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Analogue gravity models of emergent gravity: lessons and pitfalls

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Abstract. Analogue models of gravity have provided a test bed for many classical and quantum field theory effects in curve spacetime. Here we present a review of some relevant results towards their extension as toy models of emergent gravity scenarios. From these models we shall try to draw general lessons about the emergent gravity tackle on the cosmological constant problem as well as about the characteristic phenomenological signatures they suggest. Finally, we shall discuss current constraints on these signatures and the field's future perspectives.

1. Introduction

General Relativity (GR) still stands strong after 100 years from its lay out by Einstein. However, in spite of its numerous successes we cannot say we fully understand it: many odd features of the theory still baffle us and suggest that only the tip of the iceberg has been uncovered. In particular, an incomplete list of puzzling facts may comprise

- The nature of the expected singularities of classical GR.
- Critical phenomena in gravitational collapse
- Horizon thermodynamics and their problems (information loss and transplanckian issues)
- Spacetime thermodynamics: Einstein equations as equations of state.
- Thermodynamics interpretation of Einstein equations (see Padmanabhans talk and papers)
- The dark ingredients of our universe
- Faster than light and Time travel solutions
- AdS/CFT duality, holographic behaviour
- Gravity/fluid duality

These are all features that we struggle to understand nowadays and it is not encouraging to realise that while new items have been added to this list none of them has been ticked away as solved. In response to this growing evidence of our lack of a deep understanding of the nature of gravity, several fundamental approaches have been devised. In particular a new framework has been suggested which goes under the general name of "emergent gravity" (EG).

The basic idea of EG is that gravity is not a fundamental interaction and that spacetime is a composite object approximately like a fluid is. In this scenario GR is seen as a sort of hydrodynamics emerging from a deeper theory of the fundamental constituents which are not quanta of spacetime but rather abstract mathematical entities. In this sense, all sub-Planckian

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physics has to be seen as low energy physics, a phase akin to a Bose–Einstein condensate of the fundamental constituents [1]. Within such a framework also singularities change interpretation, from limits of predictability of GR to phase transitions of the fundamental theory where the hydrodynamic limit comes about (big bang) or ceases to exists (black holes or big rip).

It is obvious that such concepts, as fascinating as they can be, mays sound more as buzz words rather that a solid proposal for the ultimate nature of gravity. Furthermore, how can we test them? As usual in physics, in order to move from abstract ideas to more concrete physical intuition, it is then appropriate to introduce relatively simple systems that can act as tests beds for our conjectures. In what follows we shall consider a possible route in this direction i.e. the Analogue Gravity (AG) proposal.

2. Analogue gravity: an example of emergent spacetime

Analogue gravity stemmed from the realisation that within several condensed matter systems there are regimes in which the effective degrees of freedom are represented by fields propagating over effective pseudo-Riemannian structures [2].

For instance, in the case of perfect, irrotational and barotropic fluids, it can be proved that the perturbations in the velocity potential (i.e. the scalar function θ whose gradient gives the velocity of the fluid, $\vec{v} \propto \vec{\nabla}\theta$) do obey a massless Klein–Gordon equation in a curved effective spacetime whose metric tensor is given by the so-called acoustic metric,

$$g_{\mu\nu} = \frac{\rho}{c_s^2} \begin{pmatrix} -(c_s^2 - v^2) & \vdots & v^i \\ \cdots & \cdot & \cdots \\ v^i & \vdots & \delta_{ij} \end{pmatrix}, \tag{1}$$

where ρ is the local density of the fluid, c_s it the (local) speed of sound and v_i is the velocity field of the fluid flow. Analogue models have been used to understand (and possibly to test in a laboratory) some peculiar aspects of physics in curved spacetimes, otherwise inaccessible (e.g. Hawking radiation). For a review of the subject see [2]. For the large majority, these analogue models for gravity offer the possibility of studying some kinematical aspects of physics of curved spacetimes, leaving aside the issue of dynamics. But there are some exceptions.

2.1. BEC as a prototype model

Bose–Einstein condensates (BECs) have become the subject of extensive study as possible analogue models of general relativity (see e.g. [3, 4, 5, 6]).

Let us start by very briefly reviewing the derivation of the acoustic metric for a BEC system, and show that the equations for the phonons of the condensate closely mimic the dynamics of a scalar field in a curved spacetime. In the dilute gas approximation, one can describe a Bose gas by a quantum field $\widehat{\Psi}$ satisfying

$$\mathrm{i}\hbar \, \frac{\partial}{\partial t}\widehat{\Psi} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{ext}}(\mathbf{x}) + \kappa(a)\, \widehat{\Psi}^\dagger \widehat{\Psi} \right) \widehat{\Psi}. \tag{2}$$

Here κ parameterises the strength of the interactions between the different bosons in the gas. It can be re-expressed in terms of the scattering length a as $\kappa(a) = (4\pi a\hbar^2)/m$.

As usual, the quantum field can be separated into a macroscopic (classical) condensate and a fluctuation: $\widehat{\Psi} = \psi + \widehat{\varphi}$, with $\langle\widehat{\Psi}\rangle = \psi$. Then, by adopting the self-consistent mean field

approximation $\widehat{\varphi}^\dagger\widehat{\varphi}\widehat{\varphi} \simeq 2\langle\widehat{\varphi}^\dagger\widehat{\varphi}\rangle\ \widehat{\varphi} + \langle\widehat{\varphi}\widehat{\varphi}\rangle\ \widehat{\varphi}^\dagger$ one can arrive at the set of coupled equations:

$$\mathrm{i}\hbar\,\frac{\partial}{\partial t}\psi(t,\mathbf{x}) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{\rm ext}(\mathbf{x}) + \kappa\ n_c\right)\psi(t,\mathbf{x}) + \kappa\left\{2\tilde{n}\psi(t,\mathbf{x}) + \tilde{m}\psi^*(t,\mathbf{x})\right\}, \quad (3)$$

$$\mathrm{i}\hbar\,\frac{\partial}{\partial t}\widehat{\varphi}(t,\mathbf{x}) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{\rm ext}(\mathbf{x}) + \kappa\ 2n_T\right)\widehat{\varphi}(t,\mathbf{x}) + \kappa\ m_T\ \widehat{\varphi}^\dagger(t,\mathbf{x}), \quad (4)$$

with $n_c \equiv |\psi(t,\mathbf{x})|^2$, $m_c \equiv \psi^2(t,\mathbf{x})$, $\tilde{n} \equiv \langle\widehat{\varphi}^\dagger\ \widehat{\varphi}\rangle$, $\tilde{m} \equiv \langle\widehat{\varphi}\ \widehat{\varphi}\rangle$, $n_T = n_c + \tilde{n}$, and $m_T = m_c + \tilde{m}$. The equation for the classical wave function of the condensate is closed only when the back-reaction effect due to the fluctuations are neglected. (This back-reaction is hiding in the parameters $\tilde{m}$ and $\tilde{n}$.) This is the approximation contemplated by the Gross–Pitaevskii equation. In general one will have to solve both equations simultaneously. Adopting the Madelung representation for the wave function of the condensate $\psi(t,\mathbf{x}) = \sqrt{n_c(t,\mathbf{x})}\ \exp[-\mathrm{i}\theta(t,\mathbf{x})/\hbar]$, and defining an irrotational "velocity field" by $\mathbf{v} \equiv \boldsymbol{\nabla}\theta/m$, the Gross–Pitaevskii equation can be rewritten as a continuity equation plus an Euler equation:

$$\frac{\partial}{\partial t}n_c + \boldsymbol{\nabla}\cdot(n_c\mathbf{v}) = 0, \quad (5)$$

$$m\frac{\partial}{\partial t}\mathbf{v} + \boldsymbol{\nabla}\left(\frac{mv^2}{2} + V_{\rm ext}(t,\mathbf{x}) + \kappa n_c - \frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{n_c}}{\sqrt{n_c}}\right) = 0. \quad (6)$$

