HARD PROCESSES IN QCD PERTURBATION THEORY

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ABSTRACT

We review the basic theoretical ideas underlying the applications of QCD perturbation theory to hard inclusive processes. The physical origin and relevance of mass singularities are discussed. Factorization and universality of mass singularities are illustrated in the case of the calculation of quark structure functions in deep inelastic scattering.

RESUME

Nous présentons ici les idées de base concernant l'application de la théorie des perturbations en Chromodynamique Quantique aux processus inclusifs "durs". En particulier, nous discutons l'importance et l'origine physique des singularités de masse en théorie des perturbations. Les propriétés d'universalité et de factorisation de ces singularités sont mises en évidence dans le cas particulier du calcul des fonctions de structure des quarks.

Much work has been carried out during the last couple of years concerning the relationship between QCD perturbation theory and the naive quark-parton model in hard processes caracterized by the presence of a large invariant mass Q^2. If the parton model results are interpreted as the zeroth order approximation in the expansion in terms of the strong coupling constant α_s, then it is now quite well understood why radiative corrections associated, for example, with gluon bremhstralung by the incoming or outgoing partons are responsible for the logarithmic deviations from exact Bjorken scaling predicted by asymptotic freedom. For spacelike Q^2 the use of QCD perturbation theory reproduces of course the well known asymptotic freedom results obtained by means of renormalization group techniques(1). However, QCD perturbation theory can also be applied to precesses with time-like invariant masses - like for instance the Drell-Yang process - and one can therefore derive asymptotic freedom-like results in these cases in which renormalization group techniques are not available.(2)

The purpose of this talk is to give an introduction to the theoretical ideas underlying the use of QCD perturbation theory in hard processes. One starts by accepting the basic hypothesis that has already gained a large concensus, namely, that QCD is the correct underlying theory of strong interactions ; and therefore partons are colored quarks, anti-quarks and gluons whose dynamical behavior is governed by an $SU(3)_c$ gauge theory. Since real hadrons and the related confinement problem are still out of the reach of our computational abilities, one assumes further that they are described by soft wave functions which strongly damp the invariant masses and the transverse momenta of partons. Finally, one assumes that hard scattering cross sections off hadrons are obtained by convoluting hadronic wave functions with hard scattering cross sections off partons. These two postulates are of course the basic premises of any partons model.

One is therefore left with the problem of computing hard scattering cross sections off partons. Since there is a large invariant mass available, Q^2, one can expect that they can be computed in an improved perturbation theory in the running coupling constant $\alpha_s(Q^2)$. However, one must learn to tame large logarithms $\log Q^2$ which show up due to divergencies of infrared nature and which invalidate the improved perturbation theory. These logarithms are responsible for scale breaking effects in QCD, and they are best understood in my opinion from the point of view of mass singularities, as in Ref.(3)-(4), namely, infrared singularities that occur in perturbation theory when the quark m goes to zero.

We discuss first the physical origin and the relevance of collinear (mass) singularities in perturbation theory. We then proceed to discuss the applications to QCD jets and to the calculation of scale breaking in deep inelastic processes. Finally, the major results in the field are summarized in the last section

1 - The relevance and physical origin of mass singularities.

Let σ be a hard scattering cross section off partons. It will depend in general on the large invariant mass Q^2, the quark masses m^2, the renormalization mass μ^2, a set of Bjorken variables x_i and the strong coupling constant $\alpha_s(\mu^2)$. Because of dimensional reasons we can always write σ as :

$$\sigma(Q^2,m^2,\mu^2,x_B,\alpha_s(\mu^2)) = \frac{1}{Q^2} F\left(\frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}, x_B, \alpha_s(\mu^2)\right) \qquad (1)$$

where F is dimensionless. Since F is a physical observable, it must be independent of the renormalization mass μ^2. Therefore, one can think of choosing $\mu^2 = Q^2$ and writing F in the form

