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CP violation in the charm system

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Summary. — I review the implications of recent measurements of CP violation in D meson decays. The results are discussed in the context of the standard model (SM), as well as its extensions. The observed size of CP violation is not easily explained within the SM, although the required non-perurbative enhancements of the relevant hadronic matrix elements cannot be ruled out from first principles. On the other hand, using effective theory methods, one can derive significant constraints on the possible non-standard contributions from measurements of D^0-$\bar{D}^0$ mixing and CP violation in kaon decays (ϵ'/ϵ). Due to an approximate universality of CP violation in new physics scenarios which only break the $SU(3)_Q$ flavor symmetry of the SM, such contributions are particularly constrained by ϵ'/ϵ. Explanations of the observed effect within several explicit well-motivated new physics frameworks are briefly discussed. Finally I comment on possible future experimental tests able to distinguish standard *vs.* non-standard explanations of the observed CP violation in the charm sector.

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1. – Introduction

CP violation in charm provides a unique probe of New Physics (NP). Not only is it sensitive to NP in the up sector, in the Standard Model (SM) charm processes are dominated by two generation physics with no hard GIM breaking, and thus CP conserving to first approximation. Until very recently, the common lore was that "any signal for CP violation in charm would have to be due to NP". The argument was based on the fact the in the SM and in the heavy charm quark limit $m_c \gg \Lambda_{\rm QCD}$, CP violation in neutral D meson mixing enters at $\mathcal{O}(|\lambda_b/\lambda_s|) \sim 10^{-3}$ ($\lambda_q \equiv V_{cq}V_{uq}^*$), while CP-violating contributions to singly Cabibbo-suppressed D decays only appear at $\mathcal{O}(|\lambda_b/\lambda_s|\alpha_s(m_c)/\pi) \sim 10^{-4}$ [1].

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2. – CP violation in D^0-$\bar{D}^0$ mixing

Charm mixing arises from $|\Delta c| = 2$ interactions that generate off-diagonal terms in the mass matrix for D^0 and $\bar{D}^0$ mesons. The D^0-$\bar{D}^0$ transition amplitudes are defined as

$$\langle D^0|\mathcal{H}|\bar{D}^0\rangle = M_{12} - \frac{i}{2}\Gamma_{12}. \tag{1}$$

The three physical quantities related to the mixing can be defined as

$$y_{12} \equiv \frac{|\Gamma_{12}|}{\Gamma}, \qquad x_{12} \equiv 2\frac{|M_{12}|}{\Gamma}, \qquad \phi_{12} \equiv \arg\left(\frac{M_{12}}{\Gamma_{12}}\right), \tag{2}$$

where x_{12} and y_{12} are CP-conserving, while ϕ_{12} denotes the physical CP-violating mixing phase. HFAG has performed a fit to these theoretical quantities, (allowing also for CP violation in decays discussed below) using existing measurements, and obtained the following 95% CL regions [2]

$$\begin{aligned} &x_{12} \in [0.25,\, 0.99]\,\%, \qquad y_{12} \in [0.59,\, 0.99]\,\%, \\ &\phi_{12} \in [-7.1°,\, 15.8°]. \end{aligned} \tag{3}$$

The SM contributions to these quantities cannot be estimated reliably from first principles. On the other hand, short distance NP effects can be predicted and encoded in terms of an effective $|\Delta c| = 2$ Hamiltonian

$$\mathcal{H}^{\rm eff}_{|\Delta c|=2} = \frac{G_F}{\sqrt{2}} \sum_i C_i^{cu(\prime)} \mathcal{Q}_i^{cu(\prime)}, \tag{4}$$

where the definitions of the relevant operators $\mathcal{Q}_i^{cu(\prime)}$ can be found *i.e.* in [3]. Simply requiring such contributions to at most saturate the above experimental bounds on x_{12}, y_{12} and ϕ_{12} leads to very strong constraints on $C_i^{cu(\prime)}$ [4]. In particular, writing $\mathrm{Im}(C_i^{cu(\prime)}) = v_{\rm EW}^2/\Lambda_i^2$, constrains on CP-violating contributions to charm mixing in eq. (3) imply $\Lambda_i > 10^{3-4}$ TeV and are second in their strength only to the bounds on new contributions to ϵ_K.

