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$X(3872)$, $X(3940)$ as hybrid charmonium states?

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Abstract. We discuss the implications of the fact that the recently observed $X(3872)$ and $X(3940)$ are hybrid charmonium states. We use formalism of operator product expansion and nonrelativistic QCD to estimate their production rates in semiinclusive B-decays. We express the decay rates in terms of a few nonperturbative matrix elements which eventually could be fixed by experimental measurements or calculated on the lattice. We use a simple flux tube model to estimate them and predict the branching ratio $B \to X(3872) + X_s$.

The Belle collaboration has recently observed a narrow state $X(3872)$ decaying into $J/\psi\pi\pi$ [1], which was confirmed by BaBar, CDF and D0 collaborations [2, 3, 4]. A second state, $X(3940)$ was recently observed by Belle [5]. An interesting feature of both of those charmonium states is the fact that despite their large mass, they do not decay into a pair of open-charm mesons. Aside from the states with particular J^{PC} assignments, such as tensor 2^{--} states, this feature is expected of the hybrid charmonium states: hybrid charmonium whose mass is below $D^{**}\overline{D}$ threshold would not primarily decay into the final states containing ground state D-mesons D and D^* [6, 7]. The state $X(3872)$ was first discovered in weak decays of B-mesons, $B \to J/\psi K\pi\pi$, so a theoretical estimate of the production rate is quite important. Here we use non-relativistic QCD (NRQCD) formalism developed in [8] to estimate the production rate for these states in B-decays assuming their hybrid nature.

Existence of the hybrid states is implied by QCD, as it is possible to build a colorless state that contains both quark and gluon degrees of freedom. Yet most of hybrid's properties have been studied only within quark models[1]. This is true even of quarkonium hybrids, because, unlike ordinary quarkonium, hybrids are characterized by excitations of the nonperturbative gluon field. Even for large mass m_Q, there is no reason for a $\overline{Q}Q$ hybrid to be a compact object, and neither the lattice nor models support such a picture. Contrary to the hybrids, in the limit $m_Q \to \infty$ ordinary quarkonium admits a controlled Fock state decomposition[10], in which the leading term is a $\overline{Q}Q$ in a color singlet with fixed quantum numbers ${}^{2S+1}L_J$. Higher order terms, such as color octet configurations, are suppressed by powers of $\alpha_s(m_Q) \sim v$, where v is the relative velocity of the Q and the $\overline{Q}$. The expansion is possible because for large m_Q the quarkonium is a small state, of size $(m_Q v)^{-1}$, whose interaction with the color field is governed by a multipole expansion. This velocity power counting can be made explicit by a suitable rescaling of the fields in the effective Lagrangian of NRQCD [11]. By contrast, the minimum of the hybrid meson potential is at a separation r_0 of order $1/\Lambda_{\rm QCD}$, independent of m_Q. For large m_Q, fluctuations about r_0 are small, but the state itself is not compact. This is a generic feature

[1] Lattice or QCD sum rule analyses of hybrid masses are also available. Most studies predict hybrid chamonia masses to be above or around 4 GeV, but they could be lighter as well [9]

of hybrid models; for example, in a constituent gluon model, the Q and $\overline{Q}$ are in a color octet and repel each other at short distances. At large m_Q, one expects a situation somewhat like a heavy rigid rotor, with nearly degenerate rotational bands of states. This behavior has been observed in lattice studies [12]. We can make the argument explicit by rescaling the coordinates in the NRQCD Lagrangian by their "natural" size, $\mathbf{x} = \lambda_x \mathbf{X},\ t = \lambda_t T$, where, contrary to the quarkonium case [11] where $\lambda_x = 1/(mv)$ we have $\lambda_x = 1/\Lambda_{QCD}$. This rescaling explicitly takes into account the fact that in the limit $m_Q \to \infty$ hybrid quarkonium state remains finite. An effective Lagrangian can be written as

$$\mathcal{L}_{NRQCD} = iQ^\dagger \left[\partial^0 + ig_s A^0\right] Q + Q^\dagger \frac{\left[\nabla + ig_s \mathbf{A}\right]^2}{2m_Q} Q + \ ... \tag{1}$$

