# Colour Coherence Studies at LEP [1]

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## Abstract

We review some properties of QCD coherence that can be tested at LEP. In particular we discuss the following three topics: 1) Soft hadron distributions in QCD jets; 2) Properties of heavy flavour jets; 3) QCD drag effect in interjet hadron flows.

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With the start of SLC and LEP activity a wealth of experimental data on hadronic jets has become available for detailed tests of perturbative QCD and for reducing our domain of ignorance on the physics of confinement. The first experimental studies [1-5] of jets at LEP show that hadronic event shapes computed at the parton level agree well with the measured ones. This supports the hypothesis of "soft confinement" which leads to the "Local Parton-Hadron Duality" picture [6]. Such local duality follows naturally from the pre-confinement property of perturbative QCD [7]. On the basis of this hypothesis, a detailed analysis of perturbative QCD properties of hard collision data is possible.

One of the most interesting properties to study experimentally at LEP is the colour coherence of perturbative QCD, which should reveal itself in the structure of final hadronic states of hard processes. In this note we present a list of experimental investigation of hadron jet phenomena, which should reveal the colour structure of QCD and the local duality features. In particular we concentrate on the following three topics: 1) Soft hadron distribution in QCD jets; 2) Properties of heavy flavour jets; 3) QCD drag effect in interjet hadron flows.

## 1. Soft hadron distribution in QCD jets

A striking prediction of perturbative QCD is the coherence of soft gluon radiation, which is associated with gluon interference in the soft region. Destructive interference to leading infrared order reduces the phase space for parton radiation to an angular ordered region. This phenomenon may be called *intrajet colour coherence* (for reviews see [8,9]). It leads to clear predictions for i) Multiplicity moments and distributions and for ii) Hadron momentum distributions at small relative momenta. For a recent review of QCD predictions for some of these quantities at LEP, see Ref. [10]. Here we focus our attention on the main consequences of intrajet coherence for the quantities of the last type.

### *1.1 Single Inclusive Distribution of Soft Hadrons*

At LEP energies, the momentum spectrum of relatively soft particles ($x \ll 1$), which form the bulk of hadron multiplicity, should clearly exhibit the so called "hump-backed plateau" in the shape of the single inclusive distribution

$$D(x,E) = \frac{x}{\sigma}\frac{d\sigma}{dx} \tag{1}$$

where $E = Q/2$ is the jet energy, $x = p/E$ and $p$ is the hadron momentum. This is a clear consequence of intrajet QCD coherence which can be expressed in terms of angular ordered partonic cascades and leads to substantial suppression of soft radiation. The maximum of the distribution (1) is predicted by QCD to be at a value of the momentum $p = \bar{p}$ given by

$$\ln\frac{\bar{p}}{\Lambda} = \ln\bar{x} + \tau = \tfrac{1}{2}\tau\left(1 - \frac{\rho}{24}\sqrt{\frac{48}{\beta\tau}}\right) + O(1)\,; \qquad \tau = \ln\frac{E}{\Lambda}\,, \tag{2}$$

where $\beta = 11 - 2N_f/3$ and $\rho = 11 + 2N_f/27$ for $N_f$ flavours, $\Lambda$ being the effective QCD scale.

The inclusive parton distribution corresponding to (1) is infrared stable and its asymptoptic form can be computed in perturbative QCD by solving the parton cascade evolution equation in the angular ordered phase space. According to the local duality hypothesis, the inclusive distribution of any type of hadron will have the same asymptotic form, up to a normalization constant. To next-to-leading order, for both quark and gluon jets the distribution in the soft region is given by a distorted Gaussian in $\ln x$ [9,15,16]:

$$D(x,E) \simeq \frac{\langle n\rangle}{\sigma\sqrt{2\pi}} \exp\left[\frac{1}{8}k - \frac{1}{2}s\delta - \frac{1}{4}(2+k)\delta^2 + \frac{1}{6}s\delta^3 + \frac{1}{24}k\delta^4\right] \tag{3}$$

where $\delta = (\ln\bar{x} - \ln x)/\sigma$ and the parameters $\sigma$, $s$ and $k$ have been computed in Ref. [15,16]