3. – CP violation in D decays: Experiment *vs.* SM expectations

On the other hand, CP violation in neutral D meson decays to CP eigenstates f is probed with time-integrated CP asymmetries (a_f). These can arise from interferences between decay amplitudes with non-zero CP odd (ϕ_f) and even (δ_f) phase differences

$$a_f^{\rm dir} = -\frac{2r_f \sin\delta_f \sin\phi_f}{1 + 2r_f\cos\delta_f\cos\phi_f + r_f^2}, \tag{5}$$

where r_f is the absolute ratio of the two interfering amplitudes. Recently both the LHCb [5] and CDF [6] Collaborations reported evidence for a non-zero value of the

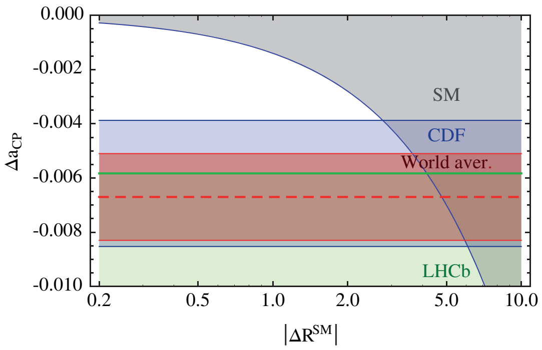

Fig. 1. – Comparison of the experimental Δa_{CP} values with the SM reach as a function of $|\Delta R^{\rm SM}|$. See text for details.

difference $\Delta a_{CP} \equiv a_{K^+K^-} - a_{\pi^+\pi^-}$. Combined with other measurements of these CP asymmetries [2], the present world average is

$$\Delta a_{CP} = -(0.67 \pm 0.16)\%. \tag{6}$$

This observation calls for a reexamination of theoretical expectations within the SM. The SM effective weak Hamiltonian relevant for hadronic singly Cabibbo-suppressed D decays, renormalized at a scale $m_c < \mu < m_b$ can be decomposed as [3]

$$\mathcal{H}^{\rm SM}_{|\Delta c|=1} = \frac{G_F}{\sqrt{2}} \sum_{q=s,d} \lambda_q \sum_{i=1,2} C_i^q \mathcal{Q}_i^q + \text{h.c.} + \dots, \tag{7}$$

where $\mathcal{Q}^q_{1,2} = [\bar{c}^\alpha \gamma_\mu (1-\gamma_5) q^{\alpha,\beta}][\bar{q}^\beta \gamma^\mu (1-\gamma_5) u^{\beta,\alpha}]$, α, β denote color indices, and the dots denote neglected penguin operators with tiny Wilson coefficients. Using CKM unitarity ($\sum_{q=d,s,b} \lambda_q = 0$), the corresponding $D^0 \to K^+K^-, \pi^+\pi^-$ decay amplitudes ($A_{K,\pi}$) can be written compactly as $A_{K,\pi} = \lambda_{s,d}(A_K^{s,d} - A_K^{d,s}) - \lambda_b A_K^{d,s}$. In the isospin limit the two different isospin amplitudes in the first term provide the necessary condition for non-zero $\delta_{K,\pi}$, while $\phi^{\rm SM}_{K,\pi} = \text{Arg}(\lambda_b/\lambda_{s,d}) \approx \pm 70°$. On the other hand $r_{K,\pi}$ are controlled by the CKM ratio $\xi = |\lambda_b/\lambda_s| \simeq |\lambda_b/\lambda_d| \approx 0.0007$. Parametrizing the remaining unknown hadronic amplitude ratios as $R^{\rm SM}_{K,\pi} \equiv -A^{d,s}_{K,\pi}/(A^{s,d}_{K,\pi} - A^{d,s}_{K,\pi})$, the SM contribution to Δa_{CP} can be written as

$$\Delta a_{CP} \approx (0.13\%)\, \text{Im}(\Delta R^{\rm SM}), \tag{8}$$

where $\Delta R^{\rm SM} = R^{\rm SM}_K + R^{\rm SM}_\pi$. Comparison of this estimate with current experimental results is shown in fig. 1. One observes that $|\,\text{Im}(\Delta R^{\rm SM})| = \mathcal{O}(2\text{–}5)$ is needed to reproduce the experimental results in eq. (6), in contrast to perturbative estimates in the heavy charm quark limit ($|R_{K,\pi}| \sim \alpha_s(m_c)/\pi \sim 0.1$) (see [1] and the more recent analyses in refs. [7]). However, ξ suppressed amplitudes in the numerator of R_i cannot be constrained by rate measurements alone, and it has been pointed out a long time ago that "$\Delta I = 1/2$ rule" type enhancements are possible [8] (see also [10]). Recently [9], an explicit estimate of potentially large $1/m_c$ suppressed contributions has been performed, yielding $\Delta a^{\rm SM}_{CP} \lesssim 0.4\%$. Although this is an order of magnitude above naïve expectations, the experimental value in eq. (6) cannot be reached.