These equations are completely equivalent to those of an irrotational and inviscid fluid apart from the existence of the so-called quantum potential

$$V_{\rm quantum} = -\frac{\hbar^2}{2m}\ \frac{\nabla^2\sqrt{n_c}}{\sqrt{n_c}}, \quad (7)$$

which has the dimensions of an energy.

Apart from the wave function of the condensate itself, we also have to account for the (typically small) quantum perturbations of the system (4). These quantum perturbations can be described in several different ways, here we are interested in the "quantum acoustic representation"

$$\widehat{\varphi}(t,\mathbf{x}) = e^{-\mathrm{i}\theta/\hbar}\left(\frac{1}{2\sqrt{n_c}}\ \widehat{n}_1 - \mathrm{i}\ \frac{\sqrt{n_c}}{\hbar}\ \widehat{\theta}_1\right), \quad (8)$$

where $\widehat{n}_1, \widehat{\theta}_1$ are real quantum fields. By using this representation Equation (4) can be rewritten as

$$\partial_t\widehat{n}_1 + \frac{1}{m}\boldsymbol{\nabla}\cdot\left(n_1\ \boldsymbol{\nabla}\theta + n_c\ \boldsymbol{\nabla}\widehat{\theta}_1\right) = 0, \quad (9)$$

$$\partial_t\widehat{\theta}_1 + \frac{1}{m}\boldsymbol{\nabla}\theta\cdot\boldsymbol{\nabla}\widehat{\theta}_1 + \kappa(a)\ n_1 - \frac{\hbar^2}{2m}\ D_2\widehat{n}_1 = 0. \quad (10)$$

Here D_2 represents a second-order differential operator obtained from linearizing the quantum potential. Explicitly:

$$D_2\,\widehat{n}_1 \equiv -\frac{1}{2}n_c^{-3/2}\ [\nabla^2(n_c^{+1/2})]\ \widehat{n}_1 + \frac{1}{2}n_c^{-1/2}\ \nabla^2(n_c^{-1/2}\ \widehat{n}_1). \quad (11)$$

The equations we have just written can be obtained easily by linearizing the Gross–Pitaevskii equation around a classical solution: $n_c \to n_c + \widehat{n}_1$, $\phi \to \phi + \widehat{\phi}_1$.

From the previous equations for the linearised perturbations it is possible to derive a wave equation for $\widehat{\theta}_1$ (or alternatively, for $\widehat{n}_1$). All we need is to substitute in Equation (9) the $\widehat{n}_1$ obtained from Equation (10). This finally leads to a wave equation for θ_1 which can be easily rewritten as

$$\partial_\mu(f^{\mu\nu}\,\partial_\nu\widehat{\theta}_1) = 0. \tag{12}$$

Where the $f^{\mu\nu}$ are *differential operators* acting on space only.

Now, if we make a spectral decomposition of the field $\widehat{\theta}_1$ we can see that for wavelengths larger than $\hbar/mc_{\rm s}$ (the so called "healing length" of the condensate), the terms coming from the linearization of the quantum potential (the D_2) can be neglected in the previous expressions, in which case the $f^{\mu\nu}$ can be approximated by (momentum independent) numbers, instead of differential operators. (This is the heart of the acoustic approximation.) Then, by identifying

$$\sqrt{-g}\;g^{\mu\nu} = f^{\mu\nu}, \tag{13}$$

the equation for the field $\widehat{\theta}_1$ becomes that of a (massless minimally coupled) quantum scalar field over a curved background

$$\Delta\theta_1 \equiv \frac{1}{\sqrt{-g}}\,\partial_\mu\left(\sqrt{-g}\;g^{\mu\nu}\,\partial_\nu\right)\widehat{\theta}_1 = 0, \tag{14}$$

with an effective metric of the form

$$g_{\mu\nu}(t,\mathbf{x}) \equiv \frac{n_c}{m\;c_{\rm s}(a,n_c)}\begin{bmatrix} -\{c_{\rm s}(a,n_c)^2 - v^2\} & \vdots & -v_j \\ \cdots\cdots\cdots\cdots & \cdot & \cdots\cdots \\ -v_i & \vdots & \delta_{ij} \end{bmatrix}. \tag{15}$$

Here the magnitude $c_{\rm s}(n_c,a)$ represents the speed of the phonons in the medium $c_{\rm s}^2(a,n_c) = \kappa(a)\;n_c/m$ With this effective metric now in hand, the analogy is fully established, and one is now in a position to start asking more specific physics questions.

2.2. Lorentz breaking in BEC models

It is interesting to consider the case in which the above "hydrodynamical" approximation for BECs does not hold. In order to explore a regime where the contribution of the quantum potential cannot be neglected we can use the so called *eikonal* approximation, a high-momentum approximation where the phase fluctuation $\widehat{\theta}_1$ is itself treated as a slowly-varying amplitude times a rapidly varying phase. This phase will be taken to be the same for both $\widehat{n}_1$ and $\widehat{\theta}_1$ fluctuations. In fact, if one discards the unphysical possibility that the respective phases differ by a time varying quantity, any time-constant difference can be safely reabsorbed in the definition of the (complex) amplitudes. Specifically, we shall write

$$\widehat{\theta}_1(t,\mathbf{x}) = {\rm Re}\left\{\mathcal{A}_\theta\;\exp(-i\phi)\right\}, \tag{16}$$

$$\widehat{n}_1(t,\mathbf{x}) = {\rm Re}\left\{\mathcal{A}_\rho\;\exp(-i\phi)\right\}. \tag{17}$$

As a consequence of our starting assumptions, gradients of the amplitude, and gradients of the background fields, are systematically ignored relative to gradients of ϕ. (Warning: What we are doing here is not quite a "standard" eikonal approximation, in the sense that it is not applied directly on the fluctuations of the field $\psi(t,\mathbf{x})$ but separately on their amplitudes and phases ρ_1 and ϕ_1.) We adopt the notation

$$\omega = \frac{\partial\phi}{\partial t}; \qquad k_i = \nabla_i\phi. \tag{18}$$

Then the operator D_2 can be approximated by the *function* $D_2 \to -n_c^{-1}k^2/2$.

