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A Minicourse on Supersymmetry

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Abstract. An immediate non-comprehensive and fast reading three hours immersion in supersymmetry. Get yourself prepared for the LHC.

1. Introduction

The series of minicourses of the Mexican Schools of Particles and Fields are thought to be an immediate, non-comprehensive, introduction to a particular field for graduate students, typically comprising three hours of lectures. Given the complexity of supersymmetry, the task seems undoable. There is a real benefit in presenting a minicourse if one at the very least attempts to make clear the main motivation of studying supersymmetry, the main ingredients necessary for it and the current status of the field, providing a damn good set of references! This is what I attempt in this contribution.

To begin with, I cite the references which I find really useful. First, the classic "A Supersymmetry Primer" [1] by S. Martin. Then the book by M. Drees, R. M. Godbole and P. Roy [2], which I think is an excellent comprehensive book for the serious supersymmetric student. There are three other references that I like. The first one of these is the very recent "Cambridge Lecures on Supersymmetry & Extra Dimensions" by F. Quevedo, S. Krippendorf and O. Schlotterer, [3]. The second one is the classic book by J. Wess and J. Bagger [4]. This one is a bit cryptic, but may be more useful for the more mathematically inclined mind. The third one is the report by M. Sohnius [5].

The real motivation of the particle physics community for studying supersymmetry is the elegant solution to the *Hierarchy Problem.* That is, if we think the Standard Model (SM) is an effective theory, broken at the Electroweak scale, and the Planck scale is a fundamental scale, why are there more than 16 orders of magnitude, in units of GeV, difference among the two scales? What would cancel the huge corrections to the Higgs mass? What fills the huge desert between the two scales? I present this motivation in Section 2, which is now, more than ever, relevant to review giving the exciting time for the possibility of discovering supersymmetric particles at the LHC (Large Hadron Collider).

The actual mathematical formalism to define entities used in supersymmetry, Superalgebra, Grassmann variables, superspace and finally supermultiplets are introduced in Sections 3, 4 and 5, respectively. These topics are superbly covered in the book by Wess and Bagger [4]. A guideline for constructing supersymmetric Lagrangians follows in Section 6. This is done in a very cryptic way, but hopefully useful enough to equip the reader for understanding the last Sections. In Section 7 I introduce the minimal supersymmetric extension of the SM (MSSM). In Section 8, I talk about mechanisms to break supersymmetry in general and in particular in

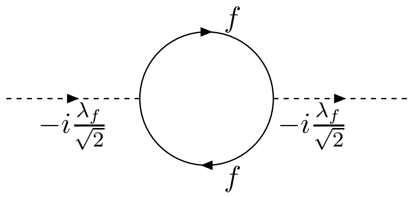

Figure 1: One loop contribution from a fermion f coupled to the Higgs field h.

the MSSM. In Section 9, I introduce the general Lagrangian that the MSSM could have after supersymmetry breaking. Here I also talk briefly about how to obtain realistic masses in the MSSM and state the current status on limits for such masses. Finally in Section 10, I mention possible extensions of the MSSM. In each section I mention further references in addition to the ones above.

2. Motivation: Hierarchy Problem & Quadratic Divergences

Let us consider the Higgs of the Standard Model (SM) [1], Φ, near its vacuum, v, where we can make the expansion $\Phi = \frac{1}{\sqrt{2}}(h+v)$. We know that $v^2 = (\sqrt{2}G_F)^{-1}$, $G_F = 1.166371 \times 10^{-5}$ GeV^{-2} and so $v \simeq 246$ GeV. The interaction with a SM fermion f is given by the Yukawa Lagrangian

$$\mathcal{L}_{\bar{f}f\Phi} = -\frac{\lambda_f}{\sqrt{2}} h\overline{f}f - \frac{\lambda_f}{\sqrt{2}} v\overline{f}f. \tag{1}$$

Via the Higgs mechanism [2], the fermion f acquires the tree level mass $m_f(0) = \lambda_f v/\sqrt{2}$ and the Higgs field obtains the tree level mass squared $m_h^2(0) = v^2/2$.

The fermionic one loop (1L) contribution to the scalar two point function reads

$$\begin{aligned}
\Pi^f_{hh}(0) &= -1\int \frac{d^4\kappa}{(2\pi)^4}\mathrm{Tr}\left[\left(-i\frac{\lambda_f}{\sqrt{2}}\right)\frac{i}{\not\kappa - m_f}\left(-i\frac{\lambda_f}{\sqrt{2}}\right)\frac{i}{\not\kappa - m_f}\right] \\
&= -2\lambda_f^2\int \frac{d^4\kappa}{(2\pi)^4}\frac{\kappa^2+m_f^2}{\left(\kappa^2-m_f^2\right)^2} = -2\lambda_f^2\int \frac{d^4\kappa}{(2\pi)^4}\left[\frac{\kappa^2-m_f^2}{\left(\kappa^2-m_f^2\right)^2} + \frac{2m_f^2}{\left(\kappa^2-m_f^2\right)^2}\right] \\
&= -2\lambda_f^2\int \frac{d^4\kappa}{(2\pi)^4}\left[\frac{1}{\kappa^2-m_f^2} + \frac{2m_f^2}{\left(\kappa^2-m_f^2\right)^2}\right].
\end{aligned} \tag{2}$$

The first term in Eq. (2) is quadratically divergent. It is particularly bad because the divergence does not depend on the mass of h. So what comes to mind is whether or not we can introduce a cut-off scale just as it is done with the logarithmic divergences. Performing integral of Eq. (2) with the substitution

$$\int \frac{d^4\kappa}{(2\pi)^4} \longrightarrow \int d\Omega \int_0^\Lambda d_{\kappa_E}\frac{\kappa^3}{8\pi^2}\frac{\kappa_E^3}{\kappa_E^2+m_f^2}, \tag{3}$$

[1] For the most part of this contribution I adopt the notation of [2].

[2] See the notes of the lectures by Poul Henrik Damgaard for an enlightening discussion of the term *spontaneous symmetry breaking.*

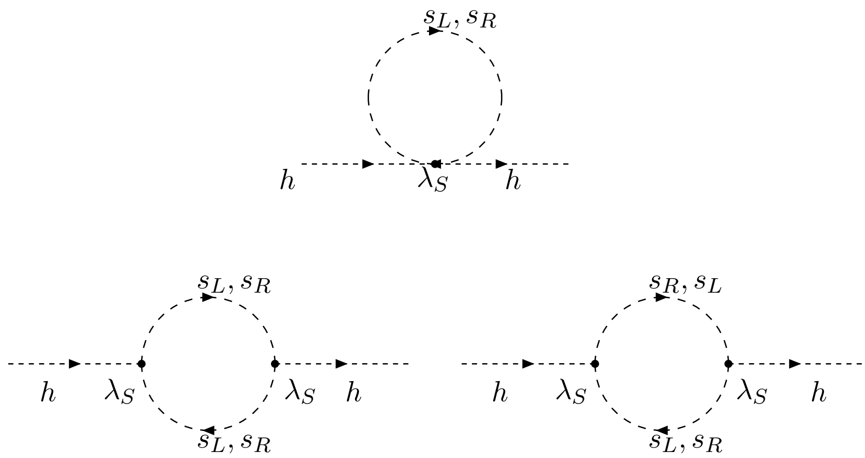

Figure 2: One loop contribution from a pair of scalars s_L and s_R coupled to the Higgs field h as in Eq. (5).

we have

$$\Pi^f_{hh}(0) = 2\lambda_f^2 \frac{\Lambda^2}{8\pi^2} i \;\Rightarrow\; m_h^2 = m_h^2(0) + 2\lambda_f^2 \frac{\Lambda^2}{8\pi^2}. \tag{4}$$

If the SM was the complete fundamental theory of particle physics, then the cut-off scale Λ could be anything. For $\Lambda = O(100)$ GeV the correction would not be orders of magnitude larger than the tree level value. However if the SM is not regarded as such and we expect to include somehow gravitational effects, we should then consider $\Lambda = O(M_{\rm P}) \simeq 2.4 \times 10^{18}$ GeV, the highest scale known in particle physics. This would be a disaster: the 1L correction to $m_h^2(0)$ would be 32 orders of magnitude larger than the tree level value. One can think also on that instead of using a momentum cut-off, Λ, we could use dimensional regularization where there would be no λ^2 piece. However, in this scheme there would enter contributions that cannot be made small if one makes an ultraviolet completion of the SM. We introduce two additional complex scalar fields s_L and s_R, with the coupling to the Higgs field

$$\begin{aligned}\mathcal{L}_{s_L s_R \Phi} &= \lambda_S |\Phi|^2 \left(|s_L|^2 + |s_R|^2\right) \;+\; \left(\lambda_f A_f \Phi s_L s_R^* + h.c.\right) \\ \longrightarrow^{\Phi = \frac{1}{\sqrt{2}}(h+v)} &= \frac{1}{2}\lambda_S h^2 \left(|s_L|^2 + |s_R|^2\right) + v\lambda_S h \left(|s_L|^2 + |s_R|^2\right). \end{aligned} \tag{5}$$

Then we can calculate for this example the scalar contributions to the self energy of h, produced by the 1L diagrams of Fig. 2. These are

$$\begin{aligned}\Pi^{s_L,s_R}_{hh}(0) &= -i\lambda_S \int \frac{d^4\kappa}{(2\pi)^4} \left[\frac{i}{\kappa^2 - m^2_{s_L}} + \frac{i}{\kappa^2 - m^2_{s_R}}\right] \\ &+ (\lambda_S v)^2 \int \frac{d^4\kappa}{(2\pi)^4} \left[\frac{1}{\left(\kappa^2 - m^2_{s_L}\right)^2} + \frac{1}{\left(\kappa^2 - m^2_{s_R}\right)^2}\right] \\ &+ |\lambda_f A_f|^2 \int \frac{d^4\kappa}{(2\pi)^4} \frac{1}{\kappa^2 - m^2_{s_L}} \frac{1}{\kappa^2 - m^2_{s_R}}. \end{aligned} \tag{6}$$