$$F = F\left(1, \frac{m^2}{Q^2}, x_B, \alpha(Q^2)\right) \qquad (2)$$

where $\alpha(Q^2)$ is the QCD running coupling constant. In the leading log approximation, where one sums all the powers of $\alpha_s(\mu^2)\log\frac{Q^2}{\mu^2}$, the running coupling constant is given by

$$\alpha(Q^2) = \frac{\alpha_s(\mu^2)}{1+b\alpha_s(\mu^2)\log(\frac{Q^2}{\mu^2})} \qquad (3)$$

where

$$b = \frac{1}{12\pi}(11N_c - 2N_f) \qquad (4)$$

N_c, N_f being the numbers of colors and flavors, respectively.

Since the running coupling constant decreases logarithmically as $Q^2 \to \infty$ (the asymptotic freedom result) one can see immediately that in this case F can be safely expanded in an improved perturbation theory in $\alpha(Q^2)$ provided that the zero mass limit $m^2 \to 0$ is regular. This will happen, as we shall soon discuss in more detail, whenever there process under consideration is fully inclusive in the sense that there are no well identified parton lines in the initial or final state. A typical example is the ratio R of hadronic to leptonic yield in e^+e^- collisions, which can be expanded as

$$R = N_c \sum_i Q_i^2 \left[1 + \frac{\alpha(Q^2)}{\pi} + \dots \right] \qquad (5)$$

Whenever there are well identified partons in the initial or final state, (as, for example, in the calculation of deep inelastic structure functions or in Drell-Yan processes) the zero mass limit $m^2 \to 0$ will not be regular in perturbation theory in the sense that logarithmic singularities of the form $\log \frac{Q^2}{m^2}$ will show up. In order to reestablish an improved perturbation theory, all leading powers fo the form $\alpha^n(Q^2)\log^n \frac{Q^2}{m^2}$ have to be summed up, non leading powers of logarithms of the form $\alpha^n(Q^2)\log^{n-\ell}(\frac{Q^2}{m^2})$ being a correction of order $\alpha^\ell(Q^2)$ with respect to the leading ones. This procedure constitutes the leading log approximation.

The physical origin of mass singularities in perturbation theory is easily understood by observing that a massless quark can emit or absorb a hard collinear gluon and remain on its mass shell. Indeed, if we consider the process quark(p)$\to$ $\to$ quark(p') + gluon(k) we obtain for the virtual mass of the outgoing quark

$$p'^2 = (p-k)^2 = -2p.k = -2p_0\, k_0(1-\cos\theta) \tag{6}$$

θ being defined as the angle of emission of the gluon with respect to the direction of the incoming quark. Clearly, if the emission is collinear ($\theta = 0$) the outgoing quark is on its mass shell. Therefore, collinear emission of hard gluons induces dangerous propagators in perturbation theory, which are responsible for the logarithmic mass singularities in transition probabilities.

Let us discuss a simple example in QED : photon bremsstrahlung by an electron in an external field. The transition amplitude corresponding to the diagram (1.a), where the photon of momentum k_μ is emitted by the outgoing electron, is given up to irrelevant factors by

$$A = \frac{2\varepsilon.p_2}{(k+p_2)^2 - m_e^2} \tag{7}$$

where ε_μ is the polarization vector of the emitted photon. We have only exhibited the electron propagator and the emission vertex $\varepsilon.p_2$ because they are the only relevant factors to understand the mass singularity when $m_e^2 \to 0$. Taking into account the mass-shell conditions $k^2 = 0$ and $p_2^2 = m_e^2$ the transition amplitude can be written, for $m_e^2 \ll \vec{p}_2^{\,2}$, as

$$A \simeq \frac{\vec{\varepsilon}.\vec{p}_2}{k\,|\vec{p}_2|(1 - \cos\theta + \frac{me^2}{2|\vec{p}_2|^2})} \tag{8}$$

