FAST BEAM-CAVITY INTERACTION AND ITS EFFECT ON BUNCH SHAPE IN STORAGE RINGS[+]

A. Papiernik, M. Chatard-Moulin, and B. Jecko

Laboratoire d'Electronique et Microondes
U.E.R. des Sciences, Limoges, France

Summary

When a bunch of stored particles passes a gap, it induces fields which affect the energy of the particles, and hence the bunch shape. An analytical formula giving the transient voltage is derived for a short bunch of any shape. The equilibrium bunch shape is calculated using Haïssinski's self consistant theory. The bunch lengthens, but comparison with experiment shows that beam-cavity fast interaction is not the main reason for bunch lengthening in rings having only one cavity.

Introduction

The longitudinal equilibrium shape of a bunch circulating in a storage ring is related to the effective accelerating voltage by[1]

$$I(\tau) = K \exp\left\{-\frac{1}{T_o H_o}\int_0^{\tau}\left[eV(y) - U_o\right] dy\right\} \qquad (1)$$

where τ is the time displacement between a particle of the bunch and an origin which will be defined below,

$I(\tau)$ is the instantaneous intensity in the bunch,

$V(\tau)$ is the effective accelerating voltage,

U_o is the average energy loss per turn,

T_o is the revolution period in the ring,

$K = I(0)$ is a constant which, as we shall see, is related to the total charge of the bunch,

H_o is a constant which depends on ring characteristics and energy.

The effective accelerating voltage $V(\tau)$ is the sum of two terms : the sinusoidal voltage $V_{RF}(\tau)$, and a transient voltage $U(\tau)$ due to the self fields induced by the bunch

$$V(\tau) = V_{RF}(\tau) + U(\tau) \ . \qquad (2)$$

$V_{RF}(\tau) = \hat{V}\sin(\omega\tau + \phi_s)$ is the accelerating voltage, taking into account sinusoidal beam loading.

The time origin $\tau = 0$ is defined such that $eV_{RF}(0)$ compensates exactly the average energy loss U_o.

Since the bunches are much shorter than the wavelength, V_{RF} may be linearized :

$$eV_{RF}(\tau) = U_o + e\,\dot{V}_{RF} \times \tau \qquad (3)$$

where $\dot{V}_{RF} = \omega\,\hat{V}\cos\phi_s = \omega\sqrt{\hat{V}^2 - (U_o/e)^2}$ is the slope of the sinusoidal voltage at origin.

At low current, the transient self fields are weak, and $U(\tau)$ may be neglected.

Under such conditions, the instantaneous equilibrium current has a gaussian shape :

$$I(\tau) = K\exp(-\tau^2/2\sigma_o^2)$$

where $\sigma_o = \sqrt{T_o H_o / e\,\dot{V}_{RF}}$ is the standard deviation of the gaussian distribution.

At higher currents, the bunch shape depends on the transient induced voltage $U(\tau)$:

$$I(\tau) = K\exp\left[-\frac{\tau^2}{2\,\sigma_o^2} - \frac{1}{\sigma_o^2\,\dot{V}_{RF}}\int_0^{\tau} U(y)\,dy\right] \ . \qquad (4)$$

The transient voltage $U(\tau)$ results from interaction between the bunch and the elements of the ring (accelerating cavity, vacuum chamber, ...). Then it depends on $I(\tau)$ and geometrical environment.

We give here below an analytical formula of $U(\tau)$ expressing only bunch-cavity interaction. Then we obtain the shape, length and position of the bunch for DCI and ACO. Finally, we compare the results with measurements on ACO.

Evaluation of the transient voltage $U(\tau)$

The detailed calculations have been reported in the reference[2]. We restrict ourselves here to the main points.

Geometry of the cavity gap

Figure 1 shows the DCI cavity. The apertures for the beam are neglected. The field front generated by the bunch propagates radially at light velocity ; the influence of the radius R_o of the gap plate appears on the axis only after a time $2R_o/c$. Thus, if the bunch is short enough, the cavity may be replaced by any cylinder of height g and radius $R > R_o$ (fig. 2).