We cancel the first two terms of Eq. (6) with the first term of the last expression in Eq. (2), provided that $-\lambda_S = \lambda_f^2$, $m_{s_L} = m_{s_R} = m_f$. Furthermore, using that $\int \frac{d^4\kappa}{(2\pi)^4} \frac{i}{\kappa^2 - m^2_{s_L}}$

$= 2\pi^2 \int_0^\Lambda \frac{d\kappa_E}{16\pi^2}\frac{\kappa_E^3}{\kappa_E^2+m_f^2}$, we can cancel all the quadratic divergences with the sum of expressions Eq. (2) and Eq. (6). The remaining logarithmic divergences in $\Pi^f_{hh}(0)+\Pi^s_{hh}(0)$ can be cancelled by replacing the logarithmic divergences by the logarithm of the square of the renormalization scale μ [3]. However by performing the integrals in the second row of Eq. (6), using the regularization of the divergent integrals as $\int \frac{d^4\kappa}{(i\pi)^2}\frac{1}{(\kappa^2-m^2)^2} \longmapsto -\ln\left[\frac{m^2}{\mu^2}\right]$,

$$\int \frac{d^4\kappa}{(2\pi)^4}\left[\frac{1}{\kappa^2-m_1^2}\frac{1}{\kappa^2-m_2^2}\right]$$
$$\longmapsto (m_1^2-m_2^2)B_0(0,m_1^2,m_2^2) = m_1^2\left(1-\ln\frac{m_1^2}{\mu^2}\right) - m_2^2\left(1-\ln\frac{m_2^2}{\mu^2}\right)$$

and summing up the fermionic and scalar contributions, where from now own we call

$$s_{L,R} \doteq \tilde{f}_{L,R}, \tag{7}$$

we obtain the total 1L correction to $m_h^2(0)$

$$\begin{aligned}
\Pi^f_{hh}(0)+\Pi^{s_L,s_R}_{hh}(0) &= -2\lambda_f^2\int\frac{d^4\kappa}{(2\pi)^4}\left[\frac{1}{\kappa^2-m_f^2}+\frac{2m_f^2}{\left(\kappa^2-m_f^2\right)^2}\right] \\
&- \lambda_{\tilde{f}}\int\frac{d^4\kappa}{(2\pi)^4}\left[\frac{1}{\left(\kappa^2-m_{\tilde{f}_L}^2\right)^2}+\frac{1}{\left(\kappa^2-m_{\tilde{f}_R}^2\right)^2}\right] \\
&+ (\tilde{\lambda}_f v)^2\int\frac{d^4\kappa}{(2\pi)^4}\left[\frac{1}{\kappa^2-m_{\tilde{f}_L}^2}+\frac{1}{\kappa^2-m_{\tilde{f}_R}^2}\right] \\
&= i\frac{\lambda_f^2}{16\pi^2}\left[-2m_f^2\left(1-\ln\frac{m_f^2}{\mu^2}\right)+4m_f^2\ln\frac{m_f^2}{\mu^2}\right. \\
&+ \left. 2m_{\tilde{f}}^2\left(1-\ln\frac{m_{\tilde{f}}^2}{\mu^2}\right)-4m_{\tilde{f}}^2\ln\frac{m_{\tilde{f}}^2}{\mu^2}\right].
\end{aligned} \tag{8}$$

We can see that indeed we need

$$\lambda_f = -\lambda_f^2, \quad m_{\tilde{f}_L} = m_{\tilde{f}_R} = m_f \tag{9}$$

to cancel exactly the 1L correction to $m_h^2(0)$. We have seen that introducing scalar particles with the same masses to those of the fermions and whose couplings are related to the couplings of the latter, Eq. (9), quadratic divergences can be cancelled at the 1L level. To cancel these kind of divergences at all orders in perturbation theory would be remarkable. Well, this is precisely what supersymmetry does. Despite the fact that we have not yet seen any experimental evidence of it, in the next sections we will explore why is so fascinating.

3. Supersymmetry Algebra

Since supersymmetric transformations change a particle with integer spin to a particle with half-integer spin, Q must be a spinor. Then we introduce properly its two spinorial indices

$$Q_A, \quad A = 1, 2. \tag{10}$$

[3] Check references [2, 1] for further comments about this.

However Q should be identified with a four dimensional (4D) quantity and it is indeed a 4D Majorana spinor, which can be decomposed as 2D Weyl spinors

$$Q_a = \left(\begin{array}{c} Q_A \\ \overline{Q}_{\dot{A}} \end{array} \right). \tag{11}$$

The natural question to ask immediately is whether or not Q_a can be a generator of the symmetries of a 4D theory. We know that the Poincaré Group generates the fullest continuous spacetime symmetries of particle interactions. We recall that its generators can be expressed as unitary operators

$$U(a) = e^{ia^\mu P_\mu}, \quad U(\Lambda) = e^{-i/2\omega^{\mu\nu} M_{\mu\nu}}, \tag{12}$$

for translations and for homogeneous Lorentz transformations, respectively. For an infinitesimal transformation on the 4D coordinates we have $x^\mu \;\rightarrow\; x^\mu = (\delta^\mu_\nu + w^\mu_\nu)x^\nu + a^\mu$.

What do we know about the properties of the generators? the Algebra of the generators which is expressed in terms of commutators, for example

$$\begin{aligned} [P_\mu, P_\nu] \;&=\; 0, \quad [M_{\mu,\nu}, P_\rho] \;=\; i\left(\eta_{\nu\rho}P_\mu - \eta_{\mu\rho}P_\nu\right), \\ [M_{\mu,\nu}, P_{\rho\sigma}] \;&=\; -i\left(\eta_{\mu\rho}M_{\nu\sigma} - \eta_{\mu\sigma}M_{\nu\rho} - \eta_{\nu\rho}M_{\mu\sigma} + \eta_{\nu\sigma}M_{\mu\rho}\right). \end{aligned} \tag{13}$$

The Coleman-Mandula theorem [6] states that the full Lie Algebra of the S-Matrix in 4D, described by $\left[T^a, T^b\right] = it^{ab}_c T^c$, is such that the generators T^a are a direct sum of P_μ and $M_{\mu,\nu}$, that is to say

$$[T^a, P_\mu] = 0 = [T^a, M_{\mu\nu}]\,. \tag{14}$$

However a supersymmetric transformation described by Q_a is transforming internal degrees of freedom from a boson into a fermion, or vice versa $Q|b, f\rangle = |f, b\rangle$, for b a boson with spin n and f a fermion with spin $m/2$. Specifically, we are making a transformation of particles of spin n to particles with spin $(n \pm 1)/2$ and particles with spin $m/2$ to particles with spin m. Thus if we want Q_a to be indeed a generator of a definite group describing the symmetries of our 4D theory then we need to specify the Algebra which is satisfied by these generators. From the Coleman-Mandula theorem we know that it cannot be a simple Lie Algebra and has the following properties: it cannot commute with rotations but commutes with translations. That the generator Q_a of these transformations cannot commute with rotations can be easily seen thinking of a rotation about an axis, described by $U(2\pi)$: $U(2\pi)|b\rangle \;=\; |b\rangle$ and $U(2\pi)|f\rangle \;=\; -|f\rangle$, then we have

$$\begin{aligned} &U(2\pi)QU(2\pi)^{-1}|f\rangle = -U(2\pi)QU(2\pi)^{-1}U(2\pi)|f\rangle = -U(2\pi)|b\rangle = -Q|f\rangle, \\ &U(2\pi)QU(2\pi)^{-1}|b\rangle = -Q|b\rangle, \end{aligned} \tag{15}$$

that is $U(2\pi)QU(2\pi)^{-1} = -Q$. But it is easy to see that it commutes with translations because the generator Q is not affecting the position of the field describing the particle. In order to get around with the use of Lie Algebras, defined in terms of commutators, then one can think about using Algebras defined also in terms of anti-commutators. Such mathematical constructions enter into the classification of the *Graded Lie Algebras* [4]. For our case, we need to expand the set of commutator relations followed by P_μ and $M_{\mu\nu}$ to include Q_a in such a way that the Poincaré Algebra is recovered and the properties described above (that it commutes with translations and

[4] For a physics perspective on Graded Lie Algebras and specifically on the Superalgebra, the book by J.F. Cornwell [7] is great.

that it cannot commute with rotations) are recovered, by means of anti-commutators. In the two component notation of Eq. (10), we have the $N = 1$ Supersymmetry Algebra in 4D

$$\begin{aligned}
&\{Q_A, \overline{Q}_{\dot{B}}\} = 2\sigma^{\mu}_{A\dot{B}} P_{\mu}, \quad \left\{\overline{Q}^{\dot{A}}, Q^B\right\} = 2\overline{\sigma}^{\mu}_{\dot{A}B} P_{\mu},\\
&\{Q_A, Q_B\} = \left\{\overline{Q}^{\dot{A}}, \overline{Q}^{\dot{B}}\right\} = 0,\\
&[Q_A, P_\mu] = \left[\overline{Q}_{\dot{A}}, P_\mu\right] = 0,\\
&[M_{\mu\nu}, Q_A] = -\left(\sigma_{\mu\nu}\right)_A^B Q_B, \qquad \left[M_{\mu\nu}, \overline{Q}^{\dot{A}}\right] = -\left(\sigma_{\mu\nu}\right)^{\dot{A}}_{\dot{B}} Q^{\dot{B}},\\
&[Q_A, R] = Q_A, \qquad \left[\overline{Q}^{\dot{A}}, R\right] = -Q^{\dot{A}}.
\end{aligned} \tag{16}$$

The last expression in Eq. (16) reflects an invariance under a *chiral* rotation, that can be thought of being generated by a $U(1)$ generator R. This will satisfy $[Q_a, R] = (\gamma_5)_{ab} Q_b$. $\sigma_{\mu\nu} = \sigma^{\mu}\overline{\sigma}^{\nu} - \sigma^{\nu}\overline{\sigma}^{\mu}$.