As desired, this has the net effect of making $f^{\mu\nu}$ a matrix of (explicitly momentum dependent) numbers, not operators. The physical wave equation (12) now becomes a nonlinear dispersion relation

$$f^{00}\,\omega^2 + (f^{0i} + f^{i0})\,\omega\,k_i + f^{ij}\,k_i\,k_j = 0. \tag{19}$$

After substituting the approximate D_2 into this dispersion relation, rearranging, and introducing the speed of sound $c_{\rm s}$ one obtains

$$\omega = v_0^i\,k_i \pm \sqrt{c_{\rm s}^2 k^2 + \left(\frac{\hbar}{2m}\,k^2\right)^2}. \tag{20}$$

Coming back to considering the form of (20) at this stage some observations are in order: It is easy to see that (20) actually interpolates between two different regimes depending on the value of the wavelength $\lambda = 2\pi/||k||$ with respect to the "acoustic Compton wavelength" $\lambda_c = h/(mc_{\rm s})$. In particular, if we assume $v_0 = 0$ (no background velocity), then for large wavelengths $\lambda \gg \lambda_c$ one gets a standard phonon dispersion relation $\omega \approx c||k||$. For wavelengths $\lambda \ll \lambda_c$ the quasi-particle energy tends to the kinetic energy of an individual gas particle and in fact $\omega \approx \hbar^2 k^2/(2m)$.

3. From emergent spacetime analogue to full emergent gravity

In order to see how some sort of gravitational dynamics is encoded in the BEC, a suitable framework must be set up in order to see how the quasiparticles backreact over the condensate.

This can be achieved by considering an improved version of the Gross–Pitaevski equation which consistently takes into account the effect of the particles out of the condensate. The Gross–Pitaevski (GP) equation is replaced by the so-called Bogoliubov–de Gennes (BdG) equation

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\nabla^2\psi - \mu\psi + \kappa|\psi|^2\psi + 2\kappa\mathfrak{n}\psi + \kappa\mathfrak{m}\psi^*, \tag{21}$$

where $\mathfrak{n}, \mathfrak{m}$ are given by the expectation values:

$$\mathfrak{n} = \langle\Xi|\widehat{\varphi}(x)^\dagger\widehat{\varphi}(x)|\Xi\rangle, \qquad \mathfrak{m} = \langle\Xi|\widehat{\varphi}(x)^2|\Xi\rangle, \tag{22}$$

where the state $|\Xi\rangle$ is the particular state one is considering. Notice that, if this state were the Fock vacuum state for particles, these expectation values would be identically zero. Notice also that one is implicitly taking a normal ordering in the particle operator, so that an unphysical (divergent) zero point energy is removed automatically.

When exploring the possibility of casting the equations for the BEC background in a gravitational form, it is clear that the non-relativistic nature of the latter implies that at most some form of Newtonian gravity could be expected to emerge. However, in order to construct some analogue of Newtonian gravity, we need massive particles as sources of the gravitational field (massless particles do not gravitate in Newtonian gravity). Therefore, the quasiparticles must not be Goldstone bosons, but instead pseudo-Goldstones: the $U(1)$ symmetry has to be broken explicitly at the level of the Hamiltonian.

In [7] an extra term was added to the Hamiltonian,

$$\widehat{H}_0 \to \widehat{H} = \widehat{H}_0 + \widehat{H}_\lambda, \qquad \widehat{H}_\lambda = -\frac{\lambda}{2}\int d^3x\left(\widehat{\Psi}(x)^2 + (\widehat{\Psi}^\dagger(x))^2\right), \tag{23}$$

where λ is a coupling constant having the dimensions of an energy in these choice of units. The breaking of the $U(1)$ symmetry has an obvious interpretation: the number of bosons is no longer a conserved charge (see [7] for concrete examples in which such system could be realized).

Preliminary analysis of the condensate wavefunction based on the extension of the GP equation to this case, leads to the homogeneous solution

$$\psi = \sqrt{n_c} e^{-i\theta/\hbar}, \qquad n_c = \frac{\mu+\lambda}{2}, \qquad \theta = 0, \tag{24}$$

where the phase $\theta = 0$ is fixed by stability of the condensate itself: for different values of θ the quasiparticles would show a tachyonic instability.

The analysis of the properties of the quasiparticles in the case of homogeneous background (see [7] for details) leads to the conclusion that the quasiparticles dispersion relation is

$$E = \left(\mathcal{M}^2 c_s^4 + p^2 c_s^2 + \frac{p^4}{4m^2} \right)^{1/2}, \tag{25}$$

where

$$c_s^2 = \frac{\mu+2\lambda}{m}, \qquad \mathcal{M}^2 = 4\frac{\lambda(\mu+\lambda)}{(\mu+2\lambda)^2} m^2. \tag{26}$$

In the case of small momenta, and when the condensate wavefunction is not exactly homogeneous (*i.e.* when $u(\mathrm{x}) \neq 0$), the Hamiltonian for the quasiparticles takes the shape

$$\widehat{H}_{q.p.} \approx \mathcal{M} c_s^2 - \frac{\hbar^2 \nabla^2}{2\mathcal{M}} + 2\frac{(\mu+\lambda)(\mu+4\lambda)}{\mathcal{M} c_s^2} u(\mathrm{x}), \tag{27}$$

which leads to the identification of a "gravitational potential":

$$\Phi_{grav}(\mathrm{x}) = \frac{(\mu+4\lambda)(\mu+2\lambda)}{2\lambda m} u(\mathrm{x}). \tag{28}$$

3.1. The emergent gravitational system

Having presented the main ideas and required tools, we pass to the results. Consider the Hamiltonian with the $U(1)$ breaking term. In the limit in which the backreaction of the condensate is small, *i.e.* in the limit in which there are few quasiparticles, when the condensate is almost homogeneous, the Bogoliubov–de Gennes equation can be rewritten in terms of the above mentioned effective Newtonian potential as

$$\left(\nabla^2 - \frac{1}{L^2} \right) \Phi_{grav} = 4\pi G_N \rho_{\mathrm{matter}} + C_\Lambda, \tag{29}$$

where

$$G_N \equiv \frac{\kappa(\mu+4\lambda)(\mu+2\lambda)^2}{4\pi\hbar^2 m \lambda^{3/2}(\mu+\lambda)^{1/2}}, \qquad C_\Lambda \equiv \frac{2\kappa(\mu+4\lambda)(\mu+2\lambda)}{\hbar^2\lambda}\left(\mathfrak{n}_\Omega + \frac{1}{2}\mathfrak{m}_\Omega \right), \tag{30}$$

$$L^2 \equiv \frac{\hbar^2}{4m(\mu+\lambda)}. \tag{31}$$

Notice the peculiar splitting of the source term. A detailed analysis [7] shows that the expectation values (22) always split into two contributions, one nonlocal term due to the quasiparticles (ρ_{matter}), and an unavoidable vacuum contribution, C_Λ, due to the inequivalence between the Fock vacuum for particles and the Fock vacuum for quasiparticles.

The reader will immediately realise that the would-be Poisson equation includes a term which makes the interaction short ranged. In particular, this range is set by the healing length L, which is an UV scale for the physics of the BEC (very much like the Planck scale in quantum gravity). This might have been guessed from the beginning, since that the healing length represents the typical scale for the dynamics of the condensate. Henceforth, this system is not an analogue for a realistic form of gravitational interaction, however it does offer some intriguing hints which we further develop in what follows.

