Creation of D9-brane–anti-D9-brane Pairs from Hagedorn Transition of Closed Strings – its application to cosmology

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Abstract

It is well known that one-loop free energy of closed strings diverges above the Hagedorn temperature. One explanation for this divergence is that a 'winding mode' in the Euclidean time direction becomes tachyonic above the Hagedorn temperature. The Hagedorn transition of closed strings has been proposed as a phase transition via condensation of this winding tachyon. But we have not known the stable minimum of the potential of this winding tachyon so far. On the other hand, we have previously calculated the finite temperature effective potential of open strings on D-brane–anti-D-brane pairs, and shown that a phase transition occurs near the Hagedorn temperature and D9-brane–anti-D9-brane pairs become stable. In this article, we present a conjecture that *D9-brane–anti-D9-brane pairs are created by the Hagedorn transition of closed strings*, and describe some circumstantial evidences. We also discuss its application to cosmology.

1 Hagedorn Transition of Closed Strings

Since the early days of string theory, it was observed that perturbative string gas has an interesting thermodynamic property. The string gas has a characteristic temperature called the Hagedorn temperature. We can compute the one-loop free energy of strings by using path integral in Matsubara method. The one-loop free energy of strings diverges above this temperature.

One explanation for this divergence is that a 'winding mode' in the Euclidean time direction becomes tachyonic above the Hagedorn temperature. Sathiapalan [1], Kogan [2] and Atick and Witten [3] have proposed the Hagedorn transition of closed strings via condensation of this winding tachyon. They advocated that the Hagedorn temperature is not really a limiting temperature but rather is associated with a phase transition. Atick and Witten argued further from the world sheet point of view. The insertion of the winding tachyon vertex operator means the creation of a tiny hole in the world sheet which wraps around the compactified Euclidean time. Thus, the addition of the winding tachyon vertex operator to the world sheet action induces the creation of a sea of such holes. At low temperature, sphere world sheet does not contribute to the free energy, since it cannot wrap the compactified Euclidean time. But if we consider the condensation of winding tachyon above the Hagedorn temperature, the sphere world sheet is no longer simply connected and it contributes to the free energy above the Hagedorn temperature. It should be noted that these modes can be interpreted as winding tachyon only in the Matsubara formalism, namely, if we perform the Wick rotation of the time direction and compactified it with period β. We cannot identify which modes condensate to what extent in Lorentzian time when this winding tachyon condensates in the Euclidean time.

Significant effort has been devoted to find out the stable minimum of the potential of this winding tachyon. But we have not known the stable minimum yet. It is difficult to compute the potential of closed string tachyon because this potential has to be calculated by closed string field theory and this theory has not been well-established.

2 Brane–anti-brane Pairs at Finite Temperature

We have previously discussed the behavior of brane-antibrane pairs at finite temperature in the constant tachyon background [4]. At zero temperature, the spectrum of open strings on these unstable branes

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contains a tachyon field T. In the brane–antibrane configuration, we have $T = 0$, and the potential of this tachyon field has a local maximum at $T = 0$. If we assume that the tachyon potential has a non-trivial minimum, it is hypothesized that the tachyon falls into it. Sen conjectured that the potential height of the tachyon potential exactly cancels the tension of the original brane-antibrane pairs [5]. This implies that these unstable brane systems disappear at the end of the tachyon condensation.

Although brane-antibrane pairs are unstable at zero temperature, there are the cases that they become stable at finite temperature. We have calculated the finite temperature effective potential of open strings on these branes based on boundary string field theory. For the D9-brane–$\overline{\text{D9}}$-brane pairs, a phase transition occurs at slightly below the Hagedorn temperature and the D9-$\overline{\text{D9}}$ pairs become stable above this temperature. On the other hand, for the Dp-brane–$\overline{\text{D}p}$-brane pairs with $p \leq 8$, such a phase transition does not occur. We thus concluded that not a lower dimensional brane-antibrane pairs but D9-$\overline{\text{D9}}$ pairs are created near the Hagedorn temperature. Let us call this phase transition brane-antibrane pair creation transition. Although we only describe the case of brane-antibrane pairs, almost the same argument holds for the case of non-BPS D-branes. This work is generalized to the case that Dp-brane and $\overline{\text{D}p}$-brane are separated by Calò and Thomas [6].

