Some features and implications of exponential gravitation

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Abstract. The paper starts with a brief recapitulation of exponential gravitation concepts, and then continue by presenting the following items: a comparison of exponential gravitation with general relativity; gravitational radiation in exponential gravitation theory; supernova formation in the frame of exponential gravitation; a possible exponential gravitation explanation to the origin of jets of Active Galactic Nuclei, quasars and microquasars.

1. Introduction

This work presents and discusses features and implications of exponential gravitation. The theory of exponential gravitation can be found in refs [1-19], although under various names.

Einstein's energy equation

$$E = mc^2 \tag{1}$$

implies that in relativity, a potential energy ΔE of a body, increases its mass by a quantity

$$\Delta m = \Delta E/c^2 \tag{2}$$

In special relativity the energy E includes all sorts of energies. Yet, in Einstein's *general* relativity, gravitational energy is not included in E.

The section: "Physical basis of finiteness of self-energy when gravitation is included" of the paper by Arnowitt, Deser and Misner [20] (ADM) deals with a variation where the gravitational energy is a part of E too.

One may consider inclusion of gravitational energy in E by two approaches:

(i) E includes all sorts of energy, including the gravitational energy

$$E_g = -\frac{mMG}{r} \tag{3}$$

where r is the distance of the particle with mass m from another body with mass M, which leads to

$$m(r) = \frac{E(r)}{c^2} = m_\infty - \frac{Gm(r)M}{c^2 r} \tag{4}$$

In (4) at infinite distance r the mass m becomes m_∞ as it should.

Solving (4) for $m(r)$ while taking the mass M as independent of r, results in the mass $m(r)$ of the particle m:

$$m(r) = \frac{m_\infty}{1 + GM/c^2 r} \tag{5}$$

and its energy E

$$E(r) = \frac{m_\infty c^2}{1 + GM/c^2 r} \tag{6}$$

Formula (5) was derived by Ben-Amots [6 ,21].
Note that the *rest mass m* turns out to be *variable*, depending on the distance to another body, with which it interacts gravitationally, and also on the mass of that body.

(ii) *Another approach* to include the gravitational energy is by finding the energy $E(r) = m(r)c^2$ whose derivative in respect with its distance r from a central much larger mass M, gives the gravitational force:

$$\frac{dE}{dr} = F = -\frac{m(r)MG}{r^2} \tag{7}$$

so that

$$\frac{d[E(r)]}{dr} = c^2 \frac{d[m(r)]}{dr} = -F(r) = \frac{Gm(r)\ M}{r^2} \tag{8}$$

The force $F(r)$ is defined as the force acting on the test particle of mass $m(r)$ in the gravitational field of the central mass M, and is equal to the derivative of the energy $E(r)$ of the test particle with respect to the distance r of the test particle from the central mass. The mass M of the central body is considered as constant in the present approximation of this theory.
Solving the equation $c^2 \frac{d}{dr}[m(r)] = Gm(r)M/r^2$ extracted from (8) for M independent of r (or equivalently solving Majerník's [14], eq. (10) below), the variation of the rest mass m with r results in:

$$m(r) = m_\infty \exp\left(\frac{-GM}{c^2 r}\right) \tag{9}$$

which is graphically represented in Figure 1.
Hereafter this second method is named *exponential gravitation*, suggested by the exponential character of the solution (9).
The corresponding gravitational force is calculated below in Eq. (20) and analyzed.
The exponential gravitation (under various names, and with different proofs) was analyzed and explained in detail by many authors, starting with Milne [1] in 1948 and followed for example by Yilmaz [2, 3], Majerník [4-5], Ben-Amots [6], Kiesslinger [7], Vankov [8-11], Turanyanin [12], Ibison [13] and recently by Majerník [14-15], Vankov [16-18] and Walker [19].
One can find further details of derivation of (8) above for example in Kiesslinger [7] or Ben-Amots [6], or equivalently in Majerník [14] who defines:

$$m_\infty = m(r) + 1/c^2 \int_\infty^r F(x)dx \tag{10}$$

where $F(x)$ is the gravitational force according to Newton's gravitation law. A different independent derivation using relativistic redshift is given by Walker [19].
While the mass of the small particle m changes in the gravitational field of the central mass M, the central mass M is also changed in the gravitational field of the small particle m. This effect gives for two *equal* variable masses $M(r) = m(r)$ in Eq. (8) the equation:

$$\frac{d[m(r)]}{d(r/2)} c^2 = \frac{G[m(r)]^2}{r^2} \tag{11}$$

Note that r at the right hand side of (11) is the *distance* between the two masses; yet at the left hand side of (11) the *movement* of the mass is toward (or away from) the mass center, which is located in this case at *half of the distance* r between the two *equal* masses.
Solving (11) results in:

$$m(r) = \frac{m_\infty}{1 + G\,m_\infty/2c^2 r} \tag{12}$$

thus showing that while the distance r approaches zero, both equal masses diminish to zero mass and finally annihilate.