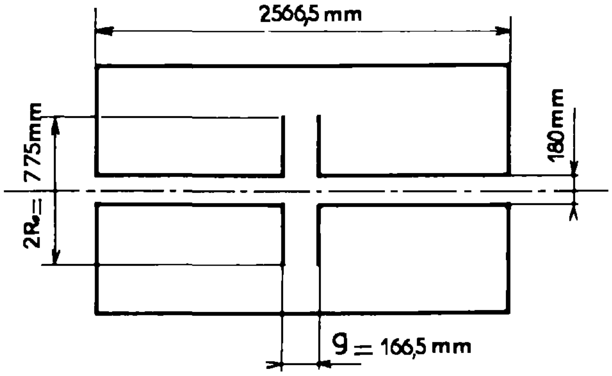

Fig. 1 : Actual geometry of the DCI cavity.

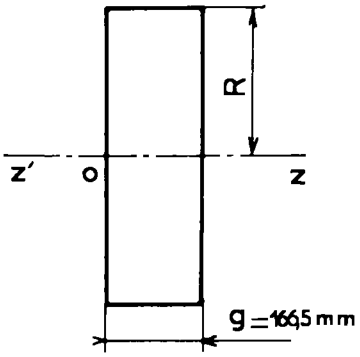

Fig. 2 : Equivalent simplified geometry.

Self field and voltage generated by the bunch are calculated by summing the response to a unit step of current

We denote by $U_H(x)$ the transient voltage acting on the particle entering the cavity at a time x later than the head of the unit step of current to which it belongs. If the actual instantaneous intensity $I(\tau)$ in the bunch is zero outside the interval $[-b \;,\; +b]$, and continuous inside, the transient voltage $U(\tau)$ is :

$$U(\tau) = -\int_0^{b-\tau} U_H(x)\,\dot{I}(\tau + x)\,dx \tag{5}$$

$U_H(x)$ is evaluated by eigenmode method[3,4,5]

We obtain :

$$U_H(x) = \frac{-1}{\pi\,\varepsilon_o c}\sum_{p=0}^{\infty}\sum_{n=1}^{\infty}\frac{\alpha_p}{v_{on}^2\;J_1^2(v_{on})\;\omega_{onp}}\left\{2\sin\omega_{onp}x - (-1)^p\sin\left[\omega_{onp}(x-\frac{g}{c})\right] - (-1)^p\sin\left[\omega_{onp}(x+\frac{g}{c})\right]\right\} \tag{6}$$

$\alpha_p = 1$ if $p = 0$ and $\alpha_p = 2$ if $p \neq 0$.

v_{on} is the nth root of Bessel function J_o and $\omega_{onp} = c\sqrt{(v_{on}/R)^2 + (p\pi/g)^2}$.

Using properties of Fourier series and Fourier-Bessel series, we find :

$$\sum_{p=0}^{\infty}\sum_{n=1}^{\infty}\frac{\alpha_p}{v_{on}^2\;J_1^2(v_{on})}\;\frac{\sin\theta\sqrt{(\frac{v_{on}}{R})^2 + (\frac{p\,\pi}{g})^2}}{\sqrt{(v_{on}/R)^2 + (p\pi/g)^2}} = \left|\begin{array}{ll} -\frac{g}{2}\ln\frac{\theta}{R} & \text{when } 0 < \theta < 2g < R \\ -\frac{g}{2}(\ln\frac{\theta}{R} + \ln\frac{\theta^2 - 4g^2}{R^2}) & \text{when } 2g < \theta < 4g < R \end{array}\right.$$

$$\sum_{p=0}^{\infty}\sum_{n=1}^{\infty}\frac{(-1)^p\;\alpha_p}{v_{on}^2\;J_1^2(v_{on})}\;\frac{\sin\theta\sqrt{(\frac{v_{on}}{R})^2 + (\frac{p\,\pi}{g})^2}}{\sqrt{(v_{on}/R)^2 + (p\pi/g)^2}} = \left|\begin{array}{ll} 0 & \text{when } 0 < \theta < g < R \\ -\frac{g}{2}\ln\frac{\theta^2 - g^2}{R^2} & \text{when } g < \theta < 3g < R \\ -\frac{g}{2}\left|\ln\frac{\theta^2 - g^2}{R^2} + \ln\frac{\theta^2 - 9g^2}{R^2}\right| & \text{when } 3g < \theta < 5g < R \end{array}\right. \tag{7}$$

