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X- AND γ-RAY PRODUCTION BY TEMPLATE MODULATED COHERENT LIGHT

PAUL L. CSONKA

Institute of Theoretical Science and Department of Physics
University of Oregon
Eugene, Oregon 97403, U.S.A.

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In this proposed device coherent electromagnetic radiation, modulated by an optical element called template, produces a field at the trajectory of charged particles which are accelerated by the field and emit electromagnetic radiation. The template may be a holograph. The spectrum of this radiation depends on the original coherent radiation, the template and the position of the particle in the field. For existing machines high intensity X- and γ-ray production is possible and the frequency and polarization spectrum can be controlled. Coherent production (between the charged particles) enhances the radiation particularly at lower energies. This suggests the possibility of producing essentially monochromatic photons. Coherent (between photons) radiation production seems feasible. For a suitably chosen field the dominant part of the radiative reaction force goes to zero near the beam axis so that it does not interfere with the earlier suggested method of particle acceleration with template modulated coherent light.

In this paper we briefly described a proposed device capable of producing X-rays and γ-rays with a tunable frequency and polarization spectrum. The device is to be used in conjunction with high energy accelerators or storage rings.

From here on we will use the phrase "γ-ray" to mean both X-rays and γ-rays, unless explicitly specified otherwise.

In the proposed device, γ-rays are to be produced by high energy charged particles traversing a suitably shaped electromagnetic field. The production mechanism is related to, but different from, the one by which high energy charged particles produce synchrotron radiation. The difference is the following. The intensity and, to some extent, the polarization spectrum of the radiation produced by the device to be described are variable and can be adjusted. By contrast, for any accelerator or storage ring and a given particle energy, the synchrotron radiation spectra are fixed and often inconvenient. The proposed device is also related to but distinct from a class of devices which we will refer to as undulators. In an undulator charged particles pass through a static magnetic field which varies as a function of position along the particle trajectory. When this magnetic field is inserted into a particle accelerator or storage ring, the field exerts a force on the charged particles and the so accelerated particles emit electromagnetic radiation. The change in the particle trajectory caused by the magnetic field can have the shape of a "break" or "wave."[1,2] The intensity of the magnetic field can be varied. The device to be described differs from an undulator, in that in it no static field is to be used, but instead a radiation field shaped by an optical template. This makes it possible to increase the power radiated in the form of γ-rays, to produce Fourier components with a much shorter wavelength than static magnetic fields would permit, and to combine and adjust Fourier components in an essentially arbitrary manner, the precise meaning of this phrase being given below. The electromagnetic field at the trajectory in turn determines the intensity and polarization spectra of the radiation produced by the device here described. By appropriately choosing the former, one can select an intensity and, to some extent, the polarization spectrum for the latter.

In the template radiator described here, high velocity (in the laboratory) charged particles are accelerated by an electromagnetic field. During acceleration, these particles radiate γ-rays. The charged particles may be electrons, protons or other particles. We need not specify their exact nature at this time, and until we do, we will refer to them simply as "particles."

The accelerating electromagnetic field is produced in two steps. In the first step, coherent electromagnetic radiation is generated by a source which may be a laser. We need not now specify the exact nature of the source, and until we do, we will refer to it simply as "light source" and to the radiation produced by it as "light." In the second step the light produced by the light source passes through a template and is then focused on the particle trajectory. The template[3] is an optical element which modulates both the intensity and the phase of the light passing through it, and it may be a holograph. By suitably choosing the template, any electromagnetic field can be constructed at the particle trajectory, provided only that it satisfies Maxwell's equations and the boundary condition that it is produced at the template by a converging monochromatic field. In practice it will sometimes be convenient not to insist on creating the most desirable field at the trajectory, but one which differs from it by the appearance of certain unwanted but relatively harmless Fourier components. When the template is a simple holograph, then these unwanted Fourier components form the so called "twin image" of the desired "correct image" and can indeed be quite harmless as will be discussed below.

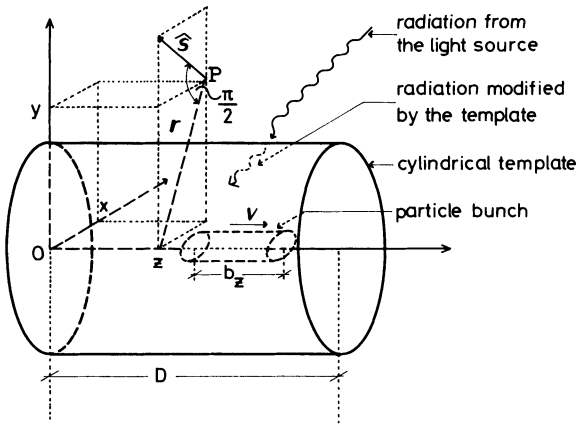

FIGURE 1 The figure shows the x, y, z Cartesian coordinates of an arbitrary point P, its radius vector $\mathbf{r}$, and the $\hat{\mathbf{s}}$ unit vector defined at P by $\hat{\mathbf{z}} \times \hat{\mathbf{r}} = \hat{\mathbf{s}}$. The electromagnetic radiation from the light source passes through a cylindrical template which modifies it so as to produce a suitable electromagnetic field. This field extends from 0 to D along the z axis and accelerates charged particles thereby inducing them to radiate. The particles enter the field with a common velocity $\mathbf{v}$, and come in bunches whose length along z is b_z, and whose horizontal and vertical diameter (not shown in figure) are b_x and b_y.