3 Creation of D9-brane–$\overline{\text{D9}}$-brane Pairs from Hagedorn Transition of Closed Strings

Let us consider the relationship between above two phase transitions. Let us return to the argument of Atick and Witten about the meaning of the condensation of the winding tachyon [3]. The insertion of the winding tachyon vertex operator corresponds to the creation of a tiny hole in the world sheet which wraps around the compactified Euclidean time, and the condensation of winding tachyon induces an infinite number of tiny holes in the world sheet. But what is the hole of closed string world sheet? Let us try to think about it from a different point of view. If we identify the boundary of a hole created by winding tachyon vertex operator with a boundary of open string on a D9-$\overline{\text{D9}}$ pair, the insertion of winding tachyon vertex operator means the insertion of the boundary of open strings, which wraps the compactified Euclidean time once, in the tiny hole limit. Then the sphere world sheet which is no longer simply connected is naturally reinterpreted as higher-loop open string world sheet. If we enlarge the size of this hole, we can describe open strings with arbitrary boundary. Therefore, we present a following conjecture :

D9-brane–$\overline{\text{D9}}$-brane pairs are created by the Hagedorn transition of closed strings.

That is, above two phase transitions are two aspects of one phase transition. In the sense that $T = 0$ is the perturbative vacuum of open strings, this is a phase transition from closed string vacuum to open string vacuum. In other words, the stable minimum of the Hagedorn transition is the open string vacuum.

4 Circumstantial Evidences

Here we describe some circumstantial evidences for this conjecture. First, if we consider the thermodynamic balance on D9–$\overline{\text{D9}}$ pairs, we can show that energy flows from closed strings to open strings and open strings dominate the total energy. This is because we can reach the Hagedorn temperature for closed strings by supplying finite energy, while we need infinite energy to reach the Hagedorn temperature for open strings on these branes. This implies that, as the temperature increases, the creation of D9–$\overline{\text{D9}}$ pairs begins before closed strings are highly excited.

Secondly, one-loop free energy of open strings on a D9-$\overline{\text{D9}}$ pair approaches to the propagator of winding tachyon in the closed string vacuum limit, as is sketched in Fig. 1. This is an example that we can identify the closed string sphere world sheet with winding tachyon vertex operators with the open string world sheet in the closed string vacuum limit. Atick and Witten consider only closed string vacuum and looking for stable minimum in winding tachyon space. But it is reasonable to look for the stable minimum in all the open string tachyon space.

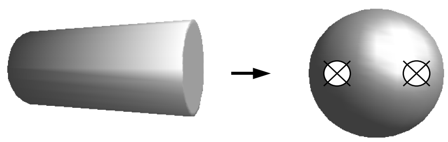

Figure 1: The cylinder world sheet (left) approaches to the sphere world sheet with two winding tachyon vertices insertion (right).

Thirdly, we can show that the finite temperature effective potential at the open string vacuum becomes the global minimum in entire space of the open string tachyon field near the Hagedorn temperature.

In the case of a D9-$\overline{\text{D9}}$ pair, the potential energy at $T = 0$ is given by

$$V \simeq -\frac{2v_9}{\pi {\beta_H}^9 (\beta - \beta_H)}, \tag{1}$$

where β_H is the inverse of the Hagedorn temperature and v_9 is the 9-dimensional volume of the system that we are considering. From this we can see that this potential energy decreases limitlessly as the temperature approaches to the Hagedorn temperature. It is natural to think that the open string vacuum becomes the global minimum near the Hagedorn temperature. This is the property that the stable minimum of the Hagedorn transition is expected to have.

