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Weak Interactions at Very High Energies: the Role of the Higgs Boson Mass

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ABSTRACT

We give an S-matrix theoretic demonstration that if the Higgs boson mass exceeds $M_c = (8\pi\sqrt{2}/3G_F)^{\frac{1}{2}}$ partial-wave unitarity is not respected by the tree diagrams for two-body scattering of gauge bosons, and the weak interactions must become strong at high energies. We exhibit the relation of this bound to the structure of the Higgs-Goldstone Lagrangian, and speculate on the consequences of strongly-coupled Higgs-Goldstone systems. Prospects for the observation of massive Higgs scalars are noted.

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I. INTRODUCTION

Unified gauge theories of weak and electromagnetic interactions provide an attractive framework for the interpretation of weak interaction phenomena.[1] Such theories are universal in the prediction that existing data explore only the low-energy tail of a spectrum of yet-to-be-discovered particles. The most familiar of the hypothetical particles are the massive vector bosons $W^{\pm}$ and Z^0 associated with the observed charged and neutral weak currents. Somewhat more obscure are the massive scalar Higgs bosons which are connected with the spontaneous breakdown of gauge symmetry. Although the Higgs bosons serve important technical functions in field-theoretic calculations, their existence and properties are less clearly indicated by low-energy phenomenology. Thus, for example, the mass M_H of the Higgs boson is the only parameter in the Weinberg-Salam model[2] that is entirely unconstrained by present experimental evidence.

Theoretical considerations[3] suggest that the Higgs boson mass must exceed about $4 \mathrm{GeV}/c^2$, and we have recently derived a conditional upper bound[4]

$$M_H \leq M_c = (8\pi\sqrt{2}/3G_F)^{\frac{1}{2}} \simeq 1 \ \mathrm{TeV}/c^2 \quad , \tag{1.1}$$

where G_F is the Fermi constant. The precise meaning of the upper bound is that if M_H exceeds the critical value M_c, weak interactions will become strong in the TeV energy regime in the sense that perturbation theory will cease to be a faithful representation of physics.

Because the Higgs self-interaction is proportional to $G_F M_H^2$, it frequently has been remarked that a large Higgs boson mass implies a strong interaction among Higgs bosons. Weinberg[5] has championed the view that $G_F^{-\frac{1}{2}}$ is a natural mass scale of nature and that, in the event of strong Higgs self-couplings, the effective ultraviolet cutoff would be at this energy. More recently Veltman[6] considered Higgs boson contributions to certain radiative corrections. He concluded that for Higgs boson masses exceeding approximately $G_F^{-\frac{1}{2}}$ the perturbation expansion of weak interactions could well break down. Our result (1.1) is in accord with these expectations.

The condition (1.1) suggests that new phenomena are to be found in the weak interactions in addition to the charged and neutral intermediate vector bosons. Either a light scalar boson (of mass well below 1 TeV) will exist, or the weak interactions above about 1 TeV will exhibit attributes of a strongly-coupled theory: resonances of intermediate vector bosons, multiple production of intermediate vector bosons, etc.

If the Higgs boson is not very massive, say with a mass between 4.5 GeV/c^2 (the Linde-Weinberg lower bound[3]) and $2M_W$, we expect it to have the properties outlined by Ellis, Gaillard, and Nanopoulos.[7] We shall explore in this paper the possibility that the Higgs boson mass lies above the thresholds for decay into intermediate boson pairs. In this regime the decays $H \rightarrow W^+W^-$ and $H \rightarrow Z^0Z^0$ are the dominant modes, with longitudinally polarized intermediate bosons increasingly favored as M_H increases. As M_H approaches the critical mass M_c, the Higgs boson width approaches its mass, signalling a strongly-coupled theory.

Because we wish to explore a regime in which the weak interactions can become strong it is natural to approach the problem from an S-matrix point of view[8] with a particular concern for unitarity.[9] Our treatment provides a systematic investigation of the minimal Weinberg-Salam theory from this point of view. In Section II we discuss and calculate in tree approximation the Weinberg-Salam model amplitudes for all two-body reactions of gauge bosons with zero total electric charge in the s-channel. We display only those terms that are potentially relevant to the question of unitarity, omitting, for example, terms which are of ordinary electromagnetic strength at all energies. Logarithmic violations of unitarity that occur at exponentially high energies $\sim M_W e^{1/\alpha}$ will be of no concern to us here.

By focusing only on those amplitudes that constitute a potential threat to unitarity one finds a remarkable simplification of the problem. The relevant amplitudes are those which involve only longitudinal gauge bosons and the Higgs boson. The system of these particles is the subject of Section III. There it is shown that at energies large compared with the intermediate boson mass, this system is a clear reflection of the underlying Higgs-Goldstone system of the Weinberg-Salam model, with the longitudinal W^+, W^- and Z^0 behaving much like the Goldstone bosons from which they sprang. Up to terms of order $M_W/\sqrt{s}$, the S-matrix for $W_L^{\pm}$, Z_L, and H (the subscript L denotes longitudinal polarization) is identical to that for the self-interactions of a complex doublet of scalar particles, as we show in detail in an Appendix. Consequently exploration of the issue of unitarity and of the strength of high-energy weak interactions reduces to the study of a set of strongly-coupled self-interacting scalar fields. In Section IV we pursue this issue using the N/D method. Section V contains some aspects of the phenomenology of heavy Higgs particles together with a discussion of our results.

II. GAUGE BOSON SCATTERING IN THE WEINBERG-SALAM MODEL

Our presentation will center on the helicity amplitudes for two-body reactions of gauge bosons with zero total electric charge in the direct channel. In this Section we shall discuss all such amplitudes, retaining those terms which are potentially relevant to the issue of unitarity. For our purposes, contributions which are manifestly of order α or less at all energies can safely be disregarded. A non-exceptional value of the weak interaction angle θ_W is inferred from experiment.[10] Thus all gauge coupling constants are assumed to be of order $\alpha^{\frac{1}{2}}$. It will be seen that neglect of the innocuous terms dramatically reduces the number of amplitudes which must be considered, so that a complete treatment becomes quite manageable.

The particle content of the Weinberg-Salam model includes three massive intermediate bosons $\{V\} \equiv (W^+, W^-, Z^0)$, a neutral Higgs particle (H), the photon (γ) and a lepton doublet (e, ν). We include the last only to dismiss it from our considerations. Counting all helicity states, there are 39 neutral two-particle channels: $W^+W^-(9)$, $ZZ(9)$, $HH(1)$, HZ(3), $\gamma\gamma(4)$, $\gamma Z(6)$, $\gamma H(2)$, $e\bar{e}(4)$, and $\nu\bar{\nu}(1)$. We will find that, within the approximations outlined, all but four of these channels decouple in the sense that the partial wave amplitudes are small at all energies (except very near the particle poles or at exponentially large energies) for any value of the Higgs boson mass. Of the surviving channels, one (HZ_L) is isolated and three ($W_L^+W_L^-$, Z_LZ_L, HH) are coupled. We will first describe the 3×3 t-matrix for the latter system. Then we demonstrate that all other channels decouple. In keeping with our S-matrix approach, all calculations in this Section are done in the unitary gauge.

1. $W_L^+W_L^- \to W_L^+W_L^-$

For this case alone we shall detail the interplay of the several Feynman graphs that make up the tree approximation to the scattering amplitude.[8]

It is convenient to classify the cancellations among the tree graphs according to the power of (q/M_W) which enters, where q is the c.m. momentum of the gauge bosons. The high-energy behavior of the individual graphs in Fig. 1 is at worst $\sim(q/M_W)^4$. Consequently the contribution of each graph to the J-th partial wave may be written as

$$a_J = A(q/M_W)^4 + B(q/M_W)^2 + C \quad , \tag{2.1}$$

where the partial-wave amplitude a_J is defined through

$$T(s, t) = 16\pi \sum_J (2J + 1)a_J(s)P_J(\cos \theta) \quad . \tag{2.2}$$

We will refer to the coefficients in (2.1) as A-, B-, and C-forces, and refer to a force as attractive or repulsive if the coefficient is positive or negative. It will be convenient to define the dimensionless weak coupling analogous to the fine structure constant

$$\alpha_W = G_F M_W^2 \sqrt{2}/\pi = \alpha/\sin^2 \theta_W \quad . \tag{2.3}$$

All the divergent high-energy behavior of the graphs in Fig. 1 is confined to the J = 0, 1, and 2 partial waves. In each case the vanishing of A-forces results from a gauge cancellation among the contact graph and the s- and t-channel (γ + Z)-exchanges. In the J = 2 partial wave the cancellation of B-forces is also pure gauge. For the J = 0 and 1 partial waves the B-force cancellations involve the Higgs boson in an essential way.

Having noted the disappearance of all high-energy divergences, we are led to consider the surviving C-force terms which have acceptable asymptotic behavior but are not necessarily small. It will be convenient for us to present here the invariant amplitudes, deferring the partial-wave projections to Section III. The C-forces contributed by the contact graph and by the γ- and Z-exchanges are of order α_W at (almost) all energies and hence zero in our approximation. In contrast, the Higgs-exchange graphs produce C-forces which can be of order unity for large enough Higgs mass. Thus for our purposes the full amplitude is given by the C-force terms of the s- and t-channel Higgs exchange graphs,

$$T(W_L^+W_L^- \rightarrow W_L^+W_L^-) = -\sqrt{2}\, G_F M_H^2 \left[\frac{s}{s - M_H^2} + \frac{t}{t - M_H^2}\right] . \quad (2.4)$$

2. $Z_L Z_L \rightarrow Z_L Z_L$

Having traced the pattern of cancellations of divergences in the graphs for $W_L^+W_L^- \rightarrow W_L^+W_L^-$, we would serve no useful function by describing in similar detail the cancellations for other processes. We focus instead on the heart of the matter, namely the convergent C-force terms. For the reaction $Z_L Z_L \rightarrow Z_L Z_L$ there are three graphs shown in Fig. 2, the s-, t-, and u-channel Higgs boson exchanges. The resulting amplitude is

$$T(Z_L Z_L \to Z_L Z_L) = -\sqrt{2}\, G_F M_H^2 \left[\frac{s}{s - M_H^2} + \frac{t}{t - M_H^2} + \frac{u}{u - M_H^2} \right] . \qquad (2.5)$$

3. $HH \to HH$

The four graphs in Fig. 3 contribute to Higgs boson elastic scattering. In this case the evaluation of the amplitude involves no cancellations. The result is

$$T(HH \to HH) = -3\sqrt{2}\, G_F M_H^2 \left[1 + \frac{3M_H^2}{s - M_H^2} + \frac{3M_H^2}{t - M_H^2} + \frac{3M_H^2}{u - M_H^2} \right] . \qquad (2.6)$$

4. $Z_L Z_L \to W_L^+ W_L^-$

The four graphs contributing to this process are shown in Fig. 4. The only non-negligible C-force arises from the s-channel Higgs exchange, which gives

$$T(Z_L Z_L \to W_L^+ W_L^-) = -\sqrt{2}\, G_F M_H^2 \frac{s}{s - M_H^2} . \qquad (2.7)$$

5. $HH \to W_L^+ W_L^-$

Each of the four graphs in Fig. 5 contributes a non-vanishing C-force. The Feynman amplitude is

$$T(HH \to W_L^+ W_L^-) = -\sqrt{2}\, G_F M_H^2 \left[1 + \frac{3M_H^2}{s - M_H^2} + \frac{M_H^2}{t - M_W^2} + \frac{M_H^2}{u - M_W^2} \right] , \qquad (2.8)$$

where the four terms emerge from the contact graph, the s-, t-, and u-channel Higgs exchanges, respectively.