The Haag, Łopuszánski, Sohnius theorem [8] states that the $N = 1$ Supersymmetry (SUSY) Algebra in 4D is a unique extension of the Poincaré Algebra in 4D relativistic Quantum Field Theory (QFT) with one generator Q_A. This is really just the statement of saying, if there is another generator Q_A of a 4D QFT, apart from the translations and homogeneous Lorentz transformations, then that will define a unique Algebra, the supersymmetry algebra, $N = 1$ in 4D. But of course this theorem allows for theories with more supersymmetries, that is, there could be more than one generator Q_B in a 4D QFT.

4. Grassmann Variables and Superspace

Supersymmetric theories can be defined in terms of anti-commuting or Grassmann variables. These are such that

$$\left\{\theta, \overline{\theta}\right\} \doteq \theta\overline{\theta} + \overline{\theta}\theta = 0 \text{ and } \theta \;=\; 0^2 \;=\; \overline{\theta}^2. \tag{17}$$

From these basic rules we can develop the properties of calculus on these variables. For example, a function can be *Taylor-Grassmann* expanded as

$$f(\theta) = f_0 + f_1\theta, \tag{18}$$

where $f_0, f_1 \in \mathbb{C}$. Then we have the nice properties

$$\int d\theta\, \theta = 1, \quad \frac{df(\theta)}{d\theta} = f_1, \quad \theta f(\theta) = \theta f_0 + \cancel{f_1\theta^2}^{\,0}. \tag{19}$$

We can see why Grassmann Variables could be so useful, the expansion Eq. (18) is finite and simple differentiation or integration can project out the components of the functions defined on θ. It happens that supersymmetric Lagrangians and their particle interactions can be written in a space which is spanned by the coordinates of our 4D space plus two Grassmann coordinates

$$z \doteq (x^{\mu}, \theta, \overline{\theta}), \tag{20}$$

this space is called the *Superspace*. However the Grassmann variables in Eq. (20) are not as simple as in Eq. (17), because they will contain two indices, A and B. The indices A and B will label two spinor indices in the $(\frac{1}{2},0)$ representation of the Lorenz group, while the indices $\dot{A}$ and $\dot{B}$ will label two spinor indices in the $(0,\frac{1}{2})$ representation. Accordingly, the tensors $\epsilon^{A,B}$ and $\epsilon_{A,B} = \text{Inv}[\epsilon^{A,B}]$ represent the metric in the space of the $(\frac{1}{2},0)$ representations, while $\epsilon^{\dot{A},\dot{B}} = \epsilon_{AB}$ and $\epsilon_{\dot{A},\dot{B}} = \text{Inv}[\epsilon^{\dot{A},\dot{B}}]$ represent the metric in the space of the $(0,\frac{1}{2})$ representations [5].

We can then rise and lower the indices as

$$\theta^A = \epsilon^{A,B}\theta_B, \quad \theta_A = \epsilon_{A,B}\theta^B, \quad \overline{\theta}^{\dot{A}} = \epsilon^{\dot{A},\dot{B}}\overline{\theta}_{\dot{B}}, \quad \overline{\theta}_{\dot{A}} = \epsilon_{\dot{A},\dot{B}}\overline{\theta}^{\dot{B}}. \tag{21}$$

The supersymmetry generators can be represented in a differential form in terms of the Grassmann variables $Q_A = -i\left(\partial_A + i\sigma^\mu_{A\dot{B}}\overline{\theta}^{\dot{B}}\partial_\mu\right)$ and $\overline{Q}^{\dot{A}} = i\left(\overline{\partial}^{\dot{A}} - i\sigma^{\mu\dot{A}B}\theta_B\partial_\mu\right)$.

Here we note that for the supersymmetry algebra, Eq. (16), to be satisfied, we need $\overline{\epsilon}\overline{Q} = (\epsilon Q)^\dagger$. Also we define $\sigma^{\mu\dot{A}B} = (1_{2\times 2}, -\overrightarrow{\sigma})$ and $\sigma^\mu_{A\dot{B}} = (1_{2\times 2}, \overrightarrow{\sigma})$. Then we define the chiral covariant and anti-chiral covariant derivatives

$$\begin{aligned} \mathcal{D}_A &= \partial_A - i\sigma^\mu_{A\dot{B}}\overline{\theta}^{\dot{B}}\partial_\mu \ \rightarrow \ \text{in the } (\frac{1}{2},0) \text{ representation}, \\ \overline{\mathcal{D}}^{\dot{A}} &= \overline{\partial}^{\dot{A}} + i\sigma^{\mu\dot{A}B}\theta_B\partial_\mu \ \rightarrow \ \text{in the } (0,\frac{1}{2}) \text{ representation}. \end{aligned} \tag{22}$$

In the superspace defined by Eq. (20), the infinitesimal supersymmetric transformations act as

$$\left(x^\mu, \theta, \overline{\theta}\right) \rightarrow \left(x^\mu - i\theta\sigma^\mu\overline{\epsilon} + i\epsilon\sigma^\mu\overline{\theta}, \theta + \epsilon, \overline{\theta} + \overline{\epsilon}\right), \tag{23}$$

where ϵ is an infinitesimal Grassmann parameter.

5. Particle Supermultiplets aka Superfields

Being the case that we have an Algebra, we need to find representations, i.e. multiplets, that satisfy the properties of the Algebra. In the case of supersymmetry, it is necessary to define them in the superspace. The two classes of such supermultiplets or superfields used in particle physics are the *Chiral superfields* and the *Vector superfields*.

Before describing each kind of superfields, we just state their general definition and properties. The definition is a generalization of Eq. (18), that is a function $\mathcal{F}$ defined in superspace [6]

$$\begin{aligned} \mathcal{F}\left(x^\mu, \theta, \overline{\theta}\right) &= f_0(x) + f_1^A(x)\theta_A + \overline{f}_{2\dot{A}}(x)\overline{\theta}^{\dot{A}} \\ &+ f_3(x)\theta\theta + \overline{f}_4(x)\overline{\theta\theta} + f_5^A(x)\theta_A\overline{\theta\theta} + \overline{f}_{6\dot{A}}(x)\overline{\theta}^{\dot{A}}\theta\theta + f_7(x)\theta\theta\overline{\theta\theta}. \end{aligned} \tag{24}$$

There are various things to note. First, since there are two indices for θ, the naive commutation relations of Eq. (17) will not hold. Instead we have

$$\theta\theta = -2\theta^A\theta^B\epsilon_{AB}, \quad \overline{\theta\theta} = -2\overline{\theta}_{\dot{A}}\overline{\theta}_{\dot{B}}\epsilon^{\dot{A}\dot{B}}. \tag{25}$$

Second, the functions $f_i(x)$ are not all of the same type, some carry spinor indices and some do not. In Eq. (24) f_0, f_3, f_4 and f_7 are scalars, f_1 and f_2 must be spinors in the $(1/2,0)$ (left) representation, while f_2 and f_6 must be spinors in the $(0,1/2)$ (right) representation.

[5] ϵ_{AB} it is the anti-symmetric tensor in two dimensions.

[6] The review by S. Martin [1] is a great introduction to these topics and in general for supersymmetric phenomenology.

Last, the θ's with no apparent indices have been left so because we have contracted explicitly all the spinor indices. Just as in § 4, we can perform simple differentiations or integrations to project out the components of the superfields, e.g.

$$\int d^4\theta \mathcal{F}\left(x^\mu, \theta, \overline{\theta}\right) = f_7(x), \quad \int d^4\theta[\theta\theta\overline{\theta\theta}] = 1. \tag{26}$$

Last, there are two terms in Eq. (24) that receive special names: the F term, which is the term that appears multiplied by $\theta\theta$, and the D term, which appears multiplied by $\theta\theta\overline{\theta\theta}$.

The supersymmetry transformations act on $\mathcal{F}$ as

$$\delta\mathcal{F} \;=\; \mathcal{F}\left(x^\mu - i\theta\sigma^\mu\overline{\epsilon} + i\epsilon\sigma^\mu\overline{\theta}, \theta + \epsilon, \overline{\theta} + \overline{\epsilon}\right) - \mathcal{F}(x, \theta, \overline{\theta}) \;=\; i(\epsilon Q + \overline{\epsilon}\overline{Q})\mathcal{F}. \tag{27}$$

From this last expression we can work out the supersymmetry transformations for each of the components of $\mathcal{F}$. We will explicitly state them in the next subsections. Finally we note that the superspace is a fascinating research field on its own. This only for its mathematical structure

5.1. *Chiral superfields*

A left chiral superfield is defined such that it depends only on y^μ and θ and a right chiral superfield such that it depends only on $\overline{y}^\mu$ and $\overline{\theta}$, where

$$y^\mu \doteq x^\mu - i\theta\sigma^\mu\overline{\theta} \quad ,\overline{y}^\mu \doteq x^\mu + i\theta\sigma^\mu\overline{\theta}. \tag{28}$$

According to this definition we can write

$$\Phi(y, \theta) = \phi(y) + \sqrt{2}\theta\xi(y) + \theta\theta F(y), \quad \Phi(\overline{y}, \overline{\theta})^\dagger = \phi^*(y) + \sqrt{2}\overline{\theta}\overline{\xi}(\overline{y}) + \overline{\theta\theta}F^*(\overline{y}), \tag{29}$$

where the factor $\sqrt{2}$ is introduced for convenience. Both expressions can be put in terms of x

$$\begin{aligned}
\Phi(x^\mu - i\theta\sigma^\mu\overline{\theta}, \theta) &= \phi(x) - i\theta\sigma^\mu\overline{\theta}\partial_\mu\phi(x) - \frac{1}{4}\theta\theta\overline{\theta\theta}\partial^\mu\partial_\mu\phi(x) \\
&+ \sqrt{2}\theta\xi(x) + \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\xi(x)\sigma^\mu\overline{\theta}, \\
\Phi(x^\mu + i\theta\sigma^\mu\overline{\theta})^\dagger &= \phi(x)^* + i\theta\sigma^\mu\overline{\theta}\partial_\mu\phi^*(x) - \frac{1}{4}\theta\theta\overline{\theta\theta}\partial^\mu\partial_\mu\phi^*(x) + \sqrt{2}\overline{\theta}\overline{\xi}(x) \\
&- \frac{i}{\sqrt{2}}\overline{\theta\theta}\theta\sigma^\mu\partial_\mu\overline{\xi}(x) + \overline{\theta\theta}F^*(x).
\end{aligned} \tag{30}$$