5 Application to Cosmology

The spacetime-filling branes are very advantageous in the sense that all the lower-dimensional D-branes in type II string theory are realized as topological defects through tachyon condensation from non-BPS D9-branes and D9-$\overline{\text{D9}}$ pairs. We can identify the topological charge as the Ramond-Ramond charge of the resulting D-branes. These D-brane charges can be classified using K-theory [7]. Thus, if non-BPS D9-branes exist in the early universe, various kinds of branes may form through tachyon condensation [8]. It would be interesting to examine the possibility that our Brane World forms as a topological defect in a cosmological context. We have studied the homogeneous and isotropic tachyon condensation as a first step towards 'Brane World Formation Scenario'. In this article, we describe only the simplest case. For other cases, see Ref. [9].

The low energy effective action for tree level closed strings is described by type IIA supergravity. For simplicity, we shall focus on 10-dimensional metric $g_{\mu\nu}$, and set the other fields to zero or some constants. Then the action is given by

$$S_E = -\frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\;\mathcal{R}. \tag{2}$$

We must also consider the action for non-BPS D9-branes. For simplicity, we only deal with the zero temperature case. The BSFT action for a linear tachyon profile in the flat spacetime is derived in Ref. [10]. Let us focus on tachyon T in the open string spectrum, as well as graviton $g_{\mu\nu}$ in the closed string one, and assume that the action in the curved spacetime is given by

$$S_T = \mu\int d^{10}x\sqrt{-g}\;e^{-\alpha T^2}\mathcal{F}\left(\lambda\nabla_\mu T\nabla^\mu T\right), \tag{3}$$

where α, λ and μ are constants, and

$$\mathcal{F}(z) = \sqrt{\pi}\,\Gamma(z+1)\left\{\Gamma\left(z+\frac{1}{2}\right)\right\}^{-1}. \tag{4}$$

The total action we consider is the sum of (2) and (3). We assume that the universe is homogeneous and isotropic, and that the spatial curvature is flat. In order to perform the numerical calculation,

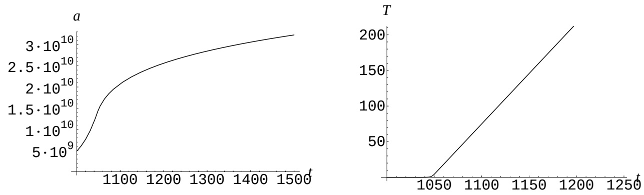

Figure 2: The time evolution of the scale factor $a(t)$ (left) and the tachyon field $T(t)$ (right) in the case of Einstein gravity coupled to tachyon field. We have set $a(0) = 1$. The initial condition for the tachyon is $T = 10^{-10}$ and $\dot{T} = 0$ at $t = 1000$. We choose such a small initial value of T in order to show the inflation phase exists before the decelerated expansion phase.

we must choose an initial condition. It is expected that, even if non-BPS D9-branes are stable near the Hagedorn temperature initially, they become unstable because the energy density decreases as the universe expands. Then the tachyon starts to roll down from the local maximum of the potential at $T = 0$. When the tachyon remains at $T = 0$, the solution for the equations of motion is de Sitter solution. Thus, it is reasonable to choose the initial condition which is close to the de Sitter solution. We calculate the numerical solution as is depicted in Fig. 2. From this we can see that the tachyon asymptotically approaches to a linear function of t. Sugimoto and Terashima have pointed out that $T \to t/\sqrt{\lambda} + \text{const.}$ as $t \to \infty$, and that it is related to tachyon matter [11]. This comes from the divergence of $\mathcal{F}(z)$ and its derivative at $z = -1$. As we can see from Fig. 2, the scale factor asymptotically approaches to a constant as $t \to \infty$. This is because the energy density of the tachyon field asymptotically approaches to zero and we are considering the case that the spatial curvature is zero. The inflation phase continues for a long time if we choose small initial value of T.

Acknowledgments

The author would like to thank the organizers of workshop for giving him an opportunity of participation.

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