$$\begin{aligned} \sigma &= \sqrt{\frac{\tau}{3}}\left(\frac{\beta\tau}{48}\right)^{\frac{1}{4}}\left(1 - \frac{\beta}{64}\sqrt{\frac{48}{\beta\tau}}\right) + O(\tau^{-\frac{1}{4}}) \\ s &= -\frac{\rho}{16}\sqrt{\frac{3}{\tau}}\left(\frac{48}{\beta\tau}\right)^{\frac{1}{4}} + O(\tau^{-\frac{5}{4}})\,; \qquad k = -\frac{27}{5\tau}\left(\sqrt{\frac{\beta\tau}{48}} - \frac{\beta}{24}\right) + O(\tau^{-\frac{3}{2}})\,. \end{aligned} \tag{4}$$

The average multiplicity $\langle n\rangle$ is predicted to have the asymptotic behaviour

$$\ln\langle n\rangle = \sqrt{48\tau\beta} - \left(\frac{1}{4} + \frac{10N_f}{27\beta}\right)\ln\tau + O(1)\,. \tag{5}$$

The multiplicities of all types of hadrons, from all hard processes at scale $Q$, should have the same asymptotic behaviour, differing only in the terms of order unity in (5), i.e. in the overall normalization.

One should note that for each type of hadron the fitted effective value of $\Lambda$ in the distribution (3) is expected to increase slightly with the hadron mass. This is due to the fact that the momentum distributions becomes stiffer as the hadron mass increases. As a consequence the position of the hump in the momentum distribution becomes higher as the mass of the hadrons increases. We should emphasise also that the form (3) is only expected to be valid in the region $x \ll 1$. An expression with better behaviour at larger $x$ has been given in [9].

Realistic theoretical predictions for hadron spectra accounting for phase space and mass effects, resonance and heavy quarks decays, etc., can be obtained by using Monte Carlo simulations with coherence [12-14]. At the partonic level this approach sums all dominant Feynman diagrams at both small and large $x$, leading to the asymptotic prediction (3) as $x \to 0$ as well as the correct leading and next-to-leading behaviour as $x \to 1$.

In order to obtain clear experimental evidence of this important perturbative feature it is useful to study not only the $x$-shape of the distribution $D(x,E)$ but also the energy dependence of the hump predicted in (2). It has been suggested [9] that, even by using the LEP data at the $Z^0$ peak, one can study the energy dependence of the inclusive distribution $D(x,E)$. This can be done by measuring the single inclusive distribution $D^{\theta}(x,E)$ obtained by counting only the hadrons which enter into a cone of aperture $\theta$ around the jet axis. Partons entering into this cone come from the QCD cascade of a primordial parton with virtuality of the order of $E_{eff} = E \sin \frac{1}{2}\theta$. Therefore the inclusive distribution $D^{\theta}(x, E)$ obtained within this cone corresponds to $D(x, E_{eff})$. More precisely, for small $x$, one should find that

$$D^{\theta}(x,E) = D(x, E \sin \tfrac{1}{2}\theta) + \left[D(x,E) - D(x, E \cos \tfrac{1}{2}\theta)\right] \tag{6}$$

where the term in square brackets accounts for partons which are emitted in the *backward* jet. Notice that for $\theta = \pi$ we obtain the full distribution. Of course the value of $E_{eff}$ should be reasonably high for perturbation theory to be applicable, so one should not take $\theta$ too small.

By this method one can also directly compare the data obtained at PETRA or PEP energies with those obtained at LEP in restricted angular cones corresponding to the same effective energies.

In this measurement $\theta$ is defined relative to the jet axis. One should note that the usual procedure of using a jet-finding algorithms may introduce some uncertainty. Actually it is possible to define the distribution $D^{\theta}(x, E)$ without finding the jet axis. This is done by introducing energy-multiplicity correlations as explained in Subsection 1.3.

### *1.2 Double Inclusive Distribution*

The two-particle inclusive distribution of hadrons in $e^+e^-$ annihilation has also been computed for small values of $x$, including next-to-leading corrections which again are of relative order $\sqrt{\alpha_S}$ [15,16].

