

# Higher Curvature Corrections to Primordial Fluctuations in Slow-roll Inflation

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## Abstract

We study higher curvature corrections to the scalar spectral index, the tensor spectral index, the tensor-to-scalar ratio, and the polarization of gravitational waves. We find that there are cases where the higher curvature corrections can not be negligible. It turns out that the tensor-to-scalar ratio could be enhanced and the tensor spectral index could be blue due to the Gauss-Bonnet term.

## 1 Introduction

According to the Wilkinson Microwave Anisotropy Probe (WMAP) data [1] and other observations, it seems plausible that the large scale structure of the universe stems from quantum fluctuations during the slow-roll inflation. Considering the accuracy of future observations, it would be worth to study the corrections due to higher curvature terms to the slow-roll inflation. To the leading order, the corrections come from the Gauss-Bonnet and the axion-like parity violating coupling terms [2]. In fact, the same type of corrections can be derived from a particular superstring theory [3]. Hence, we hope the understanding of such corrections to the inflationary scenario provides a clue to the study of superstring theory.

There have been many works concerning the Gauss-Bonnet and the parity violating corrections to the Einstein gravity [3, 4, 5]. However, since no one has systematically investigated both terms simultaneously in the context of the slow-roll inflation, the combined effect on the slow-roll inflation is not so clear. Hence, we study the slow-roll inflation with higher curvature corrections and derive the formula for the observables in terms of slow-roll parameters.

The organization of the paper is as follows: In section 2, we define slow-roll parameters in the inflationary scenario with higher curvature corrections. In section 3, we obtain the scalar and tensor spectral index and the tensor-to-scalar ratio. In section 4, the concrete example is presented as an illustration. The final section is devoted to the conclusion.

## 2 Slow-roll inflation

We consider the gravitational action with the Gauss-Bonnet and the parity violating terms

$$S = \frac{M_{\text{Pl}}^2}{2} \int d^4x \sqrt{-g} R - \int d^4x \sqrt{-g} \left[ \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi + V(\phi) \right] - \frac{1}{16} \int d^4x \sqrt{-g} \xi(\phi) R_{\text{GB}}^2 + \frac{1}{16} \int d^4x \sqrt{-g} \omega(\phi) R\tilde{R}, \quad (1)$$

where  $M_{\text{Pl}}^2 \equiv 1/8\pi G$  denotes the reduced Planck mass and  $g$  is the metric tensor. The Gauss-Bonnet term  $R_{\text{GB}}^2$  and the parity violating term  $R\tilde{R}$  are defined by

$$R_{\text{GB}}^2 \equiv R^\alpha{}_{\gamma\delta} R_\alpha{}^{\gamma\delta} - 4R^\alpha{}_{\alpha} R_\alpha{}^{\alpha} + R^2, \quad R\tilde{R} \equiv \frac{1}{2} \epsilon^{\alpha\gamma\delta} R_{\alpha\rho\sigma} R_{\gamma\delta}{}^{\rho\sigma}. \quad (2)$$

The inflaton field  $\phi$  has the potential  $V(\phi)$ . We have introduced coupling functions  $\xi(\phi)$  and  $\omega(\phi)$ . In principle, these functions should be calculated from the fundamental theory, such as superstring theory. Hence, it might be important to put constraints on these functions through observations.

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Let us take the flat Friedmann-Robertson-Walker(FRW) metric

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j, \quad (3)$$

where  $a$  is the scale factor. From the action (1), we obtain following background equations

$$3M_{\text{Pl}}^2 H^2 = \frac{1}{2}\dot{\phi}^2 + V + \frac{3}{2}H^3\dot{\xi}, \quad \ddot{\phi} + 3H\dot{\phi} + \frac{3}{2}H^2(\dot{H} + H^2)\xi + V_{,\phi} = 0, \quad (4)$$

where a dot denotes the derivative with respect to the cosmic time, and  $H$  is the Hubble parameter defined by  $H \equiv \dot{a}/a$ . Note that coupling function  $\xi$  works as the effective potential for the inflaton  $\phi$ .