It implies that ∂^0 is of the same order as $\nabla^2/2m_Q$, leading to $\lambda_t = m_Q \lambda_x^2 = m_Q/\Lambda_{QCD}^2$. The fields in (1) should also be rescaled as $Q = \lambda_Q \Psi,\ A^0 = \lambda_{A^0} \mathcal{A}^0,\ \mathbf{A} = \lambda_A \mathbf{A}$ where from the fact that the change of integration variables brings another factor of $\lambda_x^3 \lambda_t$ we obtain that $\lambda_Q^2 = \lambda_x^3$. The behavior of the gauge part of the Lagrangian $\frac{1}{4} G_{\mu\nu} G^{\mu\nu}$ is slightly more subtle, but it is easy to convince yourself that in the heavy quark limit $m_Q/\Lambda_{QCD} \to \infty$ the terms $(\nabla \times \mathbf{A})^2 \to m_Q \lambda_x^3 \lambda_A (\nabla \times \mathcal{A})^2$ and $(\nabla A^0)^2 \to m_Q \lambda_x^3 \lambda_{A^0} (\nabla \mathcal{A}^0)^2$ are the leading ones with $\lambda_A = \lambda_{A^0} = \sqrt{(m_Q \lambda_x^3)^{-1}} = \Lambda_{QCD} \sqrt{\Lambda_{QCD}/m_Q}$. This gives the rescaled NRQCD Lagrangian for hybrid quarkonia

$$\mathcal{L}^R_{NRQCD} = i\Psi^\dagger \left[\partial^0 + ig_s \sqrt{\frac{m_Q}{\Lambda_{QCD}}} \mathcal{A}^0\right] \Psi + \frac{1}{2} \Psi^\dagger \left[\nabla + ig_s \sqrt{\frac{\Lambda_{QCD}}{m_Q}} \mathcal{A}\right]^2 \Psi + \ ... \tag{2}$$

Clearly, all the fields and derivatives in (2) are dimensionless, so all suppression factors explicitly appear in front of the relevant operators providing a power counting scheme of the theory. Note that Eq.(2) implies that the Coulomb (i.e. $\mathcal{A}^0$) gluon interactions must be resummed to all orders. Hereafter we shall use conventional fields and derivatives keeping Eq.(2) in mind to maintain correct power counting of the relevant operators.

The absence of a controlled Fock state expansion for hybrid quarkonium can be understood as follows. For the ordinary heavy quarkonium, a soft glue configuration has a small overlap with the compact two-quark state. This is because the Compton wavelength of the soft gluons, of the order of Λ_{QCD}^{-1}, is much larger than the distance between the quarks. By contrast, the size of the heavy hybrid state is finite, of order Λ_{QCD}^{-1} in the heavy quark limit, allowing for a significant interaction with the soft gluonic modes.

There is, however, still a sort of Fock state expansion for quarkonium hybrids. Although the gluonic degrees of freedom are intrinsically nonperturbative, we can label the various components according to the quantum numbers of the $Q\overline{Q}$. For example, for the 0^{+-} charmonium hybrid state, we can write a decomposition of the form

$$|X\rangle = A|c\bar{c}(^3S_1)_{1,8} + g_1\rangle + B|c\bar{c}(^3P_J)_{1,8} + g_2\rangle + C|c\bar{c}(^1S_0)_{1,8} + g_3\rangle + \ldots, \tag{3}$$

where the subscripts $1, 8$ indicate the color of the $\bar{c}c$ pair and the g_i are various gluonic configurations. While one is always free to do this, here there is no model-independent hierarchy among the coefficients. This is related to the lack of a unique $\bar{c}c$ "baseline" state, and the lack of a multipole expansion to govern transitions between the different Fock components. Nonetheless, *within particular models* it is typically the case that a given hybrid is dominated by one or two $\bar{c}c$ configurations. Hence it is useful to organize our calculation of hybrid production in B decays with a decomposition such as (3).

The above ideas [8] can be used to calculate production rate of X in semiinclusive B decays, assuming that X is indeed a hybrid state. Let us first calculate the production rate in the framework of full QCD, separating hard and soft degrees of freedom. Then we compute the rate in NRQCD, and use the perturbative calculation to match the coefficients of the NRQCD matrix elements. The inclusive B meson decay rate to a hybrid charmonium state $B \to X(3872) + X_s$ is

$$\Gamma^{\rm QCD} = \frac{G_F^2\,|V_{cb}V_{cs}^*|^2\,C_2^2}{6m_b}\int\frac{d^4P}{(2\pi)^4}{\rm Im}\,i\int d^4y\,e^{iPy}\,\langle B|T\,\{j_\mu^{a\dagger}(y), j_\nu^b(0)\}|B\rangle\;C_{ab}^{\mu\nu},$$
$$C_{ab}^{\mu\nu} = \langle 0|\bar c\Gamma^\mu T^a c|X+X_{s2}\rangle\langle X+X_{s2}|\bar c\Gamma^\nu T^b c|0\rangle = \langle 0|\bar c\Gamma^\mu T^a c\,P_{\bar cc}\,\bar c\Gamma^\nu T^b c|0\rangle, \tag{4}$$