Both approaches do *not* give solutions of Einstein equations. Note that in general relativity and in Newtonian theory, the rest mass *is not* variable (is not dependent on the distance r.) Definition of the force F as the derivative of energy with respect to the distance r is used in Newtonian theory, and allowed here as quasistatic approximation.
Schwarzschild's line element [22] for mass point in vacuum is:

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 - \frac{dr^2}{1 - 2GM/c^2 r} - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 \tag{13}$$

Although both the first approach of getting variable mass $m(r)$ (5) and the exponential gravitation (9) do not assume any special geometry (except the Euclidean), if, only as a means of comparison with general relativity we were to *describe* a space giving the same gravitation force as (5) and (9) by line elements in a curved space as general relativity does, Ben-Amots obtained "pseudo" line elements for the first approach and for the exponential gravitation theory as follows:

(i) For the first approach where $m(r)$ is given by (5) and $E_g = -\frac{mMG}{r}$, Ben-Amots [21] derived:

$$ds^2 = \frac{c^2 dt^2}{1 + 2GM/rc^2} - \left(1 + \frac{2GM}{rc^2}\right) dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 \tag{14}$$

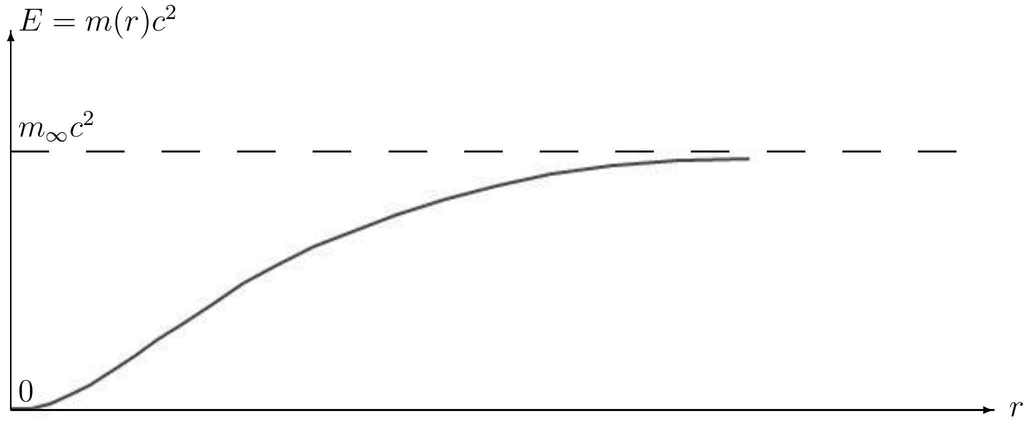

Figure 1: Energy=$m(r)c^2 = c^2 m_\infty \exp(-GM/c^2 r)$

Ben-Amots [21] also showed that (14) agrees with the present existing measurements of the three crucial tests of general relativity.

For this first approach we derive the formula for the gravitational force of the interaction between two bodies in the *quasistatic* approximation [21]:

$$\vec{F} = -\frac{dE}{dr} = -\frac{Gm_\infty M}{(r + GM/c^2)^2}\left[\frac{\vec{r}}{r}\right] \tag{15}$$

which was obtained as the derivative of the energy E of (6) in respect to r. The difference between (15) and the Newtonian gravitational force is too small to be measured with nowadays techniques.

(ii) For exponential gravitation where $m(r)$ is given by (9), Ben-Amots [6] and Majerník [14] derived the "pseudo" line element:

$$ds^2 = \exp\left(\frac{-2GM}{c^2r}\right)c^2dt^2 - \exp\left(\frac{2GM}{c^2r}\right)dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\phi^2 \tag{16}$$

The line element (16) was analyzed by Coleman [23] and Ben-Amots [6]. They showed also that the line element (16) agrees with the measurements of the three crucial tests of general relativity (see [23] and in the appendix of [6]).

In order to compare with observations, Schiff [24 ,25] analyzed the structure of the Schwarzschild's line element (13) by using a series expansion (written according to the notation here):

$$ds^2=\left[1+\alpha\frac{2GM}{c^2r}+\beta\left(\frac{2GM}{c^2r}\right)^2+...\right]c^2dt^2-\left[1+\gamma\frac{2GM}{c^2r}+\delta\left(\frac{2GM}{c^2r}\right)^2+...\right]dr^2-r^2(d\theta^2+\sin^2\theta d\phi^2) \tag{17}$$

Schiff concluded that higher terms in (17) were not subjected yet, at the time of writing [24, 25], to experimental/observational measurements. This is still correct nowadays. See Ben-Amots [21] and refs therein.

Applying on (17) the conditions that the field is spherically symmetric and that the spacetime is asymptotically flat (see, for example, Foster and Nightingale [26] and §2 below) gives

$$ds^2 = \left[1-\frac{2GM}{c^2r}+k\left(\frac{2GM}{c^2r}\right)^2+...\right]c^2dt^2 - \frac{dr^2}{1-\frac{2GM}{c^2r}+k\left(\frac{2GM}{c^2r}\right)^2+...} - r^2d\theta^2 - r^2\sin^2\theta d\phi^2 \tag{18}$$

Expanding (16) in Taylor series of $\frac{2GM}{c^2r}$ results in

$$ds^2 = \left[1-\frac{2GM}{c^2r}+\frac{1}{2}\left(\frac{2GM}{c^2r}\right)^2-\frac{1}{6}\left(\frac{2GM}{c^2r}\right)^3+...\right]c^2dt^2$$

$$-\frac{dr^2}{1-\frac{2GM}{c^2r}+\frac{1}{2}\left(\frac{2GM}{c^2r}\right)^2-\frac{1}{6}\left(\frac{2GM}{c^2r}\right)^3+...} - r^2d\theta^2 - r^2\sin^2\theta d\phi^2 \tag{19}$$

The differences between (13), (14) and (16) when expressed in the form (19) are in k and higher orders of $\frac{2GM}{c^2r}$ of (18):

- For Schwarzschild's line element (13), the coefficient k in (18) and the higher coefficients are equal to zero.
- For the pseudo line element (14), the coefficient k in (18) is $k = 1$, and higher coefficients are *not* equal to zero.