The two-particle correlation $R(x_1, x_2, E)$, defined by

$$R(x_1, x_2, E) \equiv D^{(2)}(x_1, x_2, E)/[D(x_1, E) D(x_2, E)] \,, \tag{7}$$

is a polynomial in $\ln x_i$, with coefficients that are power series in $\sqrt{\alpha_S}$. For $e^+e^-$ annihilation with $N_f = 5$ flavours one has

$$R(x_1,x_2,E) = 1.375 - 1.125 \left[\frac{\ln(x_1/x_2)}{\tau}\right]^2 - \left[1.262 + 0.877 \left(\frac{\ln(x_1 x_2)}{\tau}\right)\right] \frac{1}{\sqrt{\tau}} \,. \tag{8}$$

Thus there is a preference for $x_1 \simeq x_2$ and for both to be small. The range of these correlations is of the order of $\tau = \ln(E/\Lambda)$, so they extend over long distances in the $(\ln x_1, \ln x_2)$ plane. Thus they should be easily distinguishable from correlations due to hadronization, which would be expected to have ranges of order $\ln(Q_h/\Lambda)$, where the hadronization scale $Q_h$ is at most a few GeV.

### *1.3 Energy and Multiplicity Correlations*

To establish clear connections between theory and experiment, it is preferable [9] to work with inclusive quantities which are defined on the basis of simple jet characteristics such as energy or multiplicity flow, rather then on the basis of a given number of jets having specific directions, energies, momenta, masses, etc.

The separation of an event into individual jets is intrinsically ambiguous from the theoretical point of view. A QCD jet exhibits fractal structure, consisting of a number of sub-jets, which makes the jet definition highly artificial, especially since part of the total event multiplicity is due to soft interjet particles. These soft hadrons are distributed according to the colour properties of the event as a whole which, as a matter of principle, cannot be associated with any particular jet.

There is however a direct correspondence between the jet direction and energy flow direction, so that we may naturally study the shapes of jets and any characteristic of the hadronic system produced in a hard interaction by introducing inclusive correlations among energy flows and multiplicity flows. In this case one does not need to apply event selection procedures or jet-finding algorithms.

Inclusive correlations of this kind can be introduced in general. For instance the shape of jets can be studied by looking at the energy-energy and energy-multiplicity correlations defined by

$$\begin{aligned} C_{EE}(\theta_{12}) &= \frac{1}{E^2}\int E_1 E_2 dE_1 dE_2 \frac{d\sigma}{dE_1 dE_2 d\theta_{12}} , \\ C_{EM}(\theta_{12}) &= \frac{1}{E}\int E_1 dE_1 dE_2 \frac{d\sigma}{dE_1 dE_2 d\theta_{12}} . \end{aligned} \tag{9}$$

The energy-energy correlation is an infrared-finite quantity which requires careful treatment of collinear singularities in the small-angle region. The energy-multiplicity correlation is sensitive in addition to soft coherence effects and is given asymptotically by summing the leading Feynman diagrams, as is done in Monte Carlo programs with coherence [12-14].

Another application of this method consists is to define the "restricted cone" $x$ distribution $D^\theta(x, E)$ in (6) without the need of any specific jet-finding algorithm. As suggested in Ref. [9] this distribution can be defined in terms of the energy-multiplicity correlation as

$$D^\theta(x,E) = \frac{1}{E}\int_0^\theta d\theta_{12}\int E_1 dE_1 \int dE_2 \frac{d\sigma}{dE_1 dE_2 d\theta_{12}} \delta(x - \frac{E_2}{E}) . \tag{10}$$

*1.4 Rapidity Spectra and Jet-Finding Algorithms*

It should be stressed that QCD coherence has nothing to do with the experimentally observed dip in $y$-distributions at $y = 0$, where $y$ is the hadron rapidity (or pseudo-rapidity $\eta$) with respect to the jet axis. Actually from the point of view of the study of QCD coherence this phenomenon is *not theoretically interesting.*

Perturbative QCD predicts that the collimation of the QCD cascade around the parent parton becomes stronger with increasing energy. Moreover, the collimation of energy flow grows much faster than that of multiplicity flow. Thus, asymptotically, each event should exhibit a clear geometrical shape in which the $y$-distribution increases monotonically with decreasing $|y|$. The depletion of the region $y = 0$ is an artificial effect related to the jet-finding algorithm. For a discussion of this point see also [17].