What is the important difference between the expressions of Eq. (29) and Eq. (30)? They are two different representations of the supersymmetry Algebra, Eq. (29) is the *chiral representation* for supersymmetric generators, Eq. (30) is the so called *symmetric* representation, where $\overline{Q} = Q^\dagger$. They are completely equivalent. We can project the components of $\Phi(y, \theta)$, using equally Eq. (29) or Eq. (30) for the latter we have

$$\phi(x) = \Phi(x, \theta, \overline{\theta})|_{\theta=0=\overline{\theta}}, \quad \sqrt{2}\xi_A(x) = \mathcal{D}_A\Phi(x, \theta, \overline{\theta})|_{\theta=0=\overline{\theta}}, \quad F(x) = \frac{1}{4}\mathcal{D}\mathcal{D}\Phi(x, \theta, \overline{\theta})|_{\theta=0=\overline{\theta}}. \tag{31}$$

What is the physics that we can extract from Eq. (29)? There are four real scalar components, contained in $F(x)$ and $\phi(x)$. The spinor ξ_A contains four real fermionic fields. Thus there is an equal number of scalars and fermions. This the *off-shell* description, that means that not all the degrees of freedom are physical. F are called auxiliary fields and have trivial equations of motion, that is $\frac{\partial\mathcal{L}}{\partial F} = 0$, which can be used to eliminate them from the Lagrangian. After this, we have two physical scalar degrees of freedom and two physical fermionic degrees of freedom, both with the same mass. This sets the minimum basis for understanding § 6. To end this section we state the properties, that can be easily derived using the exposition above, of the chiral and anti-chiral superfields:

- $\overline{\mathcal{D}}_{\dot{A}}\Phi = 0, \quad \mathcal{D}^A\Phi^\dagger = 0,$
- products and linear combinations of chiral superfields are chiral superfields.

5.2. Vector superfields
A vector superfield in superspace is defined such that

$$\begin{aligned} V(x,\theta,\overline{\theta}) &= C(x) + \sqrt{2}\theta\xi(x) + \sqrt{2}\overline{\theta\xi}(x) + \theta\theta M(x) + \overline{\theta\theta}M^*(x) \\ &+ \theta\sigma^\mu\overline{\theta}A_\mu(x) + \theta\theta\overline{\theta\lambda}(x) + \overline{\theta\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\overline{\theta\theta}D(x), \end{aligned} \tag{32}$$

where $M(x)$ is a complex scalar. Here ξ and ζ are left-handed Weyl spinors and $\overline{\chi}$ and $\overline{\lambda}$ are right-handed Weyl spinors.

From the definition of a vector superfield, Eq. (32), we can easily obtain its most important property:

- $V = V^\dagger$.

6. Construction of a Supersymmetric Langrangian

6.1. General considerations
From the properties of the chiral superfields, Eq. (5.1) and Eq. (5.2), we can extract a very useful recipe for the construction of supersymmetric Lagrangians:

(i) Take all the possible products of chiral superfields which satisfy the symmetries of the proposed model, the F term it will be always part of the Lagrangian, as it transforms as a total derivative under supersymmetric transformations.
(ii) The D term of the product $\Phi\Phi^\dagger$ also transforms as a total derivative under supersymmetric transformations.

From the observations above enumerated, we see that we need to put a limit on the product of chiral superfields, because of renormalizability, obviously then we have that the quantity

$$W(\Phi_i) = h^i\Phi_i + \frac{1}{2}m^{ij}\Phi_i\Phi_j + \frac{1}{3!}y^{ijk}\Phi_i\Phi_j\Phi_k. \tag{33}$$

called *superpotential*, it is the most general combination of products of superfields that we can have. We say that the superpotential is *holomorphic* because, since each term is a chiral superfield, W can be entirely defined in terms of two different coordinates in superspace: y and θ Eq. (28).

The *holomorphicity* property of the superpotential, excludes explicitly then combinations of the form $B\overline{B}$, where $\overline{B}$ is the supermultiplet defined as $\overline{B} = B^\dagger\gamma^0$ so it not only carries the opposite quantum numbers of B but the supermultiplet $\overline{B}$ obviously contains the anti-particles of the supermultiplet B! I emphasize this because of the extended vicious notation in the supersymmetric literature where there are expressions of the form

$$W = \frac{1}{2}m^{ij}B_i\overline{B}_j. \tag{34}$$

Well the authors here abuse the notation and the harm done here it is that *we* are introducing a chiral superfield $\overline{B}$ with opposite quantum numbers to those of B but that it does not contain their anti-particle!, i.e. in these cases $\overline{B} \neq B^\dagger\gamma^0$. The careful reader will note that anyway... If we have terms in the superpotential such as

$$ABC \in W, \tag{35}$$

for A, B, C chiral superfields, then terms of the form

$$A\overline{B}C, \tag{36}$$

are also excluded because for $\overline{B} = B^\dagger\gamma^0$ this is an anti-chiral superfield and destroys explicitly the holomorphicity of W, that is why the emphasis in the notation $W(\Phi_i)$. The $\mathcal{L}$ of the theory can at most change under a SUSY transformations into itself plus a total space-time derivative. The D term of the product $\Phi_i^\dagger\Phi_j$ is

$$\left[\Phi_i^\dagger\Phi_j\right]_D = \theta\theta\overline{\theta\theta}\left(F_i^*F_j + \frac{1}{2}\partial_\mu\Phi_i^*\partial^\mu\Phi_J - \frac{1}{2}\Phi_i^*\partial_\mu\partial^\mu\Phi_j + i\xi_j\sigma^\mu\delta_\mu\overline{\xi}_i\right), \tag{37}$$

which is so special because after the supersymmetric transformations it transforms indeed as a total derivative: $\delta D = i\delta_\mu\left(\zeta\sigma^\mu\overline{\epsilon} + \overline{\lambda}\overline{\sigma}^\mu\epsilon\right)$.

We can show that $[W(\Phi_i) + h.c.]_F$ transforms as a total derivative, then we can construct the following supersymmetric Lagrangian

$$\mathcal{L} = \left[\Phi_i^\dagger\Phi_i\right]_D + \left[W(\Phi_i) + \overline{W}(\Phi_i^\dagger)\right]_F. \tag{38}$$

Would it be possible to define other quantity rather than W, such that its D or F part could supersymmetrically transform exactly into itself or up to a total derivative? The answer is no and this has profound consequences, in particular it restricts the matter content of supersymmetric theories and the way they are coupled, as we will see in § 7. Let us state this in another way. If a given Lagrangian is supersymmetric, then it can be decomposed in the form of Eq. (38) and $W(\Phi_i)$ is uniquely defined.

6.2. Interaction Free Lagrangian

The simplest supersymmetric model is the one proposed by Wess and Zumino (WZ). The action of this model can be written as

$$S = \int d^4z\left(\mathcal{L}_S + \mathcal{L}_F + \mathcal{L}_A\right), \quad \mathcal{L}_S = -\partial^\mu\phi^*\partial_\mu\phi,\ \mathcal{L}_F = i\psi^\dagger\overline{\sigma}^\mu\partial_\mu\psi,\ \mathcal{L}_A = F^*F. \tag{39}$$

The transformations of bosons into fermions and vice versa, are given by

$$\begin{aligned}
\delta\phi &= \epsilon\psi,\\
\delta\psi_A &= -i\left(\sigma^\mu\epsilon^\dagger\right)_A\partial_\mu\phi + \epsilon_A F,\\
\delta\psi^\dagger_{\dot{A}} &= i\left(\epsilon\sigma^\mu\right)_{\dot{A}}\partial_\mu\phi^* + \epsilon^\dagger_{\dot{A}}F^*,\\
\delta F &= -i\epsilon^\dagger\overline{\sigma}^\mu\partial_\mu\psi,\\
\delta F^* &= i\partial_\mu\psi^\dagger\overline{\sigma}^\mu\epsilon.
\end{aligned} \tag{40}$$

Recall that the auxiliary fields F should disappear in the expression for the *physical* Lagrangian. We can calculate the supersymmetric transformations of $\mathcal{L}_S$ and $\mathcal{L}_F$, ignoring at the moment the auxiliary fields

$$\begin{aligned}
\delta\mathcal{L}_S &= -\delta\left(\partial^\mu\phi^*\partial_\mu\phi\right),\\
&= -\left(\partial^\mu(\delta\phi^*)\partial_\mu\phi^\dagger + \partial^\mu\phi^*\partial_\mu(\delta\phi)\right),\\
&= -\epsilon^\dagger\partial^\mu\psi^\dagger\partial_\mu\phi - \epsilon\partial^\mu\psi\partial_\mu\phi^*,\\
\delta\mathcal{L}_F &= -\epsilon\sigma^\mu\overline{\sigma}^\mu\partial_\mu\psi\partial_\mu\phi^* + \psi^\dagger\overline{\sigma}^\nu\sigma^\mu\epsilon^\dagger\partial_\mu\partial_\nu\phi,
\end{aligned} \tag{41}$$

rearranging terms:

$$\delta\mathcal{L}_S + \delta\mathcal{L}_F = \epsilon\partial^\mu\Phi_\mu\phi^* + \epsilon^\dagger\partial^\mu - \partial_\mu\left(\epsilon\sigma^\nu\overline{\sigma}^\mu\psi\partial_\mu\phi^* + \epsilon\psi\partial^\mu\phi^* + \epsilon^\dagger\psi^\dagger\partial^\mu\phi\right). \tag{42}$$

It can be shown that including the auxiliary fields, the variation can also be written as a total derivative $\delta S = \int d^4x\,(\delta\mathcal{L}_S + \delta\mathcal{L}_F + \delta\mathcal{L}_A) = 0$.