Let us denote the wavelength in vacuum of the (by assumption monochromatic) light produced by the light source by λ_0, and $\omega = 2\pi c/\lambda_0$. We restrict the following discussion to the special case when the trajectory of the particle before entering the field produced by the template, is horizontal and straight, and we choose the z axis to be parallel to it. We assume that the electric field $\mathbf{E}$ produced by the template is cylindrically symmetric around the z axis. It follows that the magnetic induction, $\mathbf{B}$, is also cylindrically symmetric around the same axis. Generalization to arbitrary geometry is straightforward. We denote by $\mathbf{r}$ the radius vector measured from the z axis (see Figure 1). The horizontal and vertical Cartesian components of $\mathbf{r}$ are x and y, respectively. We choose the origin of our coordinate frame so that at the point where a particle enters the electromagnetic field produced by the template, its z coordinate is zero. To specify $\mathbf{E}$, we could now give its three Cartesian components E_x, E_y, and E_z. Instead, for any point P whose coordinates are (z, y, x) we will specify E_z, E_r and E_s, Where E_r is the E component parallel to $\mathbf{r}$, the $\hat{\mathbf{s}}$ is defined as

$$\hat{\mathbf{z}} \times \hat{\mathbf{r}} \equiv \hat{\mathbf{s}}, \tag{1}$$

and E_s is the component parallel to $\hat{\mathbf{s}}$. The $\mathbf{B}$ vector will be specified similarly. By assumption, the time dependence of $\mathbf{E}$ and $\mathbf{B}$ is given by the factor $e^{i\omega t}$, and by the assumed cylindrical symmetry $\mathbf{E}$ and $\mathbf{B}$ depend on z and r, but not on s.

The most general electromagnetic field satisfying these conditions is an arbitrary linear superposition of two types of fields. For the first type, $E_s = 0$, while for the second type $B_s = 0$. The second type can be obtained from the first type by the substitution $\mathbf{E} \leftrightarrow \mathbf{B}$, $\mathbf{B} \leftrightarrow -\mathbf{E}$. Thus it suffices to know the most general field of the first type. Since fields of the second type are of little interest for our present purposes, we will impose the condition

$$E_s = 0 \tag{2}$$

We also assume that the electromagnetic field is periodic along z with a period $2L$. Since L is arbitrary, this assumption imposes no restriction on the field, but allows us to write sums instead of integrals.

The most general electromagnetic field satisfying these four conditions (varies as $e^{i\omega t}$, cylindrical symmetry, $E_s = 0$, and $2L$ periodic along z) is

$$E_z(t, z, r) = \mathrm{Re}\left\{\frac{i\omega}{c} e^{-i\omega t} \sum_n c_n\, e^{ik_{z,n}z}\, J_0(\kappa_n r)\right\}, \tag{3a}$$

$$E_r(t,z,r) = \mathrm{Re}\left\{\frac{i\omega}{c}\,e^{-i\omega t}\sum_n c_n\,e^{ik_{z,n}z}\,\frac{-ik_{z,n}}{\kappa_n}\,J_1(\kappa_n r)\right\}, \tag{3b}$$

$$B_s(t,z,r) = \mathrm{Re}\left\{\frac{\omega}{c}\,e^{-i\omega t}\sum_n c_n\,e^{ik_{z,n}z}\,\frac{\omega/c}{\kappa_n}J_1(\kappa_n r)\right\}, \tag{3c}$$

$$E_s(t,z,r) = B_z(t,z,r) = B_r(t,z,r) = 0, \tag{3d}$$

where

$$\kappa_n \equiv \left[\left(\frac{\omega}{c}\right)^2 - k_{z,n}^2\right]^{1/2}, \tag{3e}$$

J_0 and J_1 are cylindrical Bessel functions, c is the velocity of light in vacuum, $k_{z,n} = n\pi/L$ with n some integer because the field is periodic along z, and the c_n are Fourier coefficients.

The particle moves with a velocity $\mathbf{w}(t)$ in the laboratory. The particle enters the electromagnetic field at time t_0. At that time its velocity is $\mathbf{w}(t_0) = \mathbf{v}$. By definition $\mathbf{v}$ is parallel to $\hat{\mathbf{z}}$. After t_0, the particle is accelerated by the field, and $\mathbf{w}(t)$ will change. In cases of most interest to us, the particle will oscillate while crossing the field. While oscillating, it will radiate, thereby loose energy, and slow down. We will assume that this slowing down is negligible within the time the particle crosses the field, so that the particle velocity can be written as

$$\mathbf{w}(t) = \mathbf{v} + \mathbf{u}(t), \tag{4a}$$

where $\mathbf{u}(t)$ is simply a periodic function of t, and thus the average (over one oscillation) velocity $\mathbf{v}$ is constant in time.