## 2. Properties of Heavy Flavour Jets

The study of heavy flavour jets is important not only for testing QCD [10,11] but also for the measurements [11] of heavy particle properties: lifetimes, spatial oscillations of flavor, searching for CP-violating effects in their decays, etc. Let us briefly list the main results of perturbative QCD for heavy flavour jets:

a) The main perturbative QCD properties [18,19] are connected with the bremsstrahlung factor for the radiation of a soft gluon of transverse momentum $k_t$ and energy $\omega$ from a heavy quark $Q$ of mass $M_Q$ and energy $E_Q$, given by

$$dW(k) = \frac{\alpha_S}{\pi} C_F \frac{d\omega}{\omega} \frac{k_t^2 dk_t^2}{\left[k_t^2 + \omega^2 (M_Q/E_Q)^2\right]^2} , \tag{11}$$

which leads to the depletion of soft gluon radiation at small angles $\theta$:

$$\theta \simeq \frac{k_t}{\omega} \gtrsim \frac{M}{E_Q} . \tag{12}$$

This suppression of collinear radiation by a screening mass term could prove to be a signature for jets initiated by heavy quarks.

b) Destructive interference of soft gluon radiation in hard processes involving heavy quarks can be described [19], to leading infrared order, by a Markov process: the heavy quarks undergo a QCD cascade similar to that for light quarks, but the phase space available for branching is now further reduced by the heavy quark masses, in accordance with (12). Inclusion of this effect in Monte Carlo programs is especially important in order to obtain a reliable description of heavy quark physics. This has been done in the program HERWIG [14] (versions 3 et seq.).

c) To investigate the influence of screening of collinear singularities by heavy quark masses one can compare [18,19] the shapes of light and heavy quark jets produced in $e^+e^-$ collisions. Figure 1 and 2 show such a comparison between $d$ and $b$ jets in $Z^0$ decays, generated using HERWIG. The energy-energy and energy-multiplicity correlations defined in Eq. (9) are plotted here in terms of the pseudo-rapidity $\eta = -\ln \tan \frac{1}{2}\theta_{12}$ to clarify the forward and backward regions. We see that the $b$ jets have a broader structure with less hadron flow at small angles (large $\eta$), even when heavy flavour decay products are included.

These Figures illustrate another advantage of measuring correlations rather than plotting $\eta$ relative to some jet axis: the correlations are not forward-backward symmetric and indeed give different dynamical information in the two hemispheres. In this case it is interesting to see that $C_{EE}$ in the backward direction is larger for $b$ jets (before decays) than for $d$ jets. This is due to the extra "stiffness" of the $b\bar{b}$ system, which emits less gluon bremsstrahlung.

d) The string-like phenomena appear to be the same for light and heavy quarks. This behaviour arises from the fact that the radiation from a quark at large angles, determining interjet particle flows, is insensitive to the value of the quark mass.

e) In the asymptotic regime $E \gg M \gg \Lambda$ the spectra of associated light particles with $x \ll 1$ in the heavy quark jet (within the MLLA accuracy) are approximately given by

$$D_Q(x,E) = D(x,E) - D(x,M) \tag{13}$$

where $D(x,E)$ is the usual hadron inclusive spectrum in $e^+e^-$ in (1) and (3), originating from light flavour jets. This expression is due to the absence of radiation for angles smaller than $M/E$, which implies the lack of energetic particles with energy $\omega$ larger than $E(\Lambda/M)$.

f) Because of gluon emission, the average energy fraction $x$ carried by the heavy quark is given for $Q^2 \gg M^2$ by the perturbative expression (see for instance [18] )

$$\langle x \rangle = \left(\frac{\alpha_S(Q^2)}{\alpha_S(M^2)}\right)^{\frac{32}{9\beta}} \left(1 + \frac{11}{9} C_F \frac{\alpha_S(Q^2)}{\pi} + \cdots\right) \tag{14}$$

## 3 QCD Drag Effects in Interjet Hadron Flows

A fundamental feature of the parton shower mechanism is the connection between the *colour flow* in the hard process and the observed flow of hadron multiplicity [8,9,20]. This connection was beautifully illustrated in $e^+e^-$ annihilation by the observation of the "string" or "drag" effect in three-jet final states.