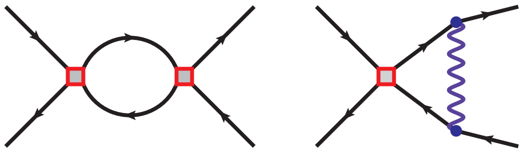

Fig. 2. – One-loop contributions of $\mathcal{H}^{\text{eff}-\text{NP}}_{|\Delta c|=1}$ (red square) to $|\Delta c| = 2$ and $|\Delta s| = 1$ operators. Weak mixing effects via a W (blue wavy line) exchange (right-hand-side diagram) and UV sensitive contributions, quadratic in $\mathcal{H}^{\text{eff}-\text{NP}}_{|\Delta c|=1}$ (left-hand-side diagram).

4. – Implications of Δa_{CP} for physics beyond SM

In the following we will therefore assume the SM does not saturate the experimental value, leaving room for potential NP contributions. These can again be parametrized in terms of an effective Hamiltonian valid below the W and top mass scales

$$\mathcal{H}^{\text{eff}-\text{NP}}_{|\Delta c|=1} = \frac{G_F}{\sqrt{2}} \sum_i C_i^{\text{NP}(\prime)} \mathcal{Q}_i^{(\prime)}, \tag{9}$$

where the relevant operators $\mathcal{Q}_i^{(\prime)}$ have been defined in [3]. Introducing also the NP hadronic amplitude ratios as $R^{\text{NP},i}_{K,\pi} \equiv G_F \langle K^+K^-, \pi^+\pi^- | \mathcal{Q}_i^{(\prime)} | D^0 \rangle / \sqrt{2}(A^{s,d}_{K,\pi} - A^{d,s}_{K,\pi})$ and writing $C_i^{\text{NP}} = v_{\text{EW}}^2/\Lambda^2$, the relevant NP scale Λ is given by

$$\frac{(10\,\text{TeV})^2}{\Lambda^2} = \frac{(0.61 \pm 0.17) - 0.12\,\text{Im}(\Delta R^{\text{SM}})}{\text{Im}(\Delta R^{\text{NP},i})}\,. \tag{10}$$

Comparing this estimate to the much higher effective scales probed by CP violating observables in D mixing and also in the kaon sector, one first needs to verify, if such large contributions can still be allowed by other flavor constraints. Within the effective theory approach, this can be estimated via so-called "weak mixing" of the effective operators (see fig. 2). In particular, time-ordered correlators of $\mathcal{H}^{\text{eff}-\text{NP}}_{|\Delta c|=1}$ with the SM effective weak Hamiltonian can, at the one weak loop order, induce important contributions to CP violation in both D meson mixing and kaon decays (ϵ'/ϵ). On the other hand, analogue correlators, quadratic in $\mathcal{H}^{\text{eff}-\text{NP}}_{|\Delta c|=1}$ turn out to be either chirally suppressed and thus negligible, or yield quadratically divergent contributions, which are thus highly sensitive to particular UV completions of the effective theory [3].

4·1. *Universality of CP violation in $\Delta F = 1$ processes.* – The strongest bounds can be derived for a particular class of operators, which transform non-trivially only under the $SU(3)_Q$ subgroup of the global SM quark flavor symmetry $\mathcal{G}_F = SU(3)_Q \times SU(3)_U \times SU(3)_D$, respected by the SM gauge interactions. In particular one can prove that their CP-violating contributions to $\Delta F = 1$ processes have to be approximately universal between the up and down sectors [11]. Within the SM one can identify two unique sources of $SU(3)_Q$ breaking given by $\mathcal{A}_u \equiv (Y_u Y_u^\dagger)_{\not{tr}}$ and $\mathcal{A}_d \equiv (Y_d Y_d^\dagger)_{\not{tr}}$, where $\not{tr}$ denotes the traceless part. Then in the two generation limit, one can construct a single source of CP violation, given by $J \equiv i[\mathcal{A}_u, \mathcal{A}_d]$ [12]. The crucial observation is that J is invariant under $SO(2)$ rotations between the $\mathcal{A}_u$ and $\mathcal{A}_d$ eigenbases. Introducing now $SU(2)_Q$ breaking NP effective operator contributions of the form $\mathcal{Q}_L = [(X_L)^{ij}\,\overline{Q}_i \gamma^\mu Q_j] L_\mu$, where L_μ

denotes a flavor singlet current, it follows that their CP-violating contributions have to be proportional to J and thus invariant under flavor rotations. The universality of CP violation induced by $\mathcal{Q}_L$ can be expressed explicitly as [11]

$$\mathrm{Im}(X_L^u)_{12} = \mathrm{Im}(X_L^d)_{12} \propto \mathrm{Tr}\,(X_L \cdot J)\,. \tag{11}$$