6. $HH \to Z_L Z_L$

The Feynman graphs and the results are identical in form to those for $HH \to W_L^+ W_L^-$:

$$T(HH \to Z_L Z_L) = -\sqrt{2}\, G_F M_H^2 \left[1 + \frac{3M_H^2}{s - M_H^2} + \frac{M_H^2}{t - M_Z^2} + \frac{M_H^2}{u - M_Z^2} \right] . \qquad (2.9)$$

7. Decoupled Channels

We shall first dispose of all channels which involve one or more transversely polarized intermediate bosons. Processes involving HZ_L and leptons will be dealt with separately. The channels with transversely polarized vector particles are unimportant for unitarity considerations because the corresponding tree graphs have softer high-energy behavior, graph by graph, than those for longitudinal polarization. By the nature of the gauge cancellations, this translates into the absence of any residual C-forces which are potentially strong (i.e. of order unity). The values of the energy and Higgs mass for which unitarity bounds may be approached or surpassed in longitudinal amplitudes are such that M_V^2/s, $M_V^2/M_H^2 \lesssim \mathcal{O}(\alpha_W)$. In this regime the transverse amplitudes are of relative order $\alpha_W^{n/2}$, where n is the number of transversely polarized particles involved in the reaction. All such amplitudes are therefore negligible under the conditions of interest to us. These assertions can easily be verified by direct calculation, but the conclusion that transverse channels decouple also follows as a corollary to the discussion in Section III. There it will be argued that our approximations are leading us back to the underlying Higgs-Goldstone system of the Weinberg-Salam model.

Turning to the HZ_L channel we observe that the elastic amplitude for $HZ_L \rightarrow HZ_L$, which is obtained by crossing from (2.9), does have non-vanishin C-force terms. However, the off-diagonal amplitudes are negligible. The HZ_L channel is thus isolated from the other channels of interest discussed earlier. To see that HZ_L decouples, note first that the reactions $HZ \rightarrow ZZ$ and $HZ \rightarrow HH$ do not occur at tree level. Evaluation of the amplitude for $HZ_L \rightarrow W_L^+ W_L^-$ involves some intricate but by now familiar cancellations among s-channel (γ + Z)-exchange and crossed-channel W-exchanges, and leads to an amplitude which is negligible in our approximation. A group theoretic argument for the isolation of HZ_L will be presented in Section III.

Concerning the leptonic channels $e\bar{e}$ and $\nu\bar{\nu}$, the only processes of conceivable relevance to the present discussion are of the general type $\ell\bar{\ell} \rightarrow V_L V_L$. We shall describe the calculation of $e^+e^- \rightarrow W_L^+ W_L^-$ in full; other reactions of the class are also inconsequential for similar reasons. For leptons of opposite helicity, the t-channel ν-exchange graph generates B-force terms which are confined to the J = 1 partial wave. These terms are exactly cancelled by the s-channel (γ + Z)-exchange.

By now it is evident that in order to obtain a potentially interesting amplitude we must produce a factor of M_H^2. Since the Higgs boson in the direct channel can play no role at all in the J = 1 partial wave, no such factor is forthcoming and the amplitude is negligible. The Higgs boson does become involved if the leptons have the same helicity. In this case the ν-exchange graph produces a divergent high-energy behavior in the J = 0 partial wave which must be cancelled by s-channel Higgs exchange. However the lepton mass factors in the Higgs-fermion-fermion coupling make the resulting amplitude utterly negligible.

We have now shown that the question of unitarity in the tree graph approximation to the Weinberg-Salam model reduces to the study of a 3×3 matrix of amplitudes for the channels $W_L^+W_L^-$, Z_LZ_L, and H_LH_L (along with the isolated HZ_L channel). In the following Section we discuss the relevant amplitudes further, derive a high-energy unitarity bound which sets a conditional upper limit on the mass of the Higgs boson, and relate the results to the Higgs-Goldstone system of the Weinberg-Salam model.

III. PARTIAL-WAVE UNITARITY, THE HIGGS BOSON MASS, AND THE HIGGS-GOLDSTONE SYSTEM

By taking partial-wave projections of the Feynman amplitudes (2.4) - (2.9), we can construct the elements of the coupled-channel t-matrix for the J = 0 partial wave. Assuming $s, M_H^2 >> M_W^2, M_Z^2$ we have

$$a_0(W_L^+W_L^- \to W_L^+W_L^-) = \frac{-G_F M_H^2}{8\pi\sqrt{2}}\left[2 + \frac{M_H^2}{s - M_H^2} - \frac{M_H^2}{s}\log\left(1 + \frac{s}{M_H^2}\right)\right]; \quad (3.1a)$$

$$a_0(Z_L Z_L \to Z_L Z_L) = \frac{-G_F M_H^2}{8\pi\sqrt{2}}\left[3 + \frac{M_H^2}{s - M_H^2} - \frac{2M_H^2}{s}\log\left(1 + \frac{s}{M_H^2}\right)\right]; \quad (3.1b)$$

$$a_0(HH \to HH) = \frac{-G_F M_H^2}{8\pi\sqrt{2}}\left[3 + \frac{9M_H^2}{s - M_H^2} - \frac{18M_H^2}{s - 4M_H^2}\log\left(\frac{s}{M_H^2} - 3\right)\right]; \quad (3.1c)$$

$$a_0(Z_L Z_L \to W_L^+W_L^-) = \frac{-G_F M_H^2}{8\pi\sqrt{2}}\left[1 + \frac{M_H^2}{s - M_H^2}\right]; \quad (3.1c)$$

$$a_0(HH \to W_L^+W_L^-) = a_0(HH \to Z_L Z_L)$$

$$= \frac{-G_F M_H^2}{8\pi\sqrt{2}}\left[1 + \frac{3M_H^2}{s - M_H^2} + \frac{4M_H^2}{\sqrt{s(s - 4M_H^2)}}\log\left(\frac{s - 2M_H^2 - \sqrt{s(s - 4M_H^2)}}{2M_H^2}\right)\right]; \quad (3.1e)$$

$$a_0(HZ_L \to HZ_L) = \tag{3.1f}$$

$$\frac{-G_F M_H^2}{8\pi\sqrt{2}}\left[1 + \frac{M_H^2}{s} - \frac{3M_H^2 s}{(s-M_H^2)^2}\log\left(1+\frac{(s-M_H^2)^2}{sM_H^2}\right) - \frac{M_H^2 s}{(s-M_H^2)^2}\log\left(\frac{s(2M_H^2-s)}{M_H^4}\right)\right] ;$$

$$a_0(HZ_L \to HH) = 0 ,$$

$$a_0(HZ_L \to Z_L Z_L) = 0 , \tag{3.1g}$$

$$a_0(HZ_L \to W_L^+ W_L^-) = 0 .$$

We first consider the effect of the elastic unitarity condition for the reaction $W_L^+ W_L^- \to W_L^+ W_L^-$,

$$|a_0(W_L^+ W_L^- \to W_L^+ W_L^-)| \leq 1 . \tag{3.2}$$

At energies far above the Higgs pole the amplitude (3.1a) approaches a constant,

$$a_0(W_L^+ W_L^- \to W_L^+ W_L^-) \underset{s \gg M_H^2}{\sim} \frac{-G_F M_H^2}{4\pi\sqrt{2}} . \tag{3.3}$$

Consequently, in order for the tree approximation to respect the unitarity bound at high energies the Higgs boson mass must satisfy

$$M_H^2 \leq \frac{4\pi\sqrt{2}}{G_F} . \tag{3.4}$$

The nature of this upper bound is to delimit the class of weak interaction theories in which low orders of perturbation theory are expected to be a reliable guide to physical phenomena.

The behavior of $|a_o(W_L^+W_L^- \to W_L^+W_L^-)|$ is shown in Fig. 6 for Higgs boson masses well below and at the critical value (3.4). If M_H is substantially less than the critical value, the magnitude of the amplitude is well within the bound (3.2) everywhere except near the Higgs boson resonance pole, where finite-width corrections are sufficient to rescue the bound. In contrast, if M_H attains or exceeds the critical value, the unitarity bound will be violated by the tree approximation at all energies above the Higgs boson pole. Higher order effects will necessarily become important at high energies, and high energy weak interactions take on considerable added richness.

It is possible to refine the bound (3.4) somewhat by considering the requirements of partial-wave unitarity on the four-channel system consisting of $W_L^+W_L^-$, $\frac{1}{\sqrt{2}}Z_LZ_L$, $\frac{1}{\sqrt{2}}HH$, and HZ_L, with amplitudes given by (3.1). For $s >> M_H^2$ each amplitude approaches a constant, so the 4×4 t-matrix takes the form

$$t_0 \underset{s >> M_H^2}{\sim} \frac{-G_F M_H^2}{4\pi\sqrt{2}} \begin{bmatrix} 1 & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} & 0 \\ \frac{1}{\sqrt{8}} & \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{\sqrt{8}} & \frac{1}{4} & \frac{3}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix} . \quad (3.5)$$

The matrix t_0 has eigenvalues $\frac{3}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}$, in units of $\frac{-G_F M_H^2}{4\pi\sqrt{2}}$, corresponding to the (unnormalized) eigenchannels $2W_L^+W_L^- + Z_L Z_L + HH$, $2W_L^+W_L^- - Z_L Z_L - HH$, $Z_L Z_L - HH$, and HZ_L. It is striking that we arrive at a t-matrix with such a simple eigenchannel structure, and we will expose the reason for this simplicity later in this Section. The most stringent unitarity bound on (3.5) is derived from the requirement that the magnitude of the largest eigenvalue not exceed unity. This is ensured by the restriction on the Higgs boson mass,

$$M_H^2 \le \frac{8\pi\sqrt{2}}{3G_F} \equiv M_c^2 \simeq (1\ \text{TeV}/c^2)^2 \quad . \tag{3.6}$$

Further assessment of the meaning of this result and the various possibilities that follow from it will take up the succeeding sections. At this point we want to clarify the reason that the four channels $W_L^+W_L^-$, $\frac{1}{\sqrt{2}}Z_L Z_L$, $\frac{1}{\sqrt{2}}HH$, and HZ_L stand so clearly apart from other neutral two-body channels for $s \gg M_W^2, M_Z^2$. The basic point can be made by direct computation. Consider the Lagrangian for the Higgs sector of the Weinberg-Salam model before the gauge couplings are turned on,

$$\mathscr{L} = (\partial^\mu \phi)^\dagger (\partial_\mu \phi) + \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2 \quad , \tag{3.7}$$

where ϕ represents a complex scalar doublet

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad . \tag{3.8}$$

Shifting the origin of fields in the usual way, we find that (3.7) describes a theory with three massless Goldstone bosons (w^+, w^-, z^0) and one massive neutral particle (h) which interact according to

$$\mathscr{L}_I = -\lambda v h(2w^+w^- + z^2 + h^2) - \tfrac{1}{4}\lambda(2w^+w^- + z^2 + h^2)^2 \quad , \qquad (3.9)$$

where $v^2 = \mu^2/\lambda$. In the language of the full Weinberg-Salam theory, v and λ are related to the Fermi constant and the Higgs boson mass by

$$\left.\begin{aligned} 1/v^2 &= G_F\sqrt{2} \quad , \\ \lambda &= \frac{G_F M_H^2}{\sqrt{2}} \quad . \end{aligned}\right\} \qquad (3.10)$$

When the gauge couplings are turned on, the Goldstone bosons mix with the longitudinal components of the vector bosons. The masses acquired by the Goldstone bosons in the course of this mixing are gauge-dependent. We find it expedient to adopt the 't Hooft-Feynman gauge,[11] in which the masses of $w^\pm$ and z are M_W and M_Z, respectively.