We consider the friction-dominated slow-roll inflation phase where the following inequalities hold:

$$\frac{\dot{\phi}^2}{M_{\text{Pl}}^2 H^2} \ll 1, \quad \frac{H^3\dot{\xi}}{M_{\text{Pl}}^2 H^2} = \frac{H\dot{\xi}}{M_{\text{Pl}}^2} \ll 1, \quad \frac{\dot{H}}{H^2} \ll 1, \quad \frac{\ddot{\phi}}{H\dot{\phi}} \ll 1. \quad (5)$$

Under these circumstances, Eqs.(4) read

$$H^2 = \frac{V}{3M_{\text{Pl}}^2}, \quad \dot{\phi} = -\frac{V_{,\phi}}{3H} - \frac{1}{2}H^3\dot{\xi}. \quad (6)$$

The inflation is driven by the potential  $V$ , whereas the dynamics of the scalar field is also determined by the Gauss-Bonnet term.

From consistency conditions, we define *five* slow-roll parameters,

$$\epsilon \equiv \frac{M_{\text{Pl}}^2 V_{,\phi}^2}{2V^2}, \quad \eta \equiv M_{\text{Pl}}^2 \frac{V_{,\phi\phi}}{V}, \quad \alpha \equiv \frac{V_{,\phi}\dot{\xi}}{4M_{\text{Pl}}^2}, \quad \beta \equiv \frac{V_{,\phi}\xi}{6M_{\text{Pl}}^2}, \quad \gamma \equiv \frac{V^2\dot{\xi}^2}{18M_{\text{Pl}}^6} = \frac{4}{9} \frac{\alpha^2}{\epsilon}, \quad (7)$$

and impose that  $\epsilon, |\eta|, |\alpha|, |\beta|, \gamma \ll 1$ . Note that, due to the Gauss-Bonnet term, the number of parameters and accompanying conditions increased. What we want to know is the effect of new parameters  $\alpha, \beta, \gamma$  on the cosmological fluctuations.

### 3 Perturbations

In this section, we show the higher curvature corrections to the scalar and tensor perturbations. We consider the scalar perturbations  $A, B, \psi, E$  and tensor perturbations  $h_{ij}$  defined by the perturbed metric

$$ds^2 = a^2(\tau) \left[ -(1 + 2A)d\tau^2 + B_{|i}d\eta dx^i + (\delta_{ij} + 2\psi\delta_{ij} + 2E_{|ij} + h_{ij})dx^i dx^j \right], \quad (8)$$

where  $\tau$  is conformal time defined by  $a(t)d\tau = dt$ , the bar denotes the spatial derivative and the prime represents the derivative with respect to the conformal time. We take the gauge,  $E = \delta\phi = 0$ .

We can calculate perturbation quantities following same procedures in the case of ordinary slow-roll inflation. Hence we skip tedious but straightforward calculations. We define power spectrum as

$$\langle 0|\hat{\psi}^\dagger\hat{\psi}|0\rangle = \int d(\log k)\mathcal{P}_\psi(k), \quad \langle 0|\hat{h}_{ij}\hat{h}^{ij}|0\rangle = \sum \int d(\log k)\mathcal{P}_T = \int d(\log k)\mathcal{P}_T, \quad (9)$$

where  $\pm$  denotes helicity of gravitational waves. The results are

$$\mathcal{P}_\psi(k) = \frac{1}{2\epsilon + 4\alpha/3 + \gamma/2} \left( \frac{H}{2\pi} \frac{\Gamma(\nu_\psi)}{\Gamma(3/2)} \right)^2 \left( \frac{-k\tau}{2} \right)^{3-2\nu_\psi} \quad (10)$$

$$\mathcal{P}_T(k) = \frac{4}{1 + \epsilon - \alpha - \gamma} \left( \frac{H}{2\pi} \frac{\Gamma(\nu_T)}{\Gamma(3/2)} \right)^2 \left( \frac{-C_T k\tau}{2} \right)^{3-2\nu_T} \left( 1 \pm \frac{\pi}{2} \frac{H}{M_c} \Omega \right), \quad (11)$$

