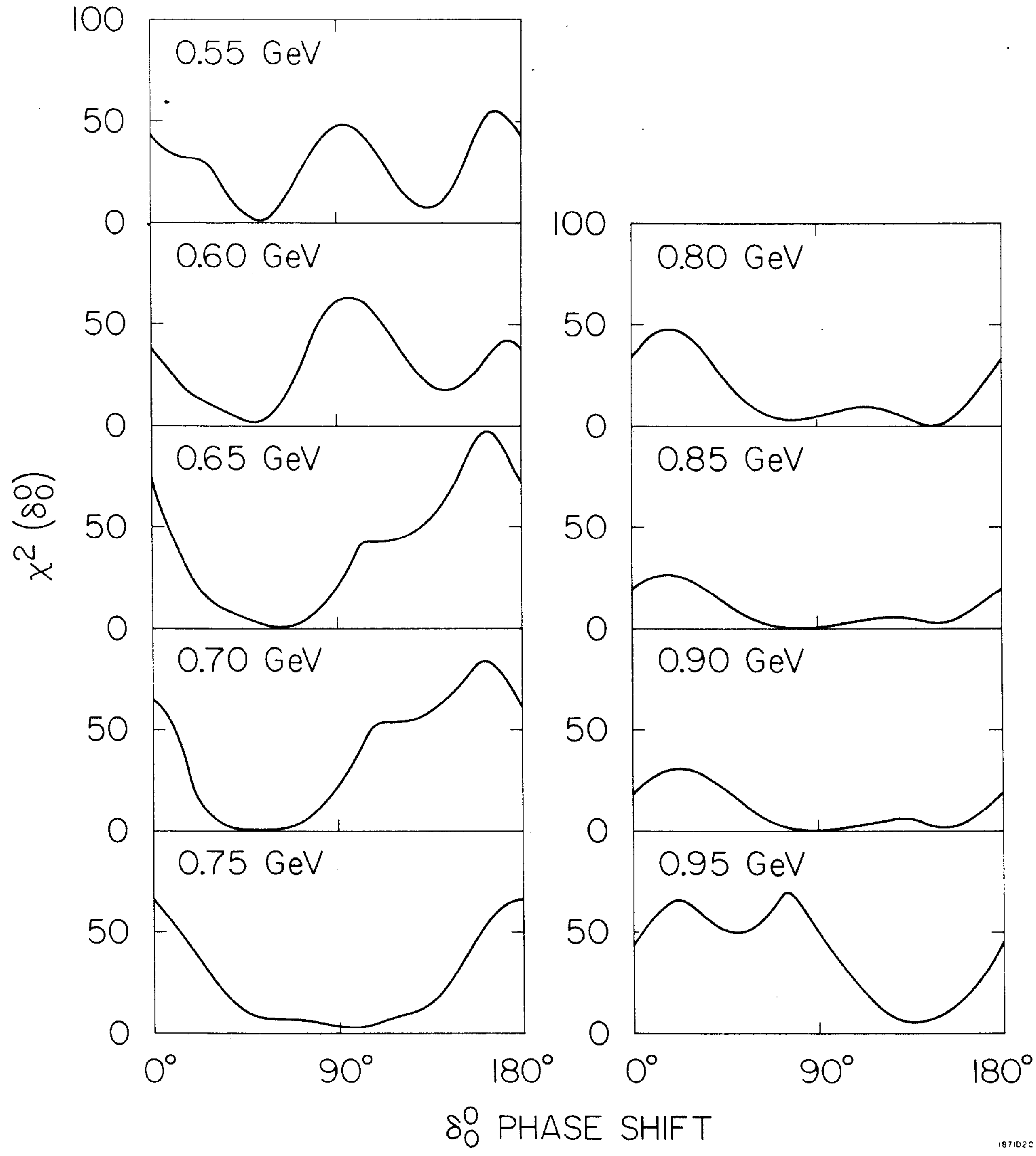

0.55 GeV
0.60 GeV
0.65 GeV
0.70 GeV
0.75 GeV
0.80 GeV
0.85 GeV
0.90 GeV
0.95 GeV
100
50
0
$\chi^2\ (\delta_0^0)$
0°
90°
180°
δ_0^0 PHASE SHIFT
1871D2C

Fig. 3