Hyperfine structure of S-states in muonic ions of lithium, beryllium and boron

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Abstract. On the basis of quasipotential method in quantum electrodynamics we make precise calculation of hyperfine structure of S-states in muonic ions of lithium, beryllium and boron. Nuclear structure and recoil corrections of orders α^5 and α^6, vacuum polarization corrections in first and second orders of perturbation theory are taken into account. Total values of hyperfine splittings are obtained which can be used for a comparison with future experimental data.

1. Introduction

The CREMA (Charge Radius Experiments with Muonic Atoms) collaboration has obtained in recent years significantly new experimental results that helped to re-examine the problem of muon bound states, posed new questions to the theory that require additional investigation [1]. The transition frequencies between the levels 2S and 2P were measured in muonic hydrogen, deuterium and ions of muonic helium using the methods of laser spectroscopy [2, 3]. They allowed to obtain the values of the nucleus charge radii with the accuracy 0.0005 fm and state the difference in proton and deuteron charge radii obtained from electron and muon atoms. Recent experiments in [4, 5] have shown that the situation remains unclear. The CREMA can be actively supported by the FAMU (Fisica Atomi MUonici) collaboration, which plans to measure the hyperfine structure (HFS) of the ground state of the muonic hydrogen atom with high accuracy [6].

One possible future activity of the CREMA collaboration may be connected with other muonic ions containing light nuclei of lithium, beryllium and boron. For these muonic ions, the description of the electromagnetic interaction of the few-nucleon systems is particularly important, and, consequently, the role of the effects of nuclear physics can be studied with greater accuracy. We may hope that the enormous interest, which experimental results of the CREMA collaboration have met over the past years can ultimately lead to a significant improvement in the theory of calculating the energy levels of muonic atoms.

2. Nuclear structure and recoil corrections

When calculating various corrections in the hyperfine structure of the spectrum, it is important to note the essential role of corrections for the structure of the Li, Be, and B nuclei. Such corrections are determined by the electromagnetic form factors of the nuclei. Basic contribution of the nuclear structure effects of order α^5 to the hyperfine splitting is determined by two-photon exchange diagrams. It is expressed in terms of electric $G_E(k^2)$ and magnetic $G_M(k^2)$ nuclear form factors in the form (the Zemach correction):

$$\Delta E^{hfs}_{str} = E_F \frac{2\mu Z\alpha}{\pi} \int \frac{d\mathbf{k}}{\mathbf{k}^4} \left[\frac{G_E(k^2) G_M(k^2)}{G_M(0)} - 1 \right]. \quad (1)$$

We have analysed numerical values of correction (1) for different parameterizations of nuclear form factors: Gaussian $G^G_E(k^2)$, dipole $G^D_E(k^2)$ and uniformly charged sphere $G^U_E(k^2)$:

$$G^G_E(k^2) = e^{-\frac{1}{6} r_N^2 k^2}, \quad G^D_E(k^2) = \frac{1}{(1+\frac{k^2}{\Lambda^2})^2}, \quad G^U_E(k^2) = \frac{3}{(kR)^3} \left[\sin kR - kR \cos kR \right], \quad (2)$$

where $R = \sqrt{5} r_N / \sqrt{3}$ is the nucleus radius, $\Lambda^2 = 12/r_N^2$. In the range $0.1 \leq k \leq 0.4$ GeV there is a difference between functions (2) which leads to different numerical values of the Zemach correction. The momentum integration in (1) can be done analytically, so that the Zemach correction with the Gaussian and uniformly charged sphere parameterizations has the form:

$$\Delta E^{hfs}_{str,G} = -E_F \frac{72}{\sqrt{3\pi}} \mu Z\alpha r_N, \quad \Delta E^{hfs}_{str,U} = -E_F \frac{72\sqrt{5}}{35\sqrt{3}} \mu Z\alpha r_N. \quad (3)$$

Among different nuclei that we consider, several nuclei have a spin $s_2 = 3/2$. So, working on the basis of quasipotential method in quantum electrodynamics we can present the amplitude of two-photon exchange interaction in the form [7, 8]:

$$i\mathcal{M}_{2\gamma} = \frac{(Z\alpha)^2}{\pi^2} \int \frac{idk}{k^4} \frac{D^{\lambda\mu}(k) D^{\rho\nu}(k)}{(p_2+k)^2 - m_2^2} \bar{u}(q_1) [\gamma_\mu \frac{\hat{p}_1 - \hat{k} + m_1}{(p_1-k)^2 - m_1^2} \gamma_\nu + \gamma_\nu \frac{\hat{p}_1 + \hat{k} + m_1}{(p_1+k)^2 - m_1^2} \gamma_\mu] \quad (4)$$