The S-matrix for the scalar theory of (3.9) is easily calculated in tree approximation. Feynman graphs for the neutral two-body channels are shown in Figs. 7 - 11. The resulting amplitudes are

$$T(w^+w^- \to w^+w^-) = -\sqrt{2}\, G_F M_H^2 \left[\frac{s}{s - M_H^2} + \frac{t}{s - M_H^2}\right] \quad ; \qquad (3.11a)$$

$$T(zz \to zz) = -\sqrt{2}\, G_F M_H^2 \left[\frac{s}{s - M_H^2} + \frac{t}{t - M_H^2} + \frac{u}{u - M_H^2}\right] ; \qquad (3.11b)$$

$$T(hh \to hh) = -3\sqrt{2}\, G_F M_H^2 \left[1 + \frac{3M_H^2}{s - M_H^2} + \frac{3M_H^2}{t - M_H^2} + \frac{3M_H^2}{u - M_H^2}\right] ; \qquad (3.11c)$$

$$T(zz \to w^+w^-) = -\sqrt{2}\, G_F M_H^{\,2} \frac{s}{s - M_H^{\,2}} \quad ; \tag{3.11d}$$

$$T(hh \to w^+w^-) = -\sqrt{2}\, G_F M_H^{\,2} \left[1 + \frac{3M_H^{\,2}}{s - M_H^{\,2}} + \frac{M_H^{\,2}}{t - M_W^{\,2}} + \frac{M_H^{\,2}}{u - M_W^{\,2}} \right] ; \tag{3.11e}$$

$$T(hh \to zz) = -\sqrt{2}\, G_F M_H^{\,2} \left[1 + \frac{3M_H^{\,2}}{s - M_H^{\,2}} + \frac{M_H^{\,2}}{t - M_Z^{\,2}} + \frac{M_H^{\,2}}{u - M_Z^{\,2}} \right] ; \tag{3.11f}$$

$$T(hz \to hz) = -\sqrt{2}\, G_F M_H^{\,2} \left[1 + \frac{3M_H^{\,2}}{t - M_H^{\,2}} + \frac{M_H^{\,2}}{s - M_Z^{\,2}} + \frac{M_H^{\,2}}{u - M_Z^{\,2}} \right] ; \tag{3.11g}$$

$$T(hz \to w^+w^-) = 0 \quad ; \tag{3.11h}$$

$$T(hz \to zz) = 0 \quad ; \tag{3.11i}$$

$$T(hz \to hh) = 0 \quad . \tag{3.11j}$$

Comparing these amplitudes with those calculated in Section II for physical gauge boson scattering we observe that the amplitudes (2.4)-(2.9) which describe the high-energy limit of the full theory are identical to the corresponding amplitudes for the scalar theory. We are led to the following

__Theorem__: If $T(W_L^{\pm}; W_L^{-}; Z_L; H)$ is an amplitude for scattering of longitudinal intermediate bosons and physical Higgs particles in the Weinberg-Salam model and if $T(w^{\pm}; w^{-}; z; h)$ is the analogous amplitude for the scalar field theory described by (3.9), in the 't Hooft-Feynman gauge, then for $s \gg M_W^{\,2}, M_Z^{\,2}$,

$$T(W_L^+;\ W_L^-;\ Z_L;\ H) \sim T(w^+;\ w^-;\ z;\ h) + \mathcal{O}(M_W/\sqrt{s}) \quad . \tag{3.12}$$

The theorem stated in (3.12) is a lemma used in showing that spontaneously broken gauge theories comprise essentially all renormalizable massive vector boson theories,[12] apart from electrodynamics with massive photons. We supply a formal demonstration in the Appendix.

The identification of the physical Weinberg-Salam model amplitudes which are dominant at high energies with the amplitudes of the underlying Higgs-Goldstone system enables us to understand the simple eigenchannel structure of the high-energy t-matrix. This structure is a manifestation of the symmetries of the Higgs-Goldstone interaction Lagrangian (3.9) which controls the high-energy limit. (In this limit the mass terms are irrelevant.) Consider first the exact O(3) symmetry of the interaction Lagrangian (3.9) in the space labelled by w_1, w_2, and z, where

$$w^{\pm} = \frac{1}{\sqrt{2}}(w_1 \pm i w_2) \quad . \tag{3.13}$$

In isospin langauge, h is isoscalar and w^+, w^-, and z are isovector. The O(3) symmetry emerges in the structure of the 2×2 t-matrix for the channels w^+w^- and $\frac{1}{\sqrt{2}}zz$ or, equivalently, $W_L^+W_L^-$ and $\frac{1}{\sqrt{2}}Z_LZ_L$. For the J = 0 partial wave, we read off from eqs. (3.1a, b, d) the form

$$t_0 = \begin{pmatrix} A(s) + B(s) & \frac{1}{\sqrt{2}}A(s) \\ \frac{1}{\sqrt{2}}A(s) & \frac{1}{2}A(s) + B(s) \end{pmatrix} \quad , \tag{3.14}$$

where

$$A(s) = \frac{-G_F M_H^2}{8\pi\sqrt{2}} \left(\frac{s}{s - M_H^2} \right) \quad , \tag{3.15}$$

$$B(s) = \frac{-G_F M_H^2}{8\pi\sqrt{2}} \left[1 - \frac{M_H^2}{s} \log \left(1 + \frac{s}{M_H^2} \right) \right] \quad . \tag{3.16}$$

The eigenvectors of the matrix (3.14) correspond to the channels $2w^+w^- + zz$ ($I = 0$) and $w^+w^- - zz$ ($I = 2$), with eigenvalues $\frac{3}{2}A + B$ and B, respectively. The isovector state is excluded by statistics from the $J = 0$ partial wave. The exact isospin symmetry of the interaction Lagrangian also explains the absence of off-diagonal processes involving hz. We assign even charge conjugation parity to all the Higgs-Goldstone particles and define a G-parity which is even for h and odd for w^+, w^-, and z. The hz channel is the only neutral two-body channel with odd G-parity and is thus isolated.

At energies very large compared with the Higgs boson mass the trilinear term in the interaction Lagrangian (3.9) becomes ineffectual (contact terms dominate pole graphs at the tree level), so the theory displays an asymptotic O(4) symmetry. The fields w_1, w_2, z, and h form a $\underline{4}$-vector in O(4) space. The Clebsch-Gordan reduction appropriate to two-particle states is

$$\underline{4} \otimes \underline{4} = \underline{9} \oplus \underline{6} \oplus \underline{1} \quad , \tag{3.17}$$

corresponding to a symmetric traceless tensor, an antisymmetric tensor, and a scalar in the O(4) space. The antisymmetric $\underline{6}$ representation does not occur in the J = 0 partial wave because of statistics. Adopting the notation

$$\Phi = \begin{pmatrix} w_1 \\ w_2 \\ z \\ h \end{pmatrix} \quad , \tag{3.18}$$

we may write the singlet representation in the two-particle basis as

$$\begin{aligned} (\underline{1}) &= \sum_{i=1}^{4} \Phi_i \Phi_i = w_1^{\,2} + w_2^{\,2} + z^2 + h^2 \\ &= 2w^+w^- + zz + hh \quad , \end{aligned} \tag{3.19}$$

and the nonet as

$$(\underline{9})_{ij} = \Phi_i \Phi_j - \tfrac{1}{4}\delta_{ij}(\underline{1}) \quad . \tag{3.20}$$

The singlet corresponds to the eigenvector of the t-matrix with the largest eigenvalue $\frac{3}{2} \times \left(\frac{-G_F M_H^{\,2}}{4\pi\sqrt{2}}\right)$. Of the elements of the nonet, three denote channels that are electrically neutral. It is convenient to choose the combinations

$$(\underline{9})_{33} + (\underline{9})_{44} = -w^+w^- + \tfrac{1}{2}zz + \tfrac{1}{2}hh \ , \tag{3.21a}$$

$$(\underline{9})_{33} - (\underline{9})_{44} = zz - hh \quad , \tag{3.21b}$$

$$(\underline{9})_{34} = hz \tag{3.21c}$$

which correspond to the three eigenvectors of the t-matrix (3.5) with common eigenvalue $\frac{1}{2} \times \left(\frac{-G_F M_H^2}{4\pi\sqrt{2}} \right)$. This completes the demonstration that the structure of the physically interesting t-matrix is a reflection of the asymptotic symmetry of the underlying Higgs-Goldstone theory.

IV. PROPERTIES OF STRONGLY-COUPLED HIGGS-GOLDSTONE SYSTEMS

We have demonstrated that the Higgs boson mass is the gauge theory parameter which governs the strength of weak interactions at high energies. If M_H is small compared with the critical mass given by (3.6), partial-wave unitarity will be respected by the tree diagrams for gauge boson scattering at (almost) all energies. Weak interactions may therefore remain weak at all but a few exceptional energies, in that higher order corrections to scattering amplitudes will be negligible. On the other hand, if M_H is comparable to or greater than the critical value of 1 TeV/c^2, weak interactions among gauge bosons necessarily become strong in the TeV regime. The proved unitarity and renormalizability of gauge theories ensure that when partial-wave unitarity is violated by tree graphs, higher-order diagrams will come to the rescue. However, the problem of sorting out the consequences of a strongly-coupled theory by field theory techniques is not one we can solve at present.

We have relied so far upon S-matrix techniques, with partial-wave unitarity playing a central role. It is therefore natural to investigate the strong-coupling situation by means of a venerable device from the S-matrix theory of strong interactions, the N/D method.[13] This technique provides a prescription according to which analytic partial-wave amplitudes which satisfy unitarity constraints can be constructed. We shall apply it to the Higgs-Goldstone theory specified by (3.9), the behavior of which should give a reliable indication of corresponding phenomena in the Weinberg-Salam model. Our rudimentary calculations are exploratory rather than definitive, but they suggest a possible viewpoint regarding the heavy Higgs alternative.

Our concern is the behavior of the theory when the Higgs boson h becomes very massive while the Goldstone particles w^+, w^-, z remain massless. In terms of the parameters of the interaction Lagrangian (3.9), this is the regime $\lambda \gtrsim 1$ with v fixed. Under these conditions the particle h is an ephemeron, being highly unstable against decays into channels with two or more Goldstone bosons. It is therefore appropriate to consider only the two coupled neutral channels w^+w^- and zz. The O(3) symmetry of the interaction Lagrangian (3.9) then separates the coupled-channel problem into two single-channel problems for $I = 0$ $(2w^+w^- + zz)$ and $I = 2$ $(w^+w^- - zz)$.

In the tree approximation the $J = 0$ partial-wave amplitudes are given by (3.14) as

$$a_0^{(I=0)}(s) = \frac{3}{2} A(s) + B(s) \quad , \tag{4.1}$$

$$a_0^{(I=2)}(s) = B(s) \quad , \tag{4.2}$$

where A(s) and B(s) are defined by (3.15) and (3.16). At low energies $s \ll M_H^2$ the isoscalar interaction

$$a_0^{(0)}(s) \underset{s \ll M_H^2}{\sim} G_F s/8\pi\sqrt{2} \tag{4.3}$$

is attractive, while the isotensor interaction

$$a_0^{(2)}(s) \underset{s \ll M_H^2}{\sim} -G_F s/16\pi\sqrt{2} \tag{4.4}$$

is repulsive and half as strong. Both amplitudes grow linearly with s until energies comparable with M_H are reached. This brings to mind the possibility that if M_H is chosen very large there may appear a scalar, isoscalar bound state which would serve as a low mass Higgs boson for phenomenological purposes. This intriguing possibility is not excluded by the crude calculations we are about to discuss, but we have no arguments for its inevitability. Indeed, our investigations suggest that unitarity forces a very massive Higgs particle pole to migrate into the complex s-plane far from the physical sheet, so the I = 0, J = 0 channel becomes nonresonant.

A number of important caveats are in order before we pursue this discussion any further. As we have applied it, the N/D technique is an implementation of <u>elastic</u> unitarity which should be limited in validity to the region $4M_W^2 < s < 16M_W^2$. Furthermore the solution we shall describe is the first-order determinantal approximation[13, 14] which is likely to be inadequate for very strong couplings. While both of these approximations could be improved, our very rough computation has revealed nothing which motivates us to undertake a more thorough exploration of the strongly-coupled system.

We display in Fig. 12 the motion in the complex s-plane of the second-sheet pole of the unitarized amplitude as a function of the input mass of the Higgs boson, with $M_W = 60$ GeV. As the input mass is increased, the output resonance migrates into the complex plane, acquiring a width comparable to its mass.

