Assisted dark energy

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Abstract

We study cosmological dynamics of a multi-field system for a general Lagrangian density having scaling solutions. This allows the possibility that scaling radiation and matter eras are followed by a late-time cosmic acceleration through an assisted inflation mechanism. Using the bound coming from Big-Bang-Nucleosynthesis (BBN) and the condition under which each field cannot drive inflation as a single component of the universe, we find the following features: (i) a transient or eternal cosmic acceleration can be realized after the scaling matter era, (ii) a "thawing" property of assisting scalar fields is crucial to determine the evolution of the field equation of state w_ϕ, and (iii) the field equation of state today can be consistent with the observational bound w_ϕ in the presence of multiple scalar fields.

1 Introduction

The constantly accumulating observational data continue to confirm the existence of dark energy responsible for cosmic acceleration today. The cosmological constant, whose equation of state is $w = -1$, has been favored by the combined data analysis of supernovae Ia, cosmic microwave background, and baryon acoustic oscillations. Meanwhile, if the cosmological constant originates from a vacuum energy associated with particle physics, its energy scale is enormously larger than the observed value of dark energy ($\rho_{\rm DE} \approx 10^{-47}\,{\rm GeV}^4$). Hence it is important to pursue an alternative possibility to construct dark energy models consistent with particle physics.

Scalar-field models such as quintessence and k-essence have been proposed to alleviate the above problem. In particular cosmological scaling solutions are attractive to alleviate the energy scale problem of dark energy because the solutions enter the scaling regime even if the field energy density is initially comparable to the background fluid density. However the condition required for the existence of scaling solutions is incompatible with the condition for the existence of a late-time accelerated solution. Hence, in the single field case, the scaling solution cannot be followed by the scalar-field dominated solution responsible for dark energy. One of the ways to allow a transition from the scaling regime to the epoch of a late-time cosmic acceleration is to consider multiple scalar fields. For a general multi-field Lagrangian density having scaling solutions, we discuss how scaling radiation and matter eras are followed by an epoch of the late-time cosmic acceleration.

2 Dynamical system

Let us start with the following 4-dimentional action

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + p(\phi, X)\right] + S_f(\phi), \tag{1}$$

where R is a scalar curvature, p is a general function in terms of the field ϕ and a kinetic term $X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$. S_f is an action for a background fluid which generally couples to the field ϕ. The existence of cosmological scaling solutions demands that the field energy density ρ_ϕ is proportional to the background fluid density ρ_f. Under this condition the Lagrangian density is restricted to take

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the following form with arbitrary function $g(Xe^{\lambda\phi})$ in the flat homogeneous and isotropic cosmological background [1, 2];

$$p(X,\phi) = Xg(Xe^{\lambda\phi})\,, \tag{2}$$

where λ is a constant. If we consider multiple scalar fields ϕ_i $(i = 1, 2, \cdots, n)$ with the Lagrangian density

$$p = \sum_{i=1}^{n} X_i g(X_i e^{\lambda_i \phi_i})\,, \tag{3}$$

the scaling solution can be followed by the accelerated scalar-field dominated point through the assisted inflation mechanism [3]. The multiple scalar fields evolve to give dynamics matching a single-field model with [3–5]

$$\frac{1}{\lambda_{\text{eff}}^2} = \sum_{i=1}^{n} \frac{1}{\lambda_i^2}. \tag{4}$$

For the Lagrangian density (3) the dynamical field equations can cast into the form of autonomous equations [6]. In the following we study the case in which one of the fields (ϕ_1) has a large slope λ_1 to satisfy a BBN bound and other fields join the scalar-field dominated attractor at late times to give rise to cosmic acceleration. We have three fixed points relevant to radiation, matter, and accelerated epochs.

First of all, the field equation of state for the radiation-dominated scaling solution is $w_{\phi_1} = 1/3$. The constraint on the field density parameter Ω_{ϕ_1} coming from the BBN is

$$\Omega_{\phi_1} = 4p_{,X_1}/\lambda_1^2 \lesssim 0.045\,, \tag{5}$$

where $p_{,X_1} \equiv \partial p/\partial X_1$.

Second, the field equation of state for the matter-dominated scaling solution is $w_{\phi_1} = 0$. The field density parameter is

$$\Omega_\phi = 3p_{,X_1}/\lambda_1^2\,. \tag{6}$$

and then $\lambda_1^2 > 3p_{,X_1}$ is required [5].

Finally, in the case of the assisted field-dominated point, the field equation of state is

$$w_\phi = -1 + \lambda_{\text{eff}}^2/3p_{,X}\,, \tag{7}$$

where $X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$ is a kinetic energy of the effective single field ϕ. The fixed point can be responsible for the late-time acceleration for $\lambda_{\text{eff}}^2 < 2p_{,X}$. Moreover, it is stable under the condition $\lambda_{\text{eff}}^2 < 3p_{,X}$ [5].