- For the exponential gravitation (16) and the pseudo line element (19), the coefficient k in (18) is $k = 1/2$, and higher coefficients are *not* equal to zero.

The difference between the solutions of exponential theory of gravitation (16), (19) and of general relativity (13) starts with the second order of $2GM/c^2r$. The experimental observations to date are not yet sufficiently accurate to distinguish between the second order terms of the solutions, as discussed by Ben-Amots [21] and refs therein, thus from the experimental point of view the question of which theory better describes the reality remains open.

Turyshev et al. [27], Maleki et al. [28], Maleki and Prestege [29] and Ben-Amots [21] discuss in detail different suggested experiments aimed to measure k in future.

For the Schwarzschild's solution (13), when $r < 2GM/c^2$, g_{00} becomes negative and g_{11} becomes positive, which give the solution the "black hole" properties (event horizon etc.).

In (16), not as for the Schwarzschild solution (13), g_{00} and g_{11} do not change signs (for any r). So, there is no black hole associated with (16) or equivalently with (9).

Substituting $m(r)$ of (9) in (7) gives the gravitational force $F(r)$ between $m(r)$ and M as:

$$F = -\frac{GMm(r)}{r^2} = -\frac{GMm_\infty}{r^2} \exp\left(\frac{-GM}{c^2 r}\right) \tag{20}$$

This force is equal to *zero* for $r = 0$, and approaches the Newtonian force $\frac{GM}{r^2}m_\infty$ as r increases to infinity (Figure 2).

For a central mass having finite dimensions in general relativity there exist Schwarzschild different exterior and interior solutions [30]. The interior solution is within a spherical body, and the different exterior solution is valid in the empty space around it. However, this exterior solution only describes the region exterior to the horizon of the "black hole."

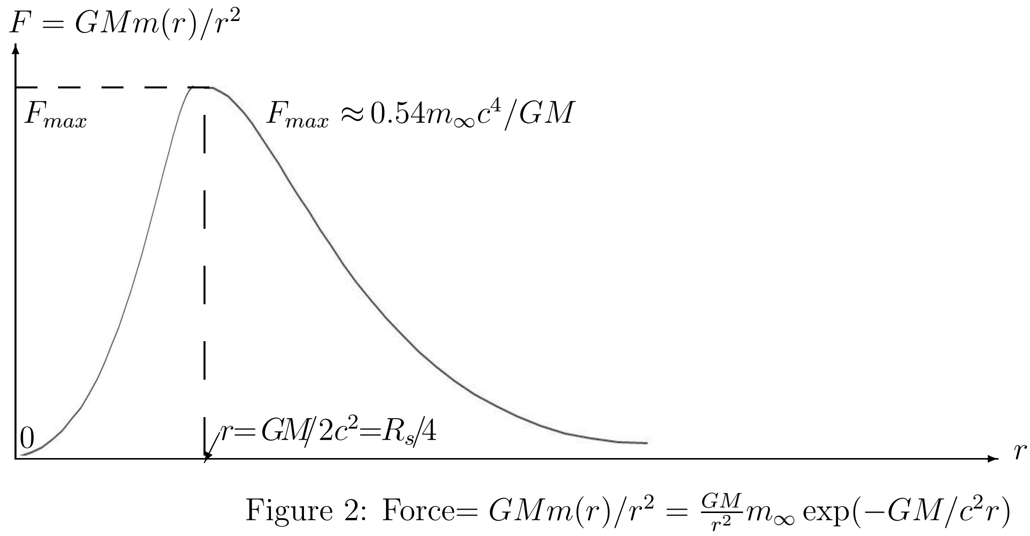

Figure 2: Force= $GMm(r)/r^2 = \frac{GM}{r^2}m_\infty \exp(-GM/c^2r)$

In Newtonian gravitation, the attraction force of a body of radius R is for the exterior case mMG/r^2 and for the interior case $mMGr/R^3$, with common value mMG/R^2 at radius $r = R$ (Figure 3). In the Newtonian solution there is a discontinuity of slope at the radius that separates between the Newtonian exterior ($r > R$) and the Newtonian interior ($r < R$) solution for the gravitational force of a spherical body (Figure 3).

While the Newtonian force is represented by *two* equations (exterior and interior, Figure 3), the force in the exponential gravitation theory has only *one* equation (Figure 2), *without the discontinuity of slope* that exists in the Newtonian force (Figure 3).