We denote the inertial frame which moves with the constant velocity $\mathbf{v}$ with respect to the laboratory by K_v. For obvious reasons, we may call this frame the average (over oscillations) restframe of the particle. We denote quantities measured in K_v by a prime, e.g. in this frame the particle velocity is

$$\mathbf{w}'(t) = \mathbf{u}'(t), \tag{4b}$$

and $\mathbf{v}' = 0$ by definition. The components of the electromagnetic field can be obtained by a Lorentz transformation from Eq. (3). Defining $v \equiv |v|$ and $\gamma \equiv (1 - v^2/c^2)^{-1/2}$,

$$E_z'(t',z',r') = E_z(t,z,r), \tag{5a}$$

$$E_r'(t',z',r') = \mathrm{Re}\left\{\sum_n c_n\,e^{-i(\omega t - k_{z,n}z)}\,\frac{\omega}{c}\left[k_{z,n} - \frac{v}{c}\frac{\omega}{c}\right]\frac{\gamma}{\kappa_n}\,J_1(\kappa_n r)\right\}, \tag{5b}$$

$$B_s'(t',z',r') = \mathrm{Re}\left\{\sum_n c_n\,e^{-i(\omega t - k_{z,n}z)}\,\frac{\omega}{c}\left[\frac{\omega}{c} - \frac{v}{c}k_{z,n}\right]\frac{\gamma}{\kappa_n}\,J_1(\kappa_n r)\right\}, \tag{5c}$$

$$E_s' = B_z' = B_r' = 0 \tag{5d}$$

where t', z' and r' are the coordinates in K_v of that space time point whose coordinates in the laboratory are t, z, and r.

We assume that while the particle is the electromagnetic field,

$$\mathbf{u}'(t) \ll c. \tag{6}$$

Then in K_v the dominant force on the particle will be electric, as opposed to magnetic, so that

$$\frac{\mathrm{d}}{\mathrm{d}t'}\,\mathbf{u}'(t',z',r') \approx q\,\frac{\mathbf{E}'(t',z',r')}{m}$$

$$\approx q\,\frac{\mathbf{E}'(t',z',r')}{m_0}, \qquad \text{if} \quad |\mathbf{u}| \ll c, \tag{7}$$

where q, m and m_0 are respectively the charge, mass and rest mass of the particle.

Denote by Δr and Δz the amplitude of particle oscillation in K_v along $\mathbf{r}$ and z respectively. Assume

$$\kappa_n\,\Delta r \ll 1, \tag{8}$$

so that during one particle oscillation along $\mathbf{r}$, the value of J_1 changes only by a small amount which we neglect. Similarly, we neglect the change in $\mathbf{E}'$ due to particle oscillations parallel to z, and can write

$$\mathbf{E}'(t', z'(t'), r'(t')] \approx \mathbf{E}'(t', z_0', r_0'), \tag{9}$$

where z_0' and r_0' are the average (over one oscillation) of the z' and r' coordinates of the particle, so that by the definition of K_v and Eq. (4b) they are constants in time.

To calculate the power, P', radiated by the oscillating particle in K_v, we use the classical formula

$$P' = \frac{2}{3}\frac{q^2}{c^3}\left[\frac{\mathrm{d}}{\mathrm{d}t'}\,u'(t')\right]^2. \tag{10}$$

This expression will be a good approximation provided that for every Fourier component of $\mathbf{E}$

$$\hbar\omega_n' \ll m_0 c^2; \qquad \text{for all } n, \tag{11}$$

is satisfied. In the cases of interest to us, Eq. (10) holds.

Substituting Eq. (7) into Eq. (10) and using approximation (9), we obtain

$$P' \approx \frac{2}{3}\frac{q^2}{m_0^2 c^3}\,[E_z'^2(t', z_0', r_0') + E_r'^2(t', z_0', r_0')]. \quad (12)$$

For any one particle the z_0' and r_0' do not change in time, by assumption. Therefore, in the laboratory frame r_0 is also constant in time, while $z = [z_0 + v(t - t_0)]\gamma$. Equation (5) shows that for fixed r_0 and linearly varying z, the fields E_z' and E_r' oscillate harmonically as a function of time due to the factor $e^{-i[\omega t - k_{z,n}z(t)]}$. Thus the time average of $E_z'^2$ is one half of the maximum value (in time) of $E_z'^2$, and similarly for $E_r'^2$. Therefore, we find from Eqs. (5) and (12) that the time average of P', is

$$\langle P' \rangle = \frac{1}{3}\frac{q^4}{m_0^2 c^3}\sum_n |c_n|^2\left(\frac{\omega}{c}\right)^2 \times \left\{J_0^2(\kappa_n r) + \left[k_{2,n} - \frac{v}{c}\frac{\omega}{c}\right]^2\left[\frac{\gamma}{\kappa_n}J_1(\kappa_n r)\right]^2\right\} \quad (13)$$

We now focus our attention on the interesting case when

$$v \approx c, \quad (13a)$$

and when γ is large enough, so that the J_0^2 term can be neglected. This last assumption is equivalent to neglecting the radiation due to E_z' in comparison with the radiation due to E_r'. In other words, we assume that the emitted radiation by a particle located at r_0, is mainly due to radial oscillations along $\mathbf{r}_0$.

$$\langle P' \rangle = \frac{1}{3}\frac{q^4}{m_0^2 c^3}\sum_n |c_n|^2\left(\frac{\omega}{c}\right)^2\left[k_{z,n} - \frac{\omega}{c}\right]^2\frac{\gamma^2}{\kappa_n^2}J_1^2(\kappa_n r). \quad (13b)$$

The field produced by the template is a superposition of several Fourier components. The component proportional to c_n describes photons which in the laboratory frame have angular frequency ω, and whose momentum along the z axis is $k_{z,n}/2\pi$. The absolute value of their momentum in the radial direction is $\kappa_n/2\pi$. Along the radial direction the momentum of half of these photons is $\kappa_n/2\pi$, and for the other half it is $-\kappa_n/2\pi$, because the cylindrical Bessel functions for large argument $\kappa_n r$ can be written as a sum of two Hankel functions, one with argument $-\kappa_n r$ and the other with argument $+\kappa_n r$, and these describe respectively a cylindrical wave converging on the z axis with radial momentum $-\kappa_n/2\pi$ and a wave diverging with radial momentum $+\kappa_n/2\pi$.