3.2. A lesson for the cosmological constant in emergent gravity?

Note that the source term in the correct weak field approximation of Einstein equations is $4\pi G_N(\rho + 3p/c^2)$. For standard non-relativistic matter, p/c^2 is usually negligible with respect to ρ. However, it cannot be neglected for the cosmological constant, since $p_\Lambda/c^2 = -\rho_\Lambda$. As a consequence $C_\Lambda = -2c_s^2\Lambda$, where Λ would be the GR cosmological constant.

Indeed it can be shown via an explicit computation in an homogeneous background, no external potential, and with the condensate is at rest it equals [8]

$$\Lambda = -\frac{20m\, g\rho_0\,(g\rho_0 + 3\lambda)}{3\sqrt{\pi}\hbar^2\lambda}\sqrt{\rho_0 a^3}\, F_\Lambda\left(\frac{\lambda}{g\rho_0}\right), \tag{32}$$

where we used that in the above regime $\mu = g\rho_0 - \lambda$ and $F_\Lambda(\lambda/g\rho_0)$ is a monotonically decreasing function ranging from $F(0) = 1$ to $F(1) = 0$.

It is important to stress that if one compares the value of Λ either with the ground-state grand-canonical energy density h or to the ground-state energy density ϵ of of the BEC, Λ does not correspond to either of them [8]. Actually, since Λ is proportional to $\sqrt{\rho_0 a^3}$, it can even be arbitrarily smaller both than h and than ϵ, if the condensate is very dilute. Furthermore, Λ is proportional only to the subdominant second order correction of h or ϵ, which is strictly related to the depletion factor of the condensate.

Indeed, several scales show up in this system, in addition to the naive Planck scale computed by combining $\hbar$ and the emergent constants G_N and c_s:

$$L_\mathrm{P} = \sqrt{\frac{\hbar c_s^5}{G_\mathrm{N}}} \propto \left(\frac{\lambda}{g\rho_0}\right)^{-3/4} (\rho_0 a^3)^{-1/4} a. \tag{33}$$

For instance, the Lorentz-violation scale $L_\mathrm{LV} = \xi \propto (\rho_0 a^3)^{-1/2} a$ differs from L_P, suggesting that the breaking of the Lorentz symmetry might be expected at scale much longer than the Planck length (energy much smaller than the Planck energy), since the ratio $L_\mathrm{LV}/L_\mathrm{P} \propto (\rho_0 a^3)^{-1/4}$ increases with the diluteness of the condensate.[1]

It is also instructive to compare the energy density corresponding to Λ to the Planck energy density:

$$\mathcal{E}_\Lambda = \frac{\Lambda c_s^4}{4\pi G_\mathrm{N}}, \quad \mathcal{E}_\mathrm{P} = \frac{c_s^7}{\hbar G_\mathrm{N}^2}, \quad \frac{\mathcal{E}_\Lambda}{\mathcal{E}_\mathrm{P}} \propto \rho_0 a^3 \left(\frac{\lambda}{g\rho_0}\right)^{-5/2}. \tag{34}$$

The energy density associated with the analogue cosmological constant is much smaller than the values computed from zero-point-energy calculations with a cut off at the Planck scale. Indeed, the ratio between these two quantities is controlled by the diluteness parameter $\rho_0 a^3$.

Taken at face value, this relatively simple model displays too many crucial differences with any realistic theory of gravity to provide conclusive evidences. However, it displays an alternative path to the cosmological constant from the perspective of a microscopic model. The analogue cosmological constant that we have discussed *cannot* be computed as the total zero-point energy of the condensed matter system, even when taking into account the natural cut-off coming from the knowledge of the microphysics [10]. Indeed, the value of Λ is related only to the (subleading) part of the zero-point energy proportional to the quantum depletion of the condensate.

The implications for gravity are twofold. First, there could be no *a priori* reason why the cosmological constant should be computed as the zero-point energy of the system. More properly, its computation must inevitably pass through the derivation of Einstein equations emerging

[1] Note also that L_LV scales with $\rho_0 a^3$ exactly as the range of the gravitational force, signalling that this model is too simple to correctly grasp all the desired features. However, in more complicated systems [9], this pathology can be cured, in the presence of suitable symmetries, leading to long range potentials.

from the underlying microscopic system. Second, the energy scale of Λ can be several orders of magnitude smaller than all the other energy scales for the presence of a very small number, nonperturbative in origin, which cannot be computed within the framework of an EFT dealing only with the emergent degrees of freedom (e.g. semiclassical gravity).

4. Quasi-particles and locality

An aspect of the physics of quasiparticles which is not often stressed concerns the issue of locality, *i.e.* whether the effective Lagrangian of the quasiparticles does obey the axioms of local quantum field theory.

In BEC, the particles ϕ and quasiparticles ω field operators are related by a Bogoliubov transformation, which have the following general structure:

$$\omega^A(\mathrm{k}) = M^A_B(\mathrm{k})\phi^B(\mathrm{k}), \tag{35}$$

when working in momentum space this relation is always linear. However, it is nonlocal in coordinate space as

$$\omega^A(\mathrm{x}) = \int d^3y\, K^A_B(\mathrm{x},\mathrm{y})\phi^B(\mathrm{y}), \tag{36}$$

where the kernel K is determined by the Bogoliubov coefficients:

$$K^A_B(\mathrm{x},\mathrm{y}) = \int d^3k\, M^A_B(\mathrm{k})e^{-i\mathrm{k}\cdot(\mathrm{x}-\mathrm{y})}. \tag{37}$$

This is the mathematical statement of the fact that quasi-particles are collective degrees of freedom. The structure of the transformation immediately implies that there is a mismatch between the notion of locality of the quasi-particle with respect to the notion of locality of the atoms. As one easily realizes, the two classes of operators ϕ, ω, separately, do obey canonical equal time commutation relations:

$$[\phi^A(\mathrm{x}), (\phi^B)^\dagger(\mathrm{y})] = \delta^{AB}\delta^3(\mathrm{x}-\mathrm{y}), \qquad [\omega^A(\mathrm{x}), (\omega^B)^\dagger(\mathrm{y})] = \delta^{AB}\delta^3(\mathrm{x}-\mathrm{y}), \tag{38}$$

which are a direct consequence of the fact that Bogoliubov transformations are preserving the algebra of the creation-annihilation operators. Therefore, as long as we use only one of the two families, there is no way in which a deviation from standard local quantum field theory can be manifest. However, the mixed commutators are nontrivial and it is straightforward to see that

$$[\phi^A(\mathrm{x}), (\omega^B)^\dagger(\mathrm{y})] = (K^{AB})^*(\mathrm{x},\mathrm{y}), \tag{39}$$

Of course, this fact becomes crucial when the effective Lagrangian describing the physics of the quasi-particles involves terms mixing particle and quasi-particle operators.

