Conformal Invariance in Beta Deformed N=4 SYM Theory

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Abstract

We claim that if by a choice of the couplings the theory can be made conformally invariant (vanishing of the beta functions) it is automatically finite (absence of UV divergencies) and vice versa. The formalism is applied to the beta deformed $\mathcal{N}=4$ SYM theory and it is shown that the requirement of conformal invariance = finiteness can be achieved for complex parameter of deformations.

1. The $\mathcal{N}=4$ supersymmetric Yang-Mills theory (SYM) attracts much attention these days providing the playground to test nonperturbative features of quantum field theory. This is related to the property of conformal invariance which is unique for four dimensional field theories [1]. Another remarkable feature of $\mathcal{N}=4$ SYM theory is that using the AdS/CFT correspondence [2] one can get deeper understanding of duality between the gauge theory and the supergravity theory. Combined information may lead to new insight in gauge theories beyond the usual PT.

Note that AdS/CFT correspondence requires from field theory to be only conformal invariant and not necessarily obtain the full $\mathcal{N}=4$ SUSY. From this point of view it is interesting to consider other SCFT's. One of such SCFT's is the so called beta-deformed $\mathcal{N}=4$ SYM theory. It is given by the action [3]

$$\begin{aligned} S &= \int d^8z Tr\left(e^{-gV}\bar{\Phi}_i e^{gV}\Phi^i\right) + \frac{1}{2g^2}\int d^6z Tr(W^\alpha W_\alpha) \\ &\quad + ih\int d^6z Tr\left(q\Phi_1\Phi_2\Phi_3 - \frac{1}{q}\Phi_1\Phi_3\Phi_2\right) + h.c., \quad q \equiv e^{i\pi\beta}, \end{aligned} \tag{1}$$

where the superfield strength tensor $W_\alpha = i\bar{D}^2(e^{-gV}D_\alpha e^{gV})$ and Φ_i with $i=1,2,3$ are the three chiral superfields of the original $\mathcal{N}=4$ SYM theory; h and β are complex numbers and g is the real gauge coupling constant. In the undeformed $\mathcal{N}=4$ SYM theory one has $h=g$ and $q=1$.

In the context of the AdS/CFT correspondence the dual supergravity background adjusted to the beta-deformed case was found in [2]. For a self-consistency of the AdS/CFT dual description it is crucial that the beta-deformed $\mathcal{N}=4$ SYM remain conformal invariant on quantum level.

2. To study this question one have to choose some regularization and renormalisation scheme and to calculate the beta functions corresponding to each coupling of a theory. In the present case, besides the gauge coupling g, one defines the couplings

$$h_1 \equiv hq, \quad h_2 \equiv h/q, \quad h_1^2 \equiv h_1\bar{h}_1, \quad h_2^2 \equiv h_2\bar{h}_2. \tag{2}$$

In general $\mathcal{N}=1$ SYM theory formulated in terms of $\mathcal{N}=1$ superfields has two types of divergent diagrams, those of the chiral field propagator and of the gauge field one. The chiral vertices are finite due to the non-renormalization theorems [4] and for the gauge vertices one can choose the so called background gauge [5] where their divergent factors coincide with the propagator ones. Thus, one has to consider the field propagators only.

This problem can be further reduced to chiral propagators since the gauge beta function can be expressed in terms of the chiral field anomalous dimensions using the explicit NSVZ gauge beta function [6]

$$\beta_g = g^2\frac{\sum T(R) - 3C(G) - \sum T(R)\gamma(R)}{1-2gC(G)}, \qquad g \equiv g^2/16\pi^2. \tag{3}$$

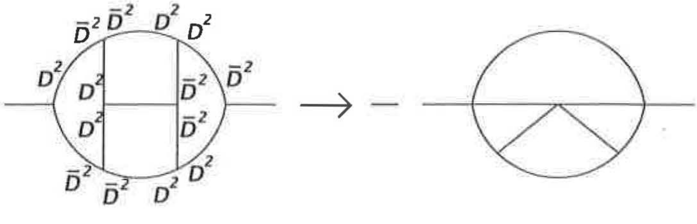
where $T(R)$ is the Dynkin index of a given representation R and $C_2(G)$ is the quadratic Casimir operator of the group. In the beta-deformed case one has the same field content as in $\mathcal{N}=4$ SYM [7] so $T(R)=C(G)$, so what one needs is to get the vanishing of the chiral field anomalous dimensions $\gamma(R)$.