Performing a Lorentz transformation to K_v, we find that the Fourier component proportional to c_n describes photons which have angular frequency and momentum components

$$\omega_n' = \gamma(\omega - vk_{z,n}) \approx \gamma(\omega - ck_{z,n}),$$
$$\frac{1}{2\pi}k_{z,n}' = \frac{1}{2\pi}\gamma\left(k_{z,n} - \frac{v}{c^2}\omega\right) \approx \gamma\frac{1}{2\pi}\left(k_{z,n} - \frac{\omega}{c}\right),$$
$$\frac{1}{2\pi}\kappa_n' = \frac{1}{2\pi}\kappa_n. \quad (14)$$

When the particle moves in the field of these photons, it will radiate photons which have in K_v angular frequency $\omega_n^{e'}$, momentum $\mathbf{k}_n^{e'}$ with components, $k_{z,n}^{e'}$ and $\kappa_n^{e'}$. By the classical approximation permitted by Eq. (11),

$$\omega_n^{e'} = \omega_n'. \quad (15)$$

The probability of photon emission by a particle located at r_0 [and, by Eq. (13b) essentially oscillating radially along r_0], in the $\mathbf{k}_n^{e'}$ direction, is proportional to $(\hat{\mathbf{k}}_n^{e'} \times \hat{\mathbf{r}}')^2$:

$$P'(\hat{\mathbf{k}}_n^{e'}) \sim (\hat{\mathbf{k}}_n^{e'} \times \hat{\mathbf{r}}_0')^2 \quad (16)$$

The polarization of the emitted photon is parallel to $(\hat{\mathbf{k}}_n^{e'} \times \hat{\mathbf{r}}_0) \times \hat{\mathbf{k}}_n^{e'}$.

So far we considered the radiation emitted by one particle only. When two particles of the same type are moving parallel to the z axis, both with velocity $\mathbf{v}'$, then they will both radiate and the radiated field emitted by them may interfere constructively or destructively at any point at $\mathbf{l}'$. Denote the position of the two particles by $\mathbf{a}_1'$ and $\mathbf{a}_2'$, $\mathbf{d}' = \mathbf{a}_2' - \mathbf{a}_1'$. The Cartesian components of $\mathbf{a}_1'$ are $a_{1x}', a_{1y}', a_{1z}'$, and $\mathbf{a}_{1r}' \equiv (a_{1x}', a_{1y}')$. Similarly for $\mathbf{a}_2'$, $\mathbf{d}'$, etc. The phase of the field at $\mathbf{a}_1'$, due to photons produced by the template with ω_n' and $\mathbf{k}_n'$ differs from the phase at $\mathbf{a}_2'$ by $(-k_{z,n}' \cdot d_z' - \kappa_n' \cdot d_r')$, i.e. the phase of the two fields emitted by the two particles differs by this much at their respective points of emission. Since the distance $|\mathbf{l}' - \mathbf{a}_1'|$ may differ from $|\mathbf{l}' - \mathbf{a}_2'|$, the phase of the radiation reaching $\mathbf{l}'$ from the two particles may be further shifted. Assuming $|\mathbf{l}'| \gg |\mathbf{d}'|$, this shift is approximately $-(\omega^{e'}/c)(\mathbf{d}' - \mathbf{l}')/|\mathbf{l}'|$ so that the phase at $\mathbf{l}'$ of the two radiations differs by a total of

$$\Delta\varphi = (-k_{z,n}'\,\mathrm{d}_z' - \kappa_n'\,\mathrm{d}_r') + \frac{\omega^{e'}}{c}(\mathbf{d}' \cdot \mathbf{l}')/|\mathbf{l}'|. \quad (17)$$

Clearly $(\mathbf{d}' \cdot \mathbf{l}')/|\mathbf{l}'| \leq |\mathbf{d}'|$, and by Eq. (3e)

$$|(k'_{z,n} \cdot \mathrm{d}'_z + \kappa'_n \, \mathrm{d}'_r)| \leq \frac{\omega'}{c} |\mathbf{d}'|,$$

so that whenever

$$\mathrm{d}' < \frac{\pi c}{2\omega}, \quad \text{then} \quad \Delta\varphi < \pi,$$

i.e. the interference is constructive everywhere. If $2\omega' \, \mathrm{d}/c > \pi$ then the interference may still be constructive when $\mathbf{l}'$ is almost parallel to $\mathbf{k}_n^{e'}$.

Similarly, if several particles of the same type are moving parallel to the z axis, all with a velocity $\mathbf{v}$, then radiation fields produced by all of them will add constructively everywhere, provided that all of them lie within a sphere of diameter

$$\mathrm{d}' \lesssim \frac{\pi c}{2\omega} \tag{18}$$

otherwise constructive interference may occur when $\mathbf{l}'$ is almost parallel to $\mathbf{k}_n^{e'}$.

The values ω_n^e and $\mathbf{k}_n^e$ in the laboratory frame can be found by Lorentz transforming $\omega_n^{e'}$ and $\mathbf{k}_n^{e'}$, and using $v \approx c$:

$$\omega_n^e = \gamma(\omega_n^{e'} + c k_{z,n}^{e'}). \tag{19}$$

The probability of emission for a photon with $\mathbf{k}_n^e$ can be obtained from Eq. (16), and depends on the values of c_n. In general, most of the γ rays will be emitted within an angle of $1/\gamma$ radians measured from the z axis. If the emitting particle is located at $\mathbf{r}_0$, the polarization of the γ rays will be predominantly parallel to $\mathbf{r}_0$.