We can see in this expression that the dangerous collinear singularity for $\theta = 0$ is in fact regulated by the finite electron mass. The factor k^{-1} is of course responsible for the usual infrared divergencies, due to the emission of soft photons. If we neglect the electron mass, the denominator will behave like θ^2 for small values of θ. However, the numerator gives also a power of θ in this limit because $\bar{\varepsilon}$ is the polarization vector of a real photon and is therefore perpendicular to $\bar{k}$. We have then :

$$\vec{\varepsilon}.\vec{p}_2 = |\vec{p}_2| \sin\theta \simeq |\vec{p}_2|\,\theta \qquad (9)$$

This vanishing of the emission vertex of a hard collinear gluon is simply understood in terms of helicity conservation : a quark that emits a collinear gluon is forced to flip its helicity. We then conclude that the amplitude A behaves both in the infrared ($k \simeq 0$) and collinear ($\theta \simeq 0$) regions as

$$A \simeq \frac{1}{k\theta} \qquad (10)$$

The transition probability for photon bremsstrahlung is then obtained by integrating over phase space,

$$\int \frac{d\vec{k}}{2k_0} |A|^2 \simeq \int \frac{k^2 dk}{k}\, d\cos\theta \,\frac{1}{k^2\theta^2} \simeq \int \frac{dk}{k} \int \frac{d\theta}{\theta} \qquad (11)$$

which exhibits the infrared and collinear singularities, of the form $\log(|p_2|/m_\gamma) . \log(|p_2|/m_e)$, m_γ being a fictition photon mass.

It is very important to keep in mind how the infrared and mass singularities are softened, that is to say, under what conditions the limit $m_\gamma \to 0$ and $m_e \to 0$ can safely be taken. This is summarized in the Kinoshita-Lee-Nauemberg Theorem[5],[6], which states essentially that infrared and mass singularities are softened when total transition rates are computed between nearly degenerate initial and/or final states. In other words, individual transition probabilities have infrared and mass singularities, but these singularities are cancelled when inclusive cross sections are computed by suming over sets of states degenerate in the appropriate zero mass limits. A typical example is again the case of electron scattering by an external field. When radiative corrections are computed by summing the Feynman diagrams of Fig.(2a,b,c,d) the result is, to lowest order in α, the Born cross section times a factor of the form

$$1 - \alpha\, C \log\frac{|\vec{p}|}{m_e} \log\frac{|\vec{p}|}{m_\gamma} \tag{12}$$

However, if one computes instead the total transition rate to a final state with finite energy resolution ΔE, to the same order of perturbation theory one must also consider the probability that a soft photon of energy $k < \Delta E$ is emitted either by the incoming or by the outgoing electron. The transition amplitudes correspond to the Feynman diagrams of Figs(1.a,b). When the inelastic cross section is added to the elastic one, infrared divergences cancel and in the final result the limit $m_\gamma \to 0$ can be safely taken. One obtains a correction factor of the form (12) except that the mass m_γ is replaced by the energy resolution ΔE.

In the same way as infrared singularities are regulated by means of an energy resolution, collinear singularities can, according to the KLN theorem[(5)], be regulated by introducing an angular resolution δ and by computing transition rates to nearly degenerate states which include the charged particles plus an arbitrary number of collinear photons, not necessarily soft. Only in this case the limit $m_e \to 0$ can be safely taken, and the total transition rate is free of mass singularities[(5)(6)].

Exactly the same arguments work in QCD. Exclusive processes containing colored partons in the initial or final state will exhibit in general mass singularities in perturbation theory. We then expect on general grounds the large logarithms $\log\frac{Q^2}{m^2}$ which invalidate perturbation theory as $Q^2 \to \infty$, unless the conditions of the KLN theorem are met and the mass singularities are softened because one is looking at an inclusive process which does not discriminate against the collinear decay of massless partons in the final state, for example. Such is the case of the ratio R of hadronic to leptonic yields in e^+e^- annihilation. It is not however the case of the quark structure functions or Drell-Yan processes because we do not include in the calculation nearly degenerate initial states containing the original partons plus an arbitrary number of hard collinear gluons.

