SLAC-PUB-5496
WIS-91/9/Mar-PH
March, 1991
(T/E)

Cosmological and Astrophysical Constraints on a Pseudo-Dirac Tau Neutrino*

LANCE J. DIXON

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94309

and

YOSEF NIR†

Physics Department, Weizmann Institute of Science, Rehovot 76100, Israel

ABSTRACT

Recent experiments suggest that ν_τ may be a pseudo-Dirac neutrino with $m_{\nu_\tau} \approx 17\ keV$. In general, if such a neutrino is to be consistent with constraints from nucleosynthesis, one needs to give up the MSW solution to the solar neutrino problem *and* the singlet-majoron solution to the energy density problem. Also, the distance required for ν_τ to oscillate into its sterile Dirac partner becomes unobservably large.

Submitted to *Physics Letters B*

★ Work partially supported by the Department of Energy, contract DE-AC03-76SF00515.
† Incumbent of The Ruth E. Recu Career Development Chair. Supported in part by the Israel Commission for Basic Research and the United States–Israel Binational Science Foundation.

The most attractive explanation of the extreme lightness of neutrinos is the see-saw mechanism. In the basis $\{\nu_L, \bar{\nu}_R\}$, the mass matrix is of the form

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}. \tag{1}$$

Assuming that R, the scale of M_R, is much higher than D, the scale of m_D, there are three *light* neutrinos with $m_l = \mathcal{O}(D^2/R)$, much smaller than a typical charged fermion mass which is $\mathcal{O}(D)$. The other three neutrinos are heavy, $m_h = \mathcal{O}(R)$, and are of no phenomenological interest if R is much higher than the electroweak breaking scale.

If the mass of ν_τ turns out to be much higher than those of ν_μ and ν_e, a possible explanation could be that the see-saw mechanism operates on only two of the three neutrinos, which is the case if M_R is of rank 2 [1]. This would give (i) two very light neutrinos, $m_l = \mathcal{O}(D^2/R)$, (ii) two intermediate neutrinos, $m_i = \mathcal{O}(D)$, and (iii) two heavy neutrinos, $m_h = \mathcal{O}(R)$.

The mass scale of the two light neutrinos can be made arbitrarily small by choosing a very high scale R. The two intermediate neutrinos form an approximate Dirac neutrino, whose mass does not depend on R. Recently, this scenario evoked renewed interest [2,3] as a possible model for the 17 keV neutrino claimed to be observed in several experiments [4]. There are several reasons for interest in the model:

a. It allows the two light neutrinos to have their masses consistent with the MSW solution to the solar neutrino problem by requiring $D^2/R \sim 10^{-3}\ eV$.

b. It allows the tau neutrino to have a mass of 17 keV by requiring $D \sim 17\ keV$. (a and b together set $R \sim 300\ GeV$.)

c. The mixing of ν_e with ν_τ could be $\sin^2\theta_{e\tau} \sim 0.01$ without violating the experimental bounds on neutrinoless double beta decay because ν_τ is an approximate Dirac neutrino.

d. The cosmological bound on the neutrino lifetime from the energy density of the universe can be satisfied because the decay of ν_τ into a majoron and a lighter neutrino is much faster than in the usual see-saw scenario.

The existence of a fourth light neutrino, the right-handed component of ν_τ, may pose a problem with the standard model of nucleosynthesis if its abundance is comparable to that of the three left-handed neutrinos at a temperature around an MeV. In particular, oscillations between $\nu_{\tau L}$ and $\nu_{\tau R}$ can populate the latter states [5,6]. This problem is evaded if ν_τ is an *exact* Dirac neutrino; that is, if the mass difference δm_{ν_τ} between the two Majorana components of ν_τ vanishes. In this work we show that, in general, to satisfy the constraints from nucleosynthesis by reducing δm_{ν_τ} sufficiently, one needs to give up the MSW solution to the solar neutrino problem *and* the singlet-majoron solution to the energy density problem.