Next we make some numerical estimates to evaluate the capabilities of such a device. We consider the special case when only one Fourier amplitude is non-zero

$$c_n = \delta_{n,n_0} c_{n_0}. \tag{20}$$

One simple way to produce this is by imprinting a cylindrically symmetric reflecting surface on a transparent one and let the reflexivity as a function of z vary as $\cos k_{z,n_0} z$. The hologram produced by this template will contain, in addition to the desired Fourier component which varies as $\exp\{k_{z,n_0} z\}$, another which varies as $\exp\{-k_{z,n_0} z\}$: the "twin image." In most cases this component will produce undesired low energy γ rays, which can be ignored. If their presence is harmful, then they have to be eliminated by constructing a more sophisticated template. Theoretically the easiest case to consider is when the light converges in the form of a cylindrical wave on the cylindrical template. In practice it will be easier to split the light beam into three or more branches and direct them onto the template from three or more directions. Of course, the electromagnetic field on the outside surface of the template depends on the manner in which the light converges, but in any case the template is so chosen as to achieve a desired field configuration in the vicinity of the z axis. Inside the template one may visualize the electromagnetic energy as being concentrated in a cylindrically converging wave train, the energy density reaching its maximum within a radius of order $2\pi/\kappa_{n_0}$ measured from the z axis.

Now Eq. (13b) reduces to

$$\langle P' \rangle = \frac{1}{3} \frac{q^4 \gamma^2}{c^3 m_0^2} \frac{\omega - c k_{z,n_0}}{\omega + c k_{z,n_0}} |c_{n_0}|^2 \left(\frac{\omega}{c}\right)^2 J_1^2(\kappa_{n_0} r) \tag{21}$$

According to Eq. (19) the maximum angular frequency

$$(\omega_{n_0}^2)_{\max} \approx \gamma \left(1 - \frac{c k_{z,n_0}^{e'}}{\omega_{n_0}^{e'}}\right)_{\max} \omega_{n_0}^{e'} = 2\gamma \omega_{n_0}^{e'}. \tag{22}$$

The average angular frequency is one half of the maximum, because in K_v all emitted photons have the same energy: their energy in the laboratory frame depends on the direction of their emission, but since $\mathbf{r}_0$ is perpendicular to $\mathbf{v}$, Eq. (16) shows that for every photon with momentum $\mathbf{k}_{z,n_0}^{e'}$ there is one with momentum $-\mathbf{k}_{z,n_0}^{e'}$. The polarization of the γ rays emitted by a particle at $\mathbf{r}_0$, is predominantly parallel to $\mathbf{r}_0$ in the laboratory. By Eq. (20), (18) and (14), the radiation emitted by those particles which in the K_v frame lie within a sphere of diameter

$$\mathrm{d}' \lesssim \frac{\pi}{2\gamma(\omega/c - k_{z,n_0})} \tag{23}$$

will add constructively everywhere.

Concerning the light source, we assume that it emits radiation impulses each of which last T_l seconds. During the pulse it emits an instantaneous power W. The light illuminates a section of the z axis whose length is D, i.e. the particle travels a distance D in the field produced by the template. (See Fig. 1.) Since the energy density of the electromagnetic field in vacuum is $\mathbf{E}^2/4\pi$, we can express the electric field in terms of W and D. Using Eqs. (3) and (20) we can then express $|c_{n_0}|$ in terms of W and D:

$$|c_{n_0}| = 2(\pi W/D\omega)^{1/2}. \tag{24a}$$

Substituting Eq. (24a) into Eq. (21), we find the value in K_v of the power radiated by one particle while it passes through the electromagnetic field.

$$\langle P' \rangle = \frac{1}{3}\frac{q^4\gamma^2}{c^3 m_0^2}\frac{\omega - ck_{z,n_0}}{\omega + ck_{z,n_0}}\frac{4\pi W\omega}{c^2 D}J_1^2(\kappa_{n_0}r),$$

(while light pulse lasts) (24)

Concerning the particles, we assume that they are produced in bunches, each bunch containing N particles. The density of the particles can be written as

$$\rho(t, z, y, x) = \rho(t, z)\rho(x, y), \qquad (25)$$

where $\rho(t, z)$ in the laboratory is a "step function moving with velocity v" along z and is nonzero over a z interval whose length is b_z. The $\rho(x, y)$ is left unspecified (see Fig. 1).

Let Δt be the time a particle spends radiating γ rays, i.e. the time the particle takes to pass through the field produced by the template. The same quantity measured in K_v is then $\Delta t' = \Delta t/\gamma$. The total energy, $\mathscr{E}'_\gamma$, radiated out by the particle during one passage through the field, is clearly measured to be $\Delta t'$. $\langle P' \rangle$ in K_v. The total momentum radiated out in K_v is zero. The total energy radiated out by the particle is measured to be $\mathscr{E}_\gamma$ in the laboratory frame:

$$\mathscr{E}_\gamma = \gamma\mathscr{E}'_\gamma = \Delta t\langle P' \rangle. \qquad (26)$$

