The Hillipop likelihood and the $A_{\rm L}$ parameter using Planck data

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The consistency of the cosmological data with the ΛCDM concordance model can be assessed through the estimation of the $A_{\rm L}$ parameter. We will show that a 2.6σ discrepancy with the expected unity value is measured using the profile likelihood method applied to the Planck public likelihoods. $A_{\rm L}$ is highly correlated to the reionisation optical depth τ and one also observes a $\simeq 2\sigma$ discrepancy between the high-ℓ and the low-ℓ Planck results on this parameter. We address those points using Hillipop, a high-ℓ likelihood, built from temperature Planck data. It makes use of foreground templates derived from the Planck data themselves, and has the interesting feature to point to a lower τ value than the official one, fully in agreement with the low-ℓ likelihood results. While the $A_{\rm L}$ value is still high using Planck data alone, we have combined Hillipop with the ACT and SPT data to further constrain the foreground nuisance parameters. We will show that this combination permits to recover an $A_{\rm L}$ value in agreement with one, keeping a τ value fully compatible between all the considered datasets and to get a coherent picture of the ΛCDM scenario. This work is further detailed in [1].

1 Appetizer: Profile likelihoods

The wide-spread approach used in Cosmology for parameter estimation is based on Bayesian inference using Monte Carlo Markov chains algorithms to explore the likelihood function. In contrast, in Particle Physics, the frequentist profile likelihood method is used in most cases[a].

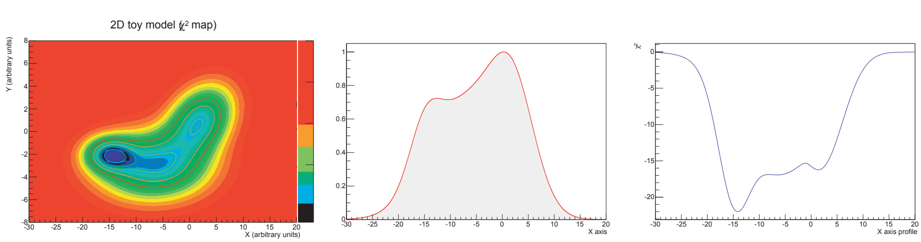

Figure 1 – χ^2 distribution in the (X, Y) plane (left), and corresponding posterior distribution (middle), and profile likelihood for X (right).

Beyond philosophical matters, a potential difference between both approaches is the so-called volume effect. Consider a problem for which we want to infer two parameters (X and Y), which likelihood function $\mathcal{L}$, or more precisely the χ^2 defined as:

$$\chi^2 = -2 \ln \mathcal{L} \tag{1}$$

[a] One can also considered the Neyman CL construction as in [5].

is shown on figure 1, and present two minima (around $X \simeq -15$ and $X \simeq 2$), and a banana shape. If one wants to estimate the mean and error on X using the Bayesian method, one would build the marginal posterior such that:

$$P(X) = \int \mathcal{L}(X,Y)p(X,Y)dY \tag{2}$$

which is represented on the middle of the figure. On the other hand, the profile likelihood (on the right hand side), is simply built for each X value as the point in the Y dimension which maximises the likelihood function:

$$\mathcal{L}(\mathcal{X}) = \max_{\mathcal{Y}} \mathcal{L}(\mathcal{X},\mathcal{Y}) \ . \tag{3}$$

One can straigforwardly show, in this precise example, that, in the Bayesian case, the mean value of the posterior distribution (around $X \simeq 1$) is far from matching the one of the best fit (blueish zone in the 2D distribution corresponding to the points where the likelihood is maximum), while, by construction, the mean value deduced from the profile likelihood method is exactly degenerated with the best fit (as the higher likelihood area of the parameter space is the "most likely" to be the true one).

Hopefully for the Planck data and the standard ΛCDM cosmology, such case does not show up [3] [2,4], but when considering extensions to ΛCDM one should be aware that such situations may occur. To ensure that the physics results do not depend on the statistical method (or at least to understand the discrepancies if any), we have build a sofware in which both approaches can be tested in a unique framework (CAMEL) that can be found here: `camel.in2p3.fr`. It makes use of CLASS [6] as Boltzman solver.

2 Planck Likelihoods

We first remind the two Planck released likelihoods [2], which are considered in the following: lowTEB, the lowℓ temperature and polarisation map based likelihood, and Plik: a gaussian high-ℓ temperature likelihood. On top of Plik, to better constrain the SZ sector (thermal and kinetic), a constraint has been used, derived from ACT data which writes:

$$A^{SZ} = Z^{kSZ} + 1.6A^{tSZ} = 9.5\mu K^2 \tag{4}$$

with a dispersion of $3\mu K^2$.

Hillipop [2] is another high-ℓ Planck likelihood ($\ell > 50$) based on cross-spectra at 100, 143, 217 and 353GHz, with temperature data, which can be used as an alternative to Plik. The main differences between them occur from the choice we have made in the likelihood building: we use 15 cross-spectra from 6 maps [7], the intercalibration coefficients are defined at the map level, we use different masks to further reduce the contamination from the foregrounds and the parameterization of the foregrounds is based on templates derived from the Planck data themselves.