First, we briefly review the constraints from the three cosmological and astrophysical considerations: the solar neutrino problem, the energy density of the universe and nucleosynthesis. Second, we explain why the three relevant neutrino properties: the masses of the light neutrinos, the mass difference between the two components of the Dirac neutrino and the coupling of the majoron to the intermediate Dirac neutrino and the light majorana neutrino, are related to each other. We then carry out a complete analysis in a two generation case. Finally, we comment on the three generation case.

(*i*) The solar neutrino problem is best explained by the non-adiabatic solution of the MSW mechanism with the following mixing parameters between ν_e and another neutrino ν_i [7]:

$$\Delta m_{ei}^2 \sin^2 2\theta_{ei} = 5 \times 10^{-8}\ eV^2. \qquad (2)$$

The mass difference could vary in the range [8]:

$$10^{-7}\ eV^2 \leq \Delta m_{ei}^2 \leq 10^{-5}\ eV^2. \qquad (3)$$

(*ii*) The requirement that a massive neutrino ν_j does not carry energy density larger than the present energy density of the universe gives a bound on its mass and lifetime (see *e.g.* [1]):

$$m_j^2 \tau_j \leq 2 \times 10^{20}\ eV^2\ \text{sec}. \tag{4}$$

Note that for a pseudo-Dirac neutrino the limit is actually stronger and depends on the mass difference between the left-handed and right-handed components. For a mass difference large enough to keep ν_R in equilibrium (until the time that ν_L decouples) through oscillations, the limit is stronger by a factor of 4. For a very small mass difference, ν_R interacts only through helicity flip and the bound is strengthened by $\sim 20\%$ (for a Dirac mass $\sim 17\ keV$).

(*iii*) The requirement that the "effective" number of neutrino generations in the time of nucleosynthesis does not exceed 3.4 [9] puts a limit on the mixing parameters Δm_{ks}^2 and $\sin 2\theta_{ks}$ of a left-handed neutrino ν_k with a *sterile* neutrino ν_s [5,6]. For a pseudo-Dirac neutrino the mixing angle is $\sin 2\theta_{ks} \approx 1$, and the bound is [6]:

$$|\Delta m_{ks}^2| \leq 10^{-3}\ eV^2. \tag{5}$$

(The bound is even stronger than this if $\nu_k = \nu_e$.) We should emphasize that the nucleosynthesis constraint we are using is subject to systematic uncertainties in the determination of the primordial fraction of ^{4}He. We assume here the validity of these observations and their interpretation within the standard cosmological model.

We are interested in the case that ν_τ is the approximate Dirac neutrino of intermediate mass. We further assume, following ref. [2], that ν_τ decays dominantly through $\nu_\tau \rightarrow J + \nu_l$ where J is a majoron and ν_l is a light majorana neutrino. Thus:

a. We identify $i = \mu$ in eq. (3),

$$10^{-7}\ eV^2 \leq \Delta m_{e\mu}^2 \leq 10^{-5}\ eV^2, \tag{6}$$

b. We identify $j = \tau$ in eq. (4),

$$\Gamma(\nu_\tau) \geq \left(\frac{m_{\nu_\tau}}{17\ keV}\right)^2\ 10^{-27}\ eV. \tag{7}$$

The width is given by

$$\Gamma(\nu_\tau \to J + \nu_l) = \frac{(Y_{\nu_l \nu_\tau})^2}{16\pi^2}\ m_{\nu_\tau}, \tag{8}$$

so we require

$$Y_{\nu_l \nu_\tau} \geq 3 \times 10^{-15} \left(\frac{m_{\nu_\tau}}{17\ keV}\right)^{1/2}. \tag{9}$$

c. We identify ν_k and ν_s of eq. (5) with the two components of the approximate Dirac neutrino,

$$|\delta m_{\nu_\tau}| \leq \left(\frac{17\ keV}{m_{\nu_\tau}}\right)\ 3 \times 10^{-8}\ eV. \tag{10}$$

We will show that, in general, to satisfy eq. (10) one needs to violate both eqs. (6) and (7). Before doing so, however, we note that eq. (10) places a direct lower limit on the oscillation length $L_{\rm osc}$ for the maximal ($\sin 2\theta_{\tau s} \approx 1$) oscillations of ν_τ into its sterile partner ν_s [2]. We have

$$L_{\rm osc} \ = \ \frac{4\pi E_\nu}{|\Delta m^2_{\tau s}|} \ > \ \left(\frac{E_\nu}{1\ GeV}\right)\ 2400\ {\rm km}. \tag{11}$$

Such a large oscillation length would make it rather difficult to observe the oscillations in terrestrial experiments.

