SLAC-PUB-216

RATIOS OF p-WAVE AMPLITUDES IN HYPERON DECAYS*

(Revised Version)

by

J. Weyers**

Stanford Linear Accelerator Center
Stanford University, Stanford, California

ABSTRACT

Assuming a current $\times$ current Hamiltonian and PCAC, it is shown that a strict pole approximation leads to predictions for the ratios of p-wave hyperon decays which are compatible with experiment.

* Work done under the auspices of the Belgian-American Educational Foundation and supported in part by the U.S. Atomic Energy Commission.

** On leave of absence from the Institut de Physique Théorique, Université de Louvain, Heverlé, Belgium.

Recently Sugawara[1] and Suzuki[2] have shown that as far as s-wave amplitudes are concerned, the universal current-current picture of weak interactions can be extended successfully to nonleptonic hyperon decays.

It is the purpose of the present note to show that, in a strict pole approximation, a very reasonable agreement with the present experimental situation can also be achieved for the relative ratios of the p-wave decay amplitudes. We emphasize the fact that most of our results are derived without a priori octet dominance.

Our assumptions will be the following:

(a) The nonleptonic part of the weak Hamiltonian is of the current-current form (universality of weak interactions)

$$H_W^{N.L.} = \frac{G}{\sqrt{2}} \frac{1}{2} \left[J_\mu^c (J_\mu^c)^+ + (J_\mu^c)^+ J_\mu^c \right] \qquad (1)$$

J_μ^c is the usual Cabibbo current[3], i.e.,

$$J_\mu^c = \cos\theta \left(j_\mu^1 + i j_\mu^2 + j_{5\mu}^1 + i j_{5\mu}^2 \right) + \sin\theta \left(j_\mu^4 + i j_\mu^5 + j_{5\mu}^4 + i j_{5\mu}^5 \right) \qquad (2)$$

the superscript is an unitary spin index and j_μ^i and $j_{5\mu}^i$ are respectively the vector and axial vector current;

(b) The axial vector current is partially conserved[4] (PCAC)

$$\partial_\mu j_{5\mu}^i = C\phi^i \qquad (i = 1, 2, 3)$$

with ϕ^i the pion field and[5]

$$C = \frac{-i\, m_N \mu_\pi^2 g_A}{g_r K^{NN\pi}(o)}$$

(c) CP invariance;

(d) The ratios of the physical p-wave amplitudes are correctly given by the ratios of the most singular contributions to each amplitude.[6]

The nonleptonic decay amplitude may be written as[7]

$$(2k_o)^{1/2} < B' \, \pi^i \left| H_W(o) \right| B > \; = \; -i \int d^4 x \; e^{-ikx} \, (\Box - \mu_\pi^2) < B' \left| \left[\phi^5(x), \; H_W(o) \right] \right| B > \theta(-x_o) \tag{4}$$

With the help of assumption (b) it is then easy to show[7] that, in the limit of a zero four-momentum pion, this amplitude becomes:

$$\frac{C}{\mu_\pi^2} \lim \, (2k_o)^{1/2} < B' \, \pi^i \left| H_W(o) \right| B > \; = \; < B' \left| \left[F_5^i(o), \; H_W(o) \right] \right| B >$$
$$- \left\{ \sum_{E_\ell = E_{B'}} < B' \left| F_5^i(o) \right| \ell >< \ell \left| H_W(o) \right| B > - \sum_{E_\ell = E_B} < B' \left| H_W(o) \right| \ell >< \ell \left| F_5^i(o) \right| B > \right\} \tag{5}$$

here $\quad F_5^i(o) = \int d^3 x \; j_{5c}^i(\vec{x}, o) \quad .$

With CP invariance, it can then be shown[2] that the first term of the r. h. s. of Eq. (5) only contributes to s-waves.

According to the soft pion emission theory developed by Nambu and Schrauner[8] the last two terms of Eq. (5) correspond to pole diagrams and they only contribute to p-wave dacays.[7]

The intermediate states $|\ell>$ should have the same mass as the initial or final particle but since we take the limit of a zero pion mass, this requirement is somewhat ambiguous.