To obtain the total energy $\mathscr{E}_{\gamma,t}$ radiated out by all the particles in a bunch we multiply $\mathscr{E}_\gamma$ by N and average J_1^2 over the cross section of the beam. We denote the resulting average of J_1^2 by

$$\bar{J}_1^2(\kappa_{n_0}r) \equiv \left(\int \rho(x, y)J_1^2(\kappa_{n_0}r)\,\mathrm{d}x\,\mathrm{d}y\right) \Bigg/ \int \rho(x, y)\,\mathrm{d}x\,\mathrm{d}y, \qquad (27)$$

and obtain from Eqs. (24), (26) and (27)

$$\mathscr{E}_{\gamma,t} = \frac{4\pi}{3}\frac{\omega - ck_{z,n_0}}{\omega + ck_{z,n_0}}\frac{q^4\gamma^2}{c^5 m_0^2}\omega\frac{W}{D}\bar{J}_1^2(\kappa_{n_0}r)N\,\Delta t. \qquad (28)$$

So far we assumed that Eq. (7) holds, i.e., that in K_v magnetic forces can be neglected in comparison with electric forces. We will now evaluate the energy radiated out by the particles as a result of their acceleration due to magnetic forces: $\mathscr{E}^M_{\gamma,t}$. Since in cases of interest to us Eq. (6) holds, the term to be evaluated will be only a small correction to $\mathscr{E}_{\gamma,t}$ given in Eq. (28). We denote by u'_z, u'_r and u'_s the corresponding components of $\mathbf{u}'$, assume

$$u'_s \ll u'_r, u'_z, \qquad (29)$$

and neglect u'_s. In evaluating $\mathscr{E}^M_{\gamma,t}$ we may proceed exactly as we did for $\mathscr{E}_{\gamma,t}$ except that we have to use Eq. (5c) instead of Eqs. (5a) and (5b), and that the force will be proportional to $|\mathbf{u}'|/c$, so that the energy radiated will be proportional to $|\mathbf{u}'|^2/c^2$. In the limit of high γ, stated after Eq. (13a), the result is simply

$$\mathscr{E}^M_{\gamma,t} = \left(\frac{u'}{c}\right)^2_{av}\mathscr{E}_{\gamma,t}, \qquad (30)$$

where $(u'/c)^2_{av}$ is the average of $(u'/c)^2$ taken over all particles, and time. (Since u' may change in time.)

DISCUSSION

1) It was suggested in another paper[3] that template modulated coherent light can be used to construct particle accelerators. The objection was raised that the high intensity electromagnetic fields in the accelerator will unavoidably lead to unwanted interactions between those photons and the accelerated particle, causing it to radiate energy and slow down. Since this effect would be more important at higher particle energies, it would make it impossible to accelerate particles to high energies by the suggested method.

We are now in a position to show that this objection is unfounded. Indeed, the field configuration suggested for the accelerator is the one given by Eq. (3). Therefore, the dominant term—i.e. the one due to electric forces in K_v—in the energy radiated out by the charged particles is given by a sum (over the various Fourier components) of terms of the form shown in Eq. (28). Assume that the particles lie within a cylinder of radius r_m from the z axis, and that $\kappa_n r_m \ll 1$. Then we may approximate $J_1^2(\kappa_n r) \approx (\kappa_n r/2)^2$, and $\mathscr{E}_{\gamma,t} \to 0$, as $r_m \to 0$. Furthermore, by Eqs. (28) and (30)

$$\mathscr{E}^M_{\gamma,t} = \left(\frac{u'}{c}\right)^2_{av}\mathscr{E}_{\gamma,t} \sim \left(\frac{u'}{c}\right)^2_{av}\bar{J}_1(\kappa_n r_m) \sim \left(\frac{u'}{c}\right)^2_{av} r_m^2. \qquad (31)$$

Here the subscript "av" means averaging over all particles and also over the time interval Δt, because u' may be a function of time. Thus, the $\mathscr{E}_{\gamma,t}$ and $\mathscr{E}^M_{\gamma,t}$ can both be made arbitrarily small provided only that r_m is small enough, i.e., provided that the particles are confined close to the z axis. Turning to the nondominant first term in the square bracket in Eq. (13), one sees that it is proportional to J_0^2, and does not go to zero as r_m does. However, this term being independent of γ, does not increase with energy. Since the forces used for acceleration in the suggested particle accelerator are largest near the z axis, while the forces which would be dangerous at high energies to to zero near the z axis, it seems that acceleration to high energies by the suggested method is indeed possible.

2) To estimate the electric fields which can be achieved by the method suggested here, we assume that the light source is a laser. For W, T_l, D and λ_0 we assume the values given in Table I. For simplicity we assume that the template produces only one Fourier component. We arbitrarily choose that component for which

$$-k_{z,n_0} = \kappa_{n_0}, \tag{32}$$

simply because for this one $|k_{z,n_0}/\kappa_{n_0}| = 1$ is an easy number to use in calculations. Substituting these values into Eq. (24a), and (5b), we evaluate $|E_r'(t', z', r')|$. Multiplying this by e, the electric charge of the electron, we obtain the absolute value of the force exerted by the field on a particle of charge e. For example, for the first column of Table I,

TABLE I

In the first part of Table I is a list of the parameters assumed for the light source which in this case is a laser; the template; and the particle beam. The last four lines give the maximum force on a particle exerted by the electromagnetic field created by the template, and some parameters of the γ radiation produced during one passage of a particle bunch through the electromagnetic field. The * means that while the dominant Fourier component has $\kappa_{n_0} = -k_{z,n_0}$, other components also exist because the field is focused by a large template on a section of only $D = 3$ cm long.