6.3. Interacting Lagrangian

We need to check how we can construct an interacting Lagrangian. The easiest example are the $U(1)$ gauge transformations, under which the superfields transform as

$$\Phi_i \longrightarrow e^{-2igt_i\Lambda(z)}\Phi_i, \quad \Phi_i^\dagger \longrightarrow \Phi_i^\dagger e^{2igt_i\Lambda^\dagger(z)}, \tag{43}$$

where g is the coupling constant, t_i are $U(1)$ charges and Λ is a complex function specifying the local gauge transformations, defined in superspace, $z = (x, \theta, \overline{\theta})$. Now we need to get a term of the sort $\Phi^\dagger V\Phi$, where V is a vector superfield, in order to have an interaction term. Then we can show that

$$\Phi_i^\dagger e^{2gt_iV}\Phi \tag{44}$$

is gauge invariant. To preserve the chirality of the gauge transformations, we need $\mathcal{D}_A\Lambda^\dagger = 0$, $\overline{\mathcal{D}}^{\dot{A}}\Lambda = 0$, that is Λ and $\Lambda^\dagger$ are chiral and anti-chiral respectively. Now defining

$$T_A \doteq -\frac{1}{4}\overline{\mathcal{D}\mathcal{D}}\mathcal{D}_AV, \quad \overline{T}_{\dot{A}} \doteq -\frac{1}{4}\mathcal{D}\mathcal{D}\overline{\mathcal{D}}_AV. \tag{45}$$

Note that T^AT_A is a left chiral superfield, then we can construct the following interacting Lagrangian

$$\mathcal{L} = \frac{1}{4}\left[T^AT_A + \overline{T}_{\dot{A}}\overline{T}^{\dot{A}}\right]_F + \left[\Phi_i^\dagger e^{2gt_iV}\Phi_i + 2\eta V\right]_D + \left[W(\phi_i) + h.c.\right]_F, \tag{46}$$

in component form, in the Wess-Zumino (WZ) gauge, once D and F are projected out, we have:

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}D^2 + \eta D + gt_i|\phi_i|^2D - \frac{1}{4}F_{\mu\nu}F^\mu\nu + i\lambda\sigma^\mu\partial_\mu\overline{\lambda} + i\xi_i\sigma^\mu\mathcal{D}^\dagger_{\mu i}\overline{\xi_i}\\ &+ \mathcal{D}_i^\mu\xi_i\mathcal{D}^\dagger_{\mu i}\overline{\xi_i} + F_i^*F_i + \left[\frac{1}{2}\xi_i\xi_jW_{ij}(\phi) + h.c.\right] - \sqrt{2}\,gt_i\left(\overline{\lambda\xi}_i + h.c.\right),\end{aligned} \tag{47}$$

where of course the superpotential $W(\phi_i)$ needs to respect gauge invariance and η is a real constant [7]. In expression Eq. (47) we have used

$$F_i^* = -W_i = \frac{\delta W}{\delta\phi_i}, \quad F_i = -\overline{W}_i = \frac{\delta W^\dagger}{\delta\phi_i^\dagger}, \quad W^i = \frac{\delta W}{\delta\Phi_i}, \quad W^{ij} = m^{ij} + y^{ijk}\Phi_i. \tag{48}$$

[7] The corresponding action to Eq. (46) is

$$\mathcal{S} = \int d^8z\left[\Phi_i^\dagger e^{2gt_iV}\Phi_i + 2\eta V\right] + \int d^6z\left[\frac{1}{4}T^AT_A + W(\Phi)\right] + \int d^6\bar{z}\left[\frac{1}{4}\overline{T}^{\dot{A}}\overline{T}_{\dot{A}} + W^\dagger(\overline{\Phi})\right].$$

We note that the D component of V is real and its gauge invariant:

$$[V]_D = \int d^2\theta V(x,\theta,\overline{\theta}) = \frac{1}{2}D(x), \tag{49}$$

exactly because V must transform under the adjoint representation of $U(1)$, which is scalar and thus $D(x)$ must be a scalar left invariant under $U(1)$. Note that this only happens in a $U(1)$ gauge theory because the expression Eq. (49) is not gauge invariant for a non-Abelian vectors superfield, since they have to be in the adjoint and so the vector field carries indices of the adjoint, $V_{IJ} = 2gV^aT^a_{ij}$.

We have that $\frac{\partial\mathcal{L}}{\partial D} = 0$ then D is another auxiliary field and it can be expressed as

$$D = -g\phi_i^* t_i \phi_i - \eta. \tag{50}$$

Then we can write the scalar potential as

$$U = W_i(\Phi_i)\overline{W}^i(\overline{\Phi}_i) + \frac{1}{2}D^2, = F_i^* F_i + \frac{1}{2}D^2, \tag{51}$$

and it is gauge invariant. When projecting the D term we have

$$\mathcal{L}_{int} = \left(-\frac{1}{2}W^{ij}\psi_i\psi_j + W^i F_i + x^{ij}F_iF_j + c.c.\right) - U, \tag{52}$$

where we can see that the left-hand side is not a symmetric quantity because the interchange of indices i,j with respect to the second term of the right-hand side, where we need to vary all possible indices k. The first part in the right-hand side of Eq. (48) is a symmetric quantity as it is explicitly given by Eq. (33).

7. The Minimal Supersymmetric Standard Model (MSSM)

The Minimal Supersymmetric Standard Model (MSSM) is as its name states the minimal possible way in one the SM can be supersymmetrized. The recipe seems obvious: just embed each of the SM particles in a supermultiplet. Since supersymmetry does commute with the gauge symmetries of the theory, the quantum numbers associated to them will be the same in the whole supermultiplet, except that the supersymmetric particles will have different spins. The consequences are then that:

- For each SM fermionic multiplet with spin 1/2, there will appear a scalar (sfermion) with spin 0.
- For each SM gauge boson, with spin 1, it will appear a fermion, better known as *gaugino*, with spin 1/2.
- Instead of having just one Higgs supermultiplet, there will be two! These supermultiplets will contain each one scalar boson of spin 0 and their fermionic superpartner called *higgsino* with spin 1/2 .

O.k. the last consequence is the non obvious part of the extension. But it is very easy to understand. From Eq. (33) we can see that the only possible way that could enter in a supersymmetric Lagrangian, Eq. (38), is to be defined as a holomorphic function of the superfields. We need to obtain from the Lagrangian of Eq. (38) the couplings of the Higgs field to both kinds of quarks: up and down. How do we do it?

We have said that each of these fields are embedded in different chiral supermultiplets

$$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \in \mathbf{Q}, \quad u_R^\dagger \in \overline{\mathbf{u}}, \quad d_R^\dagger \in \overline{\mathbf{d}} \quad \text{and} \quad H = \begin{pmatrix} H_u^+ \\ H^0 \end{pmatrix} \in \mathbf{H}, \tag{53}$$

Names		spin 0	spin 1/2	$SU(3)_C$, $SU(2)_L$, $U(1)_Y$
squarks, quarks	$\mathbf{Q}$	$(\widetilde{u}_L\ \ \widetilde{d}_L)$	$(u_L\ \ d_L)$	$(\ \mathbf{3},\ \mathbf{2}\ ,\ \frac{1}{6})$
(×3 families)	$\overline{\mathbf{u}}$	$\widetilde{u}_R^*$	$u_R^\dagger$	$(\ \overline{\mathbf{3}},\ \mathbf{1},\ -\frac{2}{3})$
	$\overline{\mathbf{d}}$	$\widetilde{d}_R^*$	$d_R^\dagger$	$(\ \overline{\mathbf{3}},\ \mathbf{1},\ \frac{1}{3})$
sleptons, leptons	$\mathbf{L}$	$(\widetilde{\nu}\ \ \widetilde{e}_L)$	$(\nu\ \ e_L)$	$(\ \mathbf{1},\ \mathbf{2}\ ,\ -\frac{1}{2})$
(×3 families)	$\overline{e}$	$\widetilde{e}_R^*$	$e_R^\dagger$	$(\ \mathbf{1},\ \mathbf{1},\ 1)$
Higgs, higgsinos	$\mathbf{H}_u$	$(H_u^+\ \ H_u^0)$	$(\widetilde{H}_u^+\ \ \widetilde{H}_u^0)$	$(\ \mathbf{1},\ \mathbf{2}\ ,\ +\frac{1}{2})$
	$\mathbf{H}_d$	$(H_d^0\ \ H_d^-)$	$(\widetilde{H}_d^0\ \ \widetilde{H}_d^-)$	$(\ \mathbf{1},\ \mathbf{2}\ ,\ -\frac{1}{2})$
		spin 1/2	spin 1	$SU(3)_C$, $SU(2)_L$, $U(1)_Y$
gluino, gluon	$\mathbf{g}$	$\widetilde{g}$	g	$(\ \mathbf{8},\ \mathbf{1}\ ,\ 0)$
winos, W bosons	$\mathbf{W}$	$\widetilde{W}^\pm\ \ \widetilde{W}^0$	$W^\pm\ \ W^0$	$(\ \mathbf{1},\ \mathbf{3}\ ,\ 0)$
bino, B boson	$\mathbf{B}$	$\widetilde{B}^0$	B^0	$(\ \mathbf{1},\ \mathbf{1}\ ,\ 0)$

Table 1: Chiral and gauge supermultiplets in the Minimal Supersymmetric Standard Model. For the chiral superfields, the spin-0 fields are complex scalars, and the spin-1/2 fields are left-handed two-component Weyl fermions. The gauginos are spin-1/2 fields which are also left-handed two-component Weyl fermions, note however that they sit in the adjoint representations of the gauge groups, as opposed the SM fermions which sit in the fundamental or trivial representations of each gauge group.