As in QED, infrared and mass singularities can be softened by introducing energy and angular resolutions. A by now classical example is the Sterman-Weinberg result[(7)] for the two-jet cross section in e^+e^- annihilation. Let us call E the total e^+e^- energy, and $\sigma(E,\theta,\varepsilon,\delta)$ the cross section for all annihilation events in which a fraction $(1-E)$ of the total energy E is emitted within a pair of oppositely directed cones of opening angle 2δ (see Fig.(3)). This means that a fraction ϵ E of the incident energy has been lost outside the cones. Then, the cross-section σ is finite in the zero-mass limit and given by :

$$\frac{1}{\pi\delta^2}\,\sigma\,(E,\theta,\varepsilon,\delta) = \frac{3\alpha^2}{4E^2}\,(1+\cos^2\theta) \tag{13}$$

$$\sum_i Q_i^2\,[1 - \frac{4\alpha_s}{3\pi}\,(3\ell n\delta + 4\ell n\delta\;\ell n\,2\varepsilon + \frac{\pi^3}{3} - \frac{5}{2})\,]$$

The reason why σ is finite in the zero mass limit is again a consequence of the KLN theorem. The $O(\alpha_s)$ term originates from virtual corrections to the $q\bar{q}$ final state and also from gluon bremsstrahlung by the outgoing q or $\bar{q}$. If, after gluon radiation by a quark emitted originally in the direction θ, one fo the decay products (quark or gluon) goes outside the cone, we know that it is neither collinear nor soft, since it has necessarily an energy ϵE. Therefore, one is not missing any nearly degenerate state in the transition rate.

2 - The quark structure functions.

Let us now consider the calculation of non-singlet flavor components in deep inelastic scattering such as, for example

$$\nu\, W_2(\text{up quark}) - \nu\, W_2(\text{down quark})$$

whose practical advantage lies in the fact that only diagrams of the kind exhibited in Fig.(4.a,b), where the electromagnetic current couples directly to the valence quark, do not cancel, since the electromagnetic charges of the up and down quark are different. Diagrams in which the electromagnetic current couples to the sea quarks, as in Fig. (4c), cancel in the difference of the two structure functions. Many authors have considered this problem in great detail (see References (2)-(4) ; (8)-(17)) and we shall limit ourselves to sketch the main lines of the calculation, following the methods of Ref.(4).

Consider first the case of physical gluon bremsstrahlung by the incoming quark (Fig. 5.a). If we call θ the angle of emission of the gluon with respect to the incoming quark, the amplitude behaves as

$$A \simeq \frac{1}{\theta^2}\cdot\theta \tag{14}$$

and therefore phase space integration will give

$$\int d\cos\theta\ |A|^2 \simeq \int \theta\ d\theta\ \frac{\theta}{\theta^2}\ \frac{\theta}{\theta^2} \simeq \int \frac{d\theta}{\theta} \simeq \log\frac{Q^2}{m^2} \tag{15}$$

where the last result follows from the fact that when a small quark mass is retained the angular integration (see Eq.(8)) the angular integration gives $\log(\frac{m^2}{|\vec{p}|^2})$, and that a simple kinematical analysis shows that $|\vec{p}|^2$ is proportional to Q^2. The constant of proportionality is of course irrelevant when one is interested in computing the leading mass singularities. We must also worry about the possibility that the gluon is radiated by the final quark. Interference diagrams of the type showed in Fig.(5.b) give a finite result. Indeed, if the gluon is collinear to the incoming quark it cannot be collinear to the outgoing one due to the hard nature of the process under consideration. Therefore, one finds for the interference diagrams

$$\int d\theta \cdot \frac{\theta}{\theta^2} = \text{finite} \tag{16}$$

In the same way it is easy to see that the emission of n gluons from the incoming quark, exhibited in Fig. 6, gives rise to a leading mass singularity of the form $(\log\frac{Q^2}{m^2})^n$. In this case it is much more convenient to trade the emission angle θ by the invariant mass t of the quark running up the multiperipheral ladder. As shown in Fig. 7.