3 Quintessence with multiple exponential potentials

We study multi-field cosmological dynamics for a quintessence model with exponential potentials [3]. This corresponds to the Lagrangian density $p_i = X_i - c_i e^{-\lambda_i\phi_i}$, i.e. the choice $g(X_i e^{-\lambda_i\phi_i}) = 1 - c_i/(X_i e^{\lambda_i\phi_i})$. Since $p_{,X_i} = 1$ in this model, the BBN bound and the condition for cosmic acceleration give

$$\lambda_1 > 9.42\,, \quad \text{and} \quad \lambda_{\text{eff}} < \sqrt{2}\,, \tag{8}$$

respectively.

Let us consider two fields ϕ_1 and ϕ_2. In Fig. 1 we plot the evolution of the background fluid density $\rho_f = \rho_r + \rho_m$ (radiation + non-relativistic matter) and the field densities ρ_{ϕ_1}, ρ_{ϕ_2} versus the redshift z for $\lambda_1 = 10$ and $\lambda_2 = 1.5$. The cases (i), (ii), (iii) correspond to the simulations for three different initial conditions of ϕ_1. Figure 1 shows that the field ϕ_1 joins the scaling regime irrespective of initial conditions of ϕ_1. Finally the system enters the epoch in which the energy density ρ_{ϕ_2} of the second field ϕ_2 dominates the dynamics. Note that the second field density ρ_{ϕ_2} is almost frozen after the initial transient period.

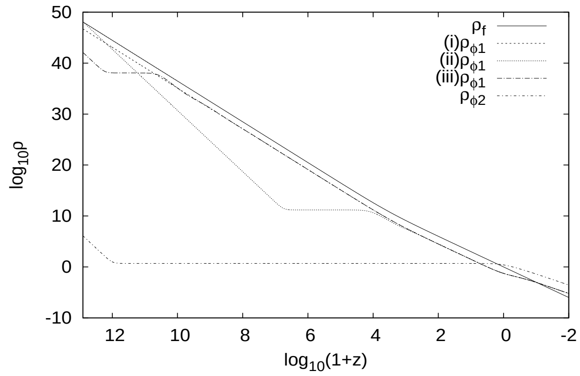

Figure 1: Example for the evolution of ρ_f, ρ_{ϕ_1}, ρ_{ϕ_2} for the qunitessence with two exponential potentials. We choose three different initial conditions for ϕ_1.

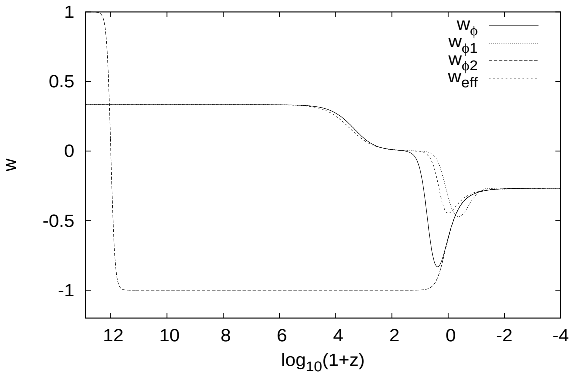

Figure 2: Example for the evolution of w_ϕ, w_{ϕ_1}, w_{ϕ_2}, and $w_{\rm eff}$ for the qunitessence with two exponential potentials.

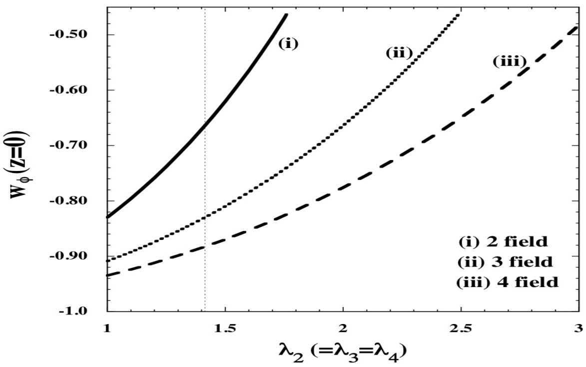

Figure 3: The equation of state w_ϕ today versus λ_i $(i \neq 1)$ for $\lambda_1 = 9.43$ in the multi-field quintessence with exponential potentials.

Figure 2 illustrates the variation of the equation of state with the same initial condition as in the case (i) of Fig. 1. Here w_ϕ and $w_{\rm eff}$ are defined by

$$w_\phi = \frac{w_{\phi_1}\Omega_{\phi_1} + w_{\phi_2}\Omega_{\phi_2}}{\Omega_{\phi_1} + \Omega_{\phi_2}}\,, \qquad w_{\rm eff} = -1 - \frac{2\dot{H}}{3H^2}\,, \tag{9}$$

where H is the Hubble expansion rate. Since $w_\phi = -0.27$ (attractor) and $w_\phi = -0.62$ $(z = 0)$ in this case, the transient acceleration occurs at the present epoch.