The above identity holds to a very good approximation even in the three-generation framework. In the SM, large values of $Y_{b,t}$ induce a $SU(3)/SU(2)$ flavor symmetry-breaking pattern [13] which allows to decompose X_L under the residual $SU(2)$ in a well-defined way. Finally, residual SM $SU(2)_Q$ breaking is necessarily suppressed by small mass ratios $m_{c,s}/m_{t,b}$, and small CKM mixing angles θ_{13} and θ_{23}.

The most relevant implication of eq. (11) is that it predicts a direct correspondence between $SU(3)_Q$-breaking NP contributions to Δa_{CP} and ϵ'/ϵ [11]. It follows immediately that stringent limits on possible NP contributions to the later, require $SU(3)_Q$-breaking contributions to the former to be below the per mile level (for $\Delta R^{\mathrm{NP},i} = \mathcal{O}(1)$).

As a corollary, one can show that within NP scenarios which only break $SU(3)_Q$, existing stringent experimental bounds on new contributions to CP-violating rare semileptonic kaon decays $K_L \to \pi^0(\nu\bar\nu, \ell^+\ell^-)$ put robust constraints on CP asymmetries of corresponding rare charm decays $D \to \pi(\nu\bar\nu, \ell^+\ell^-)$. In particular $|a_{\pi e^+e^-}^{\not{SU(3)_Q}}| \lesssim 2\%$ [11].

The viability of the remaining 4-quark operators in $\mathcal{H}^{\mathrm{eff-NP}}_{|\Delta c|=1}$ as explanations of the Δa_{CP} value in eq. (6), depends crucially on their flavor and chiral structure. In particular, operators involving purely right-handed quarks are unconstrained in the effective theory analysis but may be subject to severe constraints from their UV sensitive contributions to D mixing observables. On the other hand, QED and QCD dipole operators are at present only weakly constrained by nuclear EDMs and thus present the best candidates to address the Δa_{CP} puzzle [3].

5. – Explanations of Δa_{CP} within NP models

Since the announcement of the LHCb result, several prospective explanations of Δa_{CP} within various NP frameworks have appeared. In the following we briefly discuss Δa_{CP} within some of the well-motivated beyond SM contexts.

In the Minimal Supersymmetric SM (MSSM), the right size of the QCD dipole operator contributions can be generated with non-zero left-right up-type squark mixing contributions $(\delta_{12}^u)_{LR}$ [1, 14] (see fig. 3). Para- metrically such effects in Δa_{CP} can be written as [14]

$$|\Delta a_{CP}^{\mathrm{SUSY}}| \approx 0.6\% \left(\frac{|\,\mathrm{Im}(\delta_{12}^u)_{LR}|}{10^{-3}}\right)\left(\frac{\mathrm{TeV}}{\tilde m}\right), \tag{12}$$

where $\tilde m$ denotes a common squark and gluino mass scale. At the same time dangerous contributions to D mixing observables are chirally suppressed. It turns out however that even the apparently small $(\delta_{12}^u)_{LR}$ value required implies a highly nontrivial flavor structure of the UV theory, in particular large trilinear (A) terms and sizable mixing among the first two generation squarks (θ_{12}) are required [14]

$$\mathrm{Im}(\delta_{12}^u)_{LR} \approx \frac{\mathrm{Im}(A)\theta_{12} m_c}{\tilde m} \approx \left(\frac{\mathrm{Im}(A)}{3}\right)\left(\frac{\theta_{12}}{0.3}\right)\left(\frac{\mathrm{TeV}}{\tilde m}\right) 0.5 \times 10^{-3}.$$

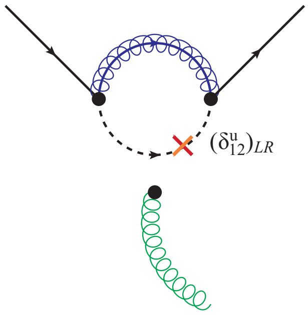

Fig. 3. – Sample one-loop squark (dashed black line)-gluino (combined straight and curly purple line) exchange diagram contributing to $|\Delta c| = 1$ QCD dipole operators in the MSSM. The (red) cross denotes an off-diagonal mass insertion ($(\delta_{12}^{u})_{LR}$). The gluon (curly green line) can be attached to any of the other (quark, squark, gluino) lines.