To illustrate this effect, consider the angular distribution of soft gluon emission from a hard massless quark-antiquark-gluon system. Denoting the quark, antiquark and hard and soft gluon directions by $\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3$ and $\mathbf{n}_4$, respectively, we have

$$W^{q\bar{q}g}(\mathbf{n}_4) \propto N_c \left(\frac{a_{13}}{a_{14}a_{43}} + \frac{a_{23}}{a_{24}a_{43}}\right) - \frac{1}{N_c} \frac{a_{12}}{a_{14}a_{42}} \tag{15}$$

where $W^{q\bar{q}g}(\mathbf{n}_4)$ represents the angular distribution of the soft gluon and $a_{ij} = 1 - \mathbf{n}_i \cdot \mathbf{n}_j$. This may be compared with the pattern for a quark-antiquark system, with a hard photon replacing the gluon jet 3 to permit the directions $\mathbf{n}_1$ and $\mathbf{n}_2$ to remain the same:

$$W^{q\bar{q}\gamma}(\mathbf{n}_4) = W^{q\bar{q}}(\mathbf{n}_4) \propto 2C_F \frac{a_{12}}{a_{14}a_{42}} . \tag{16}$$

Taking for illustration the threefold symmetric $q\bar{q}g$ or $q\bar{q}\gamma$ configuration, the ratio of emission in the direction opposite to the hard gluon or photon, $\mathbf{n}_4 = -\mathbf{n}_3$, in the two processes is

$$\frac{W^{q\bar{q}g}}{W^{q\bar{q}\gamma}} = \frac{N_c^2 - 2}{2(N_c^2 - 1)} = \frac{7}{16} , \tag{17}$$

showing the destructive interference in this region.

This depletion was confirmed both by comparison of hadron multiplicities between the jets [21] and by comparison of $q\bar{q}g$ and $q\bar{q}\gamma$ final states [22].

It will be interesting to perform the same analysis also at LEP energies where the subasymptotic corrections are less important. In the previous analyses [21] it was observed that the effect is enhanced by increasing the particle mass or $p_{tout}$. At LEP energy this enhancement should be weaker, since, according to perturbative QCD [6] it is due to finite energy corrections.

Let us mention that the string effect could be better analyzed if one could identify the quark jets.* This can be done in the case of the heavy flavour production events, in which one can tag the heavy quark decay.

We have recently proposed [23] a new method for revealing the connection between observed hadronic distributions and the colour structure of an underlying hard process. The method is related to the "string effect" analysis but has the advantage of not requiring any special event selection or jet finding. It involves measuring a ratio of energy-multiplicity correlations which is especially sensitive to colour flows in jet formation. This quantity is infrared stable and can be calculated completely perturbatively.

To illustrate the new method, consider the emission of *two* soft gluons by a hard quark-antiquark system

$$W^{q\bar{q}}(\mathbf{n}_3, \mathbf{n}_4) \propto 2C_F N_c \left( \frac{a_{12}}{a_{13}a_{34}a_{42}} + \frac{a_{12}}{a_{23}a_{34}a_{41}} - \frac{1}{N_c^2} \frac{a_{12}^2}{a_{13}a_{32}a_{14}a_{42}} \right) . \tag{18}$$

Dividing by the product of the single-gluon distributions gives the gluon-gluon correlation function

$$C^{q\bar{q}}(\eta_{34}, \phi) = \frac{W^{q\bar{q}}(\mathbf{n}_3, \mathbf{n}_4)}{W^{q\bar{q}}(\mathbf{n}_3) W^{q\bar{q}}(\mathbf{n}_4)} = 1 + \frac{N_c}{2C_F} \left( \frac{\cos\phi}{\cosh\eta_{34} - \cos\phi} \right) , \tag{19}$$

where $\eta_{34} = \eta_3 - \eta_4$ with $\eta_{3,4}$ are the pseudorapidities of the soft gluons, and $\phi = \phi_3 - \phi_4$, the corresponding azimuthal angles $\phi_{3,4}$ relative to the direction $\mathbf{n}_1 \simeq -\mathbf{n}_2$.