In the case of the BEC it is pretty clear how the underlying dynamics induces in the action for the quasi-particles an interaction term of the form

$$L_{\mathrm{int}} = -\frac{\kappa}{4}\left((\phi^A)^\dagger\phi_A\right)^2 \tag{40}$$

which explicitly involves the particle fields, rather than the quasi-particles. Hence, when computing the effects of the interaction terms, *e.g.* scattering processes between the quasi-particles, the nonlocality encoded in the kernel K will necessarily enter in the physical quantities.

5. Analogue gravity in Relativistic BEC

Bose–Einstein condensation can occur not only for non-relativistic bosons but for relativistic ones as well. The main differences between the thermodynamical properties of these condensates at finite temperature are due both to the different energy spectra and also to the presence, for relativistic bosons, of anti-bosons. These differences result in different conditions for the occurrence of Bose–Einstein condensation, which is possible, e.g., in two spatial dimensions for a homogeneous relativistic Bose gas, but not for its non-relativistic counterpart – and also, more importantly for our purposes, in the different structure of their excitation spectra.

In reference [11] an analogue model based on a relativistic BEC was studied. We summarize here the main results. The Lagrangian density for an interacting relativistic scalar Bose field $\widehat{\phi}(\mathbf{x},t)$ may be written as

$$\widehat{\mathcal{L}} = \frac{1}{c^2}\frac{\partial\widehat{\Psi}^\dagger}{\partial t}\frac{\partial\widehat{\Psi}}{\partial t} - \nabla\widehat{\Psi}^\dagger\cdot\nabla\widehat{\Psi} - \left(\frac{m^2c^2}{\hbar^2} + V(t,\mathbf{x})\right)\widehat{\Psi}^\dagger\widehat{\Psi} - U(\widehat{\Psi}^\dagger\widehat{\Psi};\lambda_i)\ , \tag{41}$$

where $V(t,\mathbf{x})$ is an external potential depending both on time t and position $\mathbf{x}$, m is the mass of the bosons and c is the light velocity. U is an interaction term and the coupling constant $\lambda_i(t,\mathbf{x})$ can depend on time and position too (this is possible, for example, by changing the scattering length via a Feshbach resonance [12]). U can be expanded as

$$U(\widehat{\Psi}^\dagger\widehat{\Psi};\lambda_i) = \frac{\lambda_2}{2}\widehat{\rho}^2 + \frac{\lambda_3}{6}\widehat{\rho}^3 + \cdots \tag{42}$$

where $\widehat{\rho} = \widehat{\Psi}^\dagger\widehat{\Psi}$. The usual two-particle $\lambda_2\widehat{\Psi}^4$-interaction corresponds to the first term $(\lambda_2/2)\widehat{\rho}^2$, while the second term represents the three-particle interaction and so on.

In this case it is convenient to use the representation

$$\widehat{\Psi} = \psi\,(1+\widehat{\chi})\ . \tag{43}$$

It is worth noticing now that the expansion in Eq. (43) can be linked straightforwardly to the previously discussed expansion in phase and density perturbations $\widehat{\theta}_1$, $\widehat{\rho}_1$, by noting that

$$\frac{\widehat{\rho}_1}{\rho} = \frac{\widehat{\chi}+\widehat{\chi}^\dagger}{2},\quad \widehat{\theta}_1 = \frac{\widehat{\chi}-\widehat{\chi}^\dagger}{2i}\ .$$

Setting $\chi \propto \exp[i\,(\mathbf{k}\cdot\mathbf{x}-\omega t)]$ one then gets from the equation of motion [11]

$$\left(-\frac{\hbar}{m}\mathbf{q}\cdot\mathbf{k} + \frac{u^0}{c}\omega - \frac{\hbar}{2mc^2}\omega^2 + \frac{\hbar}{2m}k^2\right)\left(\frac{\hbar}{m}\mathbf{q}\cdot\mathbf{k} - \frac{u^0}{c}\omega - \frac{\hbar}{2mc^2}\omega^2 + \frac{\hbar}{2m}k^2\right) - \left(\frac{c_0}{c}\right)^2\omega^2 + c_0^2k^2 = 0\ ,$$

where, for convenience we have defined the following quantities

$$u^\mu \equiv \frac{\hbar}{m}\eta^{\mu\nu}\partial_\nu\theta\ ,\quad c_0^2 \equiv \frac{\hbar^2}{2m^2}U''(\rho;\lambda_i)\rho\ ,\quad \mathbf{q}\equiv m\mathbf{u}/\hbar\ . \tag{44}$$

Here $\mathbf{q}$ is the speed of the condensate flow, c is the speed of light. For a condensate at rest ($\mathbf{q}=0$) one then obtains the following dispersion relation

$$\omega_\pm^2 = c^2\left\{k^2 + 2\left(\frac{mu^0}{\hbar}\right)^2\left[1+\left(\frac{c_0}{u^0}\right)^2\right] \pm 2\left(\frac{mu^0}{\hbar}\right)\sqrt{k^2+\left(\frac{mu^0}{\hbar}\right)^2\left[1+\left(\frac{c_0}{u^0}\right)^2\right]^2}\right\}\ . \tag{45}$$

The dispersion relation (45) is sufficiently complicated to prevent any obvious understanding of the regimes allowed for the excitation of the system. It is much richer than the non-relativistic case. For example, it allows for both a massless/gapless (phononic) and massive/gapped mode, respectively for the ω_- and ω_+ branches of (45). Nonetheless, it should be evident that different regimes are determined by the relative strength of the the first two terms on the right hand side of (45) (note that the same terms enter in the square root). This can be summarized, in low and high momentum limits respectively, for k much less or much greater than

$$\frac{mu^0}{\hbar}\left[1+\left(\frac{c_0}{u^0}\right)^2\right] \equiv \frac{mu^0}{\hbar}(1+b), \tag{46}$$

where b basically encodes the relativistic nature of the condensate (the larger b the more the condensate is relativistic).

Finally, it is also possible to recover an acoustic metric for the massless (phononic) perturbations of the condensate in the low momentum limit ($k \ll mu^0(1+b)/\hbar$) [11]

$$g_{\mu\nu} = \frac{\rho}{\sqrt{1-u_\sigma u^\sigma/c_o^2}}\left[\eta_{\mu\nu}\left(1-\frac{u_\sigma u^\sigma}{c_0^2}\right)+\frac{u_\mu u_\nu}{c_0^2}\right] . \tag{47}$$

5.1. Emergent scalar gravity in Relativistic BEC

The above discussed relativistic BEC system albeit still unrealised in laboratory is an obvious candidate for further exploring the dynamics of the emergent spacetime this time in a fully relativistic context (see [13] for a complete derivation). In order to do so let us go back to the equation of motion of the BEC

$$\left(\Box - m^2\right)\Psi - 2\lambda|\Psi|^2\Psi = 0. \tag{48}$$

This was the starting equation in ref. [11] where the acoustic metric was first derived.