The solution to the N/D equations gives no signal that a light, scalar bound state will be generated by unitarity. However, because the approximate solution becomes untrustworthy in the regime of very large input masses (hence very large couplings and very many coupled channels), the bound-state possibility has not been excluded. It goes without saying that the problem of a very strongly coupled Higgs-Goldstone system remains open.

V. DISCUSSION

Even within the restrictive framework of a minimal $SU(2) \otimes U(1)$ gauge theory there is a broad range of possibilities for the behavior of weak interactions at very high energies. In the Weinberg-Salam model, in addition to the gauge couplings and the weak interaction angle θ_W which are fixed by low-energy phenomenology, there enters as a free parameter the mass of the Higgs boson. It is natural to classify the possibilities for high-energy behavior in unified theories as light Higgs theories, with $M_H < 2M_W$, and heavy Higgs theories, with $M_H > 2M_W$.

In a light Higgs theory, the Higgs boson will decay into ordinary fermion pairs, with heavy lepton pairs and heavy quark pairs the preferred decay modes. The partial widths (well above threshold) are given by

$$\Gamma(H \rightarrow f\bar{f}) = G_F m_f^2 M_H / 4\pi\sqrt{2} \quad . \tag{5.1}$$

Decays $H \rightarrow q\bar{q}$ are expected to be observed as back-to-back jets of hadrons in the Higgs boson rest frame. The discovery of a light Higgs particle will be an indication that weak interactions remain weak at nearly all energies. In that case a perturbative treatment of interactions among leptons, intermediate bosons, and the Higgs boson is adequate to develop the consequences of the theory. An exhaustive phenomenological portrait of a light Higgs boson has been given by Ellis, Gaillard, and Nanopoulos.[7]

The conditional upper bound (1.1) on the Higgs boson mass leads us to contemplate the heavy Higgs alternative, $M_H > 2M_W$. A Higgs boson in this mass range has the striking property that it decays almost exclusively into pairs of intermediate bosons. If the mass of the Higgs boson is substantially less than the critical mass, say $2M_W < M_H \lesssim 600$ GeV/c^2, we expect that perturbative estimates of the production and decay rates should be reliable. For the intermediate boson decay modes, we find

$$\frac{\Gamma(H \to W^+W^-)}{M_H} = \frac{G_F M_W^2}{8\pi\sqrt{2}} \frac{(1-x)^{\frac{1}{2}}}{x} (3x^2 - 4x + 4) \ , \qquad (5.2)$$

$$\frac{\Gamma(H \to Z^0Z^0)}{M_H} = \frac{G_F M_W^2}{16\pi\sqrt{2}} \frac{(1-x')^{\frac{1}{2}}}{x} (3x'^2 - 4x' + 4), \qquad (5.3)$$

where $x = 4M_W^2/M_H^2$ and $x' = 4M_Z^2/M_H^2 = x/\cos^2\theta_W$. The resulting partial decay widths are shown in Fig. 13. It is amusing to note that because of its peculiar decay properties, a heavy Higgs boson may have a more distinctive experimental signature than a light one. The chain

$$H \to Z^0 Z^0, \quad Z^0 \to \ell^+\ell^-, \quad Z^0 \to \ell^+\ell^-$$

would be rather unmistakable.

A variety of production processes for the heavy Higgs boson may be considered. We have found none which promise copious production, so our discussion will be brief. The rate for Higgs formation in e^+e^- collisions is

$$\sigma_{peak}(e^+e^- \rightarrow H) \simeq \frac{4\pi}{M_H^2}\frac{\Gamma(H \rightarrow e^+e^-)}{\Gamma(H \rightarrow all)} \simeq \frac{5 \times 10^{-27}\ cm^2}{(M_H/\ 1 GeV/c^2)^2} \cdot \frac{\Gamma(H \rightarrow e^+e^-)}{\Gamma(H \rightarrow all)}, \quad (5.4)$$

a discouraging prospect (because of (5.1)) even for relatively light Higgs bosons. For colliding pp or $\bar{p}p$ beams, in which an analog of the Drell-Yan process may operate, we estimate that

$$\frac{\sigma(p^{\pm}p \rightarrow H + \text{anything})}{\sigma(p^{\pm}p \rightarrow W + \text{anything})} \approx \left(\frac{m_q}{M_H}\right)^2 \quad , \quad (5.5)$$

where the cross section for W-production is to be evaluated at "M_W" = M_H, and m_q is the quark mass. This too is likely to be a feeble rate, since the cross section for W-production is now expected[15] to be less than 10^{-32} cm^2 even for $M_W^2/s << 1$.

More promising is the production of H in association with an intermediate boson. A simple example is the reaction

$$e^+e^- \rightarrow Z_{virtual} \rightarrow ZH \quad , \quad (5.6)$$

which occurs with a cross section[16]

$$\sigma(e^+e^- \rightarrow HZ) = \frac{\pi\alpha^2}{24}\left(\frac{2K}{\sqrt{s}}\right)\frac{\left(K^2 + 3M_Z^2\right)}{\left(s - M_Z^2\right)^2}\frac{\left(1 - 4x_W + 8x_W^2\right)}{x_W^2(1 - x_W)^2} \quad , \quad (5.7)$$

where $x_W = \sin^2\theta_W$ and K is the c.m. momentum of the emerging particles. At very high energies, for which $2K \to \sqrt{s}$, the ratio

$$r \equiv \frac{\sigma(e^+e^- \to HZ)}{\sigma(e^+e^- \to \mu^+\mu^-)} \longrightarrow \frac{\left(1 - 4x_W + 8x_W^2\right)}{128x_W^2(1 - x_W)^2} \qquad (5.8)$$

from below. For $0.3 < x_W < 0.4$, the ratio is asymptotically about 9%. This result can be carried over to hadronic collisions as a rough guide. We then have, as an order-of-magnitude estimate,

$$\frac{\sigma(p^\pm p \to HZ + \text{anything})}{\sigma(p^\pm p \to \mu^+\mu^- + \text{anything})} \approx r \quad , \qquad (5.9)$$

where HZ and $\mu^+\mu^-$ production are compared at the same invariant mass.

Adopting a somewhat broader view than we have taken earlier in this article, we may envision three major possibilities for the evolution of weak interactions at very high energies.[17] The first option is that no intermediate bosons exist, in which case weak interactions among leptons or among quarks are expected to become strong at c.m. energies exceeding about 300 GeV. A second is that intermediate bosons exist, but are not described by a renormalizable theory with gauge couplings. Weak interactions among leptons or among quarks would never become strong (except at resonance poles or exponentially high energies) but unitarity at very high energies would have to be salvaged by strong interactions among the intermediate bosons.[18] The final option is the one most attractive to us, namely that weak (and electromagnetic) interactions are described by a renormalizable gauge theory. Apart from exceptional energies, the weak interactions among leptons or quarks would always remain weak. We have showed by explicit calculation of gauge boson scattering amplitudes in the Weinberg-Salam model that if the Higgs boson mass exceeds a critical value of about 1 TeV/c^2, weak interactions among $W^{\pm}$, Z, and H must become strong in the TeV energy regime. It is then likely that familiar features of the strong interaction at GeV energies such as resonance formation and multiple production would come to characterize the interactions of gauge bosons. If instead the Higgs boson mass is small compared to 1 TeV/c^2, weak interactions among all particles may remain weak at all unexceptional energies.

We interpret the bound (1.1) in much the same way as Veltman[6] has interpreted the results of his related investigation: We find it appealing to believe that new phenomena are to be found in the weak interactions at energies not much larger than 1 TeV, in addition to the anticipated discovery of the intermediate bosons. Either a light Higgs boson will exist or weak interactions will approach the richness and complexity of low-energy strong interactions.

Acknowledgments

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APPENDIX

The substance of this Appendix is implicit in Eq. (19) of reference 12. However, for completeness we present the argument in the framework of gauge field theory.

We consider the generating functional of Green's functions

$$Z\left[J_L\right] = -i\log\int\left[dV_\mu d\phi\ldots\right]\exp\left\{i(S_{eff}\left[V_\mu, \phi, \ldots\right] + \int d^4xJ_LV_L)\right\} \times \prod\delta(\partial^\mu V_\mu + iM\phi) \quad (A.1)$$

from which connected Green's functions with external longitudinally polarized vector bosons are obtained by functional differentiations with respect to the source J_L. Here we suppress the group index, so that V_μ and ϕ stand collectively for $W_\mu^\pm$, and Z_μ, and for $w^\pm$ and z, respectively, with appropriate mass M. The constraints[11]

$$\partial^\mu V_\mu + iM\phi = 0 \quad (A.2)$$

define the 't Hooft-Feynman gauge, and the effective action $S_{eff}\left[V_\mu, \phi\ldots\right]$ includes the Fadde'ev-Popov term.[19]

The longitudinal vector field V_L is defined as

$$\tilde{V}_L(k) = \epsilon_L^\mu\tilde{V}_\mu(k)\ ,\ \epsilon_L^\mu = \frac{1}{M}(|\vec{k}|,\ k_0\hat{k}) \quad , \quad (A.3)$$

where k_μ is the four-momentum carried by the vector boson, and $\tilde{V}_\mu(k)$ is the Fourier transform of $V_\mu(x)$. Eq. (A.2) states that

$$\frac{k^{\mu}}{M}\tilde{V}_{\mu}(k) = \tilde{\phi}(k) \quad , \tag{A.4}$$

while Eq. (A.3) implies

$$\begin{aligned} \tilde{V}_L(k) &= \frac{k^{\mu}}{M}\tilde{V}_{\mu}(k) + \mathcal{O}\left(\frac{M}{k_0}\right) \\ &= \tilde{\phi}(k) + \mathcal{O}\left(\frac{M}{k_0}\right) \quad . \end{aligned} \tag{A.5}$$

Thus, Eq. (A.1) may be cast in the form

$$Z\left[J_L\right] = -i\log\int\left[dV_{\mu}d\phi\ldots\right]\exp\left\{i\left(S_{eff}\left[V_{\mu}, \phi, \ldots\right] + \int d^4k\tilde{J}_L(-k)\left[\tilde{\phi}(k) + \mathcal{O}\left(\frac{M}{k_0}\right)\right]\right)\right\}$$

$$\times \prod \delta(\partial^{\mu}V_{\mu} + iM\phi) \quad . \tag{A.6}$$

In vector-boson scattering, all k_0's are of order $\sqrt{s}$, and we obtain the result of Cornwall, Levin and Tiktopoulos,[12] that

$$T(V_L\text{'s}) = T(\phi\text{'s}) + \mathcal{O}(M/\sqrt{s}) \tag{A.7}$$

for large s, $s \gg M^2$.

Inclusion of the physical Higgs mesons as external lines in the T-matrix does not alter the above discussion.

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FIGURE CAPTIONS

Fig. 1: Feynman graphs (in the unitary gauge) for the reaction $W^+W^- \to W^+W^-$.

Fig. 2: Feynman graphs (in the unitary gauge) for the reaction $ZZ \to ZZ$.

Fig. 3: Feynman graphs (in the unitary gauge) for the reaction $HH \to HH$.

Fig. 4: Feynman graphs (in the unitary gauge) for the reaction $ZZ \to W^+W^-$.

Fig. 5: Feynman graphs (in the unitary gauge) for the reactions $HH \to W^+W^-$ and $HH \to ZZ$. Here the symbol V denotes a generic intermediate boson.

Fig. 6: Sketch of the energy dependence of the $J = 0$ partial-wave amplitude for elastic scattering of longitudinally polarized W-bosons for two choices of the Higgs boson mass. For $M_H > (4\pi\sqrt{2}/G_F)^{\frac{1}{2}}$ the partial-wave unitarity bound $|a_0| \leq 1$ is violated for $s > M_H^2$.

Fig. 7: Feynman graphs for the reaction $w^+w^- \to w^+w^-$.

Fig. 8: Feynman graphs for the reaction $zz \to zz$.

Fig. 9: Feynman graphs for the reaction $hh \to hh$.

Fig. 10: Feynman graphs for the reaction $zz \to w^+w^-$.