Equation (19) provides an infrared-finite measure of the correlation between colour flows in the directions $(\eta_3, \phi_3)$ and $(\eta_4, \phi_4)$. According to the local duality hypothesis, it can be applied directly to hadronic flows. Considering, for example, the flows at the same pseudorapidity in the orthogonal ($\phi = \pi/2$) and back-to-back ($\phi = \pi$) azimuthal directions, we have

$$C^{q\bar{q}}(0, \pi/2) = 1 , \quad C^{q\bar{q}}(0, \pi) = \frac{N_c^2 - 2}{2(N_c^2 - 1)} = \frac{7}{16} . \tag{20}$$

This implies that for $\phi = \pi/2$ the hadronic multiplicities are uncorrelated, while for $\phi = \pi$ there is destructive interference, of the same magnitude as the string effect in threefold symmetric jets [Eq. (17)]. Thus measurements of hadronic flow correlations in the orthogonal and back-to-back azimuthal directions should demonstrate the same type of colour coherence as the string effect, without requiring the selection of a three-jet event sample.

Next we explain how to measure the correlated hadronic flows discussed above without having to define explicitly any jet axes for the hard process. The basic idea is to use energy-weighted correlations as discussed in Subsection 1.3. For each set of three hadrons $i$, $j$ and $k$, we define an interval of pseudorapidity for $j$ and $k$ relative to the direction of $i$, $\eta_{min} < \eta_j, \eta_k < \eta_{max}$, where $\eta_j = -\ln \tan \frac{1}{2}\theta_{ij}$. Then we define energy-multiplicity-multiplicity correlation

$$C_{EMM}(\eta_{min}, \eta_{max}, \phi) =$$
$$\frac{1}{\sigma}\int E_i\, dE_i\, dE_j\, dE_k \int_{\eta_{min}}^{\eta_{max}} d\eta_j\, d\eta_k \int_0^{2\pi} d\phi_j\, d\phi_k\, \delta(\phi - \phi_j + \phi_k) \frac{d\sigma}{dE_i\, dE_j\, dE_k\, d\eta_j\, d\eta_k\, d\phi_j\, d\phi_k}, \tag{21}$$

where $\phi_j$ and $\phi_k$ are azimuthal angles around the direction of hadron $i$. We define also the energy-multiplicity correlation

$$C_{EM}(\eta_{min}, \eta_{max}) = \frac{1}{\sigma}\int E_i\, dE_i\, dE_j \int_{\eta_{min}}^{\eta_{max}} d\eta_j \int_0^{2\pi} d\phi_j \frac{d\sigma}{dE_i\, dE_j\, d\eta_j\, d\phi_j}. \tag{22}$$

Because of the energy weighting factor $E_i$ in (21), the hadron $i$ is preferentially associated (via local duality) with the hard quark or antiquark initiating the QCD emission. For a suitable choice of the pseudorapidity interval $[\eta_{min}, \eta_{max}]$, the other two hadrons $j$ and $k$ will be associated with soft gluon emission. Thus to leading perturbative order, the integrand of (21) is given by $W^{q\bar{q}}(\mathbf{n}_3, \mathbf{n}_4)$ where we associate $\mathbf{n}_1$ with the direction of hadron $i$ and $\mathbf{n}_3$ and $\mathbf{n}_4$ with those of hadrons $j$ and $k$. Similarly the integrand of (22) is given to leading order by $W^{q\bar{q}}(\mathbf{n}_3)$.