Using (7), the double sum (6) simplifies into :

$$U_H(x) = \left|\begin{array}{ll} -\frac{1}{2\pi\varepsilon_o c}\ln\,(1 + \frac{2g}{cx}) & \text{when } 0 < x < \frac{2g}{c} \\ \\ -\frac{1}{2\pi\varepsilon_o c}\ln\,(1 + \frac{2g}{cx+2g}) & \text{when } \frac{2g}{c} < x < \frac{4g}{c} \end{array}\right. .$$

We can take for $U_H(x)$ practically without appreciable error :

$$U_H(x) = -\frac{1}{2\pi\varepsilon_o c}\ln\,(1 + \frac{2g}{cx}) \;. \tag{8}$$

Expression for $U(\tau)$

From (5) and (8) we get :

$$U(\tau) = \frac{1}{2\pi\varepsilon_o c}\int_0^{b-\tau}\ln(1+\frac{2g}{cx})\;\dot{I}\,(x + \tau)\,dx \;. \tag{9}$$

Determination of the bunch shape equilibrium

From the formulas (4) and (9), we obtain the equation satisfied by $I(\tau)$:

$$I(\tau) = K\exp\left[-\frac{\tau^2}{2\,\sigma_o^2} - \frac{1}{2\pi\,\varepsilon_o\,c\,\sigma_o^2\,\dot{V}_{RF}} \times\int_0^{\tau}\int_0^{b-y}\ln(1 + \frac{2g}{cx})\;\dot{I}\,(x + y)\,dx\,dy\right] \tag{10}$$

This equation may be written in the form $I(\tau) = F[I(\tau)]$ where F is the operator defined on the rhs of (10). $I(\tau)$ is determined by iteration as the limit of the set of functions

$$I_n(\tau) = F\left[I_{n-1}(\tau)\right]$$

where $I_0(\tau)$ is an arbitrary function ; for example $I_0(\tau) = 0$, then $I_1(\tau) = K\exp\,(-\,\tau^2/2\sigma_o^2)$.

Equation (10) is solved numerically using an IBM 1130 computer by noticing that :

(i) given σ_o and $\dot{V}_{RF}$, the solutions $I(\tau)$ of (10) depend on K. For each value of K, we can solve $I(\tau)$ and find the bunch charge

$$Q = \int_{-b}^{+b} I(\tau)\,d\tau$$

It can be noticed that K determines finally the value of Q.

(ii) according to the latter remark and to the form of (10), it follows clearly that the ratio $I(\tau)/Q$ depends only on the two parameters σ_o and $Q/\dot{V}_{RF}$.

σ_o is the standard deviation of the gaussian distribution when the self fields are disregarded. $Q/\dot{V}_{RF}$ depends on the parameters of the storage ring

$$\frac{Q}{\dot{V}_{RF}} = \frac{i\ T_o^2}{2\pi\ h\ \left[\hat{V}^2 - (U_o/e)^2\right]^{1/2}}$$

where $i = Q/T_o$ is the mean current in the ring, and h is the harmonic number.

	T_o (ns)	h	i (mA)	$\hat{V} \cos \phi_s$ (kV)	$Q/\dot{V}$ (10^{-20}CsV^{-1})
ACO	73	2	30	17.5	.0726
ADONE	350	3	60	120	.325
SPEAR	780	40	50	400	.0302
DCI	316	8	350	350	.198

Table I gives the value of the parameter $Q/\dot{V}$ for some storage rings.