Fig. 11: Feynman graphs for the reactions $hh \to w^+w^-$ and $hh \to zz$. Here the symbol v denotes a generic Goldstone boson.

Fig. 12: Trajectory in the complex s-plane of the second-sheet pole of the unitarized $I = 0$, $J = 0$ amplitude as a function of the input mass of the Higgs boson. The intermediate boson mass is fixed at 60 GeV/c^2. Tick marks along the trajectory denote the input Higgs boson mass in units of GeV/c^2; s is expressed in units of TeV^2.

Fig. 13: Partial decay widths of the Higgs boson into intermediate vector boson pairs *versus* the Higgs boson mass. For this illustration we have taken M_W = 60 GeV/c^2 and M_Z = 77 GeV/c^2.

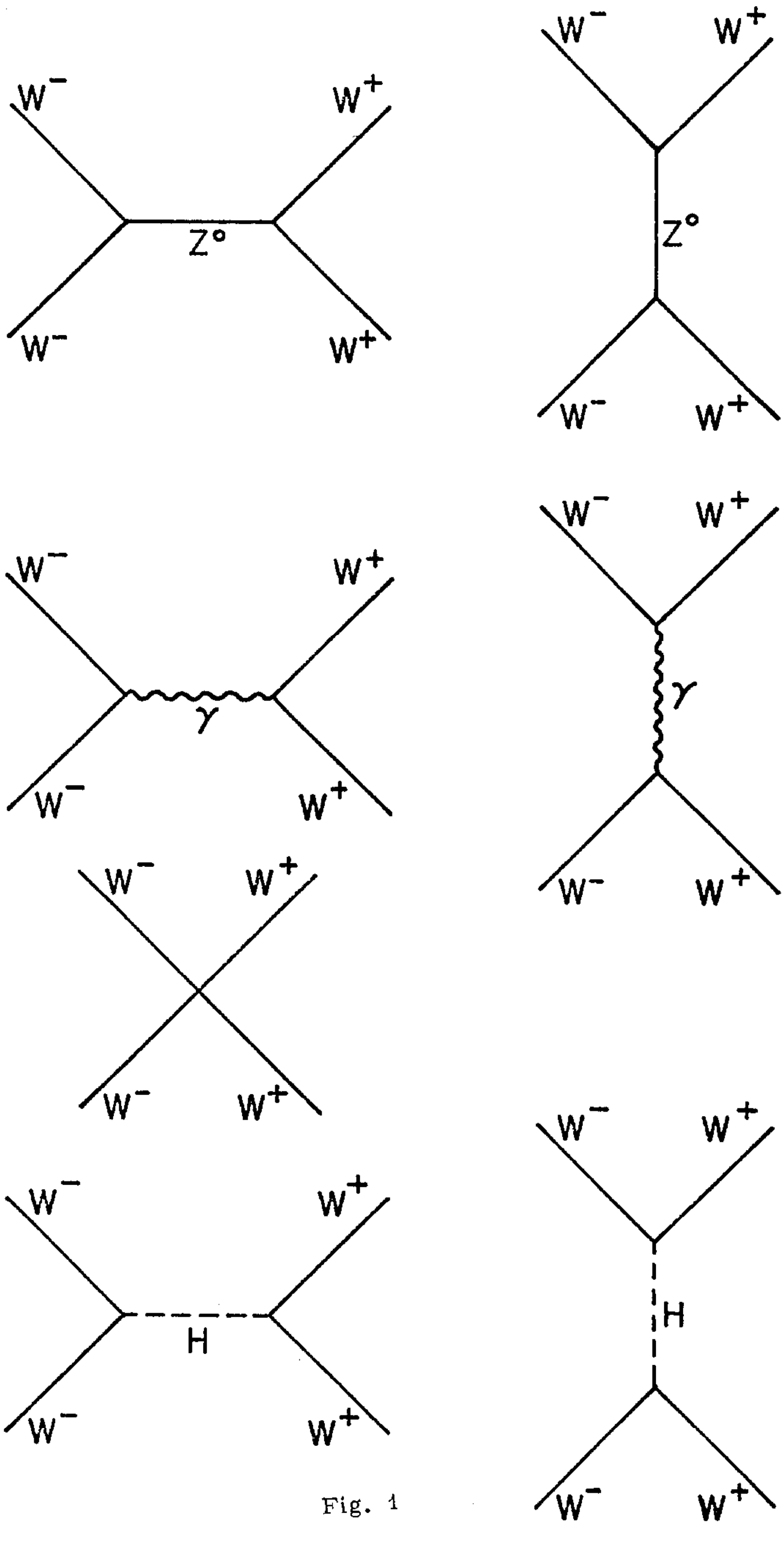
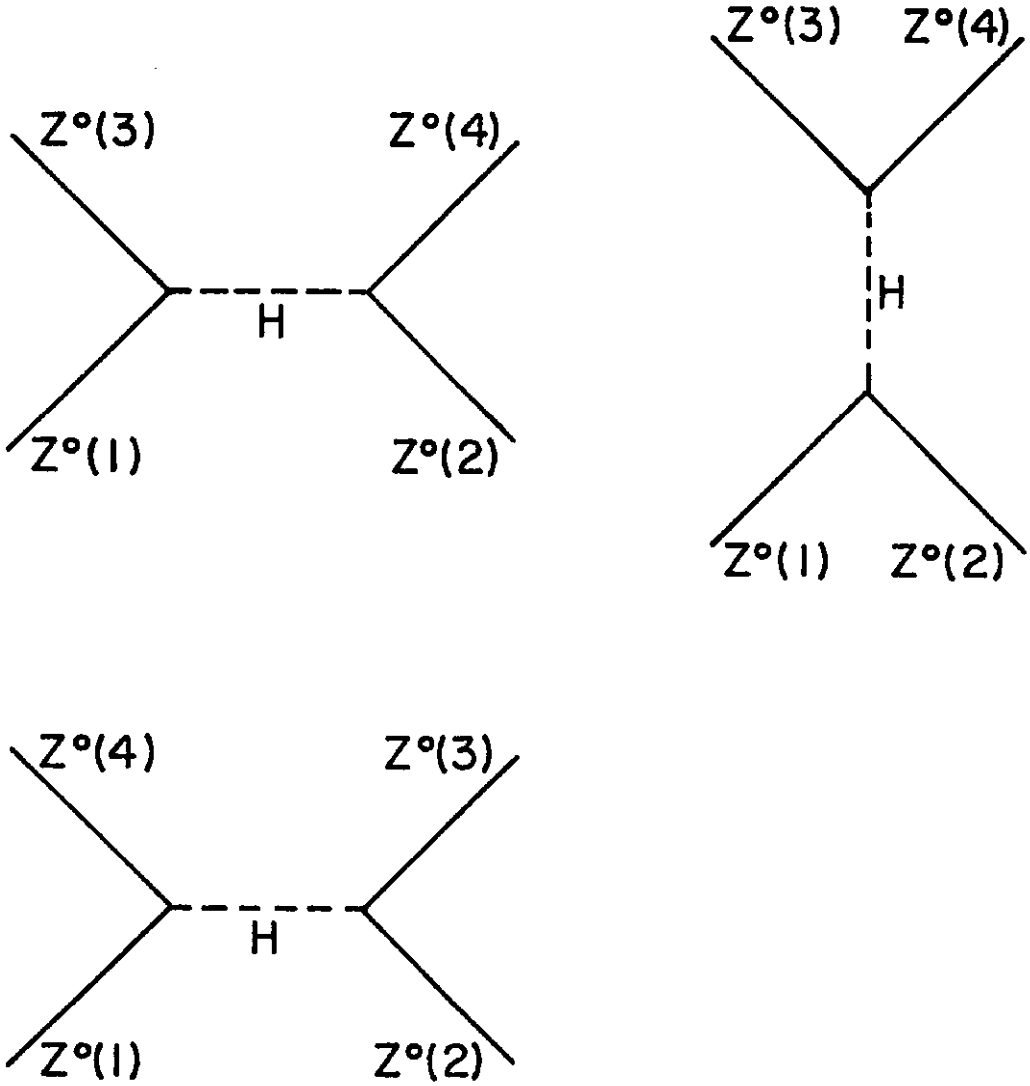


Fig. 1



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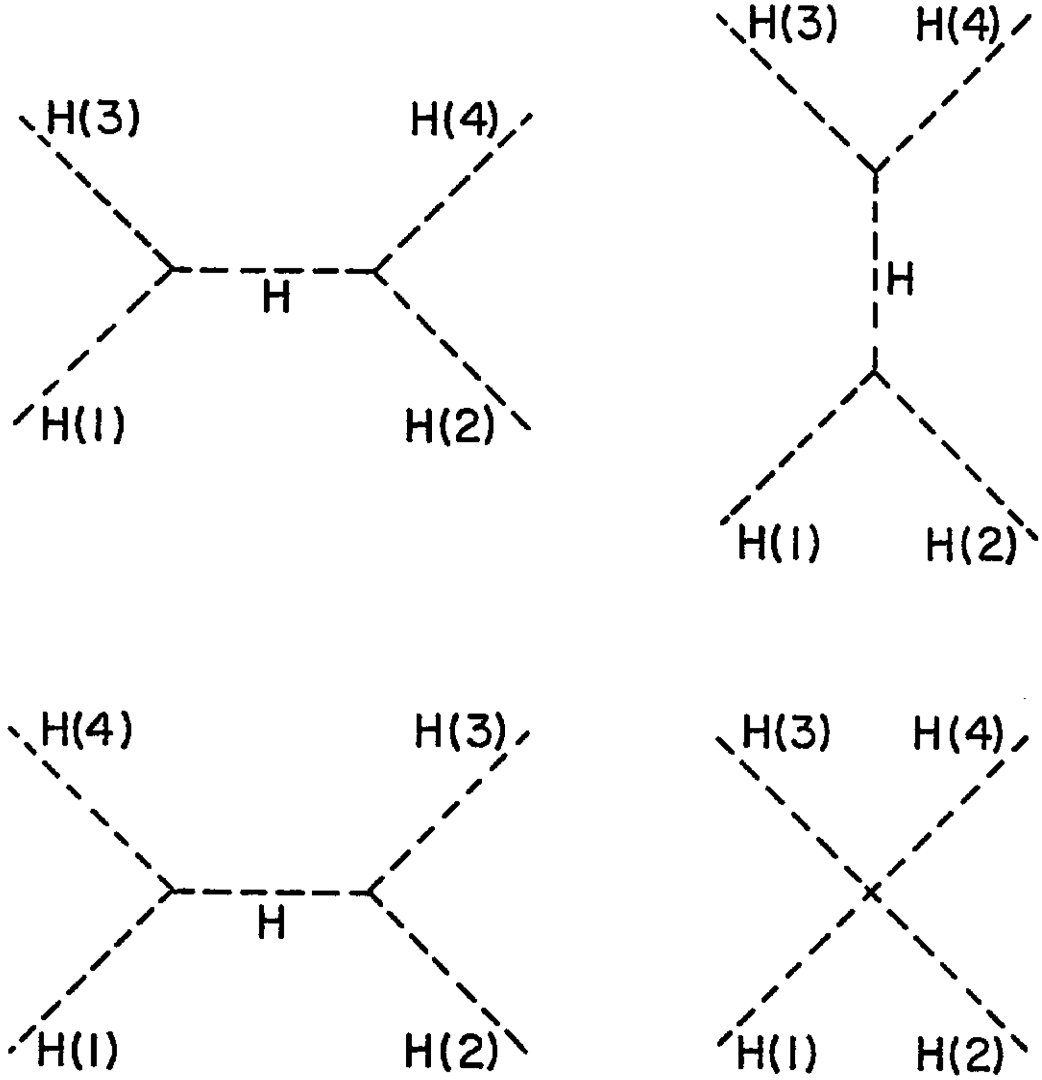