where we defined  $\nu_\psi \equiv 3/2 + 3\epsilon - \eta - \alpha/3 - \beta$  and  $\nu_T \equiv 3/2 + \epsilon + \alpha/3$ , and  $\Omega$  is another small quantity, representing effect of parity violating term, defined by

$$\Omega \equiv \frac{1}{2} \frac{M_c}{M_{\text{Pl}}} \frac{\dot{\omega}}{M_{\text{Pl}}} \simeq \text{const.}, \quad (12)$$

where  $M_c$  is the physical wave-number corresponding to the cut-off scale, *e.g.* string scale. We can read the scalar and tensor spectral indices  $n_\psi, n_T$

$$n_\psi - 1 = 3 - 2\nu_\psi = -6\epsilon + 2\eta + \frac{2}{3}\alpha + 2\beta, \quad n_T = 3 - 2\nu_T = -2\epsilon - \frac{2}{3}\alpha. \quad (13)$$

Of course, in the absence of the Gauss-Bonnet term,  $\alpha = \beta = 0$ , we recover the conventional formula.

The most impressive result is shown in Eq.(13). In the usual case, because  $\epsilon > 0$ ,  $n_T$  takes a negative value, namely, the tensor spectrum is red. However, if we incorporate the effect of the Gauss-Bonnet term, tensor spectrum could be blue, because  $\alpha$  takes either positive or negative value. Hence, detection of the blue spectrum in the gravitational waves through the observation of B-mode polarization might indicate the existence of the Gauss-Bonnet term.

From the scalar and tensor spectrum, we can deduce the tensor-to-scalar ratio  $r$ :

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_\psi} \sim 16\epsilon + \frac{32}{3}\alpha + 4\gamma = 16\epsilon + \frac{32}{3}\alpha + \frac{16}{9}\frac{\alpha^2}{\epsilon} - \frac{32}{3}(\alpha + |\alpha|). \quad (14)$$

For a negative  $\alpha$ , we have the minimum, that is,  $r = 0$ . In the case  $1 - \alpha - \epsilon$ , the last term dominates and give rise to the sizable tensor-to-scalar ratio even if  $\epsilon$  is extremely small.

Another interesting observable is the circular polarization  $\Pi$ , which is defined as difference between left and right helicity modes

$$\Pi \equiv \frac{\mathcal{P}_T^+ - \mathcal{P}_T^-}{\mathcal{P}_T^+ + \mathcal{P}_T^-} = \frac{\pi}{2} \frac{H}{M_c} \Omega. \quad (15)$$

This result for circular polarization is the same as the previous work [5], although they considered only parity violating term. Let us consider some extreme case,  $H = 10^{-4}M_{\text{Pl}}$ ,  $M_c = 10^{-3}M_{\text{Pl}}$ ,  $\Omega = 0.1$ , then, the ratio becomes  $\Pi \simeq 0.015$ . This means we have a hope to measure the sub-percent order of polarization due to the parity violating term [6]. It should be stressed that the effect of the Gauss-Bonnet term could enhance the amplitude of the primordial gravitational waves. That also enhances the detectability.

## 4 Observational implications

In the previous sections, we have derived general formula for observables in slow-roll inflation with higher curvature corrections. Here, we will discuss one model as an illustration. Let us consider chaotic inflation models with higher curvature corrections. We take the functions as

$$V = \frac{1}{2}m^2\phi^2, \quad \xi = e^{-\kappa \phi / M_{\text{Pl}}}, \quad (16)$$

where  $m$  is the mass of the inflaton and  $\kappa$  are parameters of the coupling function  $\xi$ . Then, the slow-roll parameters can be calculated as

$$\epsilon = \eta = 2\frac{M_{\text{Pl}}^2}{\phi^2}, \quad \alpha = -\frac{\kappa}{4}\frac{m^2\phi}{M_{\text{Pl}}^3}e^{-\kappa \phi / M_{\text{Pl}}}, \quad \beta = \frac{\kappa^2}{12}\frac{m^2\phi^2}{M_{\text{Pl}}^4}e^{-\kappa \phi / M_{\text{Pl}}}, \quad \gamma = \frac{\kappa^2}{72}\frac{m^4\phi^4}{M_{\text{Pl}}^8}e^{-2\kappa \phi / M_{\text{Pl}}}. \quad (17)$$