$$t = p'^2 = (p-k)^2 = -2p.k = -2|\bar{p}|.k(1-\cos\theta) \tag{17}$$

then

$$\frac{d\vec{k}}{2k_o(2\pi)^3} = \frac{k^2 dk\ d\cos\theta}{4k^2.2k} = \frac{1}{16\pi^2}\frac{dk}{|p|}dt \simeq \frac{dz.dt}{16\pi^2} \tag{18}$$

where we have written $k = zp$. This is due to the fact that in extracting the leading mass singularities one will explicitly extract from $|A_n|^2$ the θ - or t - distributions responsible for the leading logs and set whatever structure is left in $|A_n|^2$ in the collinear configuration $k = zp$. The variable z is therefore the fraction of the incoming momentum carried away by the emitted gluon. Each gluon emission will give rise to a behavior of the form $\theta_i^{-1} \simeq t_i^{-1/2}$ in the production amplitude A_n. Moreover, it follows from simple kinematical considerations that the invariant masses t_i are ordered :

$$m^2 \leq |t_1| \leq |t_2| \leq \ldots \leq |t_n| \leq Q^2 \tag{19}$$

Therefore, the angular part of the phase space integration yields a factor of the form

$$\alpha_s^n \int_{m^2}^{Q^2} \frac{dt_n}{t_n} \ldots \int_{m^2}^{t_3} \frac{dt_2}{t_2} \int_{m^2}^{t_2} \frac{dt_1}{t_1} = \frac{\alpha^n}{n!} (\log \frac{Q^2}{m^2})^n \tag{20}$$

where α_s is the strong coupling constant - not yet running - because for the moment we are computing the simplest diagrams arising from gluon radiation by the incoming quark and virtual corrections that make the coupling constant run are not yet included. It is also important to recognize that, although the ordering of invariant masses $|t_i|$ follows from kinematics, the ledaing mass singularity arises from a region of integration where the Z_i are strongly ordered, in the sense that $|t_i| \ll |t_{i+1}| \ll Q^2$. Indeed, the same result at the leading log level is obtained if $|t_i|$ is only integrated up to $\epsilon\ |t_{i+1}|$, where ϵ is an arbitrarily small but fixed number.

Equation (20) does not exhaust the phase space integration since $|A_n|^2$ still depends on the z_i's, the fraction of the momentum of the parent quark carried out by the i^{th} gluon. The z-dependence of the matrix elements is computed in detail in the literature and we shall not describe it here (see, for example, Refs. (3),(18)). Calling $\sigma_m(p)$ the spin averaged total cross-section for an incoming quark of momentum p, the final result is

$$\sigma_m(p) = \frac{1}{m!} [C_F \frac{\alpha_s}{2\pi} \log \frac{Q^2}{m^2}]^m \cdot \int_0^1 dz_1 \ldots dz_m\ P(z_1) \ldots P(z_m)\ \sigma_o((1-z_1).(1-z_m)p) \tag{21}$$

where $C_F = (N_c^2-1)/2N_c$, and $P(Z)$ are the Altarelli-Parisi probabilities

$$P(z) = \frac{1+(1-z)^2}{z} = P_{qq}(x) \ , \quad x = 1-z \tag{22}$$

which are the spin averaged probabilities of a collinear decay of a quark into quark plus gluon. In Eq.(21) σ_o represents the Born approximation to the hard process, in which no gluons are radiated by the incoming quark. The same arguments based on counting powers of θ_i given before can be used to show that interference diagrams or ladder diagrams with crossed gluon legs do not give leading mass singularities.

