SPONTANEOUS BREAKING OF CHIRAL SYMMETRY IN A VECTOR-GLUON MODEL

Toshihide Maskawa and Hideo Nakajima
Department of Physics, Kyoto University
Kyoto, Japan

Solutions of the self-consistent equation for the fermion propagator in a vector-gluon model are fully examined in the light of spontaneous breaking of chiral symmetry.[1] In other words, the question is whether the fermion mass is spontaneously generated with the axial-vector current kept conserved.[2] In our analysis, we choose a suitable gauge ("Landau-like" gauge) for the vector-gluon and make for simplicity an pole-approximation to the massive vector-gluon propagator*) and no radiative correction to the vertex function in the Schwinger-Dyson equation for the fermion self-energy part. It can be shown that these approximations together with the ladder approximations to an axial-vector vertex function and to a peudoscalar vertex function, reproduce consistently the Ward-Takahashi identity for the axial-vector current. Therefore we can safely talk about the axial-vector current conservation within these approximations. Setting $i\gamma p+m_0+\sum(p)\equiv i\alpha(p^2)$, we obtain nonlinear equations for the function α and β, characterized by a set of parameters, i.e., the coupling constant g, the bare mass of the fermion m_0, and a cutoff Λ which is introduced as a regulator mass when neccessary. The results of our analysis are as follows: (1) It can be proved that the nonlinear equations without tht cutoff have solutions only in the case of $m_0=0$ and that the number of the solutions is infinity of continuum, i.e., there is a free continuous parameter specifying the solution, if $g^2/4\pi < (16/33)^2\pi$. Apparently this situation does not come from the freedom of fixing the mass scale as is thought in massless QED since the gluon is massive. Then we argue that this situation reflects the fact that if a cutoff Λ is introduced and led to infinity, $m_0(m,\Lambda)$ tends to zero, while the value of $2m_0 j_p(=-i\partial_\mu j_{5\mu})$ at the renormalization point tends not necessarily to zero, but to a value suitably fixed by m since the renormalization constant Z_p^{-1} tends to infinity.[3] In other words, we obtain the solutions $(\beta\neq0)$ with $m_0=0$, but these are not necessarily chiral invariant; or an infinitesimal bare mass can slip into the solutions, in the cutoff version.

---

*) We do not deal with spontaneous breakdown of gauge symmetry itself here, and assume that we already have massive vector-gluon somehow.

(2) We investigate the self-consistent equations with sufficiently large but finite cutoff Λ in order to make the above argument more definite. In this case the equation $m_0=0$ certainly means chiral invariance since no infinity appears in the formalism. And we obtain one and only one solution for any fixed m_0 if $g^2/4\pi \lesssim \pi/4$. Therefore when we set $m_0=0$, we are left with the only solution $\beta(p^2)\equiv 0$, which means that no "super-conducting" solution exists for such a small value of g^2 irrespective of the value of Λ, and no Nambu-Goldstone boson appears. (3) For rather strong coupling, however, more specifically, for $g^2/4\pi \gtrsim \pi/4$, we can demonstrate that the solution for the equation with the cutoff is not unique and there does exist another solution $\beta(p^2) > 0$ ("super-conducting" solution) than the solution $\beta(p^2)\equiv 0$ ("normal-state" solution) when $m_0=0$. Further the existence of many "super-conducting" solutions, is inferred. (4) It is also found that in the region $g^2/4\pi > 8\pi$, the "normal-state" solution for the equation without the cutoff, if any, should necessarily have an unphysical singularity, i.e., the propagator should have a singurality in the space-like momentum region. This fact implies that the "normal-state" solution becomes unstable for a sufficiently large value of g^2. To the question whether the spontaneous breaking of chiral symmetry occurs in a vector-gluon model or not, we have thus answered "no for the weak coupling, and yes for the strong coupling" as far as the lowest order approximations for the vector vertex and for the fermion-antifermion scattering kernel are concerned. The latter half of our answer would be valid also for the full theory, provided that the higher order corrections do not spoil badly the applicability of our method to the proofs. For the proofs of the results stated above, we fully utilized fixed point theorems in consideration of nonlinear integral equations.

In connection with our analyses we would like to comment on some attempts of the dynamical Higgs mechanism of the references 4 and 5. In the dynamical Higgs mechanism, i.e., the Higgs mechanism without canonical Higgs' scalar, a composite Nambu-Goldstone boson (massless excitation) which supersedes the canonical Higgs scalar is necessary. Generally it is utilized the fact that if one really has spontaneous symmetry-breaking mass generation of the fermion, there appears successfully the massless excitation. However, at the point where one chooses a required solution $(\beta\neq 0)$, one should be very careful not to take a solution resulting from nonconservation of the current (in the sense stated in the part (1)), but to take a solution which makes the current conserve. Despite of careful reasoning on this point in the reference 4, solutions essentially same as the solution resulting from nonconservation of the current were finally taken in the both papers.

We have proved in the part (2) that no spontaneously symmetry-breaking solution exists if the coupling is weak. Therefore, although their formalisms are promising the solutions that they took are not suitable for their formalisms or the solutions cause internal inconsistency in the formalisms.*) Recently, there have appeared similar comments on this point in the reference 6.

REFERENCES

1) T. Maskawa and H. Nakajima, Kyoto university preprint, KUNS 311.
T. Maskawa and H. Nakajima, Prog. Theor. Phys. 52 (1974), 1326.

2) R. Haag and A.J. Maris, Phys. Rev. 132 (1963), 2325.
K. Johnson, M. Baker and R. Willey, Phys. Rev. 136 (1964), B1111.
Th.A.J. Maris, V.E. Herscovitz and G. Jacob, Phys. Rev. Letters 12 (1964), 313.
H. Pagels, Phys. Rev. Letters 28 (1972), 1482: Phys. Rev. D7 (1973), 3689.
P. Langacher and H. Pagels, phys. Rev. D9 (1974), 3413.

3) M. Baker and K. Johnson, Phys. Rev. D3 (1971), 2516.
K. Johnson, M. Baker and R. Willey, Phys. Rev. 136 (1964), B1111.

4) R. Jackiw and K. Johnson, Phys. Rev. D8 (1973), 2386.

5) J.M. Cornwall and R.E. Norton, Phys. Rev. D8 (1973), 3338.

6) J. Smit, Phys. Rev. D10 (1974), 2473.

DISCUSSION

Question (B. Schroer) Is your axial current the gauge invariant current (which in renormalized perturbation theory developes anomalies) or the chiral symmetry current?

Answer (Nakajima) In our approximation scheme, there appears no Adler-anomaly term.

---

*) In the paper of the reference 4, the equation $\partial_\mu j_{5\mu}=0$ is derived from the equation of motion $\partial_\nu F_{\mu\nu}=g' j_{5\mu}$ and the antisymmetry of $F_{\mu\nu}$, while the solution ($\beta \gtrless 0$) of the Schwinger-Dyson equation for weak coupling requires an infinitesimal bare mass in the cutoff version and therefore the operator equation, $\partial_\mu j_{5\mu}=2m_0 i\bar{\Psi}\gamma_5\Psi$ of which the r.h.s. is clearly not vanishing, follows from the equation of motion of the fermion field. On the other hand $\partial_\nu F_{\mu\nu}=g' j_{5\mu}$ is still valid even in the presence of m_0. In these situation that naive manipulation of a given Lagrangian may cause the internal inconsistency, one cannot conclude the axial-vector current conservation only because of that antisymmetry of $F_{\mu\nu}$. In other words, in order to make the original formalism successful, one should take another solution ($\beta \gtrless 0$) which requires no bare mass even in the cutoff version. The situation is exactly the same also in the reference 5.