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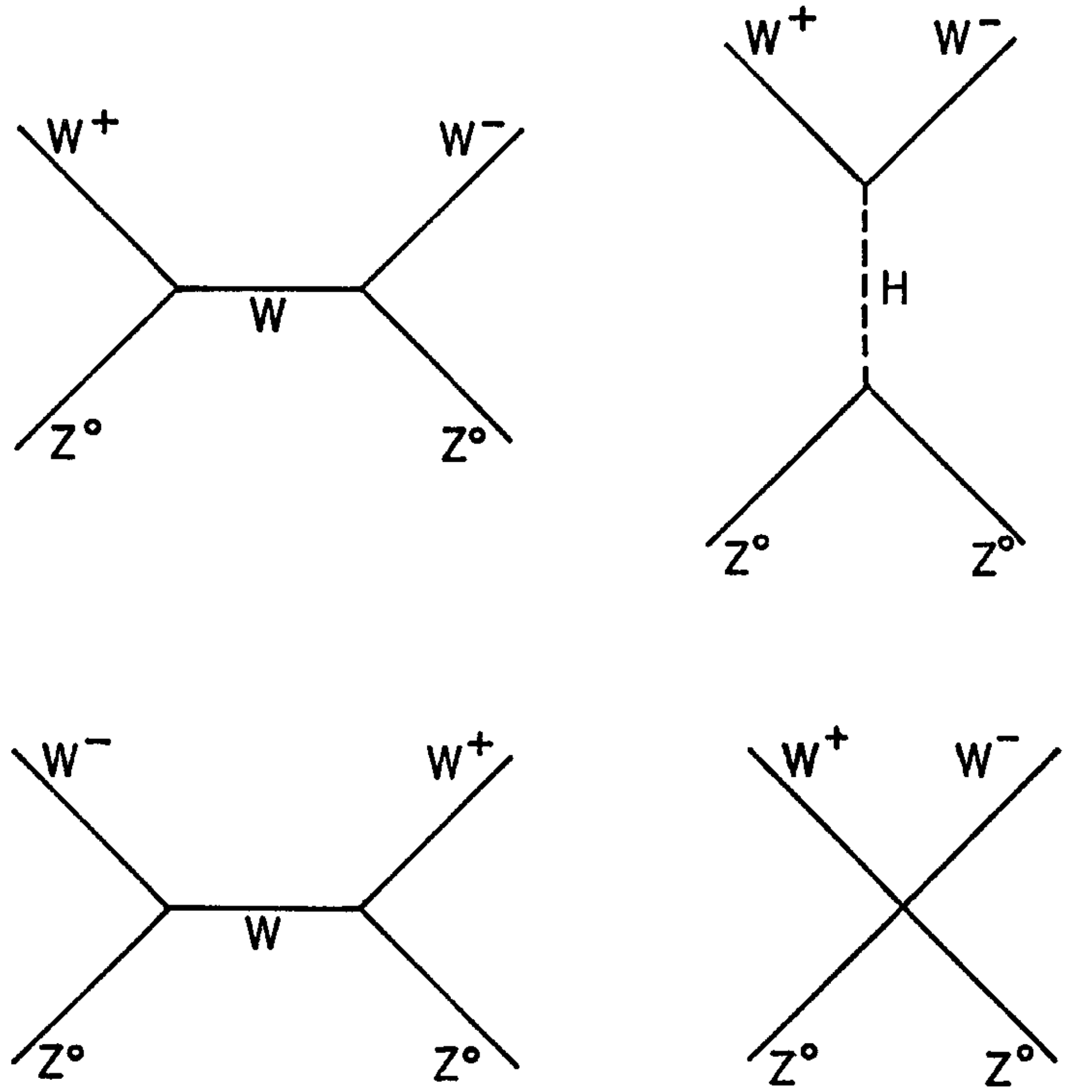

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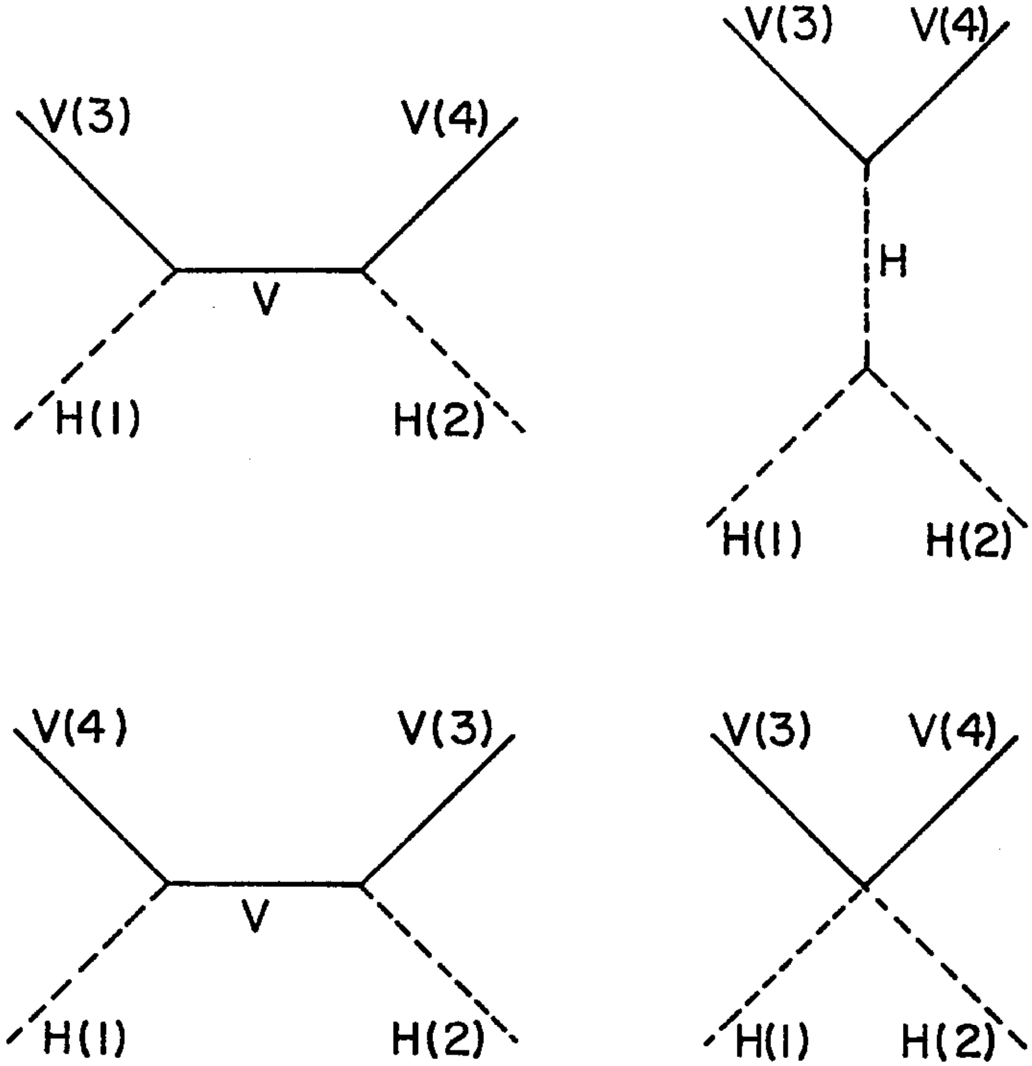
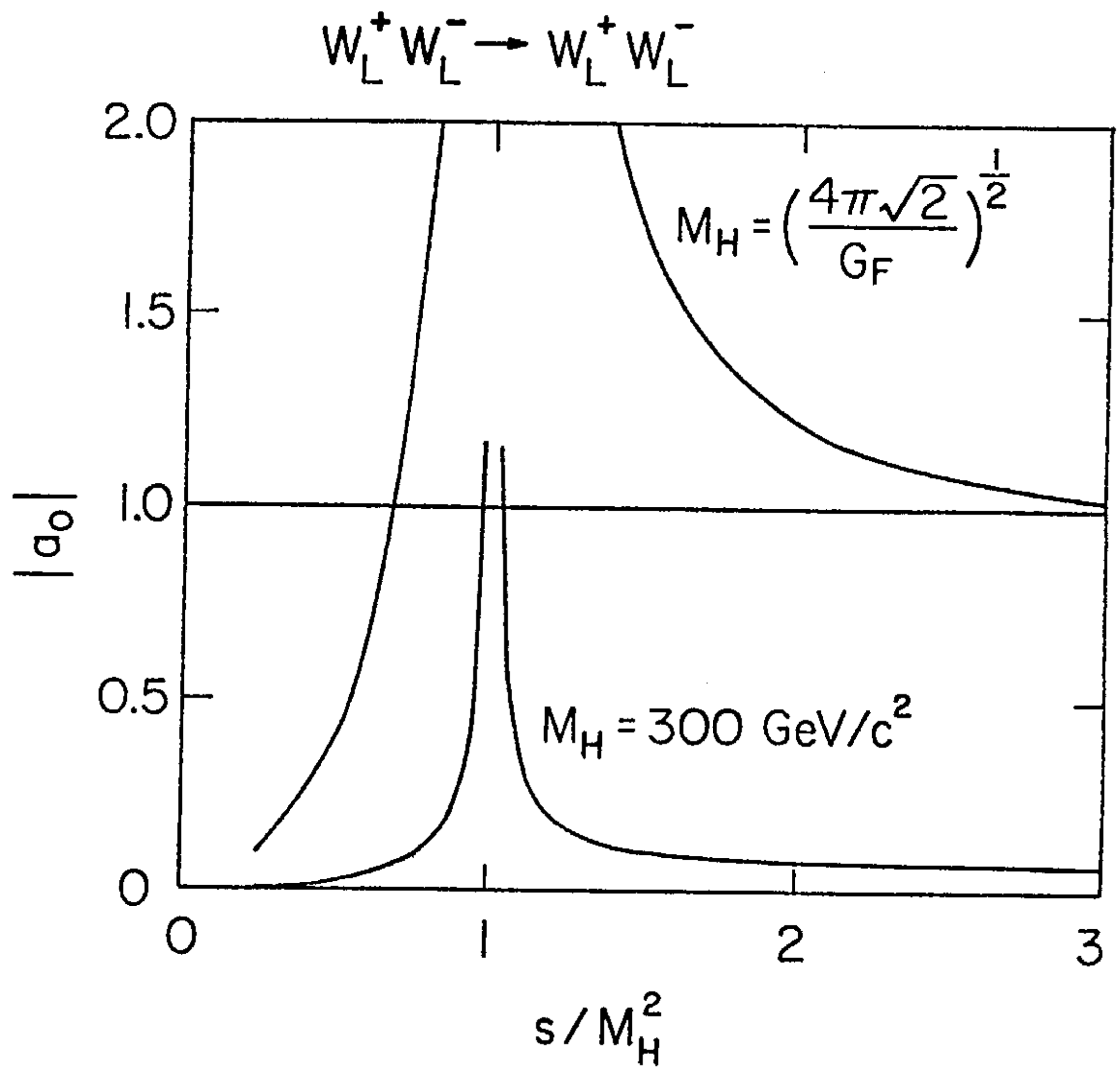


Fig. 5



$W_L^+ W_L^- \rightarrow W_L^+ W_L^-$
2.0
1.5
1.0
0.5
0
$|a_0|$
$M_H = \left(\frac{4\pi\sqrt{2}}{G_F}\right)^{\frac{1}{2}}$
$M_H = 300$ GeV/c^2
0
1
2
3
s / M_H^2