$$\bar{v}_\alpha(p_2) \mathcal{O}_{\alpha\sigma\rho}(p_2, p_2+k)(-\hat{p}_2 - \hat{k} + m_2)[g_{\sigma\tau} - \frac{1}{3}\gamma_\sigma\gamma_\tau - \frac{2}{3m_2^2}(p_2+k)_\sigma (p_2+k)_\tau +$$

$$\frac{1}{3m_2}(\gamma_\sigma (p_2+k)_\tau - \gamma_\tau (p_2+k)_\sigma)] \mathcal{O}_{\tau\beta\lambda}(p_2, p_2+k) v_\beta(q_2),$$

where p_1, p_2 are four-momenta of particles in the initial state, q_1, q_2 are four-momenta of particles in the final state, $k = q_2 - p_2 = p_1 - q_1$. $\mathcal{O}_{\alpha\mu\beta}$ is the vertex function of the spin 3/2 nucleus:

$$\mathcal{O}_{\alpha\mu\beta} = g_{\alpha\beta} \frac{(p_2+q_2)_\mu}{2m_2} F_1(k^2) - g_{\alpha\beta}\sigma_{\mu\nu} \frac{k^\nu}{2m_2} F_2(k^2) + \quad (5)$$

$$\frac{k_\alpha k_\beta}{4m_2^2} \frac{(p_2+q_2)_\mu}{2m_2} F_3(k^2) - \frac{k_\alpha k_\beta}{4m_2^2} \sigma_{\mu\nu} \frac{k^\nu}{2m_2} F_4(k^2),$$

where four form factors $F_i(k^2)$ are related to the charge G_{E0}, electroquadrupole G_{E2}, magnetic dipole G_{M1} and magnetic octupole G_{M3} form factors by the following expressions:

$$G_{E0} = \left(1 + \frac{2}{3}\tau\right) [F_1 + \tau(F_1 - F_2)] - \frac{\tau}{3}(1+\tau)[F_3 + \tau(F_3 - F_4)], \quad (6)$$

$$G_{E2} = F_1 + \tau(F_1 - F_2) - \frac{1+\tau}{2}[F_3 + \tau(F_3 - F_4)],$$

$$G_{M1} = (1 + \frac{4}{3}\tau)F_2 - \frac{2}{3}\tau(1+\tau)F_4, \quad G_{M3} = F_2 - \frac{1}{2}(1+\tau)F_4, \quad \tau = -\frac{k^2}{4m_2^2}.$$

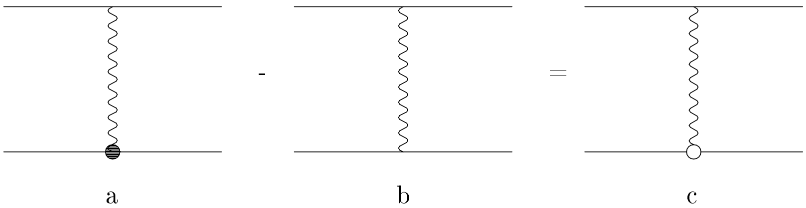

Figure 1. Nuclear structure effects in one-photon interaction.

To distinguish the contribution of the amplitude $M_{2\gamma}$ to the interaction operator of particles with total momenta $F = 2$ and $F = 1$, we use the method of projection operators, which are constructed from the wave functions of free particles in the rest frame. Thus, the projection operators on states with $F = 2$ and $F = 1$ are equal to [9]

$$\hat{\Pi}_\alpha = [u(0)\bar{v}_\alpha]_{F=2} = \frac{1+\gamma_0}{2\sqrt{2}}\gamma_\beta\varepsilon_{\alpha\beta}, \hat{\Pi}^{01}_\alpha = \frac{1+\hat{v}}{2\sqrt{2}}\gamma_5\varepsilon_\alpha, \hat{\Pi}^{11}_\alpha = \frac{1+\hat{v}}{4}\gamma_\sigma\varepsilon_{\alpha\sigma\rho\omega}v^\rho\varepsilon^\omega, \quad (7)$$

where the tensor $\varepsilon_{\alpha\beta}$ describes a muonic atom with $F = 2$, ε^ω is the polarization vector of the state with $F = 1$. The indexes (01) and (11) correspond to states with $S = 0$, $F = 1$ and $S = 1$, $F = 1$ correspondingly. We consider the state with spin 3/2 as the result of the addition of two moments $s_1 = 1/2$ and $l = 1$ and use the following expansion:

$$\Psi_{s_2=3/2,F=1,F_z} = \sqrt{\frac{2}{3}}\Psi_{S=0,F=1,F_z} + \frac{1}{\sqrt{3}}\Psi_{S=1,F=1,F_z}. \quad (8)$$