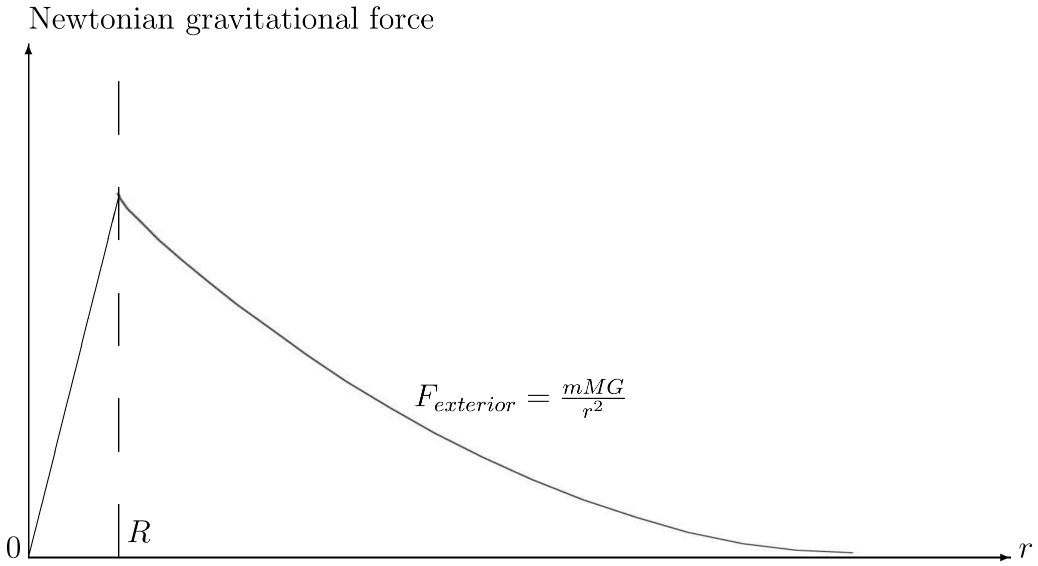

Figure 3: The Newtonian gravitational force (schematic) for a homogeneous spherical body of mass M and radius R: interior branch (left to $r = R$) and exterior branch (right). The force is zero at the center (r=0) and maximal at $r = R$

Expanding the exponential gravitation force (20) in Taylor series of $\frac{GM}{c^2 r}$ results in

$$F \approx -\frac{GMm_\infty}{r^2} + \frac{GMm_\infty}{r^2}\left(\frac{GM}{c^2 r}\right) - \frac{1}{2}\frac{GMm_\infty}{r^2}\left(\frac{GM}{c^2 r}\right)^2 + \frac{1}{6}\frac{GMm_\infty}{r^2}\left(\frac{GM}{c^2 r}\right)^3 - ... \tag{21}$$

where the first term is recognized as the well known Newtonian gravitational force for point mass.

It has been shown [6] that in the variable rest mass exponential gravity theory there cannot exist black holes, contrary to the possibility exhibited by Einstein's general relativity. §2 below returns to this point, while discussing Figure 2 and (23).

Ben-Amots [6, 21] detailed some implications of the first approach for $m(r)$ (where ΔE_g is $\Delta E_g = -\frac{mMG}{r}$. See above).

The present paper details other implications, of the *exponential gravitation* (the second approach).

It should be emphasized that the theoretical basis for the exponential gravitation was derived, analyzed and discussed in detail under various names by more than a dozen of papers [1-19] during more than 60 years. Although field equations were not given, yet any theory must be tested by experiments/observations. So far the two theories above and general relativity agree equally well with the experiments/observations.

2. Differences in basic assumptions between general relativity and exponential gravitation

First, this section mainly discusses Einstein's general relativity from a rather narrow point of view by focusing on the Schwarzschild [22] solution for Einstein's equations.

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Quoting Foster and Nightingale [26]:

> "Schwarzschild... guiding assumptions were:
> (a) that the field is static,
> (b) that the field is spherically symmetric,
> (c) that the spacetime *is empty*,
> (d) that the spacetime is asymptotically flat.
> ...Assumption (c) means that (the solution is) to be found using the empty-spacetime [Einstein's] field equations..."

Thus, assumption (c) above means that all the components of the Ricci Tensor in general relativity are null.

Still, assumption (c) of Schwarzschild is questionable since the space around a mass comprises the gravitational field of the mass, which itself is massive. (See for example Adler [31]). Thus assumption (c) affects both Schwarzschild exterior and interior solutions. The following discussion uses the fact that a gravitational field has a mass of its own, in contrast to the assumption (c) of Schwarzschild.

Also, the present author thinks that the conservation law

$$\mathrm{div}(\text{total stress energy tensor}) = 0 \tag{22}$$

(See for example Friedman [32]) can be too limiting, if the mass of the gravitational field is included in the energy. The conservation law (22) does not allow for gravitational waves to transfer energy in the first order, so that it limits the gravitational energy transfer in lowest order to quadrupolar waves. Another less limiting condition should apply instead (22).

If the claim presented here that (22) is violated because of the gravitational field being massive is correct, then bipolar gravitational waves should exist.

In the presented version of exponential gravitation theory, the gravitational radiation carries energy not only as a second order effect, but in the first order as well. This makes the mass transfer, that is the energy transfer by gravitational radiation, significant.

As stated in his lecture in Vienna in 1913, Einstein [33, 34] rejected the idea that the *rest mass* of a body can increase when its distance from another body increases. Instead, in Einstein's theory the *rest mass is constant*, while space is curved. (In the presented theory the rest mass is variable while space is Euclidean). The assumption of constant rest mass, is a constraint that Einstein imposed (firstly in his lecture in Vienna in 1913), which was used later while deriving his equations. The presented theory is free of this constraint. So, (9), which involves a *variable mass*, is not compatible with Einstein's equations. The presented solution (9) (for (8)) is close quantitatively to the solution of the Einstein's equations for weak gravitation fields (like those in the solar system, but not for neutron stars or other dense matter). For dense matter the presented solution is different from Einstein' equations solution (to be discussed below).

The exponential theory presented here complies with conditions (a), (b), (d) of Schwarzschild. Condition (d) states that the spacetime is asymptotically flat. (In the Birkhoff theorem the field should vanish at large distances, meaning here and in general relativity that the spacetime is asymptotically flat).