The correlation functions $C_{EMM}$ and $C_{EM}$ separately are not infrared finite quantities and therefore they receive leading contributions from all perturbative orders. These leading terms can be summed by replacing the soft gluons 3 and 4 by their associated parton cascades. It turns out that these leading effects of cascading factorise. We then define the correlation function

$$C(\phi) = \frac{C_{EMM}(\eta_{min}, \eta_{max}, \phi)\, C_E}{[C_{EM}(\eta_{min}, \eta_{max})]^2} \tag{23}$$

where

$$C_E = \frac{1}{\sigma}\int E_i\, dE_i \frac{d\sigma}{dE_i}. \tag{24}$$

In the correlation $C(\phi)$ the cascade factors cancel to leading order in $\sqrt{\alpha_S}$. As a result, at high energies $C(\phi)$ is determined entirely by soft gluon radiation and is asymptotically given by Eq. (19):

$$C(\phi) \sim C^{q\bar{q}}(0, \phi) \tag{25}$$

for a sufficiently narrow rapidity interval $[\eta_{min}, \eta_{max}]$.

The most important finite energy corrections to this asymptotic expression comes from three-jet events. These events contribute to $C^{q\bar{q}}(0, \phi)$ when one of the jets defines the energy flow and the other two are both recorded in the pseudorapidity interval $[\eta_{min}, \eta_{max}]$. Such a three-jet contamination, which would contribute mainly around $\phi \simeq \pi$ and for negative rapidities, can be eliminated by making it kinematically impossible for two jets in a three-jet event to enter the interval, for example by choosing $\eta_{min}$ and $\eta_{max}$ both positive. After this precaution has been taken to eliminate three-jet contributions, all remaining corrections to Eq. (25) are of relative order $\sqrt{\alpha_S}$.

To estimate the effects of finite energy corrections and hadronization, we used the Monte Carlo program HERWIG [14]. The results are shown in Fig. 3. Destructive interference, that is, $C(\phi) < 1$ for $\pi/2 < \phi < \pi$, is clearly seen in the Monte Carlo results at both the parton and hadron levels. The effects of hadronization are small at these energies. However the $\sqrt{\alpha_S}$ corrections are rather significant. $O(\sqrt{\alpha_S})$ corrections to some other multiparticle observables have been computed analytically and are also found to be large [9,16]. It would be interesting to compute the corrections to $C(\phi)$ analytically if possible.

As discussed earlier, finite energy corrections depend in general on the particle masses. It would therefore be useful to study these types of correlations for different hadron species.

An analysis of colour flow along the above lines may be performed for other correlations and hard processes. Another example is the study of azimuthal asymmetry effects [9] in $e^+e^-$ events with a hard photon ($q\bar{q}\gamma$). It is interesting to observe that in correlations with an additional multiplicity flow ($q\bar{q}g\gamma$), the asymmetry depends on $1/N_c^2$ corrections which are typically neglected in Monte Carlo simulations.

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## Figure Captions

Fig. 1. Energy-energy correlations in $Z^0 \to d\bar{d}$ and $b\bar{b}$ as a function $\eta = -\ln\tan\frac{1}{2}\theta$.

Fig. 2. Energy-multiplicity correlations in $Z^0 \to d\bar{d}$ and $b\bar{b}$ as a function $\eta = -\ln\tan\frac{1}{2}\theta$.

Fig. 3. Hadronic flow correlation defined by Eq. (23), as a function of the azimuthal angle $\phi$, for a rapidity interval $1 < \eta_{j,k} < 2$. The points are Monte Carlo predictions from the program HERWIG [14] at the parton and hadron levels, for $e^+e^-$ annihilation at $E_{cm} = 91$ GeV.The curves show the leading-order prediction (19) for various rapidity differences $\eta_{34}$.