The argument why eq. (10) is generally incompatible with eqs. (6) and (7) is very simple. With a mass matrix of the form (1) and M_R being rank 2, the various relevant quantities are of the following orders of magnitude:

$$\begin{aligned} m_{\nu_\tau} =& \mathcal{O}(D), \\ \delta m_{\nu_\tau} =& \mathcal{O}(D^2/R), \\ \Delta m^2_{e\mu} =& \mathcal{O}(D^4/R^2), \\ Y_{\nu_l\nu_\tau} =& h\ \mathcal{O}(D^2/R^2). \end{aligned} \tag{12}$$

h is a dimensionless Yukawa coupling and we will take $\frac{h^2}{4\pi} \lesssim 1$. From the MSW constraint (6) we get

$$\frac{D^2}{R} \geq 3 \times 10^{-4}\ eV. \tag{13}$$

From the energy density limit constraint (9) we get

$$\frac{D^3}{R^4} \gtrsim 5 \times 10^{-35}\ eV^{-1}. \tag{14}$$

From the nucleosynthesis constraint (10) we get

$$\frac{D^3}{R} \leq 5 \times 10^{-4}\ eV^2. \tag{15}$$

We can satisfy eq. (15) together with eq. (13) only for $D \lesssim 2\ eV$. We can satisfy eq. (15) together with eq. (14) only for $D \lesssim 0.2\ keV$. Obviously, for $D \sim 17\ keV$ eq. (15) stands in contradiction with the requirements in (13) and (14).

We now demonstrate this general argumentation in a model with only two neutrino generations, $\{\nu_l, \nu_\tau\}$. The two generation model, besides allowing a simpler calculation, is a good approximation for the case of a large mass hierarchy between the two light majorana neutrinos, say $m_{\nu_e} \ll m_{\nu_\mu}$. In this case the scale relevant to the MSW effect is that of m_{ν_μ} and it is a reasonable assumption that a non-diagonal coupling $Y_{\nu_l\nu_\tau}$ would be larger the heavier ν_l is. Thus, ν_e becomes irrelevant to the two problems that we study, namely setting the scale for the MSW effect and finding the decay rate of ν_τ. However, to study the *mixing* between the light neutrinos one would certainly need to incorporate ν_e as well, because $\sin^2\theta_{e\tau} \sim 0.01$ is implied by experiments. Moreover, as we will see later, the three generation case involves some further complications.

The matrices m_D and M_R of eq. (1) are now 2×2. Without loss of generality, we choose a basis where M_R is diagonal:

$$m_D = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}; \quad M_R = \begin{pmatrix} M & 0 \\ 0 & 0 \end{pmatrix}. \tag{16}$$

A straightforward calculation gives the following mass values:

$$m_{\nu_\tau} = (m_{22}^2 + m_{12}^2)^{1/2}, \tag{17}$$

$$\delta m_{\nu_\tau} = \frac{(m_{11}m_{12} + m_{22}m_{21})^2}{M m_{\nu_\tau}^2}, \tag{18}$$

$$m_{\nu_\mu} = \frac{(m_{11}m_{22} - m_{12}m_{21})^2}{M m_{\nu_\tau}^2}. \tag{19}$$

To find the majoron coupling to ν_τ and ν_μ, we need to rotate its 4×4 coupling matrix

$$Y = \frac{h}{M}\begin{pmatrix} 0 & 0 \\ 0 & M_R \end{pmatrix} \tag{20}$$

to the mass eigenbasis. We find

$$Y_{\nu_\mu\nu_\tau} = h\,\frac{(m_{11}m_{12} + m_{22}m_{21})(m_{11}m_{22} - m_{12}m_{21})}{M^2 m_{\nu_\tau}^2}. \tag{21}$$