Similarly, warped extra dimensional models [15] that explain the quark spectrum through flavor anarchy [15, 16] can naturally give rise to QCD dipole contributions (see fig. 4) affecting Δa_{CP} as [17]

$$|\Delta a_{CP}^{\mathrm{RS}}| \approx 0.6\% \left(\frac{Y_5}{6}\right)^2 \left(\frac{3\,\mathrm{TeV}}{m_{KK}}\right)^2, \tag{13}$$

where m_{KK} is the KK scale and Y_5 is the 5D Yukawa coupling in appropriate units of the AdS curvature. Reproducing the experimental value of Δa_{CP} requires near-maximal 5D Yukawa coupling, close to its perturbative bound [18] of $4\pi/\sqrt{N_{KK}} \simeq 7$ for $N_{KK} = 3$ perturbative KK states. In term, this helps to suppress dangerous tree-level contributions to CP violation in D-$\bar{D}$ mixing [19]. This scenario can also be interpreted within the framework of partial compositeness in four dimensions, but generic composite models typically predict even larger contributions [20].

On the other hand, in the SM extension with a fourth family of chiral fermions Δa_{CP}

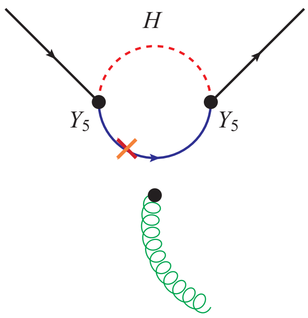

Fig. 4. – Sample one-loop Higgs (dashed red line)-(KK) quark (straight blue line) exchange diagram contributing to $|\Delta c| = 1$ QCD dipole operators in warped extra-dimensional (and similarly in partial compositeness) models. The (red) cross denotes a Dirac mass insertion. The gluon (curly green line) can be attached to any of the quark lines.

can be affected by 3×3 CKM nonunitarity and b' penguin operators

$$|\Delta a_{CP}^{\rm 4th\,gen}| \propto {\rm Im}\left(\frac{\lambda_{b'}}{\lambda_d - \lambda_s}\right). \tag{14}$$

However, due to the existing stringent constraints on the new CP-violating phases entering $\lambda_{b'}$ [21], only moderate effects comparable to the SM estimates are allowed [22].

6. – Prospects

Continuous progress in Lattice QCD methods (*c.f.* [23]) gives hope that ultimately the role of SM long distance dynamics in Δa_{CP} could be studied from first principles. In the meantime it is important to identify possible experimental tests able to distinguish standard *vs.* non-standard explanations of the observed value.

Explanations of Δa_{CP} via NP contributions to the QCD dipole operators generically predict sizable effects in radiative charm decays [24]. First, in most explicit NP models the short-distance contributions to QCD and EM dipoles are expected to be similar. Moreover, even assuming that only a non-vanishing QCD dipole is generated at some high scale, the mixing of the two operators under the QCD renormalization group implies comparable size of the two contributions at the charm scale. Unfortunately, the resulting effects in the rates of radiative $D \to X\gamma$ decays are typically more than two orders of magnitude below the long-distance dominated SM effects [17]. This suppression can be partly lifted when considering CP asymmetries in exclusive $D^0 \to P^+P^-\gamma$ transitions, where $M_{PP} = \sqrt{(p_{P^+} + p_{P^-})^2}$ is close to the ρ, ω, ϕ masses [24].

An alternative strategy makes use of (sum rules of) CP asymmetries in various hadronic D decays (necessarily including neutral mesons). It is effective in isolating possible non-standard contributions to Δa_{CP} if they are generated by effective operators with a $\Delta I = 3/2$ isospin structure [25] (which unfortunately does not include the QCD dipoles).

We note in passing that even though potential NP contributions to Δa_{CP} at short distances may respect U-spin (like the QCD dipole operators), the measured $D \to \pi\pi, K\pi, KK$ decay rates imply sizable flavor $SU(3)$ breaking due to final state long distance rescattering effects [7, 10]. Thus $a_{\pi^+\pi^-} \simeq -a_{K^+K^-}$ cannot be expected neither if the measured Δa_{CP} value is due to enhanced SM long-distance dynamics, nor if it is due to short-distance NP contributions.

Finally, correlations of non-standard contributions to Δa_{CP} with other CP-violating observables like electric dipole moments, rare top decays or down-quark phenomenology are potentially quite constraining but very NP model dependent [14, 26].

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