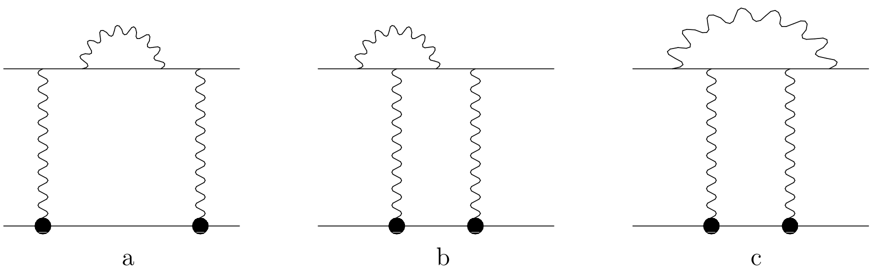

Figure 2. Direct two-photon exchange amplitudes with radiative corrections to muon line giving contributions of order $\alpha(Z\alpha)^5$ to the hyperfine structure.

As a result the value of the hyperfine splitting is determined in Euclidean space by the following formula:

$$\Delta E^{hfs}_{nS} = |\psi_{nS}(0)|^2\int d^4k V_{2\gamma}(k) = \frac{64}{9}\frac{(Z\alpha)^2}{\pi^2}|\psi_{nS}(0)|^2\int\frac{d^4k}{k^4(k^4+4m_1^2k_0^2)(k^4+4m_2^2k_0^2)}\times \quad (9)$$

$$\Big[F_1F_2\Big(k^6 - k^4k_0^2 + \frac{4}{15}\frac{k^4k_0^4}{m_2^2} - \frac{7}{10}\frac{k^6k_0^2}{m_2^2} + \frac{13}{30}\frac{k^8}{m_2^2}\Big) + F_2F_4\Big(-\frac{1}{30}\frac{k^2k_0^6}{m_2^2} + \frac{1}{15}\frac{k^4k_0^4}{m_2^2} - \frac{1}{30}\frac{k^6k_0^2}{m_2^2}\Big) +$$

$$F_2F_3\Big(-\frac{1}{15}\frac{k^2k_0^6}{m_2^2} + \frac{11}{60}\frac{k^4k_0^4}{m_2^2} - \frac{7}{60}\frac{k^8}{m_2^2}\Big) + F_1F_4\Big(-\frac{1}{5}\frac{k^2k_0^6}{m_2^2} + \frac{3}{10}\frac{k^4k_0^4}{m_2^2} - \frac{1}{10}\frac{k^8}{m_2^2}\Big) +$$

$$F_2^2\Big(\frac{1}{15}\frac{k^2k_0^6}{m_2^2}-\frac{1}{6}k^2k_0^4-\frac{2}{15}\frac{k^4k_0^4}{m_2^2}+\frac{1}{6}k^4k_0^2+\frac{23}{120}\frac{k^6k_0^2}{m_2^2}-\frac{1}{4}\frac{k^8}{m_2^2}\Big)\Big].$$

We have in (9) the main contribution (the Zemach correction) and the recoil correction m_1/m_2. The form factors $F_i(k^2)$ are expressed in terms of G_{E0}, G_{E2}, G_{M1}, G_{M3} for which the Gaussian parametrization is used in numerical calculations of integrals with respect to k. The values of the form factors at zero have the form [10]:

$$G_{E0}(0)=1,\quad G_{M1}(0)=\frac{m_2\mu_N}{m_pZ},\quad G_{E2}(0)=m_2^2Q,\quad G_{M3}(0)=\frac{m_2}{m_pZ}m_2^2\Omega. \tag{10}$$

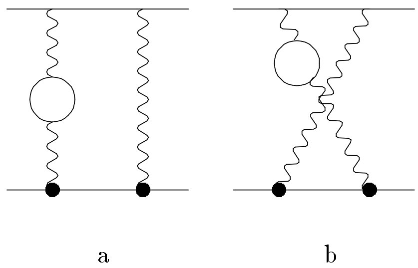

Figure 3. Two photon exchange amplitudes accounting for effects of vacuum polarization and nuclear structure. The wavy line denotes the photon.

