

Are octonions necessary to the Standard Model?

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Abstract. There have been a number of claims, going back to the 1970s, that the Standard Model of particle physics, based on fermions and antifermions, might be derived from an octonion algebra. The emergence of $SU(3)$, $SU(2)$ and $U(1)$ groups in octonion-based structures is suggestive of the symmetries of the Standard Model, but octonions themselves are an unsatisfactory model for physical application because they are antiassociative and consequently not a group. Instead, the ‘octonion’ models have to be based on adjoint algebras, such as left- or right-multiplied octonions, which can be seen to have group-like properties. The most promising of these candidates is the complexified left-multiplied octonion algebra, because it reduces, in effect, to $C(6)$, which has been identified by one of us (PR) in a number of previous publications as the basic structure for the entire foundation of physics, as well as the algebra required for the Standard Model and the Dirac equation. Though this algebra has long been shown by PR as equivalent to using a complexified left-multiplied or ‘broken’ octonion, it doesn’t need to be derived in this way, as its real origins are in the respective real, complex, quaternion and complexified quaternion algebras of the fundamental parameters of mass, time, charge and space. The ‘broken’ octonion, however, does have value in leading to the higher (and equally broken) symmetries, such as E_8 , which incorporate fermions, with their two spin states, along with gauge bosons and vacuum states into a unified scheme.

1. Introduction

The possibility that octonions could involve symmetries relating to the Standard Model was considered by Günaydin and Gürsey as early as the 1970s [1]. The idea has been taken up by a number of later authors, including Geoffrey Dixon [2, 48], John Baez [3] and Cohl Furey [4-7]. Dixon’s view is that Nature ought to privilege the four division algebras and he has attempted to base the group structure of the Standard Model on the combined algebra of real numbers (R), complex numbers (C), quaternions (H) and octonions (O) [2]. He states that these are the only algebras that combine the correct Galois sequences and quadratic residue codes. (The Galois connection is suggestive of why the fourth algebra is the last as the fifth element in a sequence tends to be the symmetry-breaker in Galois theory.) In the combined algebra he has found evidence of $U(1)$, $SU(2)$ and $SU(3)$ symmetries, and he has sought to use the antiassociativity of the octonions to lead to an explanation of the chirality of the weak interaction. Though octonions are not a group, the Freudenthal-Tits magic square suggests that their rotations and self-interactions can lead to all the groups of interest in fundamental physics up to E_8 . Though he has been able to find the gauge groups of the Standard Model in his algebra, he hasn’t been able to pinpoint just why these groups are selected from all the ones available, and he hasn’t been able to generate specific particle structures.



Baez proposed that, if particles are represented by octonions, then 10-D string theory might be represented by a 2-D string in an 8-D octonionic ‘space’ [3] (though this has the problem of introducing two time-like elements, one for the space and one for the string). Furey has argued, more recently along similar lines to Dixon, but has ended up using octonion chain multiplication (a version of left-multiplied octonions) to reduce the Dixon product to a $Cl(6)$ algebra, which she represents as a complexified left-multiplied octonion [4-7]. She finds that $Cl(6)$, which has only a quarter of the elements of $R \otimes C \otimes H \otimes O$, 64 rather than 256, is more conducive to finding particle-like structures in the mathematics. Contemporary with, and in advance of, many of these developments, one of us (PR) has produced a series of theoretical results, sometimes alone and sometimes in collaboration with other authors, which, starting from a more fundamental position, comes to a $Cl(6)$ structure for the Standard Model fermions, and the $U(1)$, $SU(2)$ and $SU(3)$ symmetries, and E_8 for the entire set of fermions and bosons, including vacuum particles and spin states [8-47]. This theory does not privilege octonions as such, because their antiassociativity prevents them from having group structure and leads to difficulties with physical interpretation, but has, from the beginning, recognised the possibility of mapping the $Cl(6)$ structure onto a complexified left- or right-multiplied octonion, particularly because of the significance of the octonions in creating the E_8 symmetry. The purpose of this paper is to examine these theories with respect to each other, as it is clear that the basis of Standard Model physics lies somewhere within them. We aim to show that, while octonions are not the *origin* of the development, as certain authors believe, and are not fundamental to physics, they have a role to play, in their left- or right-multiplied application, as an alternative representation of the more fundamental $Cl(6)$. We will also propose a new significance for the $R \otimes C \otimes H \otimes O_L$ product in connection with E_8 .

