October 22, 1963

STANDING VERSUS TRAVELLING WAVEGUIDES

M. Jakobson
Los Alamos Scientific Laboratory

Since the electron linear accelerators in operation at the present time are travelling wave machines, the question naturally arises: at what energy would a travelling wave proton accelerator be more desirable than a standing wave proton accelerator? The transition from a drift tube section to a travelling wave section could, in principle, take place as in Fig. 1.

For the conventional constant impedance linac in which the fractional power loss is constant along the guide, the power diffusion equation is

$$P = P_o e^{-\frac{z}{\ell_o}} \tag{1}$$

where ℓ_o is the power attenuation length. When the voltage gain in a tank length ℓ is maximized for a given power input by varying ℓ/ℓ_o, the condition $\ell_o \sim 0.4\ell$ is obtained. As a result ~ 10% of the rf power is available at the end of the tank. This power could be used to drive the amplifier for the next tank if the associated phase control problems were not too difficult.

For beam loading less than a percent, the decrease in the electrical gradient along the tank caused by the beam will be small. With beam loading of the order of 20% the decrease will be appreciable and will diminish the

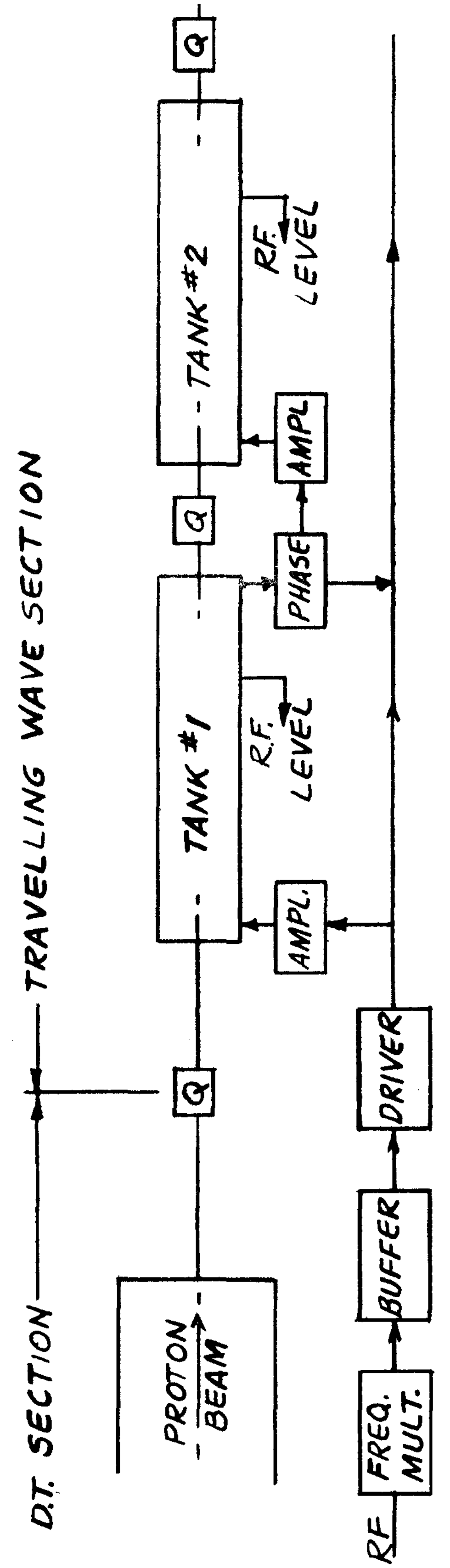
D.T. SECTION
TRAVELLING WAVE SECTION
PROTON
BEAM
Q
TANK #1
Q
TANK #2
Q
AMPL.
R.F.
LEVEL
PHASE
AMPL
R.F.
LEVEL
RF
FREQ.
MULT.
BUFFER
DRIVER

FIG 1

phase acceptance for the travelling wave section. Another difficulty that arises in the travelling wave section is associated with the damping of the phase and energy oscillations of the non-synchronous particles.

Consider the phase oscillations of the protons in the tanks of the travelling wave section. For small oscillations, the frequency of the phase oscillations in the frame of the synchronous particles is given by

$$\Omega_\psi^2 = \frac{e\epsilon_o \omega \sin\psi_s}{\gamma m_o \beta c} \tag{2}$$

Using this expression and assuming that the energy of the phase oscillations divided by their frequency is constant, the difference of the phase of a particle from the synchronous phase can be obtained:

$$\Delta\psi \propto (\epsilon_o \sin\psi_s \; \gamma^3\beta^3)^{-1/4} \tag{3}$$

If, in a travelling wave tank, the power travelling down the tank is diminished along the tank by a factor of 10, the electrical gradient will decrease by a factor of ~ 3. Assuming Eq. (3) to be valid, the ratio of the phase spread of the particles leaving the tank is

$$\frac{\Delta\psi_o}{\Delta\psi_i} \sim \left(\frac{\epsilon_{o\ out}}{\epsilon_{o\ in}}\right)^{-1/4} \left(\frac{\beta_o\gamma_o}{\beta_i\gamma_i}\right)^{-3/4} \sim 1.3 \tag{4}$$

In order for Eq. (3) to be valid, the change in field strength should take place over many oscillations. When the number of phase oscillations is computed for the energy gain from 200 to 220 MeV, only a fraction of a phase oscillation is involved. Although for this case the ratio given by Eq. (4) is incorrect, one would not expect the phase spread to be damped as rapidly as*

$$\frac{\Delta\psi_o}{\Delta\psi_i} \sim \left(\frac{\beta_o\gamma_o}{\beta_i\gamma_i}\right)^{-3/4} \tag{5}$$

Multiple feeds along a tank would eliminate these difficulties but it is an added complication that is undesirable. Another possible solution would be to construct a constant gradient travelling-wave accelerator.