Let us again decompose $\widehat{\Psi}$ as $\widehat{\Psi} = \psi(1+\widehat{\chi})$, where ψ is the condensed part of the field ($\langle\Psi\rangle = \psi$), which we now take to be real, and $\widehat{\chi}$ is the fractional fluctuation.[2] Note that $\widehat{\chi}$ is instead complex and $\langle\widehat{\chi}\rangle = 0$. It can be written in terms of its real and imaginary parts $\chi = \chi_1 + i\chi_2$ (and from now on we drop the hat notation). Substituting this decomposition in eq. (48) and taking the expectation value we get the equation of motion for the condensate

$$(\Box - m^2)\psi - 2\lambda\psi^3 - 2\lambda\psi^3\left[\,3\langle\chi_1^2\rangle + \langle\chi_2^2\rangle\,\right] = 0, \tag{49}$$

where we have assumed that the cross-correlation of the fluctuations vanish, i.e., $\langle\chi_1\chi_2\rangle = 0$. This is justified a posteriori by equations (53), which show that χ_1 and χ_2 do not interact with each other at the order of approximation we are working. Eq. (49) determines the dynamics of the condensate taking into account the backreaction of the fluctuations. It is the relativistic generalization of the Gross–Pitævskii equation.

5.2. Dynamics of perturbations: acoustic metric

Having determined the dynamics of the condensate we now want to calculate the equations of motion for the perturbations themselves. To this end, we insert $\Psi = \psi(1+\chi_1+i\chi_2)$ in eq. (48) and expand it to linear order in ψ's. Using the Gross–Pitævskii equation to that order and separating the real and imaginary parts we get the equation of motion for χ_1 and χ_2,

$$\Box\chi_1 + 2\eta^{\mu\nu}\partial_\mu(\ln\psi)\partial_\nu\chi_1 - 4\lambda\psi^2\chi_1 = 0, \tag{50a}$$

$$\Box\chi_2 + 2\eta^{\mu\nu}\partial_\mu(\ln\psi)\partial_\nu\chi_2 = 0. \tag{50b}$$

[2] The reality of the condensate order parameter is the crucial assumption here. See the relevant discussion in [13].

We therefore see that χ_2 is the massless mode, which is the Goldstone boson of the broken $U(1)$ symmetry, while χ_1 is the massive mode with mass $2\psi\sqrt{\lambda}$. We now define a "acoustic" metric, which is conformal to the background Minkowski,

$$g_{\mu\nu} = \psi^2\, \eta_{\mu\nu}. \tag{51}$$

The relation between the d'Alembertian operators for $g_{\mu\nu}$ and $\eta_{\mu\nu}$ is given by,

$$\Box_g = \frac{1}{\psi^2}\,\Box + \frac{2}{\psi^2}\,\eta^{\mu\nu}\,\partial_\mu(\ln\psi)\,\partial_\nu. \tag{52}$$

Equations (50) can be written in terms of the d'Alembertian of $g_{\mu\nu}$ as

$$\Box_g\chi_1 - 4\lambda\chi_1 = 0, \tag{53a}$$

$$\Box_g\chi_2 = 0. \tag{53b}$$

We see from eqs. (53) that the fluctuations propagate on a curved metric, called the acoustic metric, which in this case is conformal to the background Minkowski space eq. (51). Note that in this derivation there was no low-momentum approximation needed in order to derive the acoustic metric. On the other hand, they back-react on the condensate through the relativistic generalization of the Gross-Pitaevskii equation (49). It is natural to ask if it is possible to have a geometric description of the dynamics of the condensate too.

The Ricci tensor of the acoustic metric (51) can be calculated to be

$$R_g = -6\,\frac{\Box\,\psi}{\psi^3} \tag{54}$$

Dividing the relativistic Gross-Pitaevskii equation by ψ^3, eq. (49) can be written as

$$R_g + 6\frac{m^2}{\varphi_0^2} + 12\lambda = \langle T_{\rm qp}\rangle, \tag{55}$$

where we have defined $\langle T_{\rm qp}\rangle := -12\lambda\left[\,3\,\langle\chi_1^2\rangle + \langle\chi_2^2\rangle\,\right]$ and the subscript "qp" reminds us that this quantity is determined by the quasi-particle excitations of the condensate.

Eq. (55) is evidently reminiscent of the Einstein–Fokker equation describing Nordström gravity [14, 15],

$$R + \Lambda = 24\pi\frac{G_{\rm N}}{c^4}\,T, \tag{56}$$

where R and T are, respectively, the Ricci scalar and the trace of the stress-energy tensor of matter. Unfortunately, the gravitational analogy of our equation is spoiled by the mass term. Therefore we will consider our system in the zero mass limit, something doable in the presence of a suitable chemical potential (see discussion in [13]). otice that this limit does not spoil the presence of a condensate or the uniqueness of the Lorentz group for constituents and excitations found in sec.

The striking resemblance of equations (55) with zero mass term and (56) is still not enough to draw conclusions. Indeed, the dimensions of the various quantities appearing in eq. (55) are not canonical and need to be fixed for such comparison to be meaningful. This is due to the fact that, as is usual in the analogue gravity literature, our acoustic metric is a dimensional quantity because ψ is dimensional. The fractional perturbations χ_1 and χ_2, on the other hand, are dimensionless.

One therefore needs to suitably rescale fields in order to have a dimensionless metric and (mass) dimension one scalar fields propagating on the curved metric. The upshot of

this dimensional analysis is that we need to scale the field $\psi \to \frac{\mu}{\sqrt{c\hbar}}\psi$ and perturbation $\chi \to \frac{\sqrt{c\hbar}}{\mu}\chi$ [13]. Finally, using these rescaled quantities we can rewrite eq. (55) (with $m = 0$) in the form of eq. (56) as

$$R + \Lambda_{\text{eff}} = \langle T_{\text{qp}} \rangle, \tag{57}$$

and T_{qp} here and in the following is the same expression as in (55) but with the mass dimension one fields. Equations of motion of the quasi-particles (50) can also be rewritten in terms of the rescaled fields as

$$\Box_g \chi_1 - \frac{4\lambda\mu^2}{\hbar c}\chi_1 = 0, \tag{58a}$$

$$\Box_g \chi_2 = 0, \tag{58b}$$

where all quantities, including the $\Box_g$ operator, now pertain to those of the rescaled fields.

Remarkably, by defining the actual SET of the quasiparticle as the usual variation of their quadratic action w.r.t. the emergent metric one one finds [13] that this basically coincides with T_{qp} modulo a proportionality factor. Indeed one finds,

$$\langle T \rangle = -2\lambda\frac{\mu^2}{c\hbar}\left[3\langle\chi_1^2\rangle + \langle\chi_2^2\rangle\right] = \frac{1}{6}\frac{\mu^2}{c\hbar}\langle T_{qp}\rangle. \tag{59}$$

Due to this last expression one sees that the RHS of eq. (57) is actually given by $\frac{6c\hbar}{\mu^2}\langle T\rangle$ and hence our emergent Nordström gravity equation will be exactly of the form (56) with the identification $G_{\text{eff}} = \hbar c^5/(4\pi\mu^2)$. This value corresponds to an emergent analogue Planck scale $M_{\text{Pl}} = \mu\sqrt{4\pi}/c^2$.

We have thus succeeded in expressing the dynamics of the background for our rBEC analogue model in a geometric language

$$R + \Lambda_{\text{eff}} = 24\pi\frac{G_{\text{eff}}}{c^4}\langle T\rangle. \tag{60}$$

The acoustic metric itself is sourced by the expectation value of the trace of the stress-energy tensor of the perturbations of the condensate playing the role of the matter. These matter fields in turns propagate relativistically on a conformally flat acoustic metric (51) with equations (58).