Application to DCI

With the above assumption we have calculated the bunch shape in DCI using the following characteristics :

- number of accelerating cavities : 1,
- operating frequency of the cavity : 25.352 MHz (h=8),
- transit time through the gap g/c = .555 ns,
- order of magnitude of $\dot{V}_{RF}(0) \simeq 10^{13}$ V/s,
- order of magnitude of charge Q : 10^{10} to 10^{12} particles.

The exact bunch shape and the transient voltage are shown in figures 3 and 4 for the particular case $\sigma_o = 0.2 \times 10^{-9}$ s and $Q/\dot{V}_{RF} = .115 \times 10^{-20}$ C.s.V^{-1}.

In figure 3, it can be noticed that the gaussian distribution which describes the bunch shape when the self fields of the cavity are disregarded is modified : the bunch is longer and the center of charge is shifted forward. The dotted straight line on figure 4, whose slope is $\dot{V}_{RF}/Q$, allows comparison between the transient voltage and the RF voltage. We systematically study the deformation effect of the bunch versus σ_o and $Q/\dot{V}_{RF}$ by calculating the following parameters :

- half height bunch length ℓ (for the gaussian bunch, this length is $\ell_o = 2.35\ \sigma_o$),
- position τ_o of the center of charge defined by

$$\tau_o = \frac{1}{Q} \int_{-b}^{+b} \tau\ I(\tau)\ d\tau \quad ,$$

- average energy loss $< U(\tau) >$ defined by

$$<U(\tau)> = \frac{1}{Q} \int_{-b}^{+b} U(\tau)\ I(\tau)\ d\tau \quad .$$

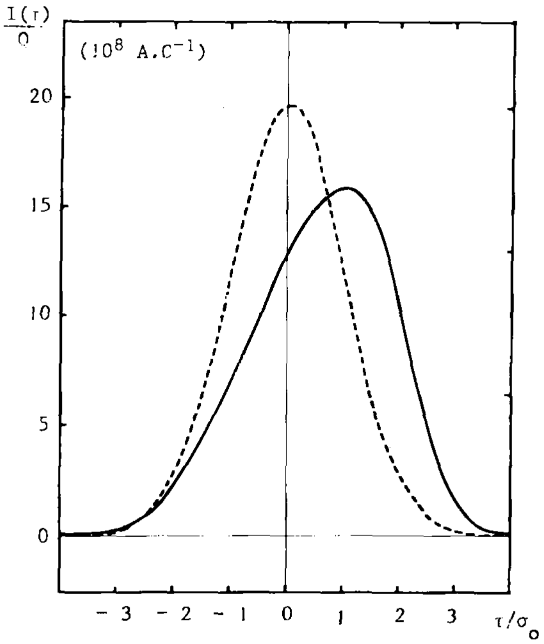

Fig. 3 : Bunch shape for σ_o = .2 ns, $Q/\dot{V}_{RF}$ = .115 × 10^{-20} C.s.V^{-1}.

The dotted curve represents the gaussian obtained by neglecting the self-fields.

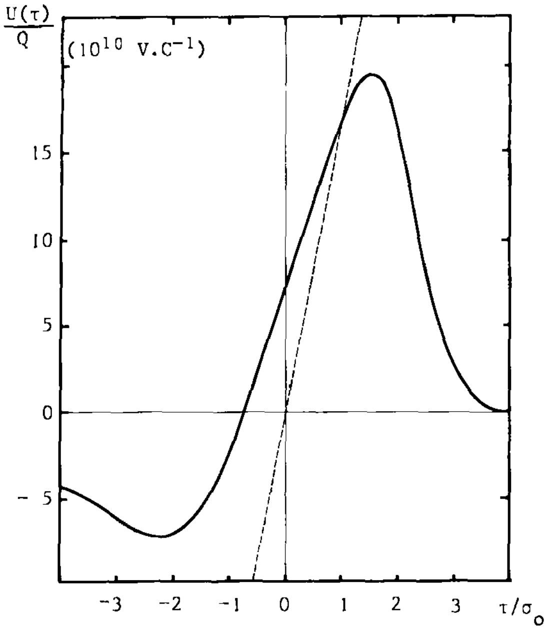

Fig. 4 : Transient voltage for σ_o = .2 ns, $Q/\dot{V}_{RF}$ = .115 × 10^{-20} C.s.V^{-1}.