Laser			
Energy output per pulse	$5 \cdot 10^4$ J	10^3 J	$5 \cdot 10^4$ J
Instantaneous power, W	10^{12} W	2.10^{10} W	10^{12} W
Duration of pulse, T_l	50 $n$sec	50 $n$sec	50 $n$sec
Wavelength in vacuum, λ_0	10^{-4} cm	10^{-4} cm	10^{-2} cm
Template			
R	15 cm	15 cm	15 cm
$(1-\alpha)^{-1}$	No optical resonance	50	No optical resonance
Length of illuminated trajectory, D	10^3 cm	10^3 cm	3 cm
$\kappa_{n_0} =$	$-k_{z,n_0}$	$-k_{z,n_0}$	dominant:* $-k_{z,n_0}$
Particles			
$(1-v^2/c^2)^{-1/2} \equiv \gamma$	$3 \cdot 10^4$		10^2
spread in γ (related to velocity spread) in bunch	$10^{-3}\gamma = 3 \cdot 10$		10^{-1}
Particle is an	electron		electron
Number of particles per bunch, N	$5 \cdot 10^{11}$		$5 \cdot 10^{10}$
Bunchlength in time, T_b	10^{-1} $n$sec		10^{-1} $n$sec
Bunchlength along z axis, b_z	3 cm		3 cm
Horizontal emittance in bunch, A_x	$\pi \cdot 1.5 \cdot 10^{-5}$ cm rad		$\pi \cdot 1.5 \cdot 10^{-7}$ cm rad
Vertical emittance in bunch, A_y	$\pi \cdot 10^{-7}$ cm rad		$\pi \cdot 10^{-9}$ cm rad
Horizontal beam diameter at waist of bunch, b_x	$1.5 \cdot 10^{-1}$ cm		10^{-3} cm
Vertical beam diameter at waist of bunch, b_y	$2 \cdot 10^{-2}$ cm		10^{-4} cm
Particle density in bunch, ρ	$5.55 \cdot 10^{13}$ cm^{-3}		$1.67 \cdot 10^{17}$ cm^{-3}
Time spent by particle in field $\Delta t = D/v$	33.3 $n$sec		10^{-1} $n$sec
(Maximum force acting on a particle exerted by field, $\lvert F_r'\rvert_{\max})/\gamma$	$2.12 \cdot 10^{-1}$ GeV/cm		$3.87 \cdot 10^{-1}$ GeV/cm
Total energy radiated while bunch traverses field once, $\mathscr{E}_{\gamma,t}$	$2.01 \cdot 10^7$ ergs		$2.26 \cdot 10^{-1}$ ergs
Maximum energy of emitted γ-ray photon, $(\hbar\omega_{n_0}^e)_{\max}$	3.81 GeV		$4.23 \cdot 10^2$ eV
Number of γ-ray photons produced while bunch traverses field once, N_γ	$6.59 \cdot 10^9$		$6.67 \cdot 10^8$

$$|F_r'(t', r', z')| = |E_r'(t', r', z') \cdot e|$$
$$= 3.64 \cdot 10^{-1}\gamma \left| J_1\left(\frac{\omega r}{\sqrt{2}c}\right) \times \cos\left[\frac{\omega}{c}\left(ct - \frac{z}{\sqrt{2}}\right)\right]\right| \frac{\text{GeV}}{\text{cm}} \quad (33a)$$

This force oscillates periodically in time with amplitude which depends on r, and reaches its maximum $|F_r'|_{\max}$, at $r = 4.05 \cdot 10^{-5}$ cm, where $J_1(\omega r/\sqrt{2}c) = 0{,}582$, and

$$|F_r'|_{\max} = 2.12 \cdot 10^{-1}\gamma \text{ GeV/cm}, \quad (33)$$

a large force by most standards.

The required power output of the light source can be reduced by the use of optical resonance. This is the same principle which is also useful in connection with particle acceleration by template modified coherent light. To achieve such resonance, one coats the inside surface of the template by a partially reflecting mirror surface whose reflectivity is α, so that a photon impinges on it $(1 - \alpha)^{-1}$ times on the average, before penetrating it. We choose the radius of the cylindrical template to be R, where R satisfies the resonance condition. Since coherence of the radiation must be preserved inside the template, the α and R can not exceed the limits imposed by the coherence length of the laser light and the maximum smoothness of the mirror surfaces. Furthermore, the field at the template must not exceed about $E = 1.5 \cdot 10^3$ cgs units depending on the template material, because higher fields would damage the template. The parameters α and R listed in Table I satisfy these conditions. Table I shows that in this manner one can achieve considerable decrease in the required W. For example, if $(1 - \alpha)^{-1} = 50$, $R = 15$ cm, then W can be reduced by a factor of 50, without reducing $|F_r'|_{\max}$.

3) The values assumed for the parameters characterizing the particle beam are listed in Table I.

For $\gamma = 3 \cdot 10^4$, the parameters are similar (but not identical) to the ones already achieved or planned for electrons with energy up to 4.5 GeV at SPEAR and up to 15 GeV at PEP, except that here we assume a simple step function density distribution (as a function of x, y and z). The quantities b_x, A_x and D are chosen so that the horizontal beam diameter varies only by a small fraction while the particle passes through the field, and so does the vertical beam diameter. We neglect these variations and consider b_x and b_y to be the corresponding diameters everywhere in the field produced by the template.