A final comment is deserved by the emergent, positive, cosmological constant term Λ_{eff}. The quantity of interest for what concern the usual cosmological constant problem is the ratio between the energy density associated to the (emergent) cosmological constant $\epsilon_{\Lambda_{\text{eff}}} \sim \left(\frac{\Lambda_{\text{eff}}c^4}{G_{\text{eff}}}\right)$ and the emergent Planck energy density $\epsilon_{\text{pl}} \sim \frac{c^7}{\hbar G_{\text{eff}}^2}$. In our case this ratio is given by

$$\frac{\epsilon_{\Lambda_{\text{eff}}}}{\epsilon_{\text{pl}}} \simeq \frac{3\lambda\hbar c}{4\pi^2}. \tag{61}$$

As one can see the ratio is proportional to $\lambda\hbar$ and so is clearly pretty small due to the presence of Planck constant and of the natural assumption of a weakly interacting system. Of course in principle this term can be "renormalised" by the vacuum contribution of the matter fields (basically the vacuum expectation value $\langle T\rangle$).

6. Going beyond: the issue of background independence

The result presented in Eq. (60) is at the same time striking and disappoint. While we succeeded in obtaining a dynamical equation for the analogue metric which reproduce a well known theory of gravity, such a theory is just a scalar theory of gravity (obviously as our fundamental fields are scalar in nature) which admits only conformally flat metrics.

Indeed, Nordström gravity is not fully background independent as the the Minkowski metric can be see then as a background structure, what one may call a prior geometry. One may hence say that diffeomorphism invariance is somewhat of a weaker form in Nordström gravity with respect the one present in general relativity. In particular, while the essence of diffeomorphism invariance in GR is encoded in the associated Hamiltonian constraints, these are not defined in the present formulation of Nordström gravity.

Another way to see this point is to resort to the famous derivation of Einstein equations from the Clausius equation ($dS = \delta Q/T$) presented by Jacobson in his 1995 seminal paper [16]. In that work, one uses the equivalence principle to construct around an arbitrary point of spacetime a Rindler wedge as observed for some accelerated observer. The wedge provides a natural notion of entropy as the entanglement entropy of the vacuum (which will be locally the Minkowski one) and of temperature through the Unruh effect. Heat fluxes can be associated to matter fluxes through the wedge. The entanglement entropy is proportional to the area which can in turn be written in terms of the expansion of the null congruence associated to the wedge. The variation of the entropy due to some flux is then associated to the variation of the expansion which is determined by the Raychaudhuri equation. The latter contains the Ricci tensor. In the end by equating the integrands on both sides of the Clausius equation and by using the conservation of the stress energy tensor (SET) one gets the Einstein equation with a cosmological constant (which come about as an integration constant) [16].

Now, it is interesting to note that if one takes an ideal system such as a perfect fluid (inviscid and irrotational), then perturbations propagate on the acoustic metric at all scales (there is no scale to provide for Lorentz breaking in the equations). In such system, all of the above argument can be reproduced: the metric is locally flat and all of the effects of quantum field theory leading to entanglement entropy and Unruh temperature still holds for phonons of the fluid. So, should we conclude that the Einstein equation can be recovered even in an (albeit ideal) analogue system? Of course this would not be compatible with that fact that the real equations, governing the fluid dynamics, are the hydrodynamic ones!

After some thinking one can realize that there is a step that even in our idealised fluid system is not granted, that is the assumption that the null congruence bounding the Rindler wedge reacts to phonon fluxes as in the true gravitational setting, i.e. ultimately, we cannot assume that phonons gravitate (affect the background geometry) via their SET. Indeed we saw explicitly in BEC systems that phonons "gravitate" in rather different ways: at best with their energy density or with the trace of their SET. So in the end the impossibility of recovering the full Einstein equations in analogue models seems to be linked to the fact that phonons will not gravitate through the right object i.e. a conserved SET, which in turns is directly linked to the background independence issue.

This point seems also strictly connected to a theorem recently demonstrated in [17]. The starting point is that background independent gravitational theories, with universal coupling to energy, are characterised by Hamiltonians that are pure boundary terms on shell. Then it is shown that in order for this to be the low energy effective description of a field theory with *local* kinematics, all bulk dynamics must be frozen and thus irrelevant to the construction.

The result indeed implies that gravitational theories emergent along the analogue gravity framework, could be truly diffeomorphism invariant only if a different notion of micro-causality for the fundamental and emergent fields is present (something not realised e.g. in BEC).

In conclusion it seems that emergent theories of gravity would be characterised either by

extra fields possibly leading to Lorentz breaking in the UV or by some degree of non-locality (which can appear or at the kinematical level or at the level of the interactions). In the last part of this contribution we shall discuss possible phenomenological consequences of these departures from standard physics and what constraints we can cast on them.

7. From analogue models to phenomenology

In closing this brief explorations on toy models of emergent gravity it is interesting to consider what we can say about the phenomenological consequences of the framework they seem to suggest. In this sense we have two main streams of investigation offered by analogue gravity ideas, i.e. we could generically expect or UV deviations from Lorentz invariance and/or intrinsic non-locality to appear both in interactions (as suggested by the phonon-atom duality in BEC) as well as in free propagation (if some for of fundamental non-locality has to be implemented as suggested by Marolf's theorem discussed above).

7.1. Lorentz breaking phenomenology

With regard of UV Lorentz breaking, both in the form of higher mass dimension dispersive or dissipative operators to be included the EFT for elementary particles, there is plenty we can say. A systematic investigation has been carried out mainly in the last 15 years based on the so called Standard Model Extension (i.e. an extension of the standard model of particle physics with Lorentz breaking operators added order by order in mass dimension and catalogues by their even or odd nature under CPT) which lead to a wealth of constraints (see e.g. [18, 19]). In particular, this approach leads to modified dispersion relations of the form

$$E^2 = p^2 + m_i^2 + \sum_{n=1}^{\infty} \eta_{i(n)} \frac{p^n}{M_{\rm Pl}^{n-2}} \ , \tag{62}$$

where we have put the low energy speed of light $c = 1$ and labelled the particle types by the i index and $M_{\rm Pl} = 1.22 \times 10^{19}$ GeV. In this parametrisation casting a strong constraints corresponds to show that the dimensionless coefficient η_i has to be much smaller than one. Most commonly the values of n considered are 3 and 4 (linear and quadratic Planck suppressed terms respectively).

It should not come as a surprise that constraints on this ansatz came mostly from high energy astrophysics and cosmology. Indeed the color dependence of the group velocity of photons can lead to observable differences of time of arrival for light emitted from very distance sources (e.g. gamma ray bursts). Putting an upper bound on color dependent delays can cast bounds on the size of the Lorentz breaking terms.

Similarly, modified dispersion relations can lead to depolarisation (vacuum birefringence), anomalous threshold reactions (e.g. vacuum Cherenkov or photon decay) as well as modify standard threshold reactions by shifting the standard threshold energy as well as by introducing "upper thresholds" (a maximum energy for which the reaction can happen). See [20, 21] for an overview.