2. Rowlands (PR), $Cl(6)$ and octonions

The fundamental theory developed by PR originates in work of the 1970s, quite separate from all the other developments. Here, physics emerges from a Klein-4 group of four fundamental parameters, mass, time, charge (in three types) and space, with respective real, complex, quaternion and multivariate vector algebras (the last being also equivalent to Pauli matrices and complexified quaternions) [8-11]. If we take these all together, with respective units 1; i ; i, j, k ; i, j, k , we find an algebra that has 64 elements [12,13], which is identical to $Cl(6)$, and which is identified as such [28]. (We note here that the vector units, i, j, k , in this abbreviated representation are commutative with the quaternion units, i, j, k , and are not their complexification.) This is a complexified double quaternion algebra, a vector quaternion algebra, a double Pauli algebra or a double vector algebra ($R \times C \times H \times H$). It is equivalent to the Clifford algebras $Cl(3, 3)$, $Cl(6, 0)$, $Cl(3, 2)$, $Cl(3, 4)$ and many others, as is stated in [28] and other sources. They are all versions of $Cl(6)$. They are equivalent to a double space or even a double space-time. They are also equivalent to the gamma algebra of the Dirac equation and this recognition has led to a new form of this equation in which the wavefunction can be represented by either an idempotent or a nilpotent. The idempotent form is more conventional but much less powerful than the nilpotent [12-13,28,40].

The nilpotent has five terms. For a free particle the amplitude is of the form

$$(\pm ikE \pm iip_x \pm jip_y \pm kip_y \pm jm) e^{-i(Et - \mathbf{p} \cdot \mathbf{r})}.$$

The idempotent forms are the same expression pre-multiplied by k, i , or j , and represent vacuum states connected with the respective weak, strong and electric interactions. The nilpotent structure situates a fermion within the entire universe; the idempotent structures restrict this to the respective vacua associated with the weak, strong and electric interactions. The coefficients of the energy, $3 \times$ momentum, and mass terms, ik, ii, ji, ki, j , can be identified as among 12 pentads, each of which can generate the entire 64 elements of the algebra [28,40]:

1	i				-1	$-i$				
ii	ij	ik	ik	j	$-ii$	$-ij$	$-ik$	$-ik$	$-j$	
ji	jj	jk	ii	k	$-ji$	$-jj$	$-jk$	$-ij$	$-i$	

ki	kj	kk	ij	i	$-ki$	$-kj$	$-kk$	$-ij$	$-i$
ii	ij	ik	ik	j	$-ii$	$-ij$	$-ik$	$-ik$	$-j$
iji	ijj	ijk	ii	k	$-iji$	$-ijj$	$-ijk$	$-ii$	$-k$
iki	ikj	ikk	ij	i	$-iki$	$-ikj$	$-ikk$	$-ij$	$-i$

The accompanying paper [47] and other publications [28,40] show that they arise from the 8 basic units, by taking each of the 3 quaternion units of charge, k , i , or j , onto the other 5:

time	space	mass	charge
i	$i \ j \ k$	1	$i \ j \ k$

This actually *creates* the conjugate concepts of quantised energy, momentum and rest mass from the more fundamental ones:

ik	$ii \ ji \ ki$	lj
energy	momentum	rest mass

The nilpotent wavefunctions or amplitudes have the property of also being creation or annihilation operators, and their combinations create the gauge boson states.