Fig. 6

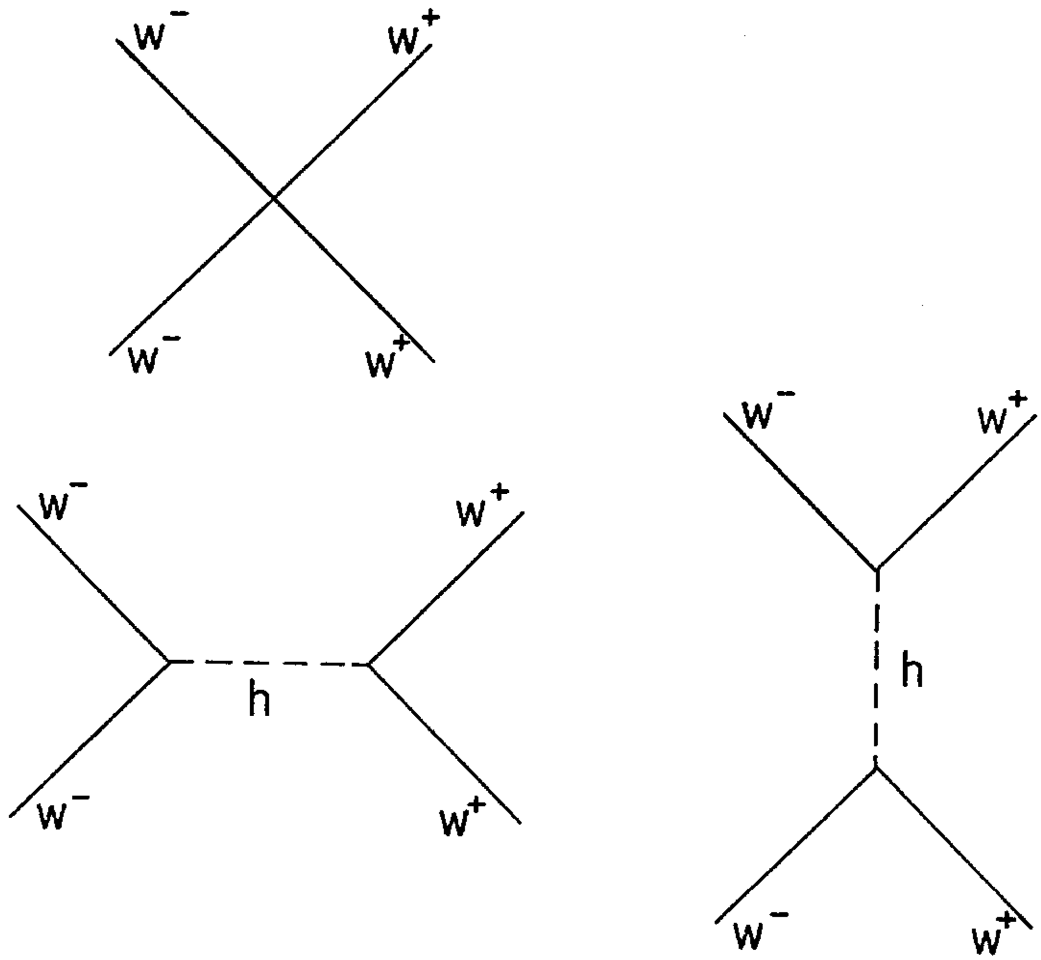


Fig. 7

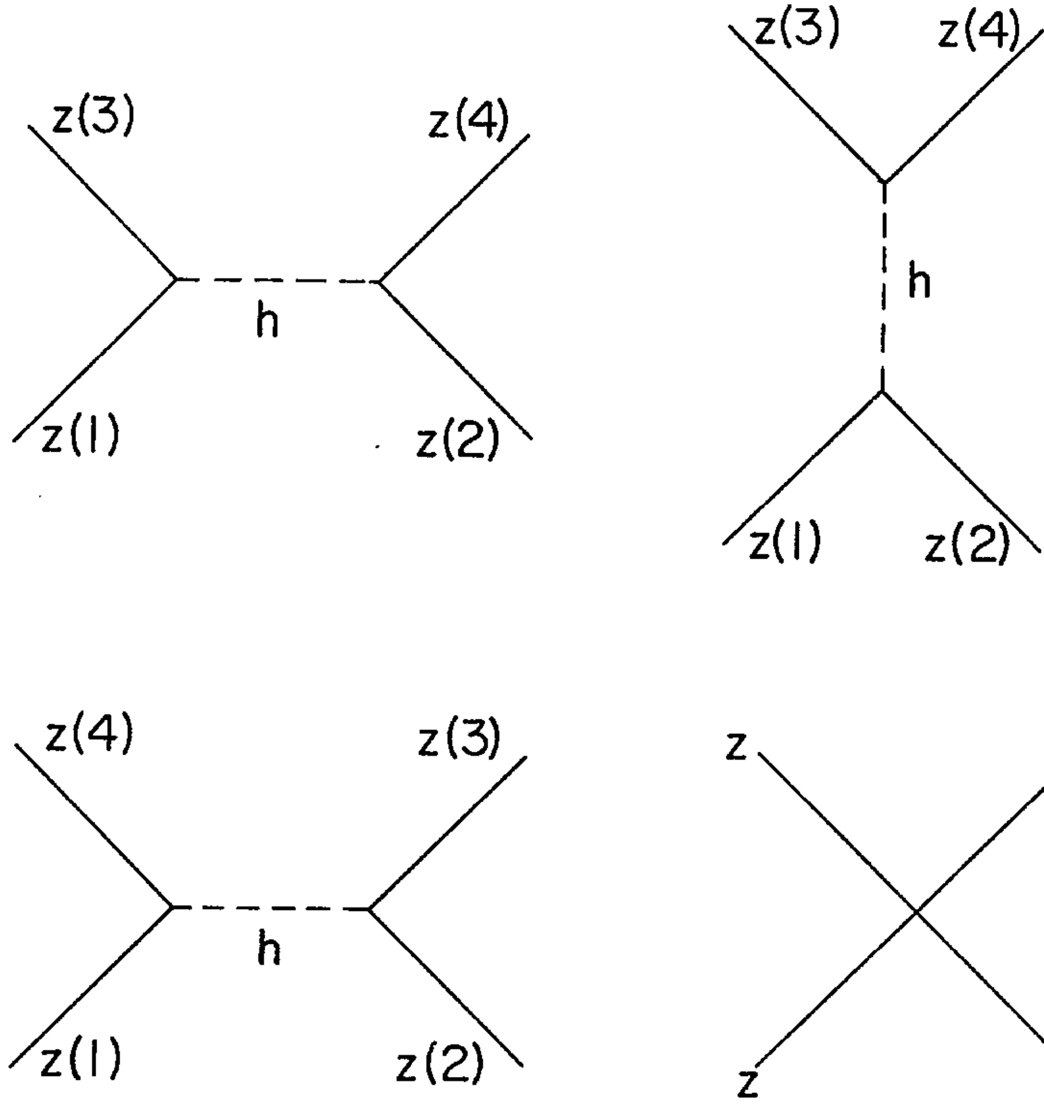


Fig. 8

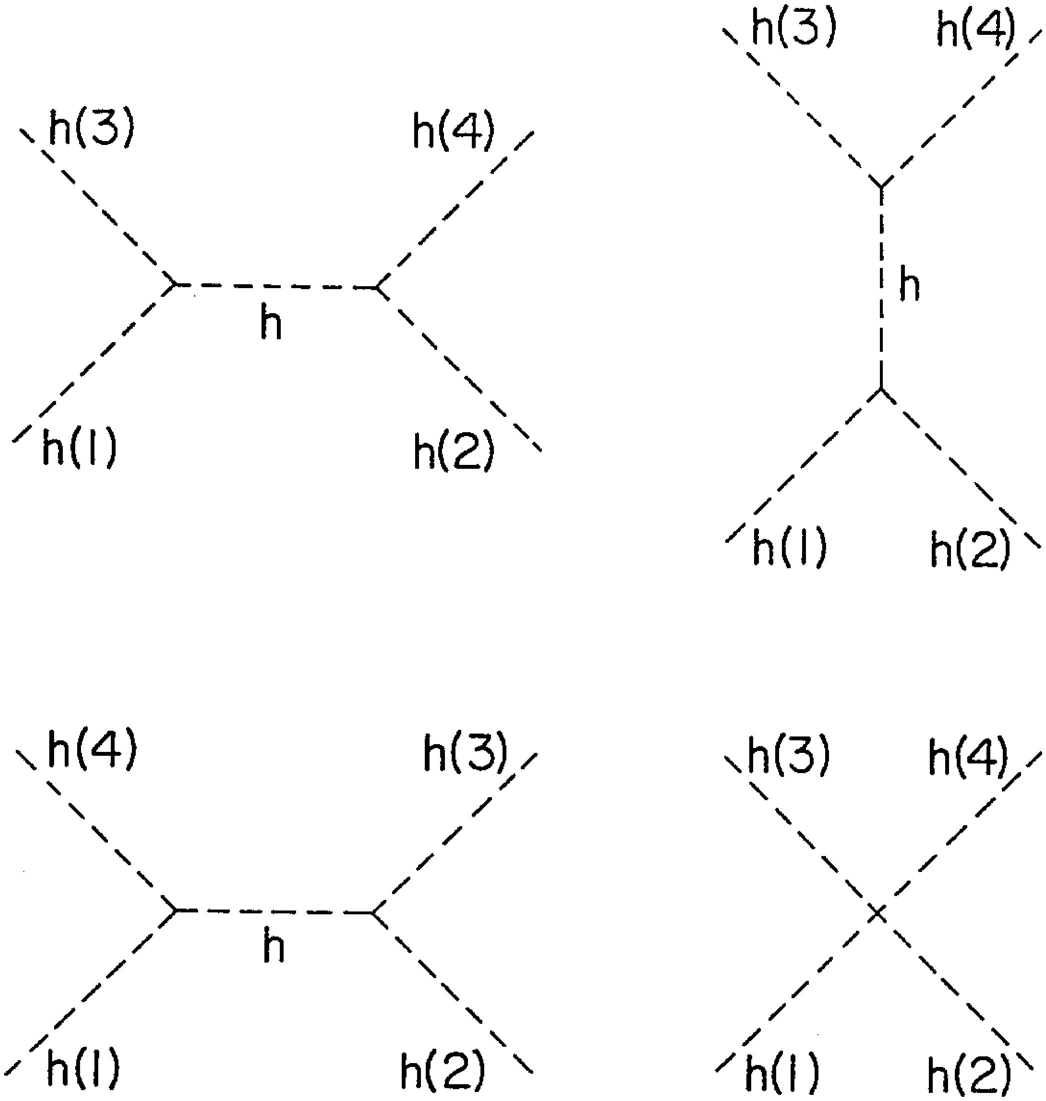


Fig. 9

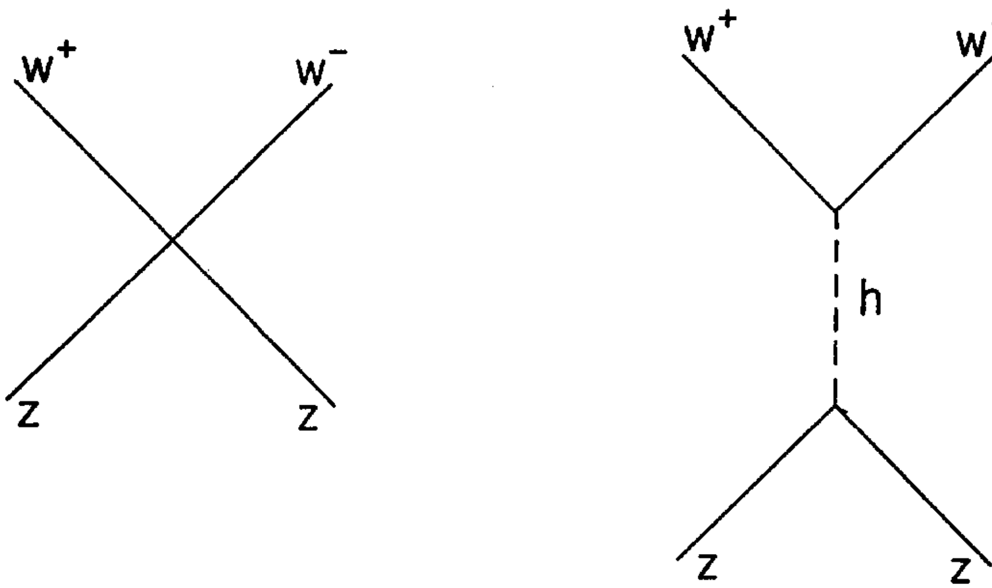


Fig. 10

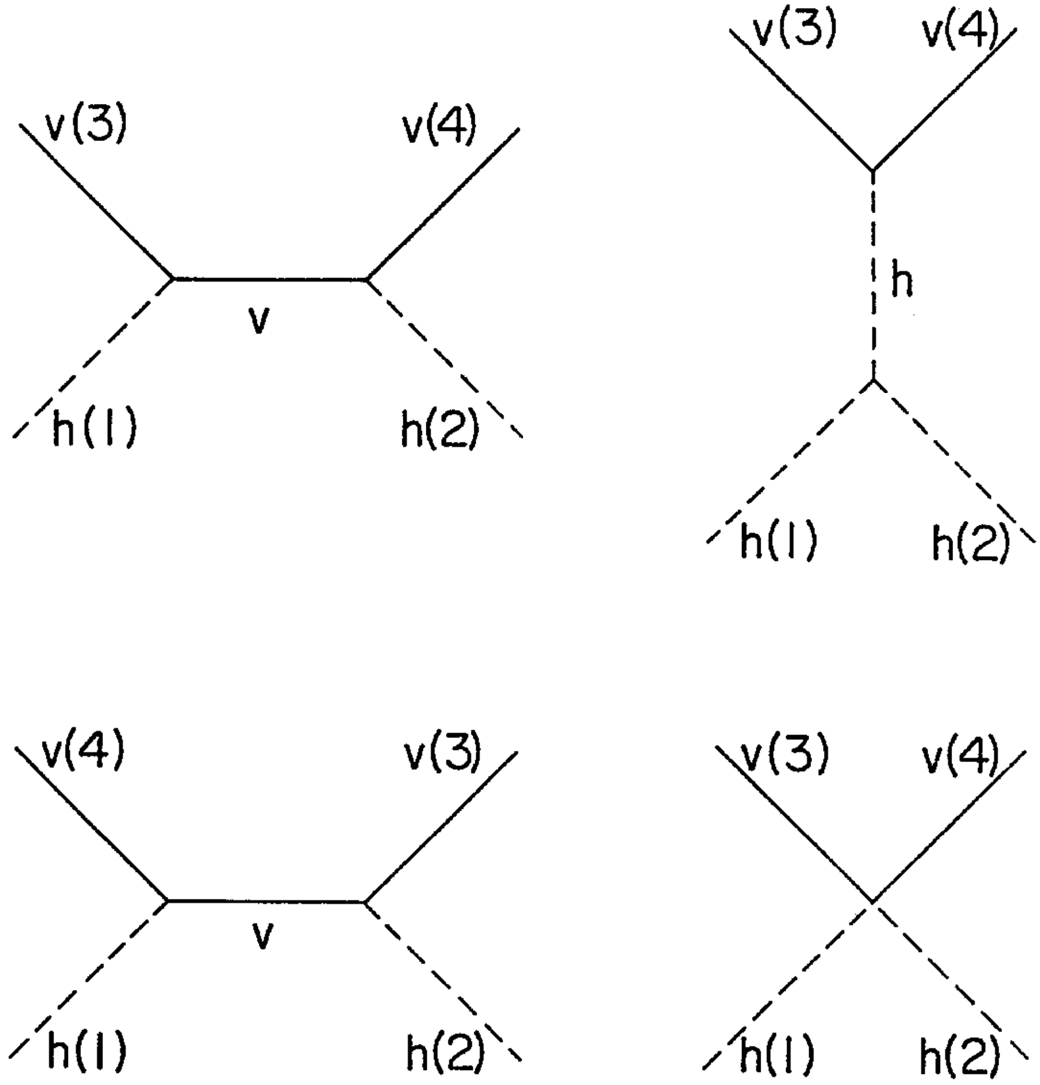


Fig. 11

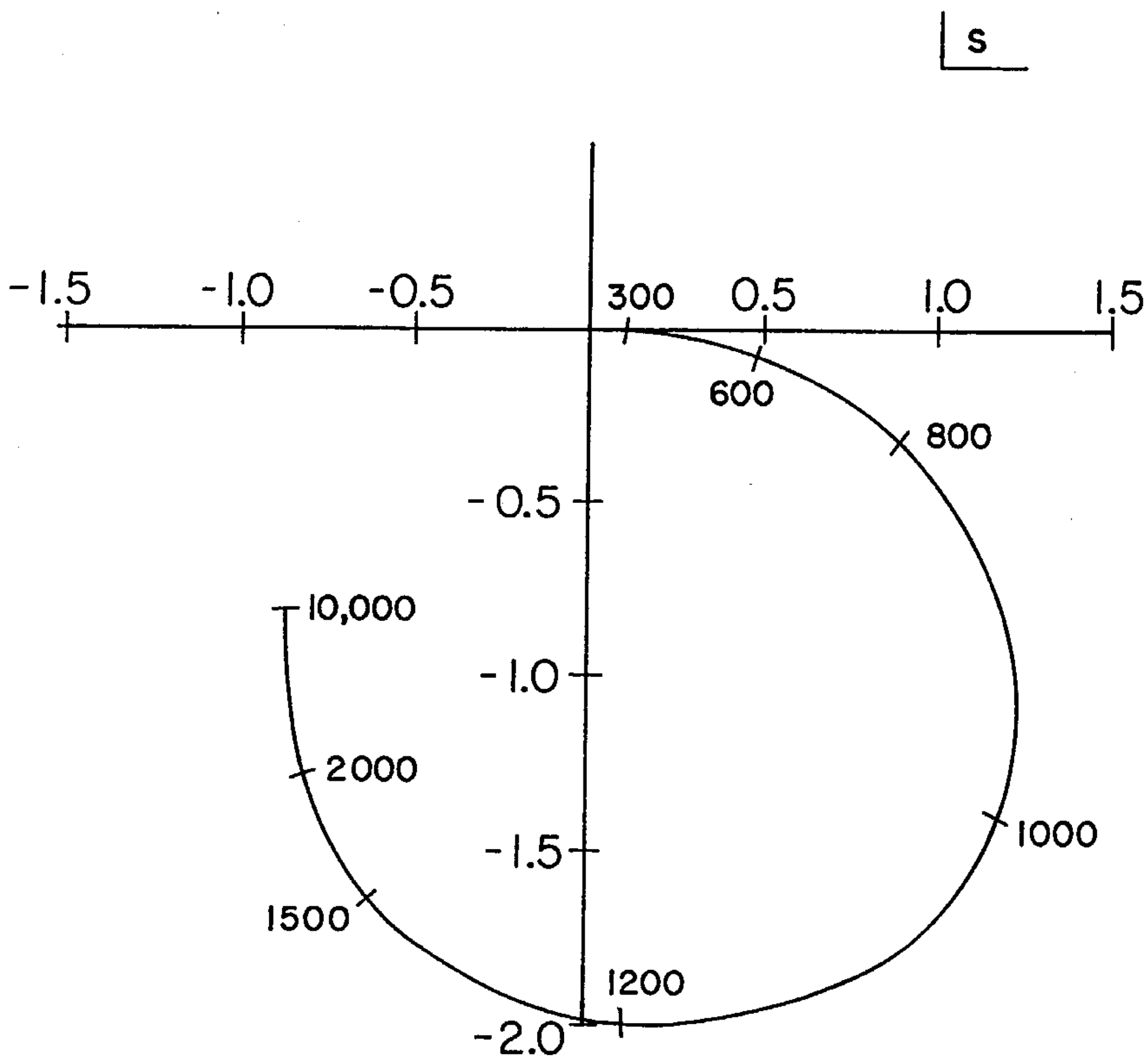