According to the theory presented here, of special importance is a region of the order of GM/c^2 close to any sufficiently dense mass (see Figure 2. Features of this figure are preserved for finite radius spherical mass distribution). This region has no discontinuities, in contrast to the black hole region of Einstein's general relativity. The physical behavior near the center of a mass (interior branch of the solution, $r < GM/2c^2$) differs from the behavior far from the mass (exterior branch of the solution), yet this behavior changes gradually and continuously in exponential gravitation theory. This continuous region replaces the sharp event horizon of the "black hole" of Einstein's general relativity. The black hole of Einstein's general relativity is characterized by the impossibility of any mass or radiation to escape from a region confined in a sharply defined event horizon. The equivalent region within the Schwarzschild radius in the theory here has different features. Electromagnetic radiation, gravitational radiation, and massive particles may escape this region, provided they have sufficient radial velocity to overcome the gravitational potential, according to the equation:

$$v > c\left[1 - \exp\left(\frac{-GM}{c^2 r}\right)\right]^{\frac{1}{2}} \tag{23}$$

While according to general relativity as currently accepted, the neutrinos are trapped when the radius of a celestial body is smaller than Schwarzchild's radius, in the theory presented here energy may escape, including neutrinos. In the future, observations on neutrinos from objects of mass and radius which by the general relativity should be black holes may determine which theory is valid.

It is reasonable to think that an adequate theory should not contain infinities, and the theory of exponential gravitation is without any infinity while Einstein's general relativity contains infinities at Schwarzschild radius and at $r = 0$.

In a broader context, the present author thinks that the equivalence principle is only an approximation that is valid for small accelerations and weak gravitational fields only.

Einstein described in ref. [35] how general relativity should be completed, and by what Born [36] described as following: "The idea is that the field produced by a body in turn reacts on the body and thus determines its world line. This is mathematically a very complicated problem the nature of which we cannot even indicate. Einstein, Infeld and Hoffmann [37] attacked it; their first investigations were so extended that only summaries could be published. Later Infeld [38] succeeded in simplifying the calculation considerably." See also Einstein and Infeld [39].

3. Gravitational radiation

3.1. Regenerated gravitational field

Van Flandern [40] asks:

> "... are the gravitational fields for a rigid stationary source frozen, or are they continuously regenerated?"

After analysis his conclusion is:

> "We conclude that the concept of frozen gravitational fields is acausal and paradoxical. Gravitational fields must continuously regenerate, like flowing waterfall. In doing so, they must consist of entities that propagate and transfer momentum."

We ask: Why doesn't the mass of the gravitational field collapse on the central mass? As already stated, the gravitational field has mass (Adler [31]), so, like any mass, it is attracted inward towards the direction of the central mass.

The answer to this question is reasoned as following: The central attraction creates an inward flux of mass of the gravitational field. The gravitational field fall should be balanced by radiation outward by the central mass with the energetic flux of field of the same mass as the falling field.

Gravitation is thus a superposition of the inward fall of massive gravitational field, and the outward radiation flow of energetic/massive gravitational field. So, some flux transfer should be allowed instead the limitation (22).

In conclusion to the above:

(i) A gravitational field that has mass, falls toward the gravitational source, implying that the source and this associated gravitational field emits continuously a counteracting gravitational field.
(ii) In addition, The present writer thinks that the conservation law (22) is too rigid, thus he allows the above mentioned gravitational waves of bipolar order, which may also be produced even by spherically symmetric sources of gravity.
(iii) To summarize the fundamental idea of exponential gravitation theory from which stems all the theory: The energy content of the gravitational potential that a body has, affects the mass of the body (as in Arnowitt et al. [20] (ADM)), just like the energy content of any other potential that the body has affects its mass.

3.2. Supernova formation

A supernova is usually assumed to be born in a bounceback of the inner layers of a star after their gravitational collapse (bounce usually thought to originate in neutrinos' pressure). This bounce back is supposed to meet the slower collapsing external layers of the star in a violent collision which ignites the fusion of the hydrogen in them and causes the explosion seen as supernova. Unfortunately spherically symmetric simulations are unable to reproduce this (Liebendörfer et al. [41]). These simulations lack sufficient outward pressure to produce the *explosion*; instead the simulations end with *implosion.*

The presented explanation of bounceback using exponential gravitation is as follows: The appropriate equivalent line element for (9) is [6]:

$$ds^2 = \exp\left(\frac{-2GM}{c^2 r}\right)c^2 dt^2 - \exp\left(\frac{2GM}{c^2 r}\right)dr^2 - r^2 d\theta^2 - r^2\sin^2\theta d\phi^2 \tag{16}$$

See Coleman [23] and Ben-Amots [6] for discussions of this line element.

Calculation of the stress-energy tensor component G_{00} (24), which is the *approximate* density of mass-energy gives (Figure 4):

$$G_{00} = -\frac{1}{r^2}\exp\left(\frac{-2GM}{c^2 r}\right)\left[1 - \left(1 + \frac{2GM}{c^2 r}\right)\exp\left(\frac{-2GM}{c^2 r}\right)\right] \tag{24}$$

(while for Schwarzschild Solution $G_{00} = 0$).

G_{00} (24) is negative, which represents attraction gravitational force (See Figure 4).

Below a certain radius when the radius approaches zero, the radial force becomes smaller and approaches zero. At radius $r = 0$ the force exerted on a test particle is zero.