Charge, in this formulation, is fundamentally quantized, as the symmetries of the Klein-4 parameter group reveal [8-10,28,40,41]. Charge quantization is a fundamental thing; it doesn't need an additional 'explanation'. The compactification also breaks the symmetry between the charge units, which now become associated with the respective pseudoscalar, vector and scalar units of time, space and continuous mass, leading to the respective $SU(2)$, $SU(3)$ and $U(1)$ symmetries associated with the weak, strong and electric interactions, as discussed in earlier publications, where the process is explained in detail leading from the abstract symmetries to the precise physical details of the interactions. So, we write

ik	$ii \ ji \ ki$	lj
weak charge	strong charge	electric charge

The double 5-dimensionality of energy-momentum-mass and the three charges, can be seen as a 10-dimensional structure, in which 6 units are conserved or 'compactified', that is, all except energy and momentum [14,28,40]. In the nilpotent form, the structure maintains the requirement for a perfect string theory, that self-duality in phase space determines vacuum selection. The only difference from Baez' octonion proposal is that a nilpotent structure eliminates the need for an actual 2-dimensional string in that the nilpotent condition makes the fifth and tenth dimensions redundant for a point-particle solution.

The 8 primary units have been mapped onto an octonionic structure in a number of ways since 1998 [15]. According to a paper of 2001: 'If the 3-dimensionality of charge and space is directly involved, the overall structure would require a quaternion and a 4-vector within another overall quaternion-type arrangement. This could be accomplished using an octonion, with sixteen members ($\pm 1m$, $\pm is$, $\pm je$, $\pm kw$, $\pm et$, $\pm fx$, $\pm gy$, $\pm hz$), though this is no longer a group. The nonassociativity of the dimensional terms in the octonion extension seems to be lost within terms which effectively cancel each other out, and are of 'no physical significance' [20]. These are octonions without antiassociativity, like left- or right-multiplied terms and involve complexification. In another paper, they are described as a 'broken octonion', which 'can be represented in a group structure through the use of left-product or right-product octonions' [23]. It is also a complexified octonion because of the presence of the three vector terms ($\pm fx$, $\pm gy$, $\pm hz$) in the physical version and the pure non-dimensional complex unit $e = i$. This early work recognised the presence of $SU(3)$ through the pseudoscalar term, $SU(2)$ through the quaternion triplet and $U(1)$ through the vector triplet [23,28].

The work significantly differs in presentation from that of other authors because it starts from a known basis, in which particles are determined by charge structures. The algebra could be arranged as

a quaternion (mass-charge) and a 4-vector (space-time), the first a conserved quantity and the second non-conserved. The double 3-dimensionality in this arrangement, and the combination of conservation and nonconservation, produce a system by which the charges (strong, electric and weak) represented by the quaternions $\mathbf{i}, \mathbf{j}, \mathbf{k}$, produce charge-structures for particles independent of space and time [8-11,28,40]. Interactions can then be purely represented through charge-structures. The rotation of quark colours in the strong interaction could be explained by rotation of the strong charge, with a discrete symmetry replacing a continuous one.

The minimal charge structure is that of the neutrino, which only has weak charge, which is a universal component of all fermions. Quarks are identified by the presence of strong charge, leptons by its absence. However, the strong charge appears in three phases or colours, a kind of subunit of charge. Charge is conserved separately in all three types. Particle types can only be changed by weak interactions, which involve simultaneous annihilations and creations of fermions and antifermions. There is no such thing as a direct excitation of a particle from vacuum, or a direct transition from one particle to another, unless it involves weak sign changes. (This property of the weak interaction in changing particle types and not being responsive to direct excitation is why it has massive boson generators and, alone among the fundamental interactions, is non-Abelian.) All other authors, including Günyadin and Gürsey, Dixon and Furey, show no knowledge of this fundamental basis and are obliged to work by reverse engineering from the presumed octonionic structure to find evidence of $SU(3)$, $SU(2)$ and $U(1)$ symmetries.

Based on many earlier works, the accompanying paper [47] shows how the charge structures have been used to define particles, independently of space or time variables, by tables and by formula, and how the electric charge values become divisible by 3 because of the perfect gauge invariance of the strong interaction. The basis of the Standard Model groups, particles and interactions, are already apparent in the tables which originally emerged in the 1970s. The formula (a single one for all fermions and antifermions) uses vector and quaternion units (from the $Cl(6)$ algebra) acting on each other to generate 1 and 0 units for each type of charge as quantum numbers [28,40]. In addition to the charge structures, there are also nilpotent wavefunctions for particles in every state, which are unique to those states, and can be derived from the charge structures, so calculations can be performed for physical interactions.