In Fig. 3 we show $w_\phi(z = 0)$ versus λ_i $(i \geq 2)$ for $\lambda_1 = 9.43$ in the presence of multiple scalar fields. This shows that, as we add more fields, we obtain smaller values of $w_\phi(z = 0)$. The observational bound $w_\phi(z = 0) < -0.8$ can be satisfied in the presence of more than two fields.

4 Multi-field dilatonic ghost condensate model

Next we proceed to the multi-field dilatonic ghost condensate model [1] with the Lagrangian density $p_i = -X_i + c_i e^{\lambda_i\phi_i} X_i^2$. This corresponds to the choice $g(X_i e^{\lambda_i\phi_i}) = -1 + c_i(X_i e^{\lambda_i\phi_i})$. In this model the BBN bound and the condition for cosmic acceleration translate into

$$\lambda_1 > 9.42\sqrt{2\tilde{Y}_1 - 1}\,, \qquad \lambda_{\rm eff} < \sqrt{6}/3\,, \tag{10}$$

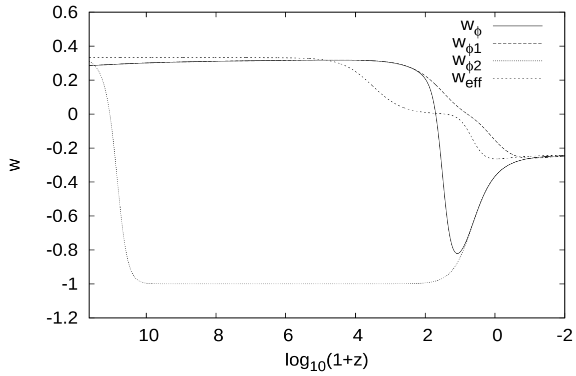

Figure 4: Example for the evolution of w_ϕ, w_{ϕ_1}, w_{ϕ_2}, and $w_{\rm eff}$ for $\lambda_1 = 40$ and $\lambda_2 = 1$ in the two-field dilatonic ghost condensate model.

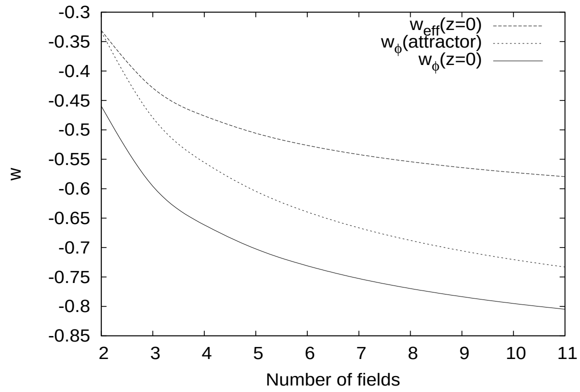

Figure 5: The equations of state versus the number n of scalar fields for $\lambda_1 = 40$ and $\lambda_i = 0.817$ $(i \geq 2)$ in the multi-field dilatonic ghost condensate model.

respectively (where $\tilde{Y}_1 = c_1 e^{\lambda_1 \phi_1} X_1$).

Figure 4 shows the evolution of the equations of state. In this model the scaling solution is absent during the radiation era (exists only in the limit $\tilde{Y}_1 \to \infty$), while it is present during the matter dominance. It takes some time for the solution to reach the scaling matter point characterized by $\tilde{Y}_1 = 1$. Hence, the period of the scaling matter era is very short. As in the case of multi-field quintessence with exponential potentials, w_ϕ first reaches a minimum and then starts to grow toward the assisted attractor. In Fig. 5 we plot $w_\phi(z=0)$, $w_{\rm eff}(z=0)$, and w_ϕ at the late-time attractor for $\lambda_1 = 40$ and $\lambda_i = 0.817$ $(i \geq 2)$. This shows that we require at least 10 scalar fields to realize the observational bound $w_\phi(z=0) < -0.8$.

5 Conclusion

We have studied cosmological dynamics of assisted dark energy for the Lagrangian density (3) that possesses scaling solutions. In the presence of multiple scalar fields the scaling matter era can be followed by the phase of a late-time cosmic acceleration as long as more than one field join the assisted attractor. Since the effective slope $\lambda_{\rm eff}$ is smaller than the slope λ_i of each field, the presence of multiple scalar fields can give rise to cosmic acceleration even if none is able to do so individually. This is a nice feature from the viewpoint of particle physics because there are in general many scalar fields (dilaton, modulus, etc) with the slopes λ_i larger than the order of unity. For quintessence with exponential potentials and the multi-field dilatonic ghost condensate model, we have shown that a thawing property of assisting multiple scalar fields allows the field equation of state w_ϕ smaller than -0.8 today.

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