note the name $\overline{\mathbf{u}}$, this is a left-chiral superfield because it contains $u_R^\dagger$. Obviously also Q is chiral and H is defined to be so, that means in superspace we have: $\mathbf{H}(y) \doteq H(y)+\sqrt{2}\theta\xi\tilde{H}(y)+\theta\theta F_H$,

where H is our well known Higgs scalar from the SM and $\tilde{H}$ is a left-chiral fermion. Then the combination

$$\epsilon_{\alpha\beta}\mathbf{Q}^\alpha\mathbf{H}^\beta\overline{\mathbf{u}} \in W \tag{54}$$

it is indeed allowed in W. Here $\epsilon_{\alpha\beta}$ is the anti-symmetric tensor in two dimensions, so the $SU(2)_L$ indices of $\mathbf{Q}$ and $\mathbf{H}$ are properly contracted. Then as we have said § Eq. (6), we cannot have at the same time a term $\epsilon_{\alpha\beta}\mathbf{Q}^\alpha\left(i\sigma^2\mathbf{H}^{*\beta}\right)\overline{\mathbf{d}}$ because $\mathbf{H}^*(y) = H^*(y)+\sqrt{2}\bar{\theta}\tilde{H}^\dagger(y)+\theta\theta F_H^*(y)$ spoils the holomorphicity that W is required to have, i.e. such that $[W+\overline{W}]_F$ could supersymmetrically transform as a total derivative. Then it happens that we need indeed another Higgs field from which eventually masses to d quarks could be obtained

$$-\epsilon_{\alpha\beta}\mathbf{Q}^\alpha\mathbf{H}_d^\beta\overline{\mathbf{d}} \in W. \tag{55}$$

This time for everything to work out properly, the hypercharge, Y, of $\mathbf{H}_d$ must be $-1/2$, since we need to preserve $U(1)_Y$ transformations and so $Y_{H_d}+Y_Q+Y_{\bar{d}} = \frac{-1}{2}+\frac{1}{6}+\frac{1}{3} = 0$.

Furthermore this is indeed the requirement that we must impose for the cancellation of the triangular anomalies in the theory. The ones relevant to the hypercharge are proportional to $\mathrm{Tr}[T_3^2Y]$ and $\mathrm{Tr}[Y^3]$, where T_3 is the third component of weak isospin. The trace is a sum over all the fermionic degrees of freedom. We know that the SM is anomaly free, that is $\mathrm{Tr}[T_3^2Y] = \mathrm{Tr}[Y^3] = 0$. Then with only one Higgs superfield, we would have only one *higgsino* with hypercharge 1/2, and since already the SM fermionic fields have zero contribution to the traces above, we would have anomalies such that $\mathrm{Tr}[T_3^2Y] \propto 1/2$ and $\mathrm{Tr}[Y^3] \propto 8$. We can see that the problem is solved with the superfield Higgs H_d of exactly $-1/2$ hypercharge.

We have identified the MSSM and for concreteness we borrow the idea from [1] of encoding its content and most commonly used notation in a Table 1. Then the most general superpotential

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that can be written with these MSSM multiplets is:

$$\begin{aligned} W &= Y_l^{ij}\epsilon_{\alpha\beta}\mathbf{H}_d^\alpha\mathbf{E}_i^c\mathbf{L}_j^\beta + Y_d^{ij}\epsilon_{\alpha\beta}\mathbf{H}_d^\alpha\mathbf{D}_i^c\mathbf{Q}_j^\beta - Y_u^{ij}\epsilon_{\alpha\beta}\mathbf{H}_u^\alpha\mathbf{U}_i^c\mathbf{Q}_j^\beta \\ &+ \mu\epsilon_{\alpha\beta}\mathbf{H}_u^\alpha\mathbf{H}_d^\beta \\ &+\frac{1}{2}\lambda^{ijk}\mathbf{L}_i\mathbf{L}_j\overline{e}_k + \frac{1}{2}\lambda'^{ijk}\mathbf{L}_i\mathbf{Q}_j\overline{\mathbf{d}}_k + \mu'^i\mathbf{L}_i\mathbf{H}_u \\ &+\frac{1}{2}\lambda''^{ijk}\overline{\mathbf{u}}_i\overline{\mathbf{d}}_j\overline{\mathbf{d}}_k, \end{aligned} \tag{56}$$

Note that we are using the so called *Left-Right* notation for the Yukawa couplings. The previous line to the last violates the lepton number L by one unit, while the last one violates the baryon number B also by one unit. These are the famous undesired couplings that make possible proton decay $p^+ \rightarrow e^+\pi^0,\ e^+K^0,\ \mu^+\pi^0,\ \mu^+K^0$, etc. Then it was invented a symmetry that could forbid the last two lines of Eq. (56), *R parity* [9] or *matter parity* [10, 11, 12, 13] such that we could remain only with the desired part of W. Models where *R parity* is violated have been some what less studied.

8. Mechanisms to Break a Supersymmetric Lagrangian

8.1. General Considerations and Spontaneous Breaking

Just as in a normal field theory, in supersymmetric field theories matter and gauge fields cannot acquire vacuum expectation values (VEV) different from zero. But as it happens in the SM, scalar fields can do the job. As usual then the supersymmetry breaking part must be coming from the scalar potential. From Eq. (51) we can see that it is in general written by D and F contributions, hence effectively one needs to make one of these non-zero when the scalars acquire VEVs different from zero.

The bad news are that for the MSSM or any of its extensions there is not a single working model where one breaks the $N = 1$ supersymmetry and generates at the same time, the mass terms and trilinear parameters for all the fields of the MSSM. The good news is that there is a way to account for all the possible terms that may appear after the breaking of supersymmetry. This is because these terms can be restricted by imposing the absence of quadratic divergences of the effective theory obtained after supersymmetry breaking. We will check out this in the next section, § 9. Here we will just state how D and F terms can be made non zero.

Spontaneous breaking of the supersymmetry occurs by definition when the vacuum is not invariant under superymmetric transformations, that is $Q_A|0\rangle \neq 0$ and $Q_A^\dagger|0\rangle \neq 0$. In supersymmetry this happens when the ground state energy is lifted from zero. This is very easy to see because for any state $|\alpha\rangle$,

$$\begin{aligned} &\mathcal{H} = \frac{1}{4}\left\{Q_1,\overline{Q}_{\dot 1}\right\} + \frac{1}{4}\left\{Q_2,\overline{Q}_{\dot 2}\right\} \implies \\ &\langle\alpha|\mathcal{H}|\alpha\rangle = \frac{1}{4}\sum_{A=1}\sum_n \left[|\langle\alpha|Q_A|n\rangle|^2\right] + \left[|\langle\alpha|\overline{Q}_{\dot A}|n\rangle|^2\right], \end{aligned} \tag{57}$$

which is a positive definite quantity except for $|\alpha\rangle = |0\rangle$, which is zero.

Is nice to break supersymmetry this way since the Lagrangian does not change its form. Once the supersymmetry is broken there is a split in the supermultiplets, generating a difference in mass between superpartners. In this case the D or F terms do not vanish in the vacuum state. Recall that the $N = 1$ supersymmetry, that we have introduced in the first sections and the one that the MSSM satisfies, is a global supersymmetry. According to Goldstone's theorem when it is broken the resulting theory will contain a Nambu-Goldstone massless particle with the same quantum numbers of the broken symmetry generator [8], Q_A. This is a two component Weyl spinor, therefore the Nambu-Goldstone *goldstino* will be a massless two component Weyl fermion.

8.2. D Term Supersymmetry Breaking

D term supersymmetry breaking happens when a D term in the scalar potential is lifted from zero. An example of this is the Fayet-Iliopoulos Mechanism, which assumes the existence of a $U(1)$ gauge factor in the theory. As we have seen from Eq. (47), in a supersymmetric $U(1)$ gauge Lagrangian, the following terms contain the scalar D

$$\mathcal{L}_{U(1)} \supset \frac{1}{2}D^2 + \eta D + g t_i |\phi_i|^2 D, \quad \mathcal{L}_{FI} \doteq \eta D, \tag{58}$$

where exaclty ηD is the Fayet-Iliopoulos part of the Lagrangian. Obviously D should be a gauge singlet. The D term itself is given by Eq. (50). In this mechanism it is assumed that there are n scalars with charges $t_i = q_i$ under $U(1)$, which may or may not have masses, m_i, coming from the superpotential. Then in general we can write

$$U = -\frac{1}{2}D^2 - \eta D - g\sum_{i=1}^{i=n} q_i|\phi_i|^2 D + m_i^2|\phi_i|^2 \;=\; \frac{1}{2}\left[-\eta - g\sum_{i=1}^{i=n} q_i|\phi_i|^2\right]^2 + m_i^2|\phi_i|^2, \tag{59}$$

where for the last expression of U we have used Eq. (50) and Eq. (51). This cannot vanish and therefore supersymmetry must be broken. Note that the scalars will have masses $m_i^2 + g q_i \eta$ while their fermion partners will have masses m_i^2.

Unfortunately the $U(1)_Y$ gauge factor of the MSSM cannot help to break supersymmetry, in this case because its fermions do not have superpotential mass terms, the m_i's. We see that even in the absence of m_i's, U (Eq. (59)), one may think to use the effective U to break supersymmetry, but it has been shown that does not and instead it does break color and electromagnetism.

However this mechanism can be used to break realistic extensions of the MSSM, which include, other than $U(1)_Y$, $U(1)$ gauge factors. It is also very useful in supersymmetry family symmetry theories where we have different $U(1)$ charges for each family of fermions and at least two scalars θ and θ' with charges 1 and -1. They can be coupled to $QH_u u_R^\dagger$ and $QH_d d_R^\dagger$, generating the hierarchical Yukawa couplings and by means of the U of Eq. (59) fix the order of the VEVs of θ and θ'.