The 12 pentad structures in the table of 64 $Cl(6)$ algebraic units above have been used to generate the *degrees of freedom* for the 48 Standard Model fermions and antifermions in at least two different ways, first by using the pentads as the particle states, with three separate (rotating) dimensions for the $\mathbf{p}$ term (quarks), or using a combined (nonrotating) $\mathbf{p}$ term (leptons), and secondly by using each of the terms as a particle, with the 60 terms dividing into 48 fermions and 12 bosons [28,32,33,40]. It is clear from various presentations that $\mathbf{i}, \mathbf{j}, \mathbf{k}$ represent colour and their absence (or their absence of separate units) the absence of colour; $\mathbf{i}, \mathbf{j}, \mathbf{k}$ in the form of CPT provide the distinguishers of generations; while $+$ and $-$ signify particles and antiparticles (because this is equivalent to conjugation of the quaternion units that all the particles require); finally, a version of complexification / noncomplexification determines weak isospin (showing that this is not a simple transition like that of colour). There are different ways to achieve these representations but one of the simplest is to define a fermion term as one containing a quaternion of some kind. We should note that mathematical degrees of freedom are not identical to particle structures, but both are available. In particular, operators that switch representations do not immediately switch particles from one state to another.

The process of obtaining 48 fermions and 12 bosons from the 12 pentads has also been extended, using two additional degrees of freedom (for left- and right-handed spin and vacuum, as well as real fermion, states) to encompass the 240 root vectors of E_8 , the final structure derived from the octonion symmetries in the Freudenthal-Tits magic square [28,32,33,40]. None of these developments comes from attempting to break down octonions or larger structures such as $R \otimes C \otimes H \otimes O$. They come from the understanding that charge is a fundamental parameter, whose intrinsic mathematical structure is derived alongside those of mass, time and space.

3. The octonion models of Furey and Dixon

Furey is motivated by trying to find an algebra in which particle characteristics do not need to be described in space and time. PR does this using charge structures, which can express interactions without using dynamical concepts, but Furey does not have the foundational concept of charge, and so opts for ‘energetically causal sets’. For Furey, particles are idempotent, minimum left ideals. For an idempotent, $A^2 = A$, where A and $1 - A$ are zero divisors. The self-interaction of a particle gives itself. Particles must be identical to exist. While $A = 1$ is a trivial example of an idempotent, nontrivial examples would include the projection operators for the chiral weak interaction $A = \frac{1}{2}(1 + \gamma_5)$ and $1 - A = \frac{1}{2}(1 - \gamma_5)$. An ideal B is a set of elements b : $b \in B$. B is a subset of a set A : $B \subset A$. $\forall a \in A, a*b \in B$. By contrast, PR makes the main representation of particles as nilpotent, with the idempotent a subsidiary one connected with vacuum. Here, $A^2 = 0$, the self-interaction of a particle vanishes, and particle must be in a unique state to exist. While $A = 0$ is a trivial example of a nilpotent (as well as idempotent), a nontrivial one would be $(ikE + \mathbf{i}p_x + \mathbf{j}p_y + \mathbf{k}ip_y \pm jm)^2 = E^2 - p^2 - m^2 = 0$.

Furey suggests that the particles of the Standard Model behave like ideals of the algebra $A = R \times C \times H \times O$. This is Dixon’s T , but she uses a complicated argument for reducing it to $Cl(6)$. Using the octonion units e_1 to e_7 in complexified form, an equivalent to the 64-component table in section 2 might be something like this:

1	e_7				-1	$-e_7$			
e_1e_3	e_1e_5	e_1e_6	ie_6e_7	ie_5	$-e_1e_3$	$-e_1e_5$	$-e_1e_6$	$-ie_6e_7$	$-ie_5$
e_2e_3	e_2e_5	e_2e_6	ie_3e_7	ie_6	$-e_2e_3$	$-e_2e_5$	$-e_2e_6$	$-ie_3e_7$	$-ie_6$
e_4e_3	e_4e_5	e_4e_6	ie_5e_7	ie_3	$-e_4e_3$	$-e_4e_5$	$-e_4e_6$	$-ie_5e_7$	$-ie_3$
$e_1e_3e_7$	$e_1e_5e_7$	$e_1e_6e_7$	ie_4e_7	ie_2	$-e_1e_3e_7$	$-e_1e_5e_7$	$-e_1e_6e_7$	$-ie_4e_7$	$-ie_2$
$e_2e_3e_7$	$e_2e_5e_7$	$e_2e_6e_7$	ie_1e_7	ie_4	$-e_2e_3e_7$	$-e_2e_5e_7$	$-e_2e_6e_7$	$-ie_1e_7$	$-ie_4$
$e_4e_3e_7$	$e_4e_5e_7$	$e_4e_6e_7$	ie_2e_7	ie_1	$-e_4e_3e_7$	$-e_4e_5e_7$	$-e_4e_6e_7$	$-ie_2e_7$	$-ie_1$

In the complexified form, one unit (here e_7) effectively loses its hypercomplex status, being reduced to the status of i . (There are a number of ways in which the seventh unit in the complexified left-multiplied octonions can be seen to lose this status. Dixon, for instance, shows how to get to $Cl(6) \simeq Cl(7,0)$ down to $Cl(3,3)$, which is another form of $Cl(6, 0)$, using the split octonions [48]).

In her thesis, Furey sees no inherent reason why $Cl(6)$ should be chosen from the infinite number of Clifford algebra alternatives to be the algebra behind the Standard Model. ‘Finally, one might ask, if we are moving to a Clifford algebraic description of octonionic multiplication via $Cl(6)$ anyway, why not just start with $Cl(6)$ in the first place? The answer to this question is two-fold. First of all, in starting only from Clifford algebras, one would be hard-pressed to know which Clifford algebras to choose. That is, an infinite number of Clifford algebras exist, and there appears to be no reason to choose one over any other.’ As PR’s work has shown, however, $Cl(6)$ is special as the fundamental algebra of mass, time, charge and space, the Klein-4 group at the heart of physics. It is also, and crucially for the Standard Model, the algebra of the Dirac equation, the quantum mechanical equation appropriate to all fermions.

Her argument continues: ‘Secondly, with an octonionic description of $Cl(6)$, we will be able to map particles into anti-particles, and vice versa, using only the complex conjugate, $i \rightarrow -i$. This is typically not the case when Clifford algebras are expressed as matrices with complex components, as was already shown in the $C \otimes H \simeq Cl(2)$ case ... for left- and right-handed Weyl spinors’ [6]. The attempt to explain the chirality of the weak interaction using the left- and right-multiplication aspects of octonions was a crucial aspect of their use by Dixon, and there is an element of this in Furey’s writing, but PR views chirality (which is built into the Dirac equation) as due to the loss of a sign degree of freedom when the nilpotent structure is created (three $\pm$ signs reducing to two $\pm$ and one $+$). In addition, in PR’s work, the antistates require the conjugates of the quaternion operators used for the weak, strong and electric charges.

While Dixon sees Clifford algebras as products of matrix operators and column / row vectors, Furey sees the Clifford algebras as a self-interacting algebra A acting on itself. Their elements are just pure algebraic objects, not matrices. For Dixon, the adjoint octonion algebra O_L is an associative matrix algebra. For Furey, left- or right-sided octonions form chains of products or maps that are associative. To all intents and purposes, they are the same as Dixon's adjoint octonion algebra O_L . For both authors, octonion anti-associativity ensures that they can never be treated as matrices only as pure algebraic objects.

Dixon and Furey divide up T in crucially different ways. For Dixon, $R \otimes C \otimes H \otimes O = P \otimes O$, where the Pauli algebra, $P = R \otimes C \otimes H$. As with Günyadin and Gürsey, he finds within O the origins of the $SU(3) \otimes SU(2) \otimes U(1)$ symmetry of the Standard Model. Furey divides T into $R \otimes C \otimes H \otimes O = (R \otimes C \otimes H) \otimes (R \otimes C \otimes O)$. She is motivated by her formulation of particle structures as energetic causal sets and this removes the space-time aspect – in effect she factorizes the Dixon algebra T into two separate factors $C \otimes H$ and $C \otimes O_L$. The first algebra can be separated out as a Pauli space-time algebra, which is not needed if particles are believed to be described by energetic causal sets. The other algebra $C \otimes O_L$ turns out to be equivalent to $Cl(6)$ which is a 64 part algebra which, as we have seen, she seemingly fails to recognise is the algebra of the Dirac equation. Since the number of fermion states is 48, not 64, this means a removal of a further 16 units, as belonging purely to space-time. No justification is given for this double reduction.