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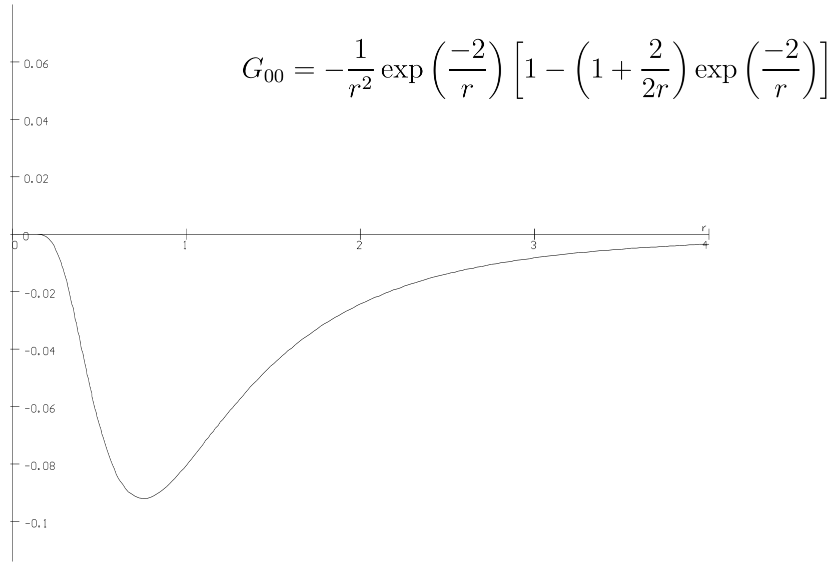
Figure 4: G_{00} (24) normalized: $G = c = M$=1



The gravitational collapse of celestial bodies compresses the matter to a very dense state. During compression the body will radiate field/mass outward with contribution to pressure given by the compressed matter. Note that at the same time the gravitational attraction decreases according to G_{00} in (24) in accord with Einstein's [42] statement that "matter cannot be concentrated arbitrarily."

This explanation based on the exponential gravitation even in the spherical symmetry context offers the above contribution to the additional pressure necessary for the bouncing outward stage which is lacking in Newtonian or general relativity models. The presented explanation will never lead to a *black hole* remnant star, because in the theory presented here such an object can never exist, and in addition a denser collapse creates an even higher outward pressure of gravitational radiation. Kiesslinger [7] arrived at this outward pressure of gravitational radiation too, but without applying to the supernova explosion.

In the presented theory, this mass/field radiation outward by the inner core during the implosion stage of a supernova, causes the core to expand outward. It stops the outer layers implosion inward, and redirects them to bounce outward. The falling outer layers collide with the expanding core and are bounced back outward. The sharply increased pressure caused by collision raises the temperature that ignites the fusion of the hydrogen and helium of the falling outer layers, and causes the final explosion of the supernova. Thus exponential gravitation theory enables a possible explanation to supernova explosions. See also §4.3 below.

Existing simulations of supernova do not consider the gravitational radiation force outward (as we suggest of being involved) so, most often the simulations of supernovas, especially spherically symmetric simulations, lead to a final implosion toward a "black hole," without bouncing outward [41], while observations imply bouncing outward[1].

General relativity or Newtonian existing model simulations fail to lead to explosion, implying that the helium and heavier elements created until the supernova formation are not dispersed in space. Introducing in the simulations the gravitational radiation force suggested by us could possibly produce the supernova explosion predicted by our model and others.

[1] Present simulations of supernovas that consider rotation either fail to reproduce observations, or fail to conserve angular momentum [41].

3.3. A possible qualitative explanation of how jets are emitted from some astronomical bodies

The approximate potential of exponential gravitation of a *non-rotating* spherically symmetric body versus a radius r, (or of a spherical body rotating along a diameter axis of rotation z) is schematically drawn in Figure 5.

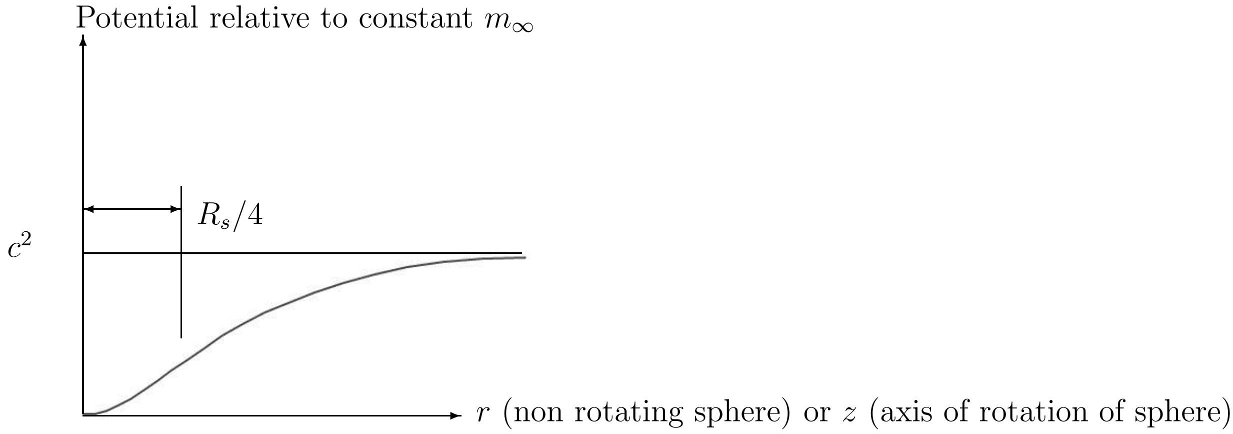


Figure 5: Schematic exponential gravitation potential along the radius r of a *non-rotating sphere*, or along the diameter z around which a spherical body rotates

The approximate potential of exponential gravitation of a *rotating* spherically symmetric body versus a radius in the equatorial plane is schematically drawn in Figure 6. This curve is for constant angular momentum along the radius. Constant angular momentum is maintained for $r > R_{max} > R_s/2$ (where R_{max} is the radius at which the potential is maximum). Continuing left of $r = R_{max}$, a constant angular momentum would bring to $v > c$.

This potential curve has a "bulge" (peaked at R_{max}) that is the contribution of centrifugal potential for exponential mass.