The calculation leading to Eq.(21) takes into account only a small subset of diagrams contributing to the quark qtructure functions, and the power counting arguments worked only because the gluons emitted were transversely polarized physical gluons. (remember that the transverse polarization was essential to obtain a power of θ in the numerator in Eq.(8)). In general one has to deal with all possible diagrams contributing to the discontinuity of the forward Compton amplitude quark (p) + γ (q) $\rightarrow$ quark (p) + γ (q). Even if one restricts oneself to gluon emission by the incoming quark, the gluons emitted can be off shell, with invariant mass k_i^2, and can further decay into massless partons. The problem of extracting leading mass singularities from QCD perturbation theory has been fully analysed by the authors mentioned before(2-4),(8-17). We shall again outline the steps leading to the general result, following Ref.(4).

a) As first noticed by Lipatov(19), a clever choice of gauge helps to minimize the number of diagrams contributing to the leading log approximation. Such a gauge is the axial gauge, defined by

$$n^\mu A_\mu^{(a)} = 0 \quad ; \quad n^\mu n_\mu \leq 0 \tag{23}$$

In this gauge, the power-counting arguments work again, and one is left with the contribution of dressed ladder diagrams exhibited in Fig. (8.a). The dotted lines and vertices represent dressed propagators and vertices. Diagrams like those exhibited in Figs (8,b,c) do not contribute to the leading log approximation.

b) Ward identities in the axial gauge together with the observation that leading mass singularities arise from the strongly ordered region $|t_i| \gg |t_{i-1}|$, k_i^2 (see Fig.(6)) allows one to prove that virtual corrections to a quark propagators cancel against virtual corrections to the vertex below it, at least in what concerns the calculation of leading mass singularities. One is then left with ladder diagrams containing dressed gluons and vritual corrections only to the external quark lines, as indicated in Fig.(9). Then the previous equation (21) is essentially correct, provided one is able to modify it by dressing the gluons and the external quark lines

c) Let us now discuss the effect of dressing the gluons. At the leading log level, the relation between the bare axial vector propagator $D_{\mu\nu}^{(0)}(k^2)$ and the dressed one $D_{\mu\nu}(k^2)$ is given by

$$\alpha_s D_{\mu\nu}(k^2) = \alpha(k^2) D^{(0)}_{\mu\nu}(k^2) \tag{24}$$

This relation is of course only true in the axial gauge, in which only virtual corrections to the gluon propagator contribute to the running coupling constant in the leading log approximation. This is clearly related to the cancellation of virtual corrections to the quark propagators and vertices. The net result of dressing the gluons according to Eq.(24) is to effect the following replacement in Eq.(21)

$$\frac{\alpha_s}{2\pi} \log \frac{Q^2}{m^2} \rightarrow \frac{1}{b} \log \left[\frac{\alpha(m^2)}{\alpha(Q^2)} \right] \tag{25}$$

The reason for this replacement is sketched in Fig.(10). One has on the left the strong coupling constant α_s and a dressed gluon of mass k^2. When one writes the dressed propagator according to Eq.(24) the running coupling constant $\alpha(k^2)$ shows up. However, one still has to compute the discontinuity in k^2 in order to obtain the cross section, and a careful analysis[4] shows that the net result is to replace $\alpha(k^2)$ by $\alpha(t_n)$, as indicated on the right of Fig. (10). Therefore, the running coupling constant has to be kept in the phase space integrations, and instead of $\frac{\alpha_s}{2\pi} \log \frac{Q^2}{m^2}$ one gets

$$\int_{m^2}^{Q^2} \alpha(t) \frac{dt}{t} = \alpha_s \int_{m^2}^{Q^2} \frac{d(\log t)}{1 + b\alpha_s \log \frac{t}{\mu^2}} = \frac{1}{b} \log \left[\frac{1+b\alpha_s \log Q^2/\mu^2}{1+b\alpha_s \log m^2/\mu^2} \right]$$

$$= \frac{1}{b} \log \left[\frac{\alpha(m^2)}{\alpha(Q^2)} \right] \tag{26}$$

which is the result announced in (25)

d) Finally, the effect of virtual corrections to the external lines is to multiply $\sigma_m(p)$ by the wave function renormalization constant Z_2. Therefore, when the sum over m - the number of emitted gluons - is performed, one obtains the final result for the cross-section for the reaction quark + $\gamma(q) \rightarrow$ anything, which can be written in the form

$$\sigma(p) = \int_0^1 dx\, F(x; \frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}) \sigma^{(0)}(xp) \tag{27}$$

