DIFFERENTIAL CERENKOV COUNTERS IN HIGH QUALITY BEAMS

D. H. White
Cornell University

ABSTRACT

The criteria for designing differential Cerenkov counters at high momenta are discussed.

1. BASIC CONSIDERATIONS

It is often said that the present DISC counters are capable of separating the known stable particles in a high momentum beam. The purpose of this note is to point out some rules of thumb that can help to design a simple differential counter that will operate satisfactorily in the secondary beams at NAL. First we note that the Cerenkov range is small and the small angle approximations are fine. θ is given by $\cos\theta = 1/\beta n$. At high energies we can express β in terms of γ, a more relevant quantity

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

$$1 - \beta^2 = \frac{1}{\gamma^2}$$

$$\beta = 1 - \frac{1}{2\gamma^2} \quad .$$

The refractive index n of the medium is close to one, so put

$$n = 1 + \delta, \qquad \delta \text{ small}$$

then

$$1 - \frac{\theta^2}{2} = \frac{1}{(1 + \delta)\beta}$$

$$= (1 + \delta)^{-1} \left(1 - \frac{1}{2\gamma^2}\right)^{-1} .$$

Expand, keeping first order terms in small quantities:

$$1 - \frac{\theta^2}{2} = (1 - \delta)\left(1 + \frac{1}{2\gamma^2}\right)$$

$$-\frac{\theta^2}{2} = -\delta + \frac{1}{2\gamma^2}$$

$$\delta = \frac{1}{2}\left(\theta^2 + \frac{1}{\gamma^2}\right) . \tag{1}$$

It is important, all other things being equal, that θ be as large as possible, for in this way we get the largest number of photons per unit length. Suppose we have fixed θ; then the variation of δ with γ should be large.

$$\delta = \frac{1}{2}\left(\theta^2 + \frac{1}{\gamma^2}\right)$$

$$d\delta = -\frac{1}{\gamma^3}\, d\gamma$$

$$\frac{d\delta}{\delta} = -\frac{2}{\gamma^3\left(\theta^2 + \frac{1}{\gamma^2}\right)}\, d\gamma$$

$$= -\frac{2}{\left(\theta^2\gamma^2 + 1\right)}\,\frac{d\gamma}{\gamma} .$$

It is important that the denominator not be allowed to grow too large, so we can assign a limit $\gamma^2\theta^2 = 1$, above which our sensitivity is reduced. We set $\gamma = 1/\theta$ therefore.

Number of Photons

An expression which yields the number of photons in the sensitive region of a photomultiplier is

$$N = 400 \sin^2\theta \times \ell ,$$

where ℓ is the length of path in the radiating medium, in cm. Using small angles,

$$\frac{N}{400} = \theta^2\, \ell ,$$

but since $\theta = 1/\gamma$

$$\ell = \gamma^2 \frac{N}{400} .$$

Given some prejudice for how many photons we think we need, the length of the counter for the limiting condition on γ is set. Moreover if we say that the photons are to be collected without reflection from the walls then the radius of the optical system that focuses the light into a ring at the aperture stop is given by

$$\frac{R}{\ell} = \theta .$$

So

$$R = \ell \frac{1}{\gamma} = \gamma \times \frac{N}{400} . \tag{2}$$

If we return to the optical density required $\delta = n - 1 = 1/\gamma^2$, setting $\theta = 1/\gamma$. Before we can settle on these parameters we must check that θ is large compared to the beam divergences we must cope with. In Table I is shown the value of θ, R, and ℓ for various values of γ under these assumptions. We can see for γ up to 100 this counter seems feasible.