For these parameters $\rho(x, y) = \rho(x)\rho(y)$, and the beam cross section is a quadrangle whose horizontal and vertical sides are respectively $1.06 \cdot 10^3\kappa_{n_0}^{-1}$ and $\sqrt{2} \cdot 10^2\kappa_{n_0}^{-1}$. We evaluate $\bar{J}_1^2$ for this $\rho(x, y)$ from Eq. (27). We use the approximation

$$\bar{J}_1(\kappa_{n_0} r) \approx \left(\frac{2}{\pi\kappa_{n_0} r}\right)^{1/2} \cos\left(\kappa_{n_0} r - \frac{3\pi}{4}\right),$$

observe that averaging over $\cos^2$ gives a factor 1/2 when $b_y \gg \kappa_{n_0}^{-1}$, and evaluate $\bar{J}_1^2$ approximately:

$$\bar{J}_1^2(\kappa_{n_0} r) \approx 10^{-3}. \quad (34)$$

Substituting Eq. (34) into (28) we find $\mathscr{E}_{\gamma,t}$, the energy produced during the time when one bunch of particles passes once through the field produced by the template. The result is listed in Table I. From Eqs. (14), (22), and (32), we find the maximum angular frequency (as measured in the laboratory frame) of the emitted γ rays. For example, when $\lambda = 10^{-4}$ cm, then

$$(\omega_{n_0}^e)_{\max} = 3.41\gamma^2\omega_{n_0} = 6.43 \cdot 10^{15}\gamma^2 \text{ sec}^{-1}, \quad (35)$$

which means that the maximum energy of the emitted γ ray photons is 3.81 GeV in the laboratory frame, and 63.5 keV in K_v. Using $\mathscr{E}_{\gamma,t}$, we find that the number, N_γ, of photons emitted while one bunch of particles traverses the field produced by the template is about $6.5 \cdot 10^9$. The γ-ray photons are emitted within a typical angle of γ^{-1} measured from the direction of the velocity of the emitting particle. According to Table I, the particle velocities themselves are parallel within an angle of 10^{-4} and $5 \cdot 10^{-6}$ radians respectively in the horizontal and vertical direction, so that the γ rays will be emitted within a typical angle of 10^{-4} radians in the horizontal direction, and $3 \cdot 10^{-5}$ radians in the vertical direction.

The parameters for $\gamma = 10^2$ are extrapolated from the parameters for $\gamma = 3 \cdot 10^3$ using the fact that the equilibrium electron beam diameters are proportional to γ. (The extrapolated beam damping time, proportional to γ^{-3} would then be about 45 minutes, but with a smaller radius of orbit curvature, it can be reduced). The N is assumed to be reduced by a factor of ten.

When $\gamma = 10^2$, then b_x, $b_y \ll \kappa_{n_0}^{-1}$ and the beam can be arranged so that it passes at a distance $r_0 \approx 1.8\kappa_{n_0}^{-1}$ from the axis, where $\bar{J}_1^2$ reaches its maximum value of 0.3387. For $\lambda_0 = 10^{-2}$ cm, the maximum energy of the emitted photons is $2.5 \cdot 10^2$ eV, and the radiation is emitted within a typical angle of 10^{-2} radians.

4) According to Eqs. (23) and (32), those particles which in K_v lie within a sphere of diameter d′ will radiate coherently, provided that

$$\mathrm{d}' \lesssim \pi[(2 + \sqrt{2})\gamma\omega/c]^{-1}. \tag{36}$$

When $\gamma = 10^2$, and $\lambda_0 = 10^{-2}$ cm, then

$$\mathrm{d}' = 1.46 \cdot 10^{-5}\ \mathrm{cm}, \tag{37}$$

and from Table I we find

$$\rho' = \gamma^{-1}\rho = 5.55 \cdot 10^{14}\ \mathrm{cm}^{-3}, \tag{38}$$

so that the number of particles inside a sphere of diameter d′, is

$$n = \frac{4\pi}{3}(\mathrm{d}')^3\rho' \approx 2.5. \tag{39}$$

Since n is proportional to λ^3, it can rapidly reach high values. These n particles will radiate coherently. The total radiated intensity coming from these particles will be about n times that which appears in Table I. This radiation will not be observable in the high energy region of the emitted radiation, unless the particle bunches have a substructure. The coherent radiation will be best observed in the high energy part of the emitted spectrum, if the substructure consists of small bunches of particles, d'/γ long in the laboratory, each containing about n particles. (If no substructure exists, then in the laboratory frame the coherent radiation will be emitted predominantly with a momentum and energy equal to the momentum and energy of the photons produced by the template).

5) The large enhancement in $\mathscr{E}_{\gamma,t}$ which can be achieved by coherent scattering by the electrons, opens up the possibility of producing essentially monochromatic γ rays. Indeed, Table I shows that for high values of n, most of the photons produced by the template are transformed into γ-ray photons near the surface of the particle beam. If the b_x and b_y beam diameters are large compared to the wavelength λ_0' measured in K_v of the photons produced by the template, then we see that the γ rays are produced simply by reflection from a fast moving mirror surface: the surface of the particle bunch. When the front of the particle bunch is sufficiently well defined, production of high energy monochromatic γ rays is possible in the direction which in the laboratory is almost parallel to $\mathbf{v}$.

6) So far we discussed coherence referring to production of γ rays by several electrons simultaneously. Next we will discuss the production of coherent γ ray radiation, that is radiation in which the γ ray photons are monochromatic and in phase with each other.

To produce coherent γ radiation, one can use the produced γ-rays (whether or not they are essentially monochromatic) to irradiate and invert the electron population of a target, by putting it into a higher, possibly metastable energy level. Choosing the γ-ray frequency appropriately, this method would be much more energy efficient than the usual method of pumping with (low energy) laser photons. This method of producing coherent γ-ray radiation will be discussed elsewhere.

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