where $P_{\bar cc} = |\bar cc + X_3\rangle\,\langle\bar cc + X_3|$ is a projection operator onto the $\bar cc$ state in the ψ_g, $j_\mu^a = \bar s\Gamma_\mu T^a b$. The Eq. (4) is then matched with the equivalent expression in NRQCD,

$$\Gamma^{\rm NRQCD} = \frac{G_F^2\,|V_{cb}V_{cs}^*|^2\,C_2^2}{m_b}\int\frac{d^4P}{(2\pi)^4}\sum_n\frac{C_n}{m_c^{d_n-4}}\,\langle O_n^{\psi_g}\rangle\,2\pi\delta(P^2 - m_\psi^2), \tag{5}$$

where $O_n^{\psi_g}$ represents a set of NRQCD operators of increasing dimension d_n

$$\begin{aligned}
\langle O_8^X(^3S_1)\rangle &= \langle 0|\chi^\dagger\sigma^j T^a\psi P_X\psi^\dagger\sigma^j T^a\chi|0\rangle,\;\; \langle O_8^{\psi_g}(^1S_0)\rangle = \langle 0|\chi^\dagger T^a\psi P_X\psi^\dagger T^a\chi|0\rangle,\\
\langle O_8^X(^3P_1)\rangle &= \frac{1}{2}\langle 0|\chi^\dagger(-\tfrac{i}{2}\mathbf{D}\times\sigma)^j T^a\psi P_X\psi^\dagger(-\tfrac{i}{2}\mathbf{D}\times\sigma)^j T^a\chi|0\rangle,\\
\langle P_8^X(^3S_1)\rangle &= \frac{1}{2}\left[\langle 0|\chi^\dagger\sigma^j(-\tfrac{i}{2}\mathbf{D})^2 T^a\psi P_X\psi^\dagger\sigma^j T^a\chi|0\rangle + \langle 0|\chi^\dagger\sigma^j T^a\psi P_X\psi^\dagger\sigma^j(-\tfrac{i}{2}\mathbf{D})^2 T^a\chi|0\rangle\right],\\
\langle P_8^X(^1S_0)\rangle &= \frac{1}{2}\left[\langle 0|\chi^\dagger(-\tfrac{i}{2}\mathbf{D})^2 T^a\psi P_X\psi^\dagger T^a\chi|0\rangle + \langle 0|\chi^\dagger T^a\psi P_X\psi^\dagger(-\tfrac{i}{2}\mathbf{D})^2 T^a\chi|0\rangle\right],\\
\langle Q_8^X(^3S_1)\rangle &= \frac{1}{2}\Big[\langle 0|\chi^\dagger(-\tfrac{i}{2}\mathbf{D})^j(-\tfrac{i}{2}\mathbf{D}\cdot\sigma)T^a\psi P_X\psi^\dagger\sigma^j T^a\chi|0\rangle\\
&\qquad + \langle 0|\chi^\dagger\sigma^j T^a\psi P_X\psi^\dagger(-\tfrac{i}{2}\mathbf{D})^j(-\tfrac{i}{2}\mathbf{D}\cdot\sigma)T^a\chi|0\rangle\Big].
\end{aligned} \tag{6}$$

Here P_X is a projection onto the hybrid state $|X(3872)\rangle$ which is normalized nonrelativistically. Note that each derivative insertion brings a power of the velocity of the heavy quark, $p/m_c \sim v$: $O_8^X(^3P_1)$ is suppressed compared to $O_8^X(^3S_1)$ by v^2. We are unable to calculate matrix elements of these operators in a model independent fashion, so as usual they are left as free parameters. Since the same matrix elements would govern the total annihilation width of the hybrid or the hybrid photoproduction cross section, one should extract the leading matrix elements from a small number of experiments.

Performing the perturbative calculation of $C_{ab}^{\mu\nu}$, boosting the relativistic four-component spinors u_c to the $\bar cc$ center of mass frame and then replacing them by nonrelativistic two-component spinors ξ and η we arrive at a simple expression