II. PARTICLE SEPARATION

Another consideration is whether we can separate π, K, and $\overline{p}$. Suppose the beam momentum is p and we have two particles of masses m_1 and m_2. Relativistically $\gamma_1 = p/m_1$, $\gamma_2 = p/m_2$. Suppose we have designed the counter optimally for particle 1 so that $\theta = 1/\gamma_1$. We calculate the difference in Cerenkov angle of the two particles

$$\Delta\theta = \sqrt{2\delta - \frac{1}{\gamma_1^2}} - \sqrt{2\delta - \frac{1}{\gamma_2^2}}$$

As a figure of merit

$$\frac{\Delta\theta}{\theta} = \frac{\sqrt{2\delta - \frac{1}{\gamma_1^2}} - \sqrt{2\delta - \frac{1}{\gamma_2^2}}}{\sqrt{2\delta - \frac{1}{\gamma_1^2}}}$$

Additionally $\delta = 1/\gamma_1^2$. Assuming the counter is set at the optimum point in the range,

$$\frac{\Delta\theta}{\theta} = \frac{\dfrac{1}{\gamma_1} - \sqrt{\dfrac{2}{\gamma_1^2} - \dfrac{1}{\gamma_2^2}}}{\dfrac{1}{\gamma_1}}$$

$$= 1 - \sqrt{2 - \left(\frac{\gamma_1}{\gamma_2}\right)^2}$$

$$= 1 - \sqrt{2 - \left(\frac{m_2}{m_1}\right)^2} .$$

If m_1 is a kaon and m_2 a pion $(m_2/m_1)^2$ is negligible and

$$\frac{\Delta\theta}{\theta} = 1 - \sqrt{2},$$

the negative sign implies that the θ for the pion is greater. If m_2 is a proton then the fact that the square root is negative implies that the protons are below threshold, which we can verify by looking at Eq. (1).

In Table I we also show values of $\Delta\theta$ for K and π as a function of the design γ of the K.

In Table I we also show the pressure of CO_2 needed in the counter for the design γ and the multiple scattering induced by the gas alone.

III. CONCLUSION

We conclude that conventional differential counters can be built to operate in the range of momenta we expect to meet at NAL. It is worth noting that a counter designed at BNL by Kycia to operate in the few GeV/c range has characteristics similar to that in line 1 of Table I (for $\gamma = 10$) as we might expect and operates very well.

Remember if we need more photons we simply make the counter longer and larger in radius in proportion. The larger radius also makes the counter less sensitive to the finite size of the beam.

It is proper to ask why should one build a DISC counter at all. It is because the care taken to correct the dispersion and careful optical design means that the counter operates well when it is set far from the point that we regard as ideal in this design and still gives large rejection ratios. It remains to be seen whether the flexibility given by low pressures and large spaces in which to set these counters offsets the possibly limited momentum range that this less ambitious engineering design (than the DISC) affords.

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Table I. Design Parameters for Gas Cerenkov Counter.

Design (γ)	length, (m)	radius (cm)	$n - 1 = \delta \times 10^{-3}$	θ (mrad)	$\Delta\theta$ π, K (mrad)	$\Delta\theta$ disp (mrad)	Pressure CO_2(atm)	θ (multiple scatt) for K, (mrad)
10	0.25	2.5	10	100	40	0.2	21.9	0.46
20	1.0	5	2.5	50	20	0.1	5.48	0.23
30	2.25	7.5	1.1	33	13	0.06	2.41	0.15
40	4.0	10.	0.63	25	10	0.05	1.38	0.11
50	6.25	12.5	0.40	20	8	0.04	0.877	0.90
60	9.00	15.	0.28	17	7	0.03	0.614	0.8
70	12.25	17.5	0.20	14	6	0.03	0.44	0.7
80	16.00	20.	0.16	12.5	5	0.03	0.35	0.6
90	20.25	22.5	0.12	11	4	0.02	0.26	0.5
100	25.00	25	0.10	10	4	0.02	0.22	0.5

NOTES

1. The parameters are calculated for the number of photons N = 100.
2. From AIP Handbook of Physics

$$(n - 1) \text{ for } CO_2 \text{ at NTP} = 0.4565 \times 10^{-3} \text{ at } 4360 \text{ Å}$$
$$\text{and} \quad 0.4579 \times 10^{-3} \text{ at } 8678 \text{ Å}$$

3. With these parameters at the design condition the counter contains 1 gm/cm^2 of CO_2 as radiator.
4. The multiple scattering angle for 1 gm of CO_2 for K mesons is

$$\theta = \frac{15}{p} \sqrt{\frac{\ell}{\ell\,\text{rad}}} = \frac{4.6}{\gamma} \text{mrad.}$$