Different parameters of light nucleus (Li, Be, B) were investigated in electron scattering experiments [10, 11]. Some of them are unknown with good accuracy, but, nevertheless, one can obtain approximate estimates of the corresponding contributions. After angular analytical integration in (9) we make numerical integration over k. Another correction for the structure of the nucleus of order α^6 is obtained as a result of the decomposition of the magnetic form factor of the nucleus (see Fig. 1(a)) in first order PT and electric form factor in second order PT. The contribution to the HFS in this case has the form:

$$\Delta E^{hfs}_{1\gamma,str}=\frac{2}{3}\mu^2Z^2\alpha^2r_M^2\frac{3n^2+1}{n^2}E_F(nS), \tag{11}$$

$$\Delta E^{hfs}_{str,sopt}(1S)=E_F(1S)\frac{R^2W^2}{4}[\frac{4}{75}(-53+15C+15\ln RW)-\frac{RW}{12}(-15+4C+4\ln RW)], \tag{12}$$

$$\Delta E^{hfs}_{str,sopt}(2S)=E_F(2S)\frac{R^2W^2}{4}[\frac{4}{75}(-107+60C+60\ln RW)+\frac{RW}{3}(17-8C-8\ln RW)], \tag{13}$$

where we present an expansions in (RW) up to terms of first order in square brackets ($RW(^6_3Li)=0.038$, $RW(^7_3Li)=0.036$, $RW(^9_4Be)=0.050$, $RW(^{10}_5B)=0.060$, $RW(^{11}_5B)=0.060$).

There are other nuclear structure corrections of order α^6 connected with the radiative insertions in the muon line (Fig. 2) and vacuum polarization effect (Fig. 3). For the amplitudes in Fig. 2 we obtained in [12] general integral expressions using which it can be possible to find numerical values in HFS. To obtain a contribution of amplitudes in Fig. 3 we can use the potential $V_{2\gamma}$ from (9) with respective modification as in [9].

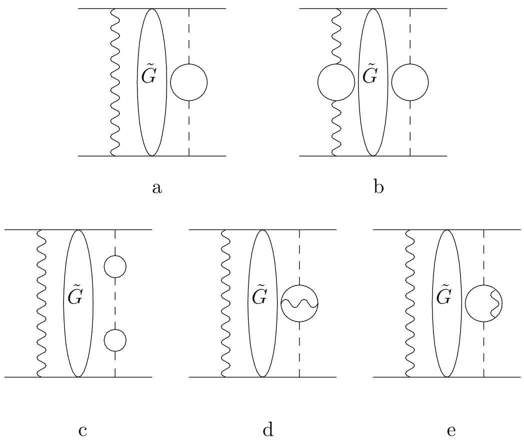

Figure 4. Effects of one- and two-loop vacuum polarization in second order PT.

3. Vacuum polarization effects

Vacuum polarization (VP) corrections represent second part of contributions which have specific form for different muonic ions. To calculate a contribution of one-loop vacuum polarization contribution in first order PT we use the potential which can be obtained in momentum representation after a standard modification of hyperfine muon-nucleus interaction [13, 14, 15]. In coordinate representation it is defined by the following integral expression:

$$\Delta V^{hfs}_{1\gamma,vp}(r)=\frac{4\alpha g_N(1+a_\mu)}{3m_1m_p}(\mathbf{s}_1\mathbf{s}_2)\frac{\alpha}{3\pi}\int_1^\infty\rho(\xi)d\xi\left(\pi\delta(\mathbf{r})-\frac{m_e^2\xi^2}{r}e^{-2m_e\xi r}\right). \quad (14)$$

Averaging (14) over bound state wave functions, we get the contribution of order α^5 to hyperfine structure of $1S-$ and $2S$-states ($a_1=m_e/W$, $W=\mu Z\alpha$):

$$\Delta E^{hfs}_{1\gamma,vp}(1S)=\frac{4\alpha^2(Z\alpha)^3\mu^3g_N(1+a_\mu)}{9m_1m_p\pi}<(\mathbf{s}_1\mathbf{s}_2)>\int_1^\infty\rho(\xi)d\xi\left[1-\frac{m_e^2\xi^2}{W^2}\int_0^\infty xdxe^{-x\left(1+\frac{m_e\xi}{W}\right)}\right]= \quad (15)$$

$$E_F(1S)\frac{\alpha(1+a_\mu)}{9\pi\sqrt{1-a_1^2}}\Big[\sqrt{1-a_1^2}(1+6a_1^2-3\pi a_1^3)+(6-3a_1^2+5a_1^4)\ln\frac{1+\sqrt{1-a_1^2}}{a_1}\Big],$$