We can rewrite eq. (21) as

$$[Y_{\nu_\mu\nu_\tau}]^2 = h^2\,\frac{\delta m_{\nu_\tau}\cdot m_{\nu_\mu}}{M^2}. \tag{22}$$

First, we note that the explicit eqs. (17)–(21) confirm our order of magnitude estimates in (12). If we try to make δm_{ν_τ} small by lowering the scale of the m_{ij}

elements or increasing M, we at the same time decrease m_{ν_μ} and $Y_{\nu_\mu\nu_\tau}$. However, it is possible to concoct a model where δm_{ν_τ} is vanishingly small while m_{ν_μ} is not. As an example we can take m_D to be simultaneously diagonal with M_R: ν_τ is an exact Dirac neutrino of mass m_{22} while $m_{\nu_\mu} = m_{11}^2/M$. Note that the mixing between ν_μ and the Dirac neutrino, $\sin\theta_{\mu\tau} = 0$ in this case. On the other hand, eq. (22) shows that it is impossible to make δm_{ν_τ} vanishingly small while keeping $Y_{\nu_\mu\nu_\tau}$ finite: for $\delta m_{\nu_\tau} < 3\times10^{-8}\ eV$ and $m_{\nu_\mu} < 3\times10^{-3}\ eV$, and assuming $M > 300\ GeV$, we get $Y_{\nu_\mu\nu_\tau} < 3\times10^{-17}\ h$, more than an order of magnitude below the lower bound in eq. (9). Also note that the coupling of the majoron to a massless neutrino vanishes [1]. This is consistent with the assumptions made above.

We now study the three generation case. We choose a basis where $M_R = \mathrm{diag}(M_1, M_2, 0)$. Eqs. (17) and (18) now become:

$$m_{\nu_\tau} = (m_{33}^2 + m_{23}^2 + m_{13}^2)^{1/2}, \tag{23}$$

$$\delta m_{\nu_\tau} = \frac{(m_{11}m_{13} + m_{21}m_{23} + m_{31}m_{33})^2}{M_1 m_{\nu_\tau}^2} + \frac{(m_{12}m_{13} + m_{22}m_{23} + m_{32}m_{33})^2}{M_2 m_{\nu_\tau}^2}. \tag{24}$$

As for the two light majorana neutrinos, we get

$$\begin{aligned} m_{\nu_e} + m_{\nu_\mu} &= \sum_{i=1}^{2} \frac{(m_{2i}m_{33} - m_{3i}m_{23})^2 + (m_{3i}m_{13} - m_{1i}m_{33})^2 + (m_{1i}m_{23} - m_{2i}m_{13})^2}{M_i m_{\nu_\tau}^2}, \\ m_{\nu_e}\cdot m_{\nu_\mu} &= \frac{(\det m_D)^2}{M_1 M_2 m_{\nu_\tau}^2}. \end{aligned} \tag{25}$$

It is obvious that in general the scales of δm_{ν_τ} and m_{ν_l} are closely related. Yet, as in the two generation case, it is possible to fine-tune the various combinations that appear in eq. (24) in such a way that δm_{ν_τ} vanishes while m_{ν_l} does not.

Additional information about the various entries of the mass matrix can be obtained from constraints on the mixing of ν_τ with the flavor eigenstates ν_e and

ν_μ. The mixing required by Simpson's experiment gives:

$$(m_{13}/m_{\nu_\tau})^2 = \sin^2\theta_{e\tau} \sim 0.01 \Longrightarrow m_{13}/m_{\nu_\tau} \sim 0.1. \tag{26}$$

The experimental bound on the mixing between ν_μ and ν_τ gives

$$(m_{23}/m_{\nu_\tau})^2 = \sin^2\theta_{\mu\tau} \leq 0.001 \Longrightarrow m_{23}/m_{\nu_\tau} \leq 0.03. \tag{27}$$