For very high angular velocity this "bulge" becomes a high peak that can be many orders of magnitude higher than the potential energy of a non-rotating body. Actually, this bulge is a potential barrier.

Substituting a central mass M of a typical galaxy nucleus of 200 million solar masses in G_{00} (24), the radius for the dip of G_{00} (Figure 4) results to be of the order of the distance of Mars from the Sun. Rotation causes the peak at R_{max} (Figure 6) to rise higher, and the radius for this peak to be smaller, depending on the angular velocity of the rotation. For more on this influence of rotation see below in Section 4.2.

Consider an explosive or another pressure creating event that increases the pressure in the core ($r < R_{max}$ in Figure 6) of a fast rotating very massive body. The particles very near the center lack sufficient angular momentum to cross the high potential barrier in direction perpendicular to the rotation axis, but they have two narrow outlets around the axis of rotation, in which the potential barrier is much lower and reaching a minimum of zero for the axis itself. If the particles are sufficiently energetic, they will eventually escape as two narrow jets through two narrow polar outlets.

Actually narrowly collimated jets are observed escaping along the rotation axis of active galactic nuclei and quasars on scales of hundreds of thousands of light years. There are similar smaller jets getting out of microquasars. The exponential gravitation presented here gives then this possible explanation of these jets (at this stage only a qualitative one). See also §4.2 below.

There are other suggested theories that try to explain the observed jets in other ways (for

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example, [44-48] and references therein), including explanations based on stellar magnetism and other phenomena.

4. Symmetry considerations for astronomical jets

4.1. Spherical symmetry

A test particle at the center of a spherically symmetric massive body of finite radius, is equally attracted to all directions. The resultant force of gravitational attraction acting on this particle is equal to zero. Figure 3 that shows this for Newtonian gravitational force at $r = 0$.

Figure 2 approximates the force along the radius r for exponential gravitation. This force has a left branch for $r < R_{max}$ representing the interior solution for the force. This interior force approaches zero when the radius r approaches zero. (When increasing the radius r, the force gradually approaches the Newtonian (exterior) solution $F = -mMG/r^2$ as shown in Figure 3).

For calculating the stress-energy G_{00} term for massive central body we use the mass (9) in Weinberg's [49] procedure for the line element. Slightly modifying Weinberg's procedure (Ben-Amots [6]) results in the pseudo line element (16). The stress-energy term G_{00} (24) (derived from (16)) is the normalized pressure, that is the normalized $\frac{\text{force}}{\text{area}}$, and incorporates the gravitational attraction force from *all directions*. The force diminishes gradually to zero near the center of the body. Thus this diminishing of the central force near the center just expresses spherical symmetry. (Compare to the Newtonian gravitational force for the spherically symmetric case described in Figure 3, which approaches zero at the center).

Schwarzschild exterior solution [22] is for a *mass point* surrounded by *empty space*, thus it lacks the diminishing of the attraction gravitational force toward zero, of the force in a massive body of finite radius. Tolman [30] correctly used *interior* solution (and not exterior solution) for the interior of bodies of *finite radius*. Einstein [42] concluded that "'Schwarzschild singularities' do not exist in physical reality," because "matter cannot be concentrated arbitrarily," but others [50], [51] later made manipulations on Schwarzschild exterior solution [22] to get insight on the solution near the center. See Thorne's review [52]. Yet using Schwarzschild exterior solution [22] for zero radius implies that the force at zero radius is infinite instead zero force as obtained in exponential gravitation.

Similarly, a point mass is surrounded by a gravitational field, that possesses energy and mass (Adler [31]), which is neglected in Schwarzschild solution. This mass of the surrounding gravitational field produces a zero resultant force at the center, because it attracts a test particle at the center symmetrically and equally from all directions. Schwarzschild solution ignores this, because of the assumption (c) (§2), that space is empty. So, the singularity and infinite values of Schwarzschild solution originates in *neglecting* this aspect of *spherical symmetry* (that the resultant force at the center is zero).

4.2. Examining symmetry for a rotating body

The exponential gravitation potential along any radius r of a spherical non-rotating body, or along the axis of rotation z passing through the center of a spherical rotating body, is shown in figure 5. For a rotating body, adding the potential of the centrifugal force along a radius perpendicular to the axis of rotation results in a centrifugal barrier as seen in figure 6. The faster the rotation is, the higher is the centrifugal barrier (Figure 7).

Still symmetry dictates a zero force component in the equatorial plane at the center.