To bypass the problematic property of the anti-associativity of the octonions O , Furey reformulates Dixon's left-octonionic algebra O_L , which is not a division algebra (unlike the octonions), possesses nontrivial zero divisors, and most importantly is associative. She uses the fact that one-sided multiplication of octonionic imaginary units multiplied by each other in specific orderings can produce maps between octonionic imaginary units that are associative.

Furey's idempotent states are excitations from vacuum, where neutrinos are the lowest vacuum state (presumably electron neutrinos). The vacuum thus represents the neutrino and not the zero-particle state; other quarks and leptons are excitation states of this neutrino vacuum state. The idempotents are eigenvalues of raising and lowering a number operator from the eigenvalues of the algebra. The process, which is partly based on Dixon's earlier work, can be compared with PR's more comprehensive algebraic generation of the quantum numbers for weak, strong and electric charges. Furey quantizes the electric charge of all fermions by dividing the number operator by 3. There is no obvious reason for this value, other than yielding the 'correct' answers, and the arbitrariness may be significant in itself [16,17, 28,40], but, as the number operator must take on integer values, this is said to be a derivation of the quantization of electric charge. The values for the number operators divided by 3 gives the electric charge structure for the fermions in a single generation. It is notable that this is only for the *electric* charge, though in a single generation this is distinctive for each particle, and is notably 0 for the neutrino, which is probably why Furey identifies this as a vacuum state.

Furey's theory concerns only Standard Model group representations. It does not represent real particles and is unable to calculate scattering amplitudes. This is not the case with PR's work, where wavefunctions and amplitudes for all fermions and bosons are explicitly represented, derived in principle from the charge structures, and where scattering amplitudes and propagators and other aspects of particle interactions are directly calculated [28,40,47]. Furey sees her minimum left ideal idempotents as subspaces of an algebra which represent the particles, but her ladder operators lifting them out of the neutrino vacuum state do not always represent real interactions. The model requires extra information to connect the algebraic operations to physical ones. Without the concept of weak, strong and electric charge units, the number structures cannot be converted into amplitudes or wavefunctions.

The two $SU(3)$'s which Furey sees in the algebra, following aspects previously pointed out by Dixon and other authors, are taken to stand for the colour symmetry of the strong force and the three generations. These are, of course, also present in all forms of PR's theory, as vectors and quaternions, though there the symmetry of the quaternions is a broken one, like that of the three generations, whereas the symmetry of the vectors is unbroken, like that of the colour interaction. The tables in this theory also contain two complete $SU(3)$'s and an incomplete one, but, once the octonion is split by complexification,

the emergence of two 3-D structures within it becomes inevitable, as well as a fragment that would have contributed to a third. Furey's model of $SU(2)$ weak isospin is somewhat less well achieved as the subtitle 'Towards weak isospin' in her most recent paper [7] might indicate and resorts to *right* multiplication and additional algebraic structures. The paper also discusses the origins of $SU(2)_L \otimes U(1)$, and $SU(5)$ in terms of Grand Unification, but these considerations go beyond identifying particle states. They can be compared with those of PR in [28] and [40] and related papers, which include the precise calculation of certain key parameters. Overall, it is clear that several of Furey's specific procedures reflect those of Dixon and other earlier authors, but it is the *restriction* to $Cl(6)$ that brings it closer to an explanation of the Standard Model, for Dixon's more inclusive algebra makes it difficult to justify excluding possibilities that are in the algebra but don't happen to coincide with the physical evidence. All the theories are weakest where algebraic procedures are selected to reproduce experimental results rather than because the mathematical structure uniquely demands it.