where $\sigma^{(0)}(xp)$ is the naive parton model cross section for a quark of scaled momentum xp and F can be interpreted as the probability of finding a quark in a quark carrying a factor x of the parent's momentum. From the previous results it is clear that the result for F is

$$F(x, \frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}) = Z_2 \left\{ \delta(1-x) + \sum_{m=1}^{\infty} \frac{1}{m!} \left[\frac{C_F}{2\pi b} \log \frac{\alpha(m^2)}{\alpha(Q^2)} \right]^m \right.$$
$$\left. \int_0^{1-\varepsilon} \prod_{i=1}^{m} dx_i \, f(x_i) \, \delta(x - \prod_{i=1}^{m} x_i) \right\} \tag{28}$$

where $x_i = 1-z_i$, and $f(x_i)$ is the Altarelli-Parisi function

$$f(x) = \frac{1 + x^2}{1 - x} \tag{29}$$

Notice that the phase-space integration over x_i cannot be carried out all the way up to $x = 1$ because $f(x)$ is singular at this point. In this case the emitted quark carries all the momentum of the parent quark, and therefore the emitted gluon is soft. This divergence is therefore the usual infrared divergence, which eventually cancelled by the contribution of virtual processes included in Z_2. Keeping for the moment an energy cutoff ε in Eq.(28), it is clear that the moments of F(x) exponentiate in a simple way. Indeed, if one considers

$$M_n(\frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}) = \int_0^1 dx \, x^{n-1} F(x, \frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}) \tag{30}$$

and defines

$$A_n^R = C_F \int_0^{1-\varepsilon} dx \, x^{n-1} f(x) \tag{31}$$

it follows from Eq.(28) that

$$M_n(\frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}) = Z_2 \left[\frac{\alpha(m^2)}{\alpha(Q^2)} \right]^{\frac{1}{2\pi b} A_n^R} \tag{32}$$

e) It remains to compute the leading mass singularities in the wave function renormalization constant Z_2. This is done in Ref. (4) by a clever application of the KLN theorem, by relating the virtual gluon corrections to the uncoming quark propagators to processes involving gluons absorbed by the quark. The result is

$$Z_2 = \left[\frac{\alpha(m^2)}{\alpha(Q^2)}\right]^{-\frac{1}{2\pi b}A^V} \tag{33}$$

where

$$A_V = C_F \int_0^{1-\varepsilon} dx \; \frac{1+x^2}{1-x} \tag{34}$$

Putting Eqs. (32) and (33) together, the result is infrared finite and the limit $\varepsilon \to 0$ can be taken. One obtains for the moments

$$M_n(Q^2) = \left[\frac{\alpha(m^2)}{\alpha(Q^2)}\right]^{-\frac{1}{2\pi b}A_n} \tag{35}$$

where A_n is the usual anomalous dimension of lowest twist operators given by :

$$A_n = C_F \int_0^1 dx \; \left(\frac{1+x^2}{1-x}\right)(x^{n-1} - 1) \tag{36}$$

Eqs. (35) and (36) are the usual results of asymptotic freedom in QCD. It is customary to write the anomalous dimension A_n as the moments of a regularized Altarelli-Parisi probability(18)

$$P_{qq}(x) = C_F \left(\frac{1+x^2}{1-x}\right)_+ \tag{37}$$

which is itself a distribution in the sense that

$$\int_0^1 dx \; \left(\frac{1+x^2}{1-x}\right)_+ g(x) = \int_0^1 dx \; \left(\frac{1+x^2}{1-x}\right)(g(x) - g(1)) \tag{38}$$

From Eq.(35) it is clear that there is a natural "evolution" variable in QCD. Indeed, by replacing Q^2 by Y defined as :

$$Y = \frac{1}{2\pi b} \log \left[1+2\pi b \; \alpha_s \; \log \frac{Q^2}{\mu^2}\right] \tag{39}$$

then the moments of the structure functions at two different values of Q^2 are given by

$$M_n(Y) = e^{A_n(Y-Y_o)} M_n(Y_o) \tag{40}$$

and this equation is just the solution of the Altarelli-Parisi equations[11] for the non-singlet components of the structure functions.