3 Parameter estimation

3.1 Hillipop combined with lowTEB

Table 1 shows a comparison of the ΛCDM parameters infered from Plik and Hillipop, using the lowTEB likelihood for the low-ℓ part.

The main differences that show up are for τ and A_s (not suprisingly since both are correlated through the fact that the measured C_ℓ spectrum scales as $\propto A_s e^{-2\tau}$). As the estimation of those parameters is mainly driven by the low-ℓ data, one may therefore wonder why they are so different.

Table 1: ΛCDM parameters estimated with the Plik+lowTEB likelihoods (middle column) and with Hillipop+lowTEB (last column).

ΛCDM parameter	Plik+lowTEB	Hillipop+lowTEB
$\Omega_b h^2$	0.02222 ± 0.00023	0.02221 ± 0.00023
$\Omega_c h^2$	0.1197 ± 0.0022	0.1192 ± 0.0022
H_0	67.31 ± 0.96	67.51 ± 0.97
τ	0.078 ± 0.019	0.072 ± 0.020
$\ln(10^{10} A_s)$	3.089 ± 0.036	3.068 ± 0.038
n_s	0.9655 ± 0.0062	0.9645 ± 0.0071
σ_8	0.829 ± 0.014	0.816 ± 0.015

The τ value derived from the lowTEB likelihood using the profile likelihood method reads:

$$\tau = 0.067^{+0.023}_{-0.021} \quad \text{lowTEB} \quad , \tag{5}$$

2.2σ away from the Plik only value:

$$\tau = 0.172^{+0.038}_{-0.042} \quad \text{Plik} \quad , \tag{6}$$

while we have with Hillipop $\tau = 0.134^{+0.038}_{-0.048}$, with similar errors than Plik, but a mean value closer to the lowTEB estimate (1.2σ). Having shown that Hillipop seems to be more coherent with the lowTEB likelihood, one can go one step further and open the ΛCDM parameter space to another parameter: $A_{\rm L}$.

3.2 $A_{\rm L}$

In Boltzman codes, weak lensing enters the prediction of the CMB spectrum through a convolution of the unlensed spectrum with the lensing potential power spectrum C_ℓ^Ψ, which, at first order, smoothes the peaks. A fudge factor $A_{\rm L}$ has been introduced [9 10], which scales C_ℓ^Ψ in the C_ℓ estimation. If its fitted value is 1, the weak lensing is well modeled and the data are in agreement with ΛCDM, on the other hand, if we found $A_L = 0$ this would lead to the conclusion that the data do prefer a theory where the weak lensing is ignored. When measuring $A_{\rm L}$ different from 1 indicates either a problem in the model, or remaining systematics in the data.

Results are summarized in table 2 for different configurations for Plik: the first line corresponds to the published result, while the second line shows the systematics linked to the MCMC and the Boltzman solver, finally a comparison between the last and second lines shows the impact of the profile analysis wrt the Bayesian one, pushing $A_{\rm L}$ to even higher values.

Table 2: $A_{\rm L}$ values obtained for different Boltzman solvers and/or statistical methods as stated in the last column for the Plik+lowTEB likelihoods.

$A_{\rm L} = 1.22 \pm 0.10$	Plik+lowTEB, camb/MCMC [8]
$A_{\rm L} = 1.24 \pm 0.10$	Plik+lowTEB, CLASS/MCMC
$A_{\rm L} = 1.26^{+0.11}_{-0.10}$	Plik+lowTEB, CLASS/profile

One can go through the same exercise with Hillipop, and one would get:

$$A_{\rm L} = 1.22^{+0.11}_{-0.10} \quad [\text{Hillipop} + \text{lowTEB}, \text{CLASS/profile}]. \tag{7}$$

This result is of the same order of magnitude than the Plik result, still with a lower τ value as shown before. It has to be noted also that, at this stage, one observes a correlation between $A_{\rm L}$ and A^{kSZ}. For this reason, instead of using an ad-hoc prior to mimic the very-high-ℓ ACT and SPT data (as in Eq. 4), we do prefer to directly use those data to further consider the full correlation matrix of cosmological and nuisance parameters.

3.3 Combining SPT and ACT data with Hillipop

The data considered here are the following ones: ACT [11], SPTHigh [12] and SPTLow [13], with the corresponding nuisance parameters (cf. [1] for more details). They are called VHL in the following. When extending the foreground templates up to those high-ℓ regions and proceeding with a profile likelihood fit, we end up with:

$$A_{\mathrm{L}} = 1.03 \pm 0.08 \quad \text{Hillipop} + \text{lowTEB} + \text{VHL} \tag{8}$$

fully compatible with 1, and passing successfully the test. This allows to get a coherent picture of ΛCDM which parameters are shown in table 3.

Table 3: Estimates of cosmological parameters using MCMC techniques for the six ΛCDM parameters, when combining lowTEB, Hillipop, and VHL likelihoods.

Parameter	Hillipop+lowTEB+VHL
$\Omega_b h^2$	0.02200 ± 0.00019
$\Omega_c h^2$	0.1200 ± 0.0020
$100\theta_{\mathrm{s}}$	1.04200 ± 0.00040
τ	0.059 ± 0.017
n_s	0.9630 ± 0.0054
$\ln(10^{10} A_s)$	3.045 ± 0.032
σ_8	0.811 ± 0.013

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