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Light-Front Methods and Non-Perturbative QCD *

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1 Introduction

In this talk I will briefly discuss some features of light-front quantization methods for nonperturbative QCD in comparison with traditional lattice methods. Some of the novel features and new directions are illustrated in the transparency file: http://www.slac.stanford.edu/grp/th/lectures/BNLlattice.pdf.

A central focus of non-perturbative light-front methods in QCD is the set of light-front Fock state wavefunctions $\psi^H_n(x_i, k_{i,\perp,}\lambda_i)$, which represent a hadron in terms of its quark and gluon degrees of freedom [1]. Here $x_i = \frac{k^+_i}{P^+} = \frac{k^0_i+k^z_i}{P^0+P^z}$ is the boost-invariant light-cone momentum fraction with $\sum_{i=1}^n x_i = 1$. The λ_i represent values of the spin-projections S^z_i of the constituents. In principle, the light-front wavefunctions of hadrons can be computed by diagonalizing the QCD Hamiltonian H_{LF} quantized at fixed $\tau = t + z/c$ in light-cone gauge $A^+ = 0$ [2]. The set of ψ^H_n wavefunctions are then obtained as the projections of the hadron's eigenstate on the $n-$ particle Fock states. Remarkably, the light-front wavefunctions are frame independent; *i.e.*, independent of the hadron's total momentum P^+ and $P_\perp$. Given these amplitudes, one can calculate many hadronic processes from first principles. The sum of squares of the light-front wavefunctions give the quark and gluon distributions $q_H(x, Q)$ and $g_H(x, Q)$ at resolution Q, including all spin measures and correlations, as well as the unintegrated distributions $q_H(x, k_\perp)$ and $g_H(x, k_\perp)$ controlling inclusive reactions. Form factors and exclusive weak decay matrix elements have exact representations. Similarly, the deeply virtual Compton amplitude $\gamma p \to \gamma p'$ can be expressed as overlap integrals $n = n', n = n'+2$ of the initial and final light-front proton wavefunctions [3]. The hadron distribution amplitudes $\phi^H(x_i, Q)$ which control hard exclusive processes, including exclusive B decays, are the transverse momentum integrals of the lowest particle number valence Fock state wavefunction [4].

The light-front quantization of QCD can be carried out with rigor using the Dirac method to impose the light-cone gauge constraint and eliminate dependent degrees of freedom [5]. Unlike the case in equal time quantization, the vacuum remains trivial. One can verify the QCD Ward identities for the physical light-cone gauge and compute the QCD β function. Recently, Srivastava and I [6] have extended the light-front quantization procedure to the Standard Model. The spontaneous symmetry breaking of the gauge symmetry is due to a zero mode of the scalar field rather than vacuum breaking. The Goldstone component of the scalar field provides mass to the $W^\pm$ and Z^0 gauge bosons as well as completing its longitudinal polarization. The resulting theory is free of Faddeev-Popov ghosts and is unitary and renormalizable.

The light-front Hamiltonian has been diagonalized for a number of $1+1$ theories including QCD(1+1) [7] and supersymmetric theories [8] using the discretized light-cone quantization (DLCQ) method, which discretizes the momentum variables $k^+_i = \frac{2\pi}{L}n_i, P^+ = \frac{2\pi}{L}K$ with $\sum n_i = K$ truncates the Fock space, while retaining the essential Lorentz symmetries of the theory. The continuum limit is approached as the harmonic resolution $K \to \infty$ Model $3+1$ theories are also being solved using DLCQ and Pauli-

Villars fields as regulators [9]. A new Pauli-Villars regularization method for QCD has been developed by Franke *et al.* [10].

Thus light-front quantization of QCD can provide a rigorous alternative to lattice methods. It provides nonperturbative solutions to the hadronic bound state and continuum solutions to the spectrum and the corresponding wavefunctions in Minkowski space. There are no fermion-doubling problems or finite-size effects. However, the diagonalization of the LF Hamiltonian is computationally challenging for QCD, an area which could greatly benefit from the expertise and computer technology of the lattice community. A possible alternative is to use variational methods to minimize the expectation value of the light-front Hamiltonian. The trial wavefunction can be constrained by noting that the numerator structure of the individual Fock state wavefunctions are largely determined by the relative orbital angular momentum and J^z conservation. Ladder relations relate Fock states differing by one or two gluon quanta [11]. Other alternative methods have been developed including the transverse lattice which combines DLCQ(1+1) with a lattice in transverse space [12, 13, 14].