$$\Delta E^{hfs}_{1\gamma,vp}(2S)=\frac{\alpha^2(Z\alpha)^3\mu^3g_N(1+a_\mu)}{18m_1m_p\pi}<(\mathbf{s}_1\mathbf{s}_2)>\int_1^\infty\rho(\xi)d\xi\times \quad (16)$$

$$\left[1-\frac{4m_e^2\xi^2}{W^2}\int_0^\infty x\left(1-\frac{x}{2}\right)^2dxe^{-x\left(1+\frac{2m_e\xi}{W}\right)}\right]=$$

$$E_F(2S)\frac{\alpha(1+a_\mu)}{18\pi(4a_1^2-1)^{5/2}}\Big\{\sqrt{4a_1^2-1}[11+2a_1^2(-29+8a_1(-22a_1+48a_13-3\pi(4a_1^2-1)^2))]+$$

$$12(1-10a_1^2+66a_1^4-160a_1^6+256a_1^8)\arctan\sqrt{4a_1^2-1}\Big\}.$$

Obtained expressions (15)-(16) demonstrate the general structure of analytical corrections. The VP contribution of order α^6 is represented by two-loop VP amplitudes in 1γ interaction. They have the form of a double and a single spectral integral in coordinate space [15, 16]:

$$\Delta V^{hfs}_{1\gamma,vp-vp}(r)=\frac{4\pi\alpha g_N(1+a_\mu)}{3m_1m_p}(\mathbf{s}_1\mathbf{s}_2)\left(\frac{\alpha}{3\pi}\right)^2\int_1^\infty\rho(\xi)d\xi\int_1^\infty\rho(\eta)d\eta\times \quad (17)$$

$$\times\left[\delta(\mathbf{r})-\frac{m_e^2}{\pi r(\eta^2-\xi^2)}\left(\eta^4 e^{-2m_e\eta r}-\xi^4 e^{-2m_e\xi r}\right)\right],$$

$$\Delta V^{hfs}_{1\gamma,2-loop\ vp}(r)=\frac{8\alpha^3 g_N(1+a_\mu)}{9\pi^2 m_1 m_p}(\mathbf{s}_1\mathbf{s}_2)\int_0^1\frac{f(v)dv}{1-v^2}\left[\pi\delta(\mathbf{r})-\frac{m_e^2}{r(1-v^2)}e^{-\frac{2m_e r}{\sqrt{1-v^2}}}\right], \quad (18)$$

where two-loop spectral function $f(v)$ is written in [15, 16]. The matrix elements of (17)-(18) can be obtained as in (15)-(16). More complicated part of the calculation is related with one-loop and two-loop contributions in second order perturbation theory (PT). They are presented in Fig. 4. These corrections to the energy spectrum are determined by the reduced Coulomb Green's function $\tilde{G}_n(\mathbf{r},\mathbf{r}')$ which was obtained in compact form in [17]. Corresponding matrix elements with $\tilde{G}_n(\mathbf{r},\mathbf{r}')$ are calculated analytically and numerically over spectral parameters.

4. Conclusion

In this work we carry out a calculation of S-states hyperfine splittings in a number of muonic ions. We consider that light muonic ions of the lithium, beryllium and boron can be used in experiments of the CREMA collaboration. Our precise calculation of the HFS includes various corrections of the fifth and sixth orders in α, which were previously taken into account also in the study of the hyperfine structure of the spectrum of other muonic atoms [15, 16]. One significant difference between these calculations and the previous ones is due to the fact that in this paper we investigate the nuclei of spin 1, 3/2, 3. Total values of hyperfine splittings are the following: $\Delta E^{hfs}(1S)({}^6_3Li)=1325.02$ meV, $\Delta E^{hfs}(2S)({}^6_3Li)=164.65$ meV, $\Delta E^{hfs}(1S)({}^7_3Li)=4693.40$ meV, $\Delta E^{hfs}(2S)({}^7_3Li)=583.38$ meV, $\Delta E^{hfs}(1S)({}^9_4Be)=-4080.71$ meV, $\Delta E^{hfs}(2S)({}^9_4Be)=-505.82$ meV, $\Delta E^{hfs}(1S)({}^{10}_5B)=10233.86$ meV, $\Delta E^{hfs}(2S)({}^{10}_5B)=1265.03$ meV, $\Delta E^{hfs}(1S)({}^{11}_5B)=17572.70$ meV, $\Delta E^{hfs}(2S)({}^{11}_5B)=2172.56$ meV. A precise measurement of the HFS in muonic ions of lithium, beryllium and boron, taking into account the obtained theoretical results, will allow obtaining more accurate values of the Zemach radius.

Acknowledgments

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