Gauge/String Duality, etc.
(Pomeron and Odderon at Strong Coupling)

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We discuss conformal Pomeron and Odderon using AdS/CFT, or, equivalently, Pomeron and Odderon in N = 4 SYM, in strong coupling. We explore the relation between the 'Basso-expansion' and the Delta-j curve in strong coupling, and we demonstrate how it can be applied to other Regge intercepts in addition to Pomeron. In particular, we focus on 'Odderons' which are the leading crossing-odd, C = -1, Regge singularities. From the perspective of AdS/CFT, while Pomeron can be identified with a Reggeized Graviton, Odderons correspond to Reggeized anti-symmetric AdS_5 Kalb-Ramond tensor-fields.

1 Introduction

The Pomeron is the leading order exchange in total cross sections at high energies (up to unitarity corrections once saturation is reached) and in many exclusive processes in the Regge limit as well. Its importance has been confirmed experimentally for example at HERA, where it was shown that cross sections for many different processes (DIS, DVCS, VM production...) show a power growth with $1/x$, and that the same, universal gluon distribution functions describe these processes, and gluons dominate at small x. The odderon is the less well known cousing of the Pomeron, which is odd under charge conjugation. It dominates in Regge limit processes with the quantum numbers of the vacuum and $C = -1$. The exchange of the Pomeron has a long history of study from the weak coupling side using the BFKL equation. We present an alternative to the study of Pomeron and Odderon based on the AdS/CFT correspondence[1], which allows us to study these processes at strong coupling. The AdS/CFT framework also presents an alternative way to study the saturation region.

2 Pomeron at Strong Coupling

At strong coupling the Pomeron was first introduced by Brower, Polchinski, Strassler and Tan[2]. They show that the Pomeron emerges as the Regge trajectory of the graviton. We can introduce a vertex operator for the Pomeron

$$\mathcal{V}_P(j,\pm) = (\partial X^{\pm}\bar{\partial}X^{\pm})^{\frac{j}{2}} e^{\mp ik\cdot X}\phi_{\pm j}(r). \qquad (1)$$

As other vertex operators corresponding to states in string theory, it must satisfy the on-shell condition

$$[\frac{j-2}{2} - \frac{\alpha'}{4}\Delta_j]e^{\mp ik\cdot X}\phi_{\pm j}(r) = 0 \tag{2}$$

where $\Delta_j = (r/R)^j(\Delta_0)(r/R)^{-j}$, and Δ_0 is the scalar Laplacian in curved space. The equation above can then be used to determine the function $\phi_{\pm j}(r)$ for the Pomeron vertex operator. In [2] they show how we can expand the differential operator to order $1/\sqrt{\lambda}$ around $j = 2$

$$[j - 2 - \frac{\alpha' t}{2}e^{-2u} - \frac{1}{2\sqrt{\lambda}}(\partial_u^2 - 4)]\phi_{\pm}(u) = 0, \tag{3}$$

where $u = -\log r_0/r = -\log z/z_0$ with $z = R^2/r$. From here we can obtain the intercept

$$j_0 = 2 - \frac{2}{\sqrt{\lambda}}. \tag{4}$$

The above equation can also be used to calculate the propagator for Pomeron exchange [2,3]

$$\chi(\tau, L) = (\cot(\frac{\pi\rho}{2}) + i)g_0^2 e^{(1-\rho)\tau}\frac{L}{\sinh L}\frac{\exp(\frac{-L^2}{\rho\tau})}{(\rho\tau)^{\frac{3}{2}}} \tag{5}$$

where, due to conformal invariance, χ is a function of only two variables, $L = \log(1 + v + \sqrt{v(2+v)})$ and $\tau = \log(\frac{\rho}{2}zz's)$. L can be thought of as related to the 3 dimensional impact parameter (v is the chordal distance in H_3).

3 Applications

We can apply these methods to calculate the amplitude for any process where Pomeron exchange dominates. As mentioned in the introduction, those include many processes studied at HERA. However, single Pomeron exchange leads to an asymptotic power behavior in s, thus leading to the violation of the Froissart bound. Therefore effects beyond single Pomeron exchange eventually become important. One way to take them into account in a unitary way is to use the eikonal approximation, which sums multiple Pomeron exchanges to all order, but ignores the interactions between the Pomerons. In AdS this leads to [3]

$$A(s,t) = 2is\int d^2l e^{-i\mathbf{l}_\perp\cdot\mathbf{q}_\perp}\int dzd\bar{z}\, P_{13}(z)P_{24}(\bar{z})(1 - e^{i\chi(s,b,z,\bar{z})}). \tag{6}$$

Single Pomeron exchange would correspond to expanding the above to first order in χ. To study different processes, we just provide different wavefunction for the external states, and it has already been applied to (and presented at previous Moriond conferences) DIS [4], DVCS [5], vector meson production [6] and DIS using the soft wall model [7].