The dotted line represents the sinusoidal voltage of the cavity.

The results are summed up in the curves of figures 5, 6 and 7 obtained by solving (10) for different values of σ_o and $Q/\dot{V}_{RF}$. These curves stop where the iteration method becomes divergent and does not permit solutions. Notice in these conditions that the relative lengthening $(\ell - \ell_o)/\ell_o$ is at most 30 % and the relative center of charge shift τ_o/ℓ_o can reach 30 %.

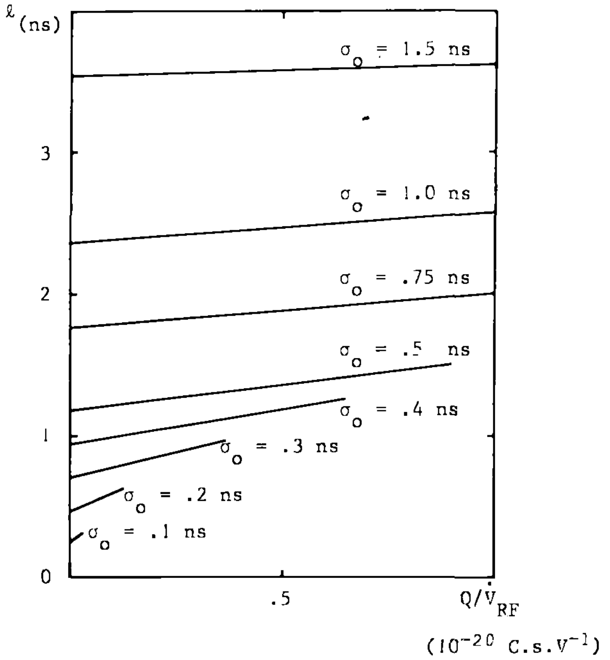

Fig. 5 : Evolution of bunch half-height length.

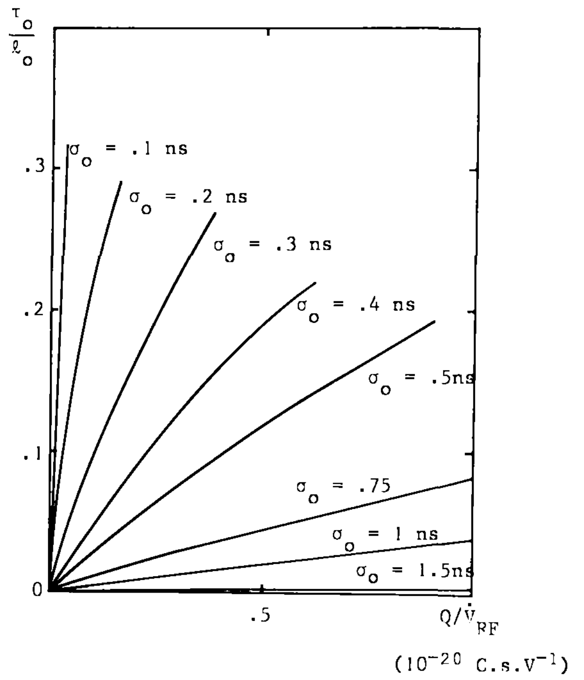

Fig. 6 : Relative shift of the center of charge.