We can evaluate the spectral index and the tensor-to-scalar ratio. The results are shown in FIG.1. We have also plotted constraints from WMAP 5-year result, combined with baryon acoustic oscillations (BAO) and Type I supernova (SN). We see these observables are sensitive to higher curvature corrections. Hence, it implies that the higher curvature corrections are relevant to precision cosmology. Here, we notice that there exist parameter regions represented by blue lines which leads to the blue spectrum in tensor modes. We plotted some typical values of  $n_T$  in Fig.1.

## 5 Conclusion

We have studied the slow-roll inflationary scenario with the leading corrections, namely, the Gauss-Bonnet and the parity violating terms. We have obtained the higher curvature corrections to the scalar spectral

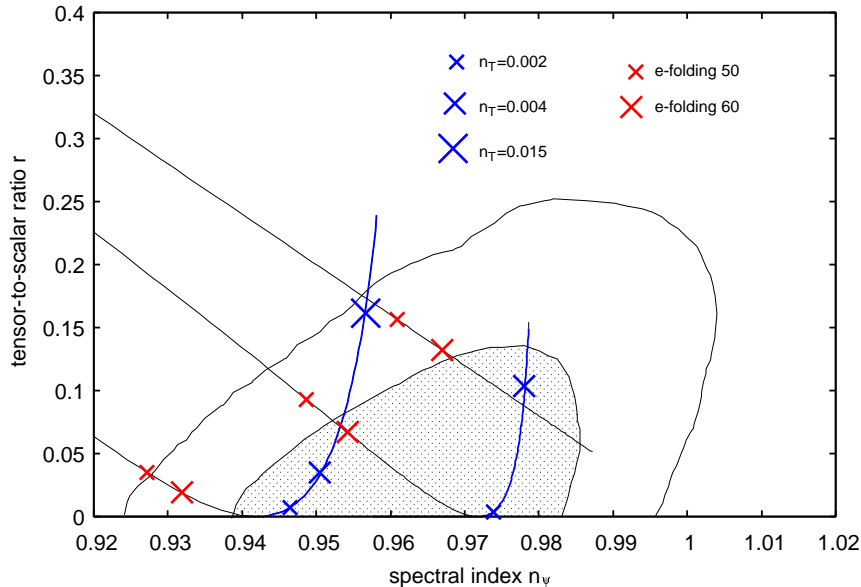


Figure 1: Expected spectral index  $n_\psi$  and tensor-to-scalar ratio  $r$  of the model (16) are shown. We have also plotted constraints from WMAP 5-year result, combined with BAO and Type I SN. The contours denote 68% and 95% confidence level. We used the out-put from Cosmological Parameters Plotter at LAMBDA [7]. Each line corresponds to the parameter (i)  $m = 10^{-5} \times M_{\text{Pl}}$ ,  $\alpha = 0$ ,  $\kappa = 0$  (ordinary slow-roll), (ii)  $m = 10^{-5} \times M_{\text{Pl}}$ ,  $\alpha = 10^{10}$ ,  $\kappa = 0.003$  and (iii)  $m = 10^{-5} \times M_{\text{Pl}}$ ,  $\alpha = 10^{10}$ ,  $\kappa = 0.01$ , from upper to lower, taking  $\phi > 0$ . Blue-colored lines denote the region that generate blue spectrum in tensor modes, and we plot some values of  $n_T$ .

index, the tensor spectral index, and the tensor-to-scalar ratio. We found that the higher curvature corrections can not be negligible in the dynamics of the scalar field, although they are negligible in the Friedmann equation. It turned out that the tensor-to-scalar ratio could be enhanced because of the modified dynamics of the scalar field. We have found the tensor spectral index could be blue due to the Gauss-Bonnet term. We have also calculated the degree of circular polarization of gravitational waves generated during the slow-roll inflation. We discussed that the circular polarization can be observable due to the Gauss-Bonnet and parity violating terms. We revealed that the observables are sensitive to the higher curvature corrections.

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