8.3. F Term Supersymmetry Breaking

F term supersymmetry breaking is easier to achieve because one only requires that there exist at least one $F*_i = \frac{\delta W}{\delta \phi_i} \neq 0$, among n possible F_i terms. If supersymmetry needs to be broken in the global minimum of the potential [9] then $F_j = 0\ \forall j \neq$ cannot simultaneously be solved for $F_i = 0$. In this case supersymmetry is indeed broken because $U = \sum_{k=1}^n F_k^* F_k \neq 0$. A consequence of $F_i = 0$ is that any superpotential without a linear term in Φ, first term in Eq. (33), cannot produce an F type symmetry breaking because given that $F_k^* = m_{kj}\phi_j + \frac{1}{3} f_{kst}\phi_s\phi_t$ can be made zero, once we take VEVs, $\langle F_k \rangle = 0$, $\forall j$ for $\langle \phi_j \rangle = 0$, then we will have $U = 0$

An example of a successful F type supersymmetry breaking is the O'Raifeartaigh model. In this model one needs a minimum of three chiral fields, Φ_i with the superpotential of the form

$$W = m\Phi_2\Phi_3 + \lambda\Phi_1(\Phi_3^2 - \mu^2), \tag{60}$$

[8] We are used to bosonic symmetry generators because of the standard $U(1)$ bosonic example of a complex scalar field. After symmetry breaking the Nambu-Goldstone massless particle is a boson.

[9] Local minima can generate the so called *metastable* vacua which can have very large lifetimes and therefore under some conditions are suitable for candidates of the "observed" vacua of a given theory.

for μ a given mass parameter. If m, μ, and λ are real then $F_1^* = \lambda(\phi_3^2 - \mu^2)$, $F_2^* = -m\phi_3$ and $F_3^* = -m\phi_2 - 2\lambda\phi_1\phi_3$. Then as O'Raifeartaigh showed, the system of equations $F_i = 0$, $i = 1, 2, 3$, has no simultaneous solution. The minimum of the potential U in this model has two different behaviours when $\mu^2 < m^2/2\lambda^2$ and $\mu^2 > m^2/2\lambda^2$. It is often easy to find global minima that break supersymmetry in models which are characterized by the first case.

8.4. Mechanisms for Supersymmetry Breaking in the MSSM

It so happens that just with the superfields of the MSSM it is not possible to achieve neither D nor F term breakings. Then additional superfields that break supersymmetry must be included, but since many fermionic components can induce dangerous couplings [10] with the MSSM fields, these couplings should be really small. The set of fields that break the MSSM supersymmetry in this way, is called the *Hidden Sector*. Now, it is not enough to break the supersymmetry, it is also necessary to obtain a realistic mass spectrum for the supersymmetric particles. There are two popular mechanisms to attempt it, which we describe in what it follows.

In the **Gravity Mediated Supersymmetry Breaking** mechanism, the hidden sector is communicated only through gravitational interactions to the MSSM fields. That is, the breaking or any coupling to MSSM fields can only be felt via gravity.

Another mechanism to attempt supersymmetry breaking in the MSSM is the **Gauge Mediated Supersymmetry Breaking**, this is just as the gravity mediated supersymmetry breaking but this time the interactions are the same gauge interactions of the MSSM. In order construct realistic models in this scenario, the couplings of the hidden sector must come from loop effects.

Is sad to say it, but up to date there is not a single full working model for supersymmetry Breaking in the MSSM.

9. Soft SUSY Breaking Lagrangian and Mass Spectrum of the MSSM

9.1. Soft SUSY Breaking

As we have mentioned in the previous section, with the MSSM fields alone, it is not possible to break the $N = 1$ supersymmety. However, given the particle content of the MSSM, it is possible to write down all the possible terms that, after symmetry breaking, could exist. These terms can be restricted by imposing the absence of quadratic divergences of the effective theory. Having said so, in the MSSM with R parity, the most general soft supersymmetric Lagrangian that can be written is

$$\begin{aligned} -\mathcal{L}_{\text{soft}} &= \tilde{q}_{Li}^\dagger (m_{\tilde{Q}}^2)^{ij} \tilde{q}_{Lj} + \tilde{u}_{Rj} (m_{\tilde{u}}^2)^{ji} \tilde{u}_{Ri}^* + \tilde{d}_{Rj} (m_{\tilde{d}}^2)^{ji} \tilde{d}_{Ri}^* \\ &\quad + \tilde{l}_{Li}^\dagger (m_{\tilde{L}}^2)^{ij} \tilde{l}_{Lj} + \tilde{e}_{Rj} (m_{\tilde{e}}^2)^{ji} \tilde{e}_{Ri}^* + \tilde{\nu}_{Rj} (m_{\tilde{\nu}}^2)^{ji} \tilde{\nu}_{Ri}^* + m_{h_d}^2 h_d^\dagger h_d + m_{h_u}^2 h_u^\dagger h_u \\ &\quad + (b\, h_d h_u + \text{h.c.}) + \Big(-a_d^{ij} h_d \tilde{d}_{Ri}^* \tilde{q}_{Lj} + a_u^{ij} h_u \tilde{u}_{Ri}^* \tilde{q}_{Lj} - a_l^{ij} h_d \tilde{e}_{Ri}^* \tilde{l}_{Lj} + a_\nu^{ij} h_u \tilde{\nu}_{Ri}^* \tilde{l}_{Lj} \\ &\quad + \frac{1}{2} M_1 \widetilde{B} \widetilde{B} + \frac{1}{2} M_2 \widetilde{W}^a \widetilde{W}^a + \frac{1}{2} M_3 \widetilde{G}^a \widetilde{G}^a + \text{h.c.} \Big), \end{aligned} \tag{61}$$

where SU(2) indices are not written explicitly. They are contracted by $\epsilon_{\alpha\beta}$ and $\delta_{\alpha\beta}$, respectively, i.e. $AB := \epsilon_{\alpha\beta} A^\alpha B^\beta$ and $A^\dagger A := A_\alpha^\dagger A^\alpha$ for SU(2) doublet fields A, B. As we see it is an incredible task to study constraints on all the parameter space of the general MSSM, counting the number of parameters we arrive at an incredible number of 109! Not only this but also the fact that we need to set up typically three main scales

$$M_{\text{UV}} > M_{\not S} > M_{\text{EW}}, \tag{62}$$

[10] That, for example, could mediate decay of particles at a rate that is not observed, etc.

which describe the scales at which one should put boundary conditions of all the parameters in Eqs. (56) and (61). $M_{\rm UV}$ represents the *Ultra Violet completion* (UV) of the model, coming from a definite supersymmetric model, $M_{\not S}$ is the scale at which supersymmetric particles decouple where we need to match to SM parameters and $M_{\rm EW}$ the Electroweak (EW) scale. On then should employ the appropriate Renormalization Group Equations (RGE) between two different scales and work out appropriate matching conditions [11]. This is a formidable numerical task and there are several computer codes that have been developed for addressing this enterprise [15, 16, 17] and several people managing their own code.

Even when the RGE evolution can be done numerically, there are still formidable hitches to worry about. The strategy followed in the last more than twenty years to tackle this problem is to simplify the number of parameters based on the following guidelines.

(i) *Minimal MSSM.* General couplings in Eq. (61) will easily induce flavour violating processes [12] at a rate that is much above the experimental bounds or measured values. Then we can think a set up where

$$\begin{aligned} m^2_{\tilde{Q}ij} &= m^2_{\tilde{u}ij} = m^2_{\tilde{d}ij} = m^2_{\tilde{e}ij} = \delta_{ij} m_0^2, \\ m^2_{H_d} &= m^2_{H_u} = m_0^2, \quad b = B\mu, \\ a_u^{ij} &= AY_u^{ij},\ a_d^{ij} = AY_d^{ij},\ a_e^{ij} = AY_e^{ij}, \\ M_1 &= M_2 = M_3 = M_{1/2}. \end{aligned} \tag{63}$$

The Constrained MSSM (**CMSSM**) adopts this strategy [13]. The conditions Eq. (63) are set up as a boundary condition at $M_{\rm UV}$ identified with the GUT scale, $M_{\rm G}$. Then the bread and butter free parameters of the CMSSM community are

$$\mathrm{sign}[\mu],\ A,\ m_0,\ M_{1/2} \text{ and } \tan\beta. \tag{64}$$

The parameters B and μ are obtained by the requirement of the Electroweak symmetry breaking conditions. That is to say, they are fitted such that with the spectra obtained from the running of the CMSSM from $M_{\rm G}$ to $M_{\rm EW}$ (passing through $M_{\not S}$) it so happens that there is a minimum for the scalar potential of the theory such that the EW symmetry is broken. The only degrees of freedom that are run from $M_{\rm G}$ down to $M_{\rm EW}$ are those of the MSSM, any theory that was above $M_{\rm G}$ is assumed to be decoupling exactly at that scale.

(ii) Minimal Supergravity **mSUGRA**
Here we are entering into a fascinating realm of supersymmetry that we have not yet mentioned, that of local supersymmetric transformations ([14]). An $N = 1$ local supersymmetry transformation turns out to describe a theory which contains a particle of spin 2 that can be identified with a *graviton* and its superpartner, of spin 3/2, the *gravitino.* This local supersymmetry is called *Supergravity* (SUGRA) [].
Supergravity theories apart from having a superpotential, have two other unique functions that are relevant. First, the so called *Kähler potential*, K, which it is a real function of scalar

[11] Two loop beta functions of the MSSM can be found in [14].
[12] Among the most pressing ones are the mixing in the $K^0 - \overline{K^0}$ system whose CP violation parameter, ϵ_K, has been precisely measured, the leptonic decays $\ell_i^- \to \ell_j^- \gamma$, the decay $b \to s\gamma$, known to a good precision.
[13] In practice, since the Yukawa matrices cannot be uniquely determined, when one comes across a *CMSSM* analysis, it is common that the analysis is assuming particular forms of the Yukawa matrices. The most used is that all Yukawa couplings, except those of the heaviest families, are zero. That is, it just takes into account the RGE running of the heaviest fermionic families, i.e. instead of running the 27 parameters from a_u^{ij}, a_d^{ij} and a_e^{ij} and the 27 from Y_u^{ij}, Y_d^{ij} and Y_e^{ij}, it just runs the 3 parameters $A_{u,d,e}$ and the three Yukawa couplings $y_{t,b,\tau}$.
[14] Check out [18] for a great introduction.