The light-front partition function, summed over exponentially-weighted light-front energies, has simple boost properties which may be useful for studies in heavy ion collisions [15].

One of the important tests of Lorentz invariance is the vanishing of the anomalous gravitomagnetic moment $B(0)$ for any spin-half system; *i.e.*, the ratio of the spin precession frequency of a particle to its Larmor frequency is exactly 2 if the magnetic field is replaced by a gravitational field. This is a consequence of the equivalence theorem of general relativity [16]. The $B(q^2)$ form factor is defined from the spin-flip matrix element of the energy momentum tensor. Hwang, Ma, Schmidt and I have shown that $\sum_i^n B_i(0) = 0$, Fock state by Fock state, in the light-front representation [17]. It is an important challenge to lattice gauge theory to verify $B(0) = 0$ for a proton, since any nonzero result will provide a diagnostic of finite lattice size and other systematic errors.

The light-front method also suggests the possibility of developing an "event amplitude generator" by calculating amplitudes for specific parton spins using light-front time-ordered perturbation theory [18]. The positivity of the k^+ light-front momenta greatly constrains the number of contributing light-front time orderings. The renormalized amplitude can be obtained diagram by diagram by using the "alternating denominator" method [19] which automatically subtracts the relevant counterterm. The resulting amplitude can be convoluted with the light-front wavefunctions to simulate hadronization and hadron matrix elements.

Recently, Hoyer, Marchal, Peigne, and Sannino and I [20] have challenged the common view that structure functions measured in deep inelastic lepton scattering are determined by the probability of finding quarks and gluons in the target. We show that this is not correct in gauge theory. Gluon exchange between the fast, outgoing partons and target spectators, which is usually assumed to be an irrelevant gauge artifact, affects the leading twist structure functions in a profound way.

This observation removes the apparent contradiction between the projectile (eikonal) and target (parton model) views of diffractive and small $x_{\rm Bjorken}$ phenomena. The diffractive scattering of the fast outgoing quarks on spectators in the target in turn causes shadowing in the DIS cross section. Thus the depletion of the nuclear structure functions is not intrinsic to the wave function of the nucleus, but is a coherent effect arising from the destructive interference of diffractive channels induced by final state interactions. This is consistent with the Glauber-Gribov interpretation of shadowing as a rescattering effect.

It is an interesting question whether the moments of structure functions obtained from lattice gauge theory can account for the nuclear shadowing phenomenon, considering that shadowing depends in detail on the phase structure of diffractive and deep inelastic scattering amplitudes.

Recent measurements from the HERMES and SMC collaborations show a remarkably large azimuthal single-spin asymmetries A_{UL} and A_{UT} of the proton in semi-inclusive pion leptoproduction $\gamma^*(q)p \to \pi X$. Recently, Dae Sung Hwang and Ivan Schmidt and I [21] have shown that final-state interactions from gluon exchange between the outgoing quark and the target spectator system lead to single-spin asymmetries in deep inelastic lepton-proton scattering at leading twist in perturbative QCD; *i.e.*, the rescattering corrections are not power-law suppressed at large photon virtuality Q^2 at fixed x_{bj}. The existence of such single-spin asymmetries requires a phase difference between two amplitudes coupling the proton target with $J^z_p = \pm\frac{1}{2}$ to the same final-state, the same amplitudes which are necessary to produce a nonzero proton anomalous magnetic moment. We show that the exchange of gauge particles between the outgoing quark and the proton spectators produces a Coulomb-like complex phase which depends on the angular momentum L^z of the proton's constituents and is thus distinct for different proton spin amplitudes. The single-spin asymmetry which arises from such final-state interactions does not factorize into a product of distribution function and fragmentation function, and it is not related to the transversity distribution $\delta q(x, Q)$ which correlates transversely polarized quarks with the spin of the transversely polarized target nucleon. These effects highlight the unexpected importance of final and initial state interactions in QCD observables.

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