4 Odderon at Strong Coupling

In AdS/CFT, the Odderon was identified as the Regge trajectory of the Kalb-Ramond $\tilde{B}_{\mu\nu}$ field [8]

$$\mathcal{V}_O(j,\pm) = (\partial X^{\pm}\bar{\partial}X^{\perp} - \partial X^{\perp}\bar{\partial}X^{\pm})(\partial X^{\pm}\bar{\partial}X^{\pm})^{\frac{j-1}{2}}e^{\mp ik\cdot X}\phi_{\pm j\perp}(r). \tag{7}$$

They must satisfy the on-shell condition.

$$[\frac{j+1}{2} - \frac{\alpha'}{4}\Delta_{O,j}]\phi_{\pm j\perp}(r) = \phi_{\pm j\perp}(r) \tag{8}$$

where $\Delta_{O,j} = (r/R)^{-(j-1)}(\Delta_{O,1})(r/R)^{(j-1)}$. To determine the differential operator for Odderon, $\Delta_{O,1}$, we can match the EOM at $j = 1$, appropriate in the infinite λ limit. In the case of the Odderon, in the supergravity limit we have two equations

$$(\Box_{Maxwell} - (k+4)^2)\tilde{B}^{(1)}_{IJ} = 0 \ , \quad (\Box_{Maxwell} - k^2)\tilde{B}^{(2)}_{IJ} = 0 \tag{9}$$

This will give us for the physical state condition

$$[j - 1 - \frac{\alpha' t}{2} e^{-2u} - \frac{1}{2\sqrt{\lambda}}(\partial_u^2 - m_{AdS}^2)]\phi_{\pm\perp}(u) = 0 \tag{10}$$

where m_{AdS}^2 is either 16 or 0, depending on which of the two solutions we are considering. The corresponding intercepts are

$$j_0^a = 1 - \frac{8}{\sqrt{\lambda}} \quad j_0^b = 1 \ . \tag{11}$$

This is analagous to weak coupling results.

5 Beyond Leading Order

We can also find the intercept as the minimum of the $j(\Delta)$ curve, which occurs at $\Delta = 2$. Let's look at the expansion of the inverse curve

$$(\Delta - 2)^2 = \tau^2 + \beta_1(\lambda)S + \beta_2(\lambda)S^2 + \beta_3(\lambda)S^3 + \cdots \tag{12}$$

($S = j - 2$ for Pomeron, $j - 1$ for odderon; τ is the 'twist' $\tau_P = 2 + k$, $\tau_a = 4 + k$, $\tau_b = k$). We can expand S_0 into powers of $1/\sqrt{\lambda}$

$$S_0 = \frac{c_1}{\sqrt{\lambda}} + \frac{c_2}{\lambda} + \frac{c_3}{\lambda^{3/2}} + \frac{c_4}{\lambda^2} + \cdots \tag{13}$$

Allowing us to solve iteratively

$$\begin{aligned} c_1 &= -b_{(10)}^{-1}\tau^2 \\ c_2 &= -b_{(10)}^{-1}b_{(11)}c_1 \\ c_3 &= -b_{(10)}^{-1}[b_{(11)}c_2 + b_{(12)}c_1 + b_{(20)}c_1^2] \ , \end{aligned}$$

and so on, and b_{ni} are coefficients in the expansion of

$$\beta_n = \lambda^{\frac{2-n}{2}}(b_{n0} + \frac{b_{n1}}{\lambda^{1/2}} + \frac{b_{n2}}{\lambda} + \frac{b_{n3}}{\lambda^{3/2}} + \cdots). \tag{14}$$

To find the Odderon intercept we assume that in the diffusion limit $\beta_1 = 2\sqrt{\lambda} + \mathcal{O}(1)$ so that we recover our result from the last chapter. Using this result and expanding S in powers of $1/\sqrt{\lambda}$ this will give us for the first few terms

$$\begin{aligned} c_1(k) &= \left(\frac{-1}{b_{1,0}}\right)k^2 \ , \\ c_2(k) &= \left(\frac{b_{1,1}}{b_{1,0}^2}\right)k^2 \ , \\ c_3(k) &= \left(\frac{-b_{1,1}^2}{b_{1,0}^3} + \frac{b_{1,2}}{b_{1,0}^2}\right)k^2 + \left(\frac{-b_{2,0}}{b_{1,0}^2}\right)k^4 , \end{aligned}$$

and so on. When $k = 0$ we can show

$$j_0^b = 1 \ , \tag{15}$$

i.e. the intercept for the type b odderons stays fixed at 1 regardless of the coupling strength. This is consistent with up-to-date weak coupling findings. For the type a Odderon we need to take some further assumptions. Recent work by Basso [9] shows that in an expansion

$$\Delta(S,k) = k + \alpha_1(\lambda,k)S + \alpha_2(\lambda,k)S^2 + \cdots \tag{16}$$

the first term is $\alpha_1(\lambda,k) = \frac{\lambda}{k} Y_k(\sqrt{\lambda})$ which give us $\beta_1 = 2\sqrt{\lambda} Y_\tau(\sqrt{\lambda})$ We assume that, for Odderons, the conformal dimensions can be matched with the one-loop calculation for the leading BKP "folded strings", as done for the Pomeron. With these assumptions we get [a]

$$j_0^{(-,a)} = 1 - \frac{8}{\lambda^{1/2}} - \frac{4}{\lambda} + \frac{13}{\lambda^{3/2}} + \frac{96\zeta(3)+41}{\lambda^2} + \frac{288\zeta(3)+\frac{1249}{16}}{\lambda^{5/2}} + \frac{-720\zeta(5)+192\zeta(3)+\frac{159}{4}}{\lambda^3} + \cdots . \tag{17}$$

A similar procedure to find the Pomeron intercept to higher order has been done first in [10], and later extended to higher order in [11,12]. To go to our order in $1/\sqrt{\lambda}$ we make use of some of the coefficients b_{ni} calculated in [12]. The presentation here has been brief due to space constraints, a detailed explanation will be presented in [13], to appear very soon.

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[a] In the talk we only presented the result up to order $1/\lambda^{5/2}$, and there was a typo in the last term.