*Note added after the meeting:

A computer has been used to check the damping of the phase and energy oscillations for a 185-800 MeV travelling wave accelerator in which the electrical gradient decreased linearly by a factor of three along the tanks. If one half of a phase oscillation occurs in a tank, instability of the beam occurs. If the tanks are long enough so that one quarter of a phase oscillation or more occurs in one tank, the expected adiabatic damping of Eq. (5) does not occur. If the tanks are reduced in length to ~ 1/8 (or less) of a phase oscillation per section, the expected adiabatic damping occurs except near the limits of the stable region. A computer program is being set up to obtain the decrease in phase acceptance due to beam loading.

For a TW accelerator, $P = Wv_g$ where P is the power flow along the tank, W is the energy unit length and v_g is the velocity with which energy is propagated along the tank. If the gradient is to be maintained constant along the tank as the power flow along the tank decreases due to dissipative losses, the guide velocity v_g must decrease in proportion. The increase in loading necessary to reduce the guide velocity will reduce the amplitude of the fundamental accelerating component so that the particles will be accelerated by a wave of decreasing amplitude, as was the case for the constant impedance accelerator. Minimizing the power required for a given energy gain in a tank will result in a v_g/c that decreases from $\sim .02$ at the tank input to $\sim .002$ at the output. As a result, for an iris-loaded guide, the amplitude of the fundamental component will be reduced such that $\epsilon_{o\ final}/\epsilon_{o\ initial} \sim 0.9$.[1]

If the tanks are sufficiently long such that Eq. (3) is valid, the phase damping along the travelling wave section will be reduced from that expected from Eq. (5). Effectively, the restoring force for the phase oscillations is diminished along each tank.

The tanks could be designed for a constant fundamental accelerating component but not for both the loaded and unloaded condition. If the gradient is constant for the beam-loaded accelerator, the gradient will increase as the beam load diminishes. The relative energy spread will be dependent upon the increasing gradient with a dependence that can be calculated if the gradient increases during a number of phase oscillations.

Let T_s be the kinetic energy of a synchronous particle. In the laboratory

$$T = T_s + \gamma\beta c P' \tag{6}$$

where P' is the momentum of the nonsynchronous particle in the frame moving with speed βc. Then $T = T_s \pm \gamma\beta c \; 2m_o W$ and the relative energy spread is

$$\frac{\Delta T}{T_s} = \frac{(T - T_s)}{T_s} \propto \left[\frac{\epsilon_o \sin\psi_s}{\beta^5} \gamma^3\right]^{1/4} \tag{7}$$

In a section such that Eq. (7) is valid, if ϵ_o increases more rapidly than β^5, there will be an increase in the relative energy spread. For tanks of lengths such that Eqs. (3) and (7) are valid, and for decreasing relative energy spread and decreasing amplitude of the phase oscillations, the amplitude of the accelerating component should be constant along the tank for the unloaded and beam loaded conditions. Essentially, this can be achieved for a resonant tank with standing waves.

COURANT: I don't understand what has happened to your computation of phase space, in the case where the adiabatic change of parameters causes the growth of the phase width, then you must be getting a decrease in energy spread conversely.

JAKOBSON: If $\Delta\psi \sim (\epsilon_o \sin\psi_s \gamma^3 \beta^3)^{-1/4}$ then ΔT should be proportional to $(\Delta\psi)^{-1}$. Equation (7) is for the relative energy spread $\Delta T/T$. So the energy spread $\Delta T \sim (\epsilon_o \sin\psi_s \gamma^3 \beta^3)^{+1/4}$ is in agreement with your comment.

COURANT: Are the individual sections long compared to a phase oscillation?

JAKOBSON: I think if one is worried about heavy loading and power requirements, you're forced to sections which are long compared with the attenuation length. It would be best to dump power of the order of only 1/10th of that which was put in.

GLUCKSTERN: Are you talking about a 10 meter section now?

JAKOBSON: Yes.

GLUCKSTERN: What is the wavelength of a phase oscillation?

JAKOBSON: Much longer.

COURANT: In that case, the adiabatic approximation for the phase bunch damping doesn't mean very much, because, from the point of view of beam dynamics, each of these sections acts as an impulse accelerator.

JAKOBSON: What you're saying is that if you had a machine with an increasing electrical gradient which did a certain amount of damping, and you then cut it in to sections, (which is really a sawtooth), you will get a different answer.

COURANT: Very little, if the sawteeth are short compared to one phase oscillation.

JAKOBSON: I believe that they're very nearly the same.

LEISS: Look at it another way. Say that you do an harmonic analysis in terms of the disturbances, where your units are in the length of a period of phase oscillation. What you're saying is that the 10th or 20th harmonic is important, and this is really quite unusual.

GLUCKSTERN: I think those two problems are different.

KNOWLES: Why does the accelerating field decrease in a constant-gradient accelerator?

JAKOBSON: The problem is that you have to decrease v_g which means you must increase the loading, or decrease the coupling of adjacent cavities. Essentially, v_g goes as $(a/b)^4$, in which a is the iris aperture radius and b the cavity radius. You have to decrease (a/b) which means that the unwanted components increase and the component doing the accelerating decreases. Unwanted spatial components expend power, but do not accelerate.

BERINGER: I gather from your comments that it is still an open question as to whether standing wave accelerators will pay for themselves because of the power expenditure.

References

(1) R. Neal, BNL 6511, p. 154.