This can be achieved by choosing the Yukawa couplings h_i $(i=1,2)$ in the form of perturbation series over g [8]:

$$h_i^2 = \alpha_{0i}g^2 + \alpha_{1i}g^4 + \alpha_{2i}g^6 + \dots \,. \tag{4}$$

If the anomalous dimensions of the chiral fields vanish, so does the gauge and Yukawa beta functions and the theory is conformally invariant. One can also think that this chose of α_{ki} will cancel all singularities in $Z_{<\Phi\bar{\Phi}>}$ and the theory will be finite.

3. The naive implementation of this program to the planar complex beta-deformed $\mathcal{N}=4$ SYM in dimensional regularization (reduction) [3, 9] meets the following problem: in 4 loops one cannot simultaneously cancel contributions by some new nontrivial supergraph to the anomalous dimension γ and all poles in the $Z_{<\Phi\bar{\Phi}>}$ factor. So one may think that there is contradiction between conformal invariance and finiteness. However,



(a) (b)

Figure 1: The new nontrivial 4 loop supergraph and its scalar counterpart

this is the result of mistreatment of dimensional regularization. In case of dimensional regularization and $\overline{MS}$ renormalization scheme (we ignore the problem of inconsistency of dimensional reduction in higher orders [10] assuming that it is adjusted by finite corrections) to achieve simultaneously the conformal invariance and finiteness one has to use the two fold expansion [11] instead of one fold series (4)

$$\begin{aligned} h_1^2 &= g^2(a+\alpha_0^{(3)}\varepsilon^3)+g^4\alpha_1^{(2)}\varepsilon^2+g^6\alpha_2^{(1)}\varepsilon+g^8\alpha_3^{(0)}+\dots , \\ h_2^2 &= g^2(b+\beta_0^{(3)}\varepsilon^3)+g^4\beta_1^{(2)}\varepsilon^2+g^6\beta_2^{(1)}\varepsilon+g^8\beta_3^{(0)}+\dots, \end{aligned} \tag{5}$$

both in gauge coupling constant g^2 and parameter of dimensional regularization ε. (Note the total power of expansion terms equals 4 that reflects the fact that the first non-vanishing contribution comes in 4 loops.) The existing freedom of choice of the coefficients $\alpha_i^{(m)}$ and $\beta_k^{(l)}$ is enough to get *simultaneously* the vanishing of the anomalous dimensions (read *conformal invariance*) and the pole terms in $Z_{<\Phi\bar{\Phi}>}$ factor (read *finiteness*).

The explicit calculations up to for loops leads to the following result:

$$\begin{aligned} & h_1^2+h_2^2=\bar{h}h(\bar{q}q+1/\bar{q}q) \qquad (6) \\ & = g^2\left\{2+\frac{5}{18}\zeta_5\delta^4\varepsilon^3+\frac{5}{3}\zeta_5\delta^4(\frac{g^2N}{4\pi^2})\varepsilon^2+5\zeta_5\delta^4(\frac{g^2N}{4\pi^2})^2\varepsilon+10\zeta_5\delta^4(\frac{g^2N}{4\pi^2})^3+\dots\right\}, \end{aligned}$$

where we denoted $a-b\equiv\delta$, $a+b=2$. Case $\delta=0$ corresponds to the real β [3,9]. So if $\alpha_i^{(m)}$ and $\beta_k^{(l)}$ satisfy (6) condition up to 4 loops then the anomalous dimension γ and all poles in $Z_{<\Phi\bar{\Phi}>}$ vanishes and the theory is conformal invariant and finite up to 4 loops. Notice that if $\alpha_i^{(m)}$ and $\beta_k^{(l)}$ satisfy eq.(6) all the poles except a simple one in 5 loops in $Z_{<\Phi\bar{\Phi}>}$ also vanish. To cancel a simple pole in 5 loops one have to add new terms in (5). Adding new terms in (5) one can obtain conformal invariant and finite theory at any given order of P.T.

Conclusion

We conclude that properly treated β deformed $\mathcal{N}=4$ SYM theory can be made (at least perturbatively) *simultaneously* conformal invariant and finite for any *complex* value of the deformation parameter β since these two requirements are *identical*. This can be achieved by adjusting the Yukawa couplings order by order in PT. In the framework of dimensional regularization (reduction) this requires the double series over the gauge coupling g^2 and the parameter of dimensional regularization ε. For the bare coupling, on the contrary, only the one fold series over ε is enough. The whole procedure depends on regularization (for bare quantities) and renormalization scheme (for the renormalized ones). In the other regularization techniques it looks differently but the main conclusion remains the same.

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