The approximation which was used in previous attempts[7,9] for describing p-wave decays was the exact SU(3)-limit. This is not quite consistent: indeed, if one neglects the $\Sigma - \Lambda$ mass difference it is completely unjustified, in the limit $\mu_\pi \to o$, to neglect the contributions of, for example, the so-called meson poles.[10]

In this letter we examine what happens when mass differences are seriously[11] taken into account ($m_N \neq m_\Lambda \neq m_\Sigma \neq m_\Xi$). From Eq. (5) the meaning of our model is evident: only the most singular contributions to each amplitude are retained. In this limit, we cannot calculate the magnitude of the amplitudes since we have neglected too many contributions but, if assumption (d) is correct we may still predict the ratios of the various amplitudes.

In our model, the p-wave amplitudes are thus given by

$$\Lambda^o_o = - \frac{\mu_\pi^2}{2\sqrt{2}\, C} < n \left| F_5^3(o) \right| n > < n \left| H_W(o) \right| \Lambda >$$

$$\Lambda^o_- = - \frac{\mu_\pi^2}{4C} < p \left| F_5^+(o) \right| n > < n \left| H_W(o) \right| \Lambda >$$

$$\Xi^o_o = - \frac{\mu_\pi^2}{2\sqrt{2}\, C} \left[- < \Lambda \left| H_W(o) \right| \Xi^o > < \Xi^o \left| F_5^3(o) \right| \Xi^o > \right]$$

$$\Xi^-_- = - \frac{\mu_\pi^2}{4C} \left[- < \Lambda \left| H_W(o) \right| \Xi^o > < \Xi^o \left| F_5^+(o) \right| \Xi^- > \right]$$

$$\Sigma^-_- = - \frac{\mu_\pi^2}{4C} \left[- < n \left| H_W(o) \right| \Sigma^o > < \Sigma^o \left| F_5^+(o) \right| \Sigma^- > \right]$$

$$\Sigma_{+}^{+} = -\frac{\mu_{\pi}^{2}}{4C}\left[<n\left|F_{5}^{-}(o)\right|p><p\left|H_{W}(o)\right|\Sigma^{+}> - <n\left|H_{W}(o)\right|\Sigma^{o}><\Sigma^{o}\left|F_{5}^{-}(o)\right|\Sigma^{+}>\right]$$

$$\Sigma_{o}^{+} = -\frac{\mu_{\pi}^{2}}{2\sqrt{2}\,C}\left[<p\left|F_{5}^{3}(o)\right|p><p\left|H_{W}(o)\right|\Sigma^{+}> - <p\left|H_{W}(o)\right|\Sigma^{+}><\Sigma^{+}\left|F_{5}^{3}(o)\right|\Sigma^{+}>\right]$$

As usual superscripts refer to the charge of the decaying particle and subscripts to the charge of the emitted pion.

We define reduced matrix elements by

$$<B'\left|F_{5}^{i}(o)\right|B> = a^{i} f + b^{i} d$$

$$<B'\left|H_{W}(o)\right|B> = a\, A_{27} + b\, A_{8s} + c\, A_{8a}$$

f and d are the usual antisymmetric and symmetric parts of the $\bar{B}$ B M vertex

It is easy, then, to obtain:

$$\sqrt{2}\,\Lambda_{o}^{o} + \Lambda_{-}^{o} = 0 \qquad (6)$$

$$\sqrt{2}\,\Xi_{o}^{o} - \Xi_{-}^{-} = 0 \qquad (7)$$

Eqs. (6) - (8) coincide, with the predictions of the $\Delta I = \frac{1}{2}$ rule. We stress the fact that they have been obtained without the assumption of octet dominance.

For the Σ's we obtain

$$\Sigma_{-}^{-} + \Sigma_{+}^{+} - \sqrt{2}\,\Sigma_{o}^{+} = \frac{\mu_{\pi}^{2}}{2C} <p\left|H_{W}\right|\Sigma^{+}> <\Sigma^{+}\left|F_{5}^{3}(o)\right|\Sigma^{+}>$$

Thus the Σ triangle should not close. However the contributions of the $\Sigma^- \Sigma^0 \pi^-$ and $\Sigma^+ \Sigma^+ \pi^0$ vertices are of pure f type and thus much smaller than those of the d part of the $\bar{B}$ B M vertex.

Therefore we predict a very small p-wave amplitude for Σ^-_- which seems to be confirmed experimentally[12] and an <u>almost</u> closed triangle (it is closed for the d part of the vertex).