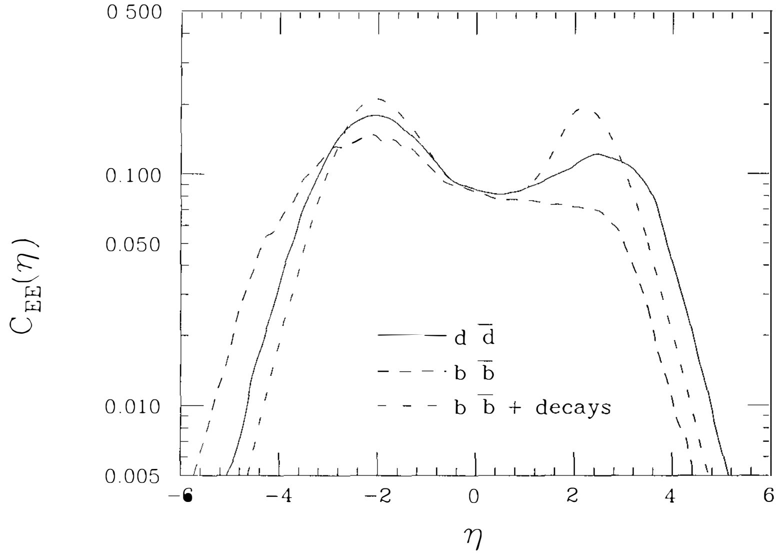
0 500
0.100
0.050
0.010
0.005
$C_{EE}(\eta)$
d $\overline{d}$
b $\overline{b}$
b $\overline{b}$ + decays
−6
−4
−2
0
2
4
6
$\eta$

Fig. 1

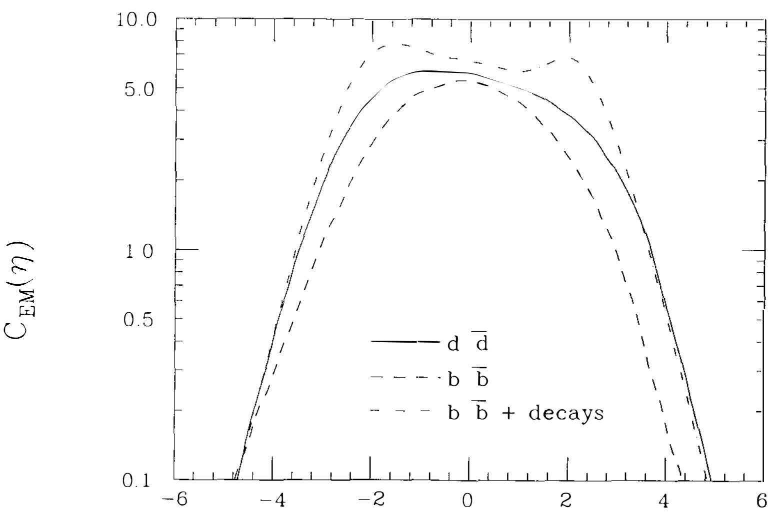
10.0
5.0
1 0
0.5
0.1
$C_{EM}(\eta)$
d $\overline{d}$
b $\overline{b}$
b $\overline{b}$ + decays
−6
−4
−2
0
2
4
6
$\eta$

Fig. 2

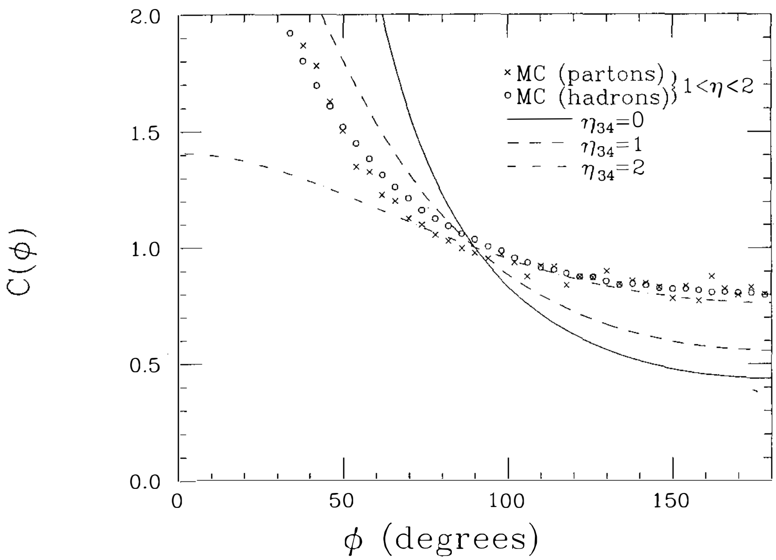
2.0
1.5
1.0
0.5
0.0
0
50
100
150
C(φ)
φ (degrees)
× MC (partons)
○ MC (hadrons)
1<η<2
η34=0
η34=1
η34=2

Fig. 3