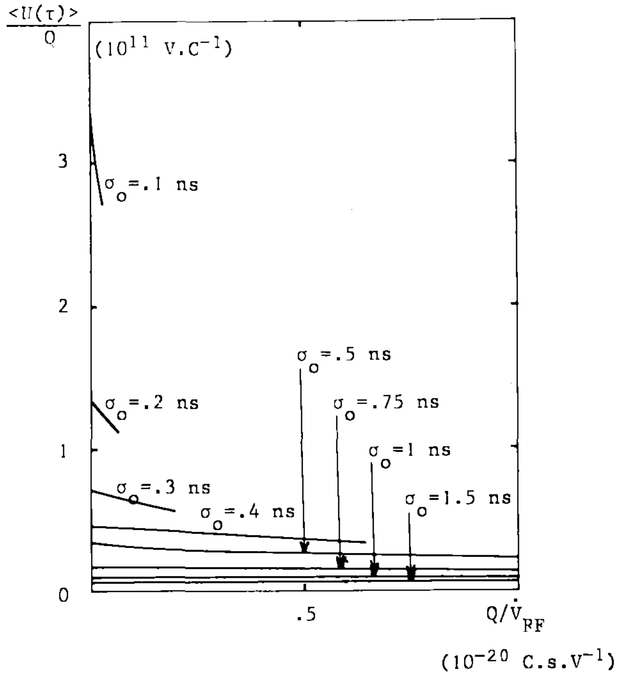

Fig. 7 : Average energy loss.

Application to ACO

The gap shape of ACO is quite complicated and can hardly be regarded as a cylindrical cavity.

However, since the expression of $U(\tau)$ only depends on the transit time g/c, we determined the lengthening on ACO using g/c = 0.35 ns.

Comparing with experimental results[6] (fig. 8), we notice that the bunch-cavity interaction does not explain more than 25 to 40 % of the total lengthening.

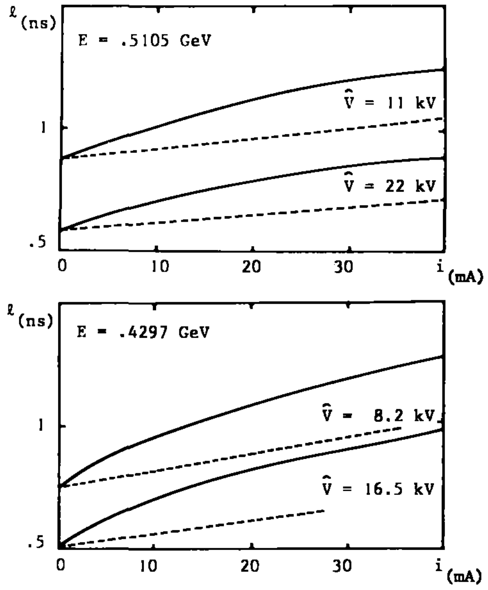

Fig. 8 : Evolution of the ACO bunch length versus *beam intensity*.

——— experimental length
----- calculated length

Conclusion

The bunch-cavity interaction is not the main cause of bunch lengthening observed at high intensity in storage rings using only one cavity. Actually, at each discontinuity of the vacuum chamber, the bunch loses energy and a transient voltage similar to those we have studied appears. The geometry of the vacuum chamber being very complicated, even a simplified calculation is difficult. However, when storage rings are equipped with several accelerating cavities, the bunch-cavity interactions can generate important bunch lengthening.

Acknowledgments

We wish to express our thanks to Professor Haïssinski, Dr P. Brunet, Dr D. Potaux and Dr H. Zyngier for their encouragement and helpful suggestions.

---

† Work supported by the Laboratoire de l'Accélérateur Linéaire d'Orsay.

References

1. J. Haïssinski, Il Nuovo Cimento 18 B, N° 1, pp 72-82 (1973)
2. M. Chatard-Moulin, B. Jecko and A. Papiernik, Rapport 73/A-1 Laboratoire d'Electronique et Micro-ondes, Université de Limoges, France (1973)
3. K. Kurokawa, IEEE Transactions on Microwave Theory and Techniques, MTT-6, pp. 178-187 (1958)
4. Van Bladel, Electromagnetic Field (Mc Graw-Hill), p. 308
5. M. Chatard-Moulin, B. Jecko and A. Papiernik, Rapport 73/A-2, Laboratoire d'Electronique et Micro-ondes, Université de Limoges, France (1973)
6. Groupe "Anneaux de Collisions", RT/34-69, Laboratoire de l'Accélérateur Linéaire, Orsay (1969)