$$\begin{aligned}
\Gamma^{\rm QCD} &= \frac{G_F^2\,|V_{cb}V_{cs}^*|^2\,C_2^2}{m_b}\int\frac{d^4P}{(2\pi)^4}\frac{2\pi m_b^2}{9\mu_X}(1-\mu_X)(1+2\mu_X)\delta((p_b-P)^2)\times 4m_c^2\\
&\times\;\Bigg\{\left(1+\frac{\mathbf{q}'^2+\mathbf{q}^2}{2m_c^2}\right)\xi'^\dagger\sigma^j T^a\eta'\eta^\dagger\sigma^j T^a\xi + \frac{1}{m_c^2}\xi'^\dagger(\mathbf{q}'\times\sigma)^j T^a\eta'\eta^\dagger(\mathbf{q}\times\sigma)^j T^a\xi\\
&\qquad + \frac{3\mu_X}{4\mu_c(1+2\mu_X)}\left(1-\frac{\mathbf{q}'^2+\mathbf{q}^2}{2m_c^2}\right)\xi'^\dagger T^a\eta'\eta^\dagger T^a\xi\\
&\quad - \frac{1}{2m_c^2}\left[\xi'^\dagger\mathbf{q}'^j(\mathbf{q}'\cdot\sigma)T^a\eta'\eta^\dagger\sigma^j T^a\xi + \xi'^\dagger\sigma^j T^a\eta'\eta^\dagger\mathbf{q}^j(\mathbf{q}\cdot\sigma)T^a\xi\right] + \ldots\Bigg\},
\end{aligned} \tag{7}$$

where $\mu_c = m_c^2/m_b^2$ and $\mu_X = m_{X(3872)}^2/m_b^2$. Eq. (7) expresses the decay rate in terms of matrix elements of operators of increasing dimension.

To perform the matching, we have to calculate $\Gamma(B \to X(3872)+X_s)$ in perturbative NRQCD using Eqs. (5) and (6) and then evaluate the NRQCD coefficients by matching full NRQCD states to perturbative ones. The partial decay rate is

$$\begin{aligned} \Gamma &= \Gamma_0 \frac{4(1-\mu_X)^2(1+2\mu_X)}{m_b^2 m_X} \left[\langle O_8^X(^3S_1)\rangle + \frac{3\mu_X}{4\mu_c(1+2\mu_X)} \langle O_8^X(^1S_0)\rangle + \frac{2}{m_c^2}\langle O_8^X(^3P_1)\rangle \right. \\ &\left. + \frac{1}{m_c^2}\langle P_8^X(^3S_1)\rangle - \frac{3\mu_X}{4\mu_c(1+2\mu_X)m_c^2}\langle P_8^X(^1S_0)\rangle - \frac{1}{m_c^2}\langle Q_8^X(^3S_1)\rangle + \dots \right]. \end{aligned} \tag{8}$$

where we have defined $\Gamma_0 = C_2^2 |V_{cb}V_{cs}^*|^2 G_F^2 m_b^5/144\pi$. Since the Fock space decomposition of hybrid states is not controlled by a multipole expansion, the number of matrix elements important for production processes can be reduced by keeping those that are leading in powers of $1/m_c$. The formula (8) determines the decay rate of B into a given hybrid charmonium state in terms of *universal* parameters which must be fixed by other experiments. Using the flux tube model [13] to estimate the matrix elements (6) we find [8]

$$\Gamma(B \to X(3872) + X_s) \leq (1 \times 10^{-3})\, \Gamma(b \to c\bar{c}s)\,. \tag{9}$$

Thus we estimate that the production of this hybrid charmonium is more than a factor of ten lower than that of typical ordinary charmonium states. A very similar estimate is expected for the $X(3940)$ state.

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References

[1] S. K. Choi *et al.* [Belle Collaboration], Phys. Rev. Lett. **91**, 262001 (2003).
[2] B. Aubert *et al.* [BABAR Collaboration], arXiv:hep-ex/0406022.
[3] D. Acosta *et al.* [CDF II Collaboration], Phys. Rev. Lett. **93**, 072001 (2004).
[4] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **93**, 162002 (2004).
[5] S. Olsen, these proceedings.
[6] F. E. Close and P. R. Page, Nucl. Phys. B **443**, 233 (1995).
[7] P. R. Page, Phys. Lett. B **402**, 183 (1997).
[8] G. Chiladze, A. F. Falk and A. A. Petrov, Phys. Rev. D **58**, 034013 (1998).
[9] T. E. Browder, S. Pakvasa and A. A. Petrov, Phys. Lett. B **578**, 365 (2004).
[10] G.T. Bodwin, E. Braaten and G.P. Lepage, Phys. Rev. **D51**, 1125 (1995).
[11] M. Luke and A.V. Manohar, Phys. Rev. **D55**, 4129 (1997).
[12] T. Manke *et al.*, Phys. Rev. **D57**, 3829 (1998).
[13] N. Isgur and J. Paton, Phys. Lett. **124B**, 247 (1983); Phys. Rev. **D31**, 2910 (1985); J. Merlin and J. Paton, J. Phys. **G11**, 439 (1985).