f) Finally, it is not difficult to generalize the calculation to include the mixing of sea and gluons. In this case the structure function $F^{ab}(x,\frac{Q^2}{\mu^2},\frac{m^2}{\mu^2})$ represents the probability of finding the parton b in parton a, carrying a fraction x of the parent's momentum. There are four Altarelli-Parisi probabilities corresponding to the four basic processes indicated in Figs.(11,a,b,c,d)

$$P_{qq}(x) = C_F \left(\frac{1+x^2}{1-x}\right)_+ \tag{41.a}$$

$$P_{gq}(x) = C_F \frac{1+(1-x)^2}{x} \tag{41.b}$$

$$P_{qg}(x) = \frac{N_f}{2} [x^2 + (1-x)^2] \tag{41.c}$$

$$P_{gg}(x) = 2C_A \left[\frac{1-x}{x} + \left(\frac{x}{1-x}\right)_+ + x(1-x) - \frac{1}{12}\,\delta(x-1)\right] - \frac{N_f}{3}\,\delta(x-1) \tag{41.d}$$

and, if the anomalous dimension matrix A_n^{ba} is defined as

$$A_n^{ba} = \int_0^1 dx^{n-1}\, P(a \to b(x) + c(1-x)) \tag{42}$$

the evolution of the moments M_n^{ba} is given by the matrix relation

$$M_n^{ba}(Y) = [e^{A_n(Y-Y_o)}]_{bc}\, M_n^{ca}(Y_o) \tag{43}$$

3 - General results.

We have discussed in the previous section how mass singularities can be handled, in the leading log approximation, in the case of a quark initiated hard process. One of the most important results is the one shown in Eq.(27), where it can be seen that mass singularities factorize away from the naïve parton model cross section, since they are entirely included in the multiplicative factor $F(x, \frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2})$. Moreover, they are universal in the sense that the function F is independent of the particular hard process under consideration. Had we considered an arbitrary process with several non collinear partons in the initial state, we would have obtained a multiplicative factor containing all mass singularities which is jsut the product of F functions, one for each parton line.

The same result is true for hard processes containing well identified partons in the final state. In this case the universal functions that factorize the mass singularities are interpreted as parton decay functions, namely, the probability that a parton of momentum xp decays into a parton of momentum p. At the leading log level, structure and decay functions are the same except for the different kinematical limits for the x variable.

Parton cross sections as given for example by Eq.(27) must still be convoluted with hadronic wave functions. Because of factorization, the effect of mass singularities can be folded into Q^2 dependent hadron structure functions, and the naive parton model results are recovered. The Q^2 dependence of hadronic structure functions is of course known from the previous results.

The problem of factorization beyond the leading log approximation has been investigated in Refs. (4),(11)and (16).Mass singularities do indeed factorize in the same way as before in the scaling part of parton cross sections. However, beyond the leading log approximation structure and decay functions are not necessarily the same. The residual cross section $\sigma^{(0)}$ can be expanded in an improved perturbation theory in $\alpha(Q^2)$.

Finally, it must be mentionned that a large amount of work has been done in analysing the structure of quark and gluon jets in the leading log approximation. These results are discussed at length by other speakers at this Conference[(20), (21),(22)].

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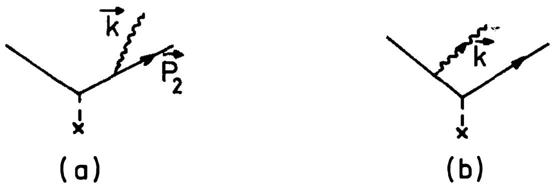

Fig.1

Fig. 1. Lowest-order Feynman diagrams for radiation by an electron in the presence of an external field .

Fig. 2

Fig. 2. Lowest order radiative corrections to electron scattering by an external field.