To study the majoron couplings and their relation to the neutrino mass spectrum, we can neglect the mixing of ν_τ with the light neutrinos and put $m_{13} = m_{23} = 0$. (This condition may also be imposed by an appropriate unitary transformation on the left-handed neutrinos, so no generality is lost as long as we do not discuss mixing.) The above equations simplify to:

$$m_{\nu_\tau} = m_{33}, \tag{28}$$

$$\delta m_{\nu_\tau} = \frac{m_{31}^2}{M_1} + \frac{m_{32}^2}{M_2}, \tag{29}$$

$$\begin{aligned} m_{\nu_e} =& c_\gamma{}^2\left(\frac{m_{11}^2}{M_1}+\frac{m_{12}^2}{M_2}\right) + s_\gamma{}^2\left(\frac{m_{21}^2}{M_1}+\frac{m_{22}^2}{M_2}\right) + 2c_\gamma s_\gamma\left(\frac{m_{21}m_{11}}{M_1}+\frac{m_{22}m_{12}}{M_2}\right), \\ m_{\nu_\mu} =& s_\gamma{}^2\left(\frac{m_{11}^2}{M_1}+\frac{m_{12}^2}{M_2}\right) + c_\gamma{}^2\left(\frac{m_{21}^2}{M_1}+\frac{m_{22}^2}{M_2}\right) - 2c_\gamma s_\gamma\left(\frac{m_{21}m_{11}}{M_1}+\frac{m_{22}m_{12}}{M_2}\right). \end{aligned} \tag{30}$$

where we defined $c_\gamma \equiv \cos\gamma$ and $s_\gamma \equiv \sin\gamma$ and

$$\tan(2\gamma) = -2\,\frac{m_{21}m_{11}/M_1 + m_{22}m_{12}/M_2}{(m_{21}^2 - m_{11}^2)/M_1 + (m_{22}^2 - m_{12}^2)/M_2}. \tag{31}$$

The majoron couplings are given by:

$$\begin{aligned} Y_{\nu_e\nu_\tau} =& \frac{m_{31}(c_\gamma m_{11} + s_\gamma m_{21})}{M_1 V} + \frac{m_{32}(c_\gamma m_{12} + s_\gamma m_{22})}{M_2 V}, \\ Y_{\nu_\mu\nu_\tau} =& \frac{m_{31}(c_\gamma m_{21} - s_\gamma m_{11})}{M_1 V} + \frac{m_{32}(c_\gamma m_{22} - s_\gamma m_{12})}{M_2 V}, \end{aligned} \tag{32}$$

where V is the VEV of the majoron, $V = M_1/h_1 = M_2/h_2$. In general, δm_{ν_τ} is small enough if m_{31}^2/M_1, $m_{32}^2/M_2 = \mathcal{O}(10^{-8})\ eV$. In such a case, $Y_{\nu_l\nu_\tau}$ is too small to satisfy the requirement from the energy density limit (9). It is, however, possible to fine-tune $\delta m_{\nu_\tau} = \frac{m_{31}^2}{M_1} + \frac{m_{32}^2}{M_2} \ll \frac{m_{31}^2}{M_1}, \frac{m_{32}^2}{M_2}$ and avoid this relation. (If one incorporates the requirements on the *mixing* between ν_e and ν_μ that follow from the MSW constraint (2), the fine-tuning of δm_{ν_τ} looks even more contrived.)

In this work we did not discuss further astrophysical constraints on a 17 keV pseudo-Dirac ν_τ. In particular, it has been shown that the helicity flip of left-handed into right-handed neutrinos provides a mechanism for fast cooling of supernovae. The observation of neutrinos from SN1987A on a time scale of seconds puts an upper bound on the Dirac mass [11] which is at best only marginally compatible with $m_{\nu_\tau} \sim 17\ keV$. In contrast to our previous considerations, for this bound the distinction between an exact Dirac or a pseudo-Dirac neutrino is unimportant: the MSW effect in the supernova induces a very large mass difference between $\nu_{\tau L}$ and the sterile $\nu_{\tau R}$, so that there are no oscillations between the two.