All this new physics led to severe constraints on different particle species illustrated schematically in Table 1 (taken from [19] so updated to 2013). These seem indeed very tight constraints, but a caveat is in order: everywhere you see a (CR) label the constraints is performed using observations of Ultra High Energy Cosmic Rays (UHECR). The status of these observations and in particular the actual observation of the so called GZK cutoff is subject of intense debate nowadays and recent evidence from the leading experiment in the field, AUGER, seems to strongly hint in disfavour of this claim. This in particular makes the reliability of the constraints on the $n = 4$ Lorentz breaking corrections very weak and in strong need of further investigations.

Order	photon	e^-/e^+	Protrons	Neutrinos[a]
n=2	N.A.	$O(10^{-16})$	$O(10^{-20})$ (CR)	$O(10^{-8} \div 10^{-10})$
n=3	$O(10^{-16})$ (GRB)	$O(10^{-16})$ (CR)	$O(10^{-14})$ (CR)	$O(40)$
n=4	$O(10^{-8})$ (CR)	$O(10^{-8})$ (CR)	$O(10^{-6})$ (CR)	$O(10^{-7})^*$ (CR)

Table 1. Summary of typical strengths of the available constrains on the SME at different n orders for rotational invariant, neutrino flavour independent LIV operators. GRB=gamma rays burst, CR=cosmic rays. [a] From neutrino oscillations we have overwhelming constraints on the difference of LIV coefficients of different flavors. * Expected constraint from future experiments.

Even the possibility of dissipative dispersion relations has been considered (inspired by analogue gravity investigations for viscous fluids, see e.g. [22]). For example, we can conjecture a dispersion relation for photons of the form [23]

$$\omega^2 = c^2k^2 - i\sigma_2 c^2 \frac{k^3}{M_{\rm Pl}}\,, \tag{63}$$

where $\sigma_2 = (4\nu_2 M_{\rm Pl})/3c$ is the dimensionless coefficient controlling the magnitude of the Lorentz violation (LV) and ν is the "viscosity" coefficient of the spacetime fluid. Using the observed 80 TeV photons from the Crab nebula which is at a distance $D_{\rm Crab} \simeq 1.9$ Kpc one obtains$\sigma_2 \approx 1.3 \times 10^{-26}$. similar strengths constrains can be obtained for neutrinos. If spacetime emergent as a fluid, it better has to be a superfluid.

It is worth stressing that higher order dissipative terms can and in principle should be considered. For example, nothing forbids such terms in superfluids (which have zero viscosity) to be non-zero. Similarly, if some fundamental, custodial, symmetry of the underlying, quantum gravitational system would forbid the above mentioned "spacetime viscosity" term still one could expect non-zero dispersive $O(k^4)$ and dissipative $O(k^5)$ terms to appear. These are sufficiently high energy modifications for which we do have relatively weak constraints on dispersion and basically no constraints on dissipation [23]. Casting strong constraints on this higher order dissipative terms would be very informative as some sort of dissipative effect should be expected at higher order even for superfluids.

7.2. Non-locality phenomenology

As said, an alternative element that seems to be required for a consistent emergent gravity scenario " a' la analogue gravity" appears to be some form of non-locality and indeed there are QG scenarios resorting to different forms of emergence where Lorentz invariance is held as a guiding principle, while spacetime is seen as emergent from more fundamental, planckian structures. Typical scenarios in this sense are those of String Field Theory (SFT) [24] and Causal Set Theory (CST) [25]. But what kind of equations of motions one should expect in this case?

A general approach to the problem could consist in considering for example the dynamics of a free scalar field in flat spacetime, i.e. the standard Klein-Gordon (KG) equation $(\Box + m^2)\phi(x) = 0$. The simplest modification to this equation, which preserves Lorentz invariance, is one which generalises the KG operator to some function thereof: $(\Box + m^2) \to f(\Box + m^2)$. Furthermore, in order to avoid Ostrogradsky-like [26] instabilities, which arise in general whenever the dynamics contains more than two time derivatives, such an equation must be non-local in both space *and* time, in the sense of possessing an infinite number of spatial and temporal derivatives. Generically, the definition of f will contain a characteristic, covariantly defined non-locality length scale l_k, which allows for a suitable power law expansion characterising the deviation from the standard local field equations.

Not surprisingly, the above expectations are fulfilled at least by two QG models which implements Lorentz invariance at the fundamental level, namely String Field and Causal Set theories. Indeed, in String Field theory one finds that the KG equation in four dimensions is modified to [27]

$$f(\Box + m^2) = (\Box + m^2)\, \exp\left[l_k^2(\Box + m^2)\right], \tag{64}$$

with f therefore an analytic function. On the other hand, in Causal Set Theory one finds that $f(\Box + m^2)$ is generally non-analytic, and in four dimensions can be expanded as [28, 29]

$$f(\Box + m^2) = (\Box + m^2) - \frac{3l_k^2}{2\pi\sqrt{6}}(\Box + m^2)^2 \left[3\gamma - 2 + \ln\left(\frac{3l_k^4(\Box + m^2)^2}{2\pi}\right)\right] + \dots \tag{65}$$

where γ is Euler-Mascheroni's constant.

It is important to note that the non-locality length scale l_k, in both models needs not to be related to the quantum/discreteness scale normally associated to QG, i.e. the Planck scale. In fact, this scale is a mesoscopic one possibly lying somewhere between the TeV scale and the Planck scale; a fact which is of particular relevance within the context of casting phenomenological constraints.

The phenomenology associated to this kind of theories is in general different depending on the form of the function f. For non-analytic forms, like the one emergent from studies in CAUSET theory, one gets in $4d$ Green functions for a massless scalar field with non-zero support inside the light cone, i.e. the theory fails to satisfy Huygens' principle, i.e. there is a "leakage" inside the light cone of the field emitted from a delta function source [28]. This could prove fruitful in testing the theory, since one can envisage performing high precision tests of radiation emitted by very localised sources to check if such afterglow is present.

Alternative the case of analytical f functions can be studied by looking at the non-relativistic limit of the Klein-Gordon equation. This provides a non-local Schrödinger evolution which can be solved in a perturbative way. In particular, it was shown in [30, 31] that the corresponding non-local evolution of opto-mechanical quantum oscillators is characterised by a spontaneous periodic squeezing that cannot be generated by environmental effects. Quite surprisingly future experiments (already under construction) will either see such effects or otherwise cast severe bounds on the non-locality scale of order $l_k \lesssim 10^{-26}m$ (well beyond the current limits set by the Large Hadron Collider $l_k \lesssim 10^{-19}m$).

8. Conclusions

In summary, we think that this brief explorations serves to show the rich landscape of scenarios and phenomenological implications offered by emergent gravity ideas. Analogue gravity models are probably not going to give us a definite answer about the actual viability of an emergent gravity framework for recovering general relativity. However, we have seen that they do provide a set of toy models that may serve as test beds for our conjectures and suggest testable predictions of emergent gravity scenarios. While the route to a consistent picture is still long, we do hope that the results presented here will encourage more researchers to explore this largely unbeaten path.

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