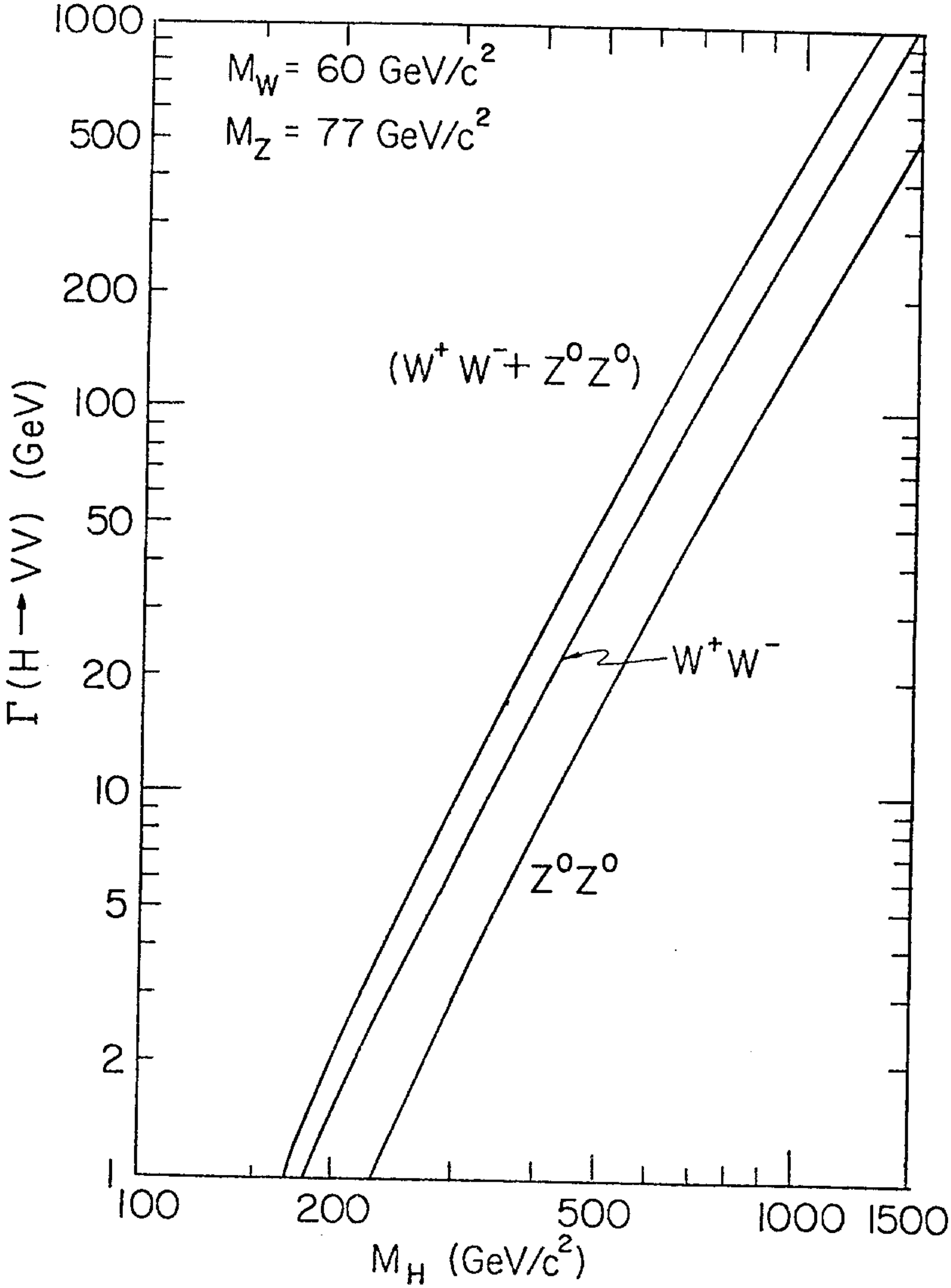

s
-1.5
-1.0
-0.5
0.5
1.0
1.5
-0.5
-1.0
-1.5
-2.0
300
600
800
1000
1200
1500
2000
10,000

Fig. 12



1000
500
200
100
50
20
10
5
2
1
M_W = 60 GeV/c^2
M_Z = 77 GeV/c^2
$(W^+W^- + Z^oZ^o)$
W^+W^-
Z^oZ^o
$\Gamma(H \rightarrow VV)$ (GeV)
100
200
500
1000
1500
M_H (GeV/c^2)

Fig. 13