4. Conclusion

Despite the claims of a number of authors, octonions are not used directly in physics. All uses of 'octonions' refer to left- or right-multiplied restricted versions, which have different properties to pure octonions and are not division algebras, although, unlike octonions, they are associative and have group multiplication properties. Using O_L or O_R , though it preserves group properties, defeats the object of invoking O as a division algebra, as Dixon requires. Though she uses Dixon's $T = R \otimes C \otimes H \otimes O$, Furey is less concerned with the intrinsic significance of division algebras and more concerned with a pragmatic realisation of the Standard Model. The key, as she sees it, is to find that the algebra for the Standard Model is more like $C \otimes O_L$ than Dixon's O_L , and that this can be achieved by removing $C \otimes H$ from Dixon's T through the concept that particles might be energetically causal sets (though this is nowhere demonstrated). Furey's theory doesn't preserve octonions any more than Dixon's and her octonionic chain multiplication is just another way of removing inconvenient octonionic properties using left-multiplied octonions. It just happens that her $C \otimes O_L$ is isomorphic to $Cl(6)$, which is the key algebra for the Standard Model, as previously realised by PR. It is also, as the algebra of mass, time, charge and space, the fundamental algebra for the whole of physics.

For PR, $Cl(6)$ may be realised in a complexified left- or right-octonionic form to keep the 8-dimensional structure of the 4 fundamental parameters, but, because the 8 is composed of units with diverse algebraic characteristics, based on smaller groupings of units, this is a broken symmetry, such as is often found in the higher groupings in Nature. The Standard Model is full of diverse groupings which can only be seen as a totality in a broken symmetry. In general, broken symmetries do not emerge from some 'symmetry breaking' mechanism, but from the creation of larger umbrella groupings in which the separate elements are diverse but composed of more perfect symmetries. In this case, the significance of octonions lies in the way they lead up to the larger groupings, such as E_8 , which can extend from the fermions to include bosons and vacuum states.

In view of the fact that mathematical development often appears to follow physical imperatives, it may be that the deep significance of a mathematics using 8 units for the creation of physics from a fundamental basis, and the significance of complex numbers and quaternions in making integers possible [28,40], might be the driver for the creation of a mathematical structure which can only be accessed in Nature in a broken form. According to PR, writing in 2009: 'The symmetries between the parameters show that all the properties can be conjugated in one cycle of the algebra, which seemingly imparts a special meaning to an algebra of 8 units (1, i , j , k , i , j , k , i , j , k), as is apparent in the mathematics. The algebras of the parameters are each subalgebras of a single Clifford algebra of this dimension, but because these subalgebras are commutative, the combination becomes equivalent to a double Clifford algebra, which, in mathematical terms, seems to allow the creation of an 8-component cycle on which it can map. Mathematics does this by creating another property, antiassociativity, which allows us to define an 8-unit algebra, octonions, on which we can map the units of mass, time, charge and space, giving the impression that this

mathematical structure arises from a physical imperative. Significantly, if we retain the identities of the parameters in the mapping, the antiassociative aspects remain in the areas which have no physical meaning' [34].

A final consideration is that the $R \otimes C \otimes H \otimes O_L$ algebra has 256 units. If we restore the terms removed by Furey in creating $C \otimes O_L$, we might follow the logic of removing that part of the algebra that is only concerned with space, time and mass, and not with particles of any description (the 16 units of $R \otimes C \otimes H$). We are then left with 240 terms, the E_8 root vector algebra seemingly containing fermions and antifermions in both spin states, and bosons, together with their vacuum realisations [32,33,40]. (Of course, this is also a broken symmetry, like that of the complexified left-multiplied octonions, and is only a projection onto the exact group known to mathematics, and not in any way a statement that the structure 'really' is E_8 any more than the Standard Model symmetry is octonion.) The algebra is also equivalent to the one identified by Dixon as $Cl(9, 1)$, a larger copy of the $Cl(3, 1)$ Pauli algebra of 3-D space [2], which is close to the concept of 10-dimensional string theory and equivalent also to a triple space. (The addition of the scalar unit 1, suggestive of the mass term, would extend the 'dimensions' to 11, adding 3 'operational' units to the 8 of mass-time-charge-space.) Perhaps Nature has found a significance for $R \otimes C \otimes H \otimes O_L$, though it is different from the one originally expected.

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