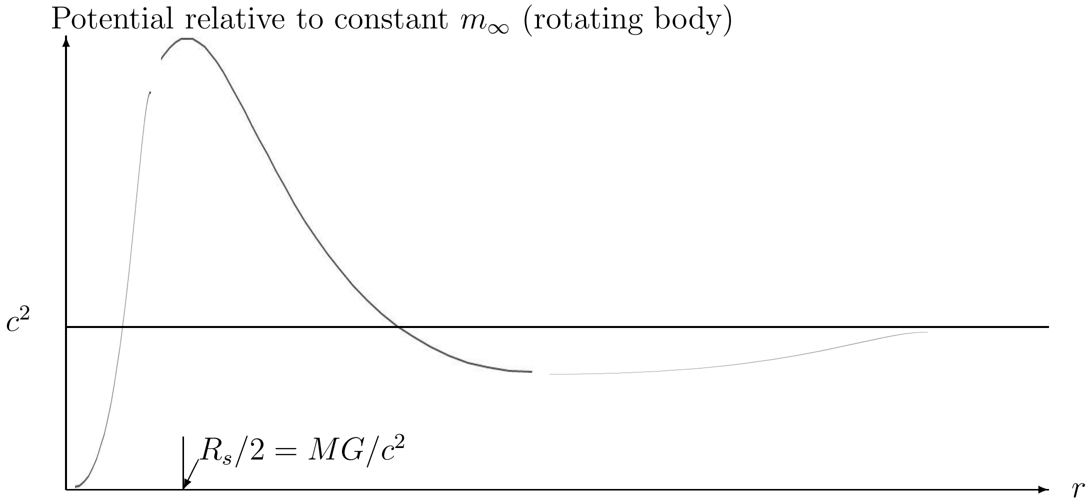

Figure 6: Schematic potential along a radius r in a section perpendicular to the axis of *rotation* passing through the center of a spherical body, where the angular momentum is constant for $r > R_{max}$. (For smaller radius the graph represents a solution for transient motion, because then there is no solution that conserves the angular momentum)

Examining the whole rotating body, centrifugal forces exist in any plane parallel to the equatorial plane, creating a potential barrier, and leaving a "tunnel" of low potential along the axis of rotation z (Figure 7).

A pressure caused by any event inside this "tunnel," cannot supply any massive particle sufficient angular momentum or the high energy necessary to overcome the high energy barrier in the radial direction. A pressure that cannot cause the explosion of the body, can be released by mass transfer only along the "tunnel" along the axis of rotation z, as two jets expelled from the poles.

In a way, the jets can be seen as caused by the symmetry of rotation, which allows for a centrifugal barrier only *around* (but not on) the rotation axis. So, this could be a possible cause that contributes to production of astrophysical jets in the rotation axis of celestial bodies.

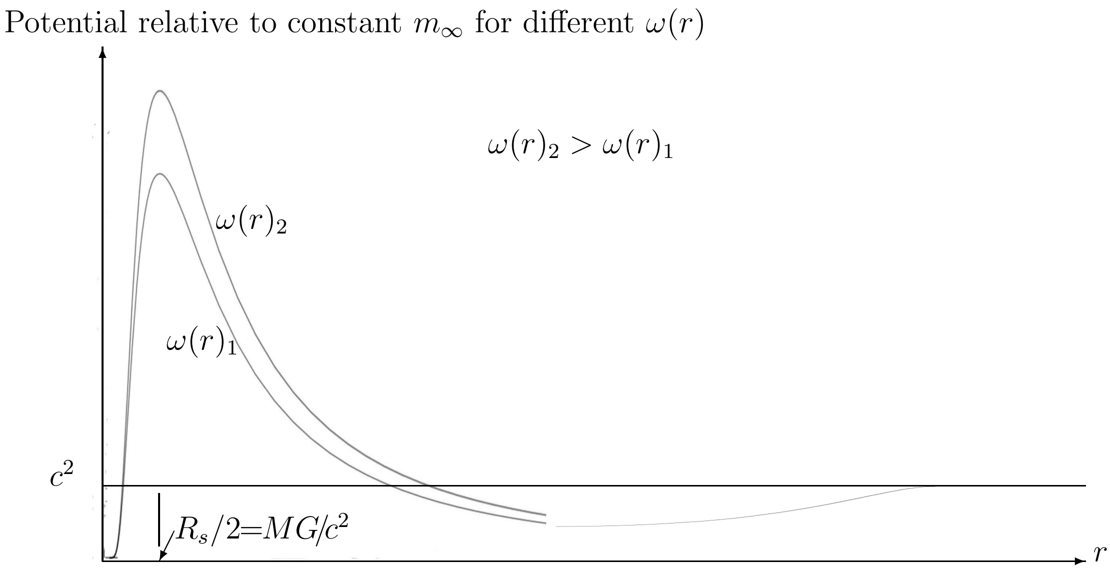

Figure 7: As in Figure 6 but the upper curve is for higher angular velocity of the body

4.3. The repelling force
The gravitational field of a spherically symmetric body has mass by itself and like any mass it is attracted inward towards the direction of the central mass. The central attraction creates an inward flux of mass of the gravitational field. Yet this cannot be the whole picture. The gravitational field fall should be balanced by radiation outward by the central mass with the energetic flux or field of the same mass as the falling field. The falling of the gravitational field, and the counteracting radiation outward by the central mass, have spherical symmetry.

Further, an implosion of a celestial spherical body of sufficiently large mass (which is a non-equilibrium event) compresses the inner core near the center, causing an increase of the gravitational radiation outward, and with it an increase of the *repelling* pressure of this radiation. The increase of the repelling pressure together with the pressure of the neutrinos, (created in the implosion stage of supernova formation), stop the implosion of upper layers and contributes to an eventual bounceback outward in the process of supernovae formation. This effect possibly contributes too to the explosion of supernovas.

5. Conclusions

The possibility to experimentally distinguish between the predictions of general relativity and exponential gravitation theories [27-29] may cause the implications of the exponential gravitation to be more significant for gravity research.

The above predictions of the exponential gravitation are solvable for two bodies, where the massive one is approximated as having constant rest mass M, and the rest mass $m(r)$ of the light body is variable and depending on the distance r between these two bodies.

Predictions based on assumption of two bodies whose both *equal* rest masses $m(r)$ are variable and depending on the distance r between these two bodies, are also obtained by the approach of exponential gravitation above, resulting in solution without exponential character, though.

Predictions based on assumption of two bodies whose both *arbitrary* rest masses $m(r)$ and $M(r)$ are variable and depending on the distance r between these two bodies, are obtained by the approach of Eq. (4) [6, 21].

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