Stronger bounds on the lifetime of ν_τ can be obtained from more model-dependent considerations of the large-scale structure formation in the early universe [12]. They cannot be satisfied with the singlet-majoron decay mode (independently of nucleosynthesis considerations) and thus we leave them out of our discussion.

Before concluding, we mention that there are other ways to theoretically allow the existence of a 17 keV ν_τ [13]. In some of these models, the majoron couples much more strongly than in the singlet-majoron model discussed above. In this case the lifetime of the ν_τ no longer poses a significant constraint. However, the majoron itself may contribute significantly to the energy density of the universe at the time of nucleosynthesis; its contribution is equivalent to 4/7 of a neutrino generation if it is in thermal equilibrium.

To summarize, if the see-saw mechanism is the reason for the extreme lightness of ν_e and ν_μ but it does not operate on ν_τ, rendering ν_τ a pseudo-Dirac neutrino with $m_{\nu_\tau} \sim 17\ keV$, then oscillations between the two components of this Dirac neutrino will increase the effective number of neutrino generations at the time of nucleosynthesis to 4.0, in contradiction with the standard model of nucleosynthesis.

If these oscillations are suppressed because the mass difference between the two components of ν_τ is very small, then it is highly unlikely that the mass difference between ν_μ and ν_e is large enough to explain the solar neutrino problem and it is almost impossible that the majoron coupling to ν_τ is large enough to satisfy constraints from the energy density limit. In addition, the period for oscillations of ν_τ into its sterile partner is so long as to make the oscillations unobservable by terrestrial experiments. We leave it to the reader to decide whether or not the first two results make the existence of a 17 keV ν_τ unlikely or cast doubts on the standard picture of nucleosynthesis.

ACKNOWLEDGMENTS: We gratefully acknowledge useful conversations with Josh Burton, Dave Schramm and especially Shmuel Nussinov.

REFERENCES

1. H. Harari and Y. Nir, Nucl. Phys. B292 (1987) 251.
2. S. Glashow, Harvard preprint HUTP-90/A075.
3. M. Fukugita and T. Yanagida, Kyoto preprint YITP/K-907 (1991).
4. J.J. Simpson, Phys. Rev. Lett. 54 (1985) 1891;
   J.J. Simpson and A. Hime, Phys. Rev. D39 (1989) 1825;
   A. Hime and J.J. Simpson, Phys. Rev. D39 (1989) 1837;
   B. Sur et al., LBL preprint, LBL-30109 (1990);
   A. Hime and N.A. Jelley, Oxford preprint, OUNP-91-01 (1991).
5. R. Barbieri and A. Dolgov, Phys. Lett. B237 (1990) 440, Nucl. Phys. B349 (1991) 743.
6. M.J. Thomson and B.H.J. McKellar, Melbourne preprint UM-P-90/44 (1990).
7. J.N. Bahcall and H.A. Bethe, Phys. Rev. Lett. 65 (1990) 2233.
8. A.J. Baltz and J. Weneser, Phys. Rev. Lett. 66 (1990) 520.

9. K.A. Olive, D.N. Schramm, G. Steigman and T.P. Walker, Phys. Lett. B236 (1990) 454.

10. N. Ushida et al., Phys. Rev. Lett. 57 (1986) 2897;
L.A. Ahrens et al., Phys. Rev. D31 (1988) 2732.

11. G. Raffelt and D. Seckel, Phys. Rev. Lett. 60 (1988) 1793;
R. Gandhi and A. Burrows, Phys. Lett. B246 (1990) 149.

12. G. Steigman and M.S. Turner, Nucl. Phys. B253 (1985) 375.

13. M.J. Dugan, A. Manohar and A.E. Nelson, Phys. Rev. Lett. 55 (1985) 170;
K.S. Babu and R.N. Mohapatra, Maryland preprint UMD-PP-91-186 (1991);
K.S. Babu, R.N. Mohapatra and I.Z. Rothstein, Maryland preprint UMD-HEP 91-203 (1991);
E. Carlson and L. Randall, Harvard preprint HUTP-91/A008 (1991).