fields of the theory. Its *metric*, $K_{ij^*} = \partial^2 K/(\partial\phi_i \partial\phi_j^*)$, appears in terms like $K_{ij^*}\overline{\psi}^{i*}\gamma^\mu\partial_\mu\psi^j$. Second, the *gauge kinetic functions*, f_{ab}, which appear in the supergravity Lagrangian, for example in the terms: $\mathcal{L}_{SUGRA} \propto \text{Re}[f_{ab}]F^a_{\mu\nu}F^{b\mu\nu} + i\text{Im}[f_{ab}]F^a_{\mu\nu}\tilde{F}^{b\mu\nu}$. The gauge kinetic functions are arbitrary analytic functions of the scalar fields of the theory.
There are two remarkable features of this theory. The first is that it is not renormalizable! The second is that a limit, where $M_{\text{P}} \to \infty$ and the mass of the gravitino ($m_{3/2} \propto 1/M_{\text{P}}$) is fixed, produces a supersymmetric theory, free of quadratic divergences and such that those parameters which were free in the soft-supersymmetry breaking Lagrangian of Eq. (61), can be calculated in terms of scalar fields of the hidden sector of the theory, the same on which K_{ij} and f_{ab} depend on.
Minimal Supergravity (mSUGRA) really refers to the choice

$$\begin{aligned} K_{ij} &= \delta_{ij}, \quad \forall \quad \text{fields in the theory,} \\ f_{ab} &= \delta_{ab}, \quad \forall \quad \text{gauge fields in the theory.} \end{aligned} \tag{65}$$

Independently of the choice of a particular superpotential W, there will be simplifications among the quantities of Eq. (61), in fact those of Eq. (63) plus relations among A, $M_{1/2}$, B and $m_{3/2}$.
Now, the CMSSM is a framework to study what happens when parameters in Eq. (64) are varied, while in mSUGRA those parameters should be identified with some underlying supergravity model. Explicitly in the context of the MSSM, mSUGRA, encompasses all the models which propose a definite superpotential for the hidden sector of the theory. An example of these is the **Polonyi Model** [15]. The defining characteristic of these models is obviously the superpotential of the hidden sector. For example, the Polonyi superpotential is $W = m^2(\Phi + \beta)$, both m and β are mass terms. In particular the choice $\beta = (2 - \sqrt{3})M_{\text{P}}$ produces the following simplifications

$$A = (3 - \sqrt{3})m_{3/2}, \quad m_0^2 = m_{3/2}^2, \quad M_{1/2} = O(m_{3/2}). \tag{66}$$

For the different approaches mentioned here, a good state of the art can be found in [20, 21, 22] and references therein.

(iii) Supersymmetric Family Symmetries They put specific relations among all the parameters in Eqs. (56) and (61) such that apart from reproducing the hierarchy of masses and mixing in the quark and lepton sector, they minimize the processes that change flavour. Generically they do not produce simple relations as Eq. (63), but the couplings of the supersymmetric theory are controlled also by the Family Symmetry [23].

9.2. MSSM Mass Spectrum

Having a look at Table 1 we can classify the following sets of supersymmetric particles: [i] scalar particles ($\widetilde{u}^*_R$, $\widetilde{d}^*_R$, $\widetilde{e}^*_R$, $\widetilde{Q}^*_L$ and $\widetilde{Q}^*_R$), [ii] charged, under $U(1)_{em}$, fermions ($\widetilde{H}^+_u$, $\widetilde{H}^-_d$, $\widetilde{W}^\pm$), [iii] neutral fermions ($\widetilde{H}^0_d$, $\widetilde{H}^0_u$, $\widetilde{B}^0$, $\widetilde{W}^0$) and [iv] the very special gluino, $\widetilde{g}$. Due to the couplings that can be derived from the superpotential of Eq. (56), once the MSSM Lagrangian is constructed via a generalization of Eq. (46) and the contribution from the soft breaking Lagrangian, Eq. (61), there will be mixings among some particles in each set.

Once the mass matrices form the mixing states and we obtain the mass eigenstates of these sets are called *Charginos* ($\chi^\pm_i$) for set [ii] and *Neutralinos* (χ^0_i) for set [iii]. For the set [i], there are indeed 6 $\widetilde{u}^*_i$ (coming from $\widetilde{u}^*_R$ and $\widetilde{Q}_L$) different particles which will mix, the same for $\widetilde{d}^*_i$ and analogously for $\widetilde{e}^*_i$. The mass eigenstates of these different particles are called respectively *up-type squarks* (aka *s-ups*) , *down-type squarks* (aka *s-downs*) and *s-electrons.*

[15] For more examples, check out [19].

The masses of these particles must be RGE evolved from $M_{\rm UV}$ down to $M_{\rm EW}$. After this is achieved we can obtain tree level masses of the *physical* mass-eigenstates mentioned above. That is not all, we have to calculate loop corrections to these masses, fortunately for some cases, there are computer programmes to do so.

Note that a crucial part of obtaining a realistic MSSM mas spectrum is to obtain EW symmetry breaking. The MSSM scalar potential is a bit more complicated than that of the SM. Assuming that at the minimum of the potential, the charged parts of the Higgs scalars are zero, as in the SM, $H_u^+ = 0$, at the minimum of the potential ($\partial U_H/\partial H_u^+ = 0$), we must also have $H_d^- = 0$. Then at the minimum of the potential electromagnetism is unbroken, since the charged components of the Higgs scalars do not get VEVs. With these constraints the MSSM Higgs scalar potential can be written as

$$U_H = (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 - (bH_u^0 H_d^0 + c.c.) + \frac{(g^2 + g'^2)}{8}(|H_u^0|^2 - |H_d^0|^2)^2. \quad (67)$$

It can be shown that a minimum of this potential requires that b, H_u^0 and H_d^0 to be real and positive, so $\langle H_u^0\rangle$ and $\langle H_d^0\rangle$ must have opposite phases. We can therefore use a $U(1)_Y$ gauge transformation to make them both real and positive without loss of generality, since H_u and H_d have opposite weak hypercharges ($\pm 1/2$). Then CP cannot be spontaneously broken by the Higgs scalar potential, since the VEVs and b can be simultaneously chosen real. This means that the Higgs scalar mass eigenstates can be assigned well-defined eigenvalues of CP, at least at tree-level. CP-violating phases in other couplings can induce loop-suppressed CP violation in the Higgs sector. After the spontaneous symmetry breaking the VEVs of the CP even Higgs, $v_u = \langle H_u^0\rangle$, $v_d = \langle H_d^0\rangle$, need to satisfy $v_u^2 + v_d^2 = 246^2/2$ GeV^2. The ratio of these VEVs is known as $\tan\beta = v_u/v_d$. In the SM we will have just one real Higgs, h after spontaneous symmetry breaking, in the MSSM there are four other physical Higgs particles: the other CP even Higgs, H^0, a CP odd neutral Higgs, A^0 and two charged Higgs fields, $H^\pm$. The phenomenology of this zoo on its own is fascinating, for a great book check out [24, 25].

As we have said in the previous section it is a phenomenal task to analyse the full parameter space of the MSSM. As the LHC is running, we would like to have an idea of some reasonable assumptions about the parameters defining the MSSM. The following assumptions are a good starting point: (i) χ_i^0 are the lightest supersymmetric particles, (ii) R-parity is conserved, (iii) with the exception of $\tilde{t}$ and $\tilde{b}$ all squarks are degenerate in mass and the masses of the superpartners of left-handed quarks and right-handed quarks are equal at $M_{\rm G}$ and (iv) there is gaugino mass unification at $M_{\rm G}$.

Then some bounds [26], all at $CL = 95\%$, can be obtained on the masses for the sparticles:

(i) for the lightest neutralino χ_1^0 (coming from mixtures of $\widetilde{B}^0$, $\tilde{Z}^0$, and $\tilde{H}_i^0$), $m_{\chi_1^0} > 46$ GeV;

(ii) for $\widetilde{\chi}_i^\pm$ (mixtures of $\widetilde{W}^\pm$ and $\widetilde{H}^\pm$ and $\tan\beta < 40$, $m_{\widetilde{\chi}_1^\pm} - m_{\widetilde{\chi}_1^0} > 3$ GeV), $m_{\widetilde{\chi}_i^\pm} > 94$ GeV;

(iii) $m_{\widetilde{e}} > 107$ GeV, $m_{\widetilde{\mu}} > 94$ GeV (here the bounds are assuming that the left parts of the sleptons do not play a significant rôle and that $1 \leq \tan\beta \leq 40$, $m_{\widetilde{\mu}} - m_{\widetilde{\chi}_1^0} > 10$ GeV) and $m_{\widetilde{\mu}} > 82$ GeV ($m_{\widetilde{\tau}} - m_{\widetilde{\chi}_1^0} > 15$ GeV);

(iv) $m_{\widetilde{b}} > 89$ GeV ($m_{\widetilde{b}_1} - m_{\widetilde{\chi}_1^0} > 8$ GeV);

(v) $m_{\widetilde{t}} > 96$ GeV (obtained from the decay $\widetilde{t} \to c\chi_i^0$, $m_{\widetilde{t}} - m_{\widetilde{\chi}_1^0} > 10$ GeV) and

(vi) $m_{\widetilde{g}} > 308$ GeV in general (assuming gauge coupling unification at $M_{\rm G}$). There are more specific bounds for particular relations among other sparticles.

10. Glimpse of Extensions of the MSSM

During the last decade neutrino physics, after the observation of their oscillations [27], has become a central arena for studies beyond the SM. They are massive, though we do not yet know their overall mass, so any model BSM should worry about them. There are ways to think that they cannot affect the basic behaviour of the MSSM. For example in scenarios where the see-saw mechanism explains through heavy right-handed neutrino masses, the small masses of oscillating energy neutrinos, the right-handed neutrinos could decouple before M_G. There are other models in which it is interesting to decouple right-handed neutrinos after M_G. Then the running of the mass spectrum of the MSSM is altered and all the sector which produces neutrino masses should also be taken into account in the running. I find cute the generic name νMSSM for this kind of models.

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