Similarly, instead of the Lee-Sugawara triangle,[13] we obtain

$$\Lambda^0_- - 2\, \Xi^-_- + \sqrt{3}\, \Sigma^+_0 = \lambda\, d\, A_{27} + \alpha f$$

(λ is a number and α a linear combination of the reduced matrix elements of the Hamiltonian).

Thus, again, we expect the deviations from a closed triangle to be small since explicit calculations[14] for s-waves have shown the ratio $\frac{A_{27}}{A_{8s}}$ is of the order of 5-10% (octet dominance is a dynamical effect) and just as for the Σ's, the f part of the $\bar{B}$ B M vertex is small.

We insist on the fact that relations (6) and (7) have definitely been confirmed by experiment while the p-wave triangles are in a much more ambiguous state.[12,15]

Finally it is interesting to speculate about the values of d and f in this strict pole approximation. Since our model takes mass differences seriously into account, we have to estimate the $\bar{B}$ B M vertex for a zero pion mass. In this limit, the axial vector current is not to be renormalized and thus $d + f = 1$. On the other hand it has been suggested[16] that in this limit, the $\bar{B}$ B M vertex might be of pure d type. Therefore a crude estimate would be[17] $d \simeq 1$, $f \simeq 0$

with these values, we predict

$$\Sigma_{-}^{-} \simeq 0$$

$$\sqrt{2}\, \Sigma_{o}^{+} \simeq \Sigma_{+}^{+}$$

which are roughly verified experimentally.

We insist once more on the fact that except for the Lee-Sugawara triangle our results have been derived without octet dominance. With this additional assumption built in, we get in our model, the following relation

$$\Lambda_{-}^{o} - 2\, \Xi_{-}^{-} + \sqrt{3}\, \Sigma_{o}^{+} + \frac{1}{\sqrt{6}} \left(\Sigma_{+}^{+} + \Sigma_{-}^{-} - \sqrt{2}\, \Sigma_{o}^{+} \right) = 0$$

which is also very well verified experimentally.[15]

Part of this work was done at the Case Institute of Technology and at the University of Washington. It is a pleasure to express my gratitude to Professor L. Foldy and to Professors E. Henley and B. Jacobsohn for their warm hospitality. Very enlightening discussions with Professors D. Speiser and C. Sommerfield are also gratefully acknowledged. I wish also to thank Professors W. Panofsky, H. P. Noyes and S. Drell for the opportunity of joining the Stanford Linear Accelerator Center.

REFERENCES AND FOOTNOTES

1. H. Sugawara, Phys. Rev. Letters 15, 870, 997 (1965).
2. M. Suzuki, Phys. Rev. Letters 15, 986 (1965).
3. N. Cabibbo, Phy. Rev. Letters 10, 531 (1963).
4. M. Gell-Mann and M. Levy, Nuovo Cimento 16, 705 (1960), Y. Nambu, Phys. Rev. Letters 4, 380 (1960).
5. S. Adler, Phys. Rev. 137B, 1022 (1965).
6. This is a "smoothness" assumption which we can not justify a priori.
7. Y. Hara, Y. Nambu and J. Schechter, Phys. Rev. Letters 16, 380 (1966).
8. Y. Nambu and E. Schrauner, Phys. Rev. 128, 862 (1962).
9. L. S. Brown and C. M. Sommerfield, Phys. Rev. Letters 16, 751 (1966).
10. C. Itzykson and M. Jacob (preprint - August 1966).
11. Electromagnetic mass differences are of course neglected.
12. R. O. Bangerter *et al.*, (Berkeley preprint - June 1966).
13. H. Sugawara, Prog. Theor. Phys. 31, 213 (1964), B. W. Lee, Phys. Rev. Letters 12, 83 (1964).
14. Y. T. Chiu and J. Schechter, Phys. Rev. Letters 16, 1022 (1966).
15. N. P. Samios, Proceedings of the International Conference on Weak Interactions, Argonne National Laboratory, 1965, p. 129.
16. M. Gell-Mann, Physics 1, 63 (1964).
17. This is to be compared with semileptonic decays where $d + f = 1.18$ and $\frac{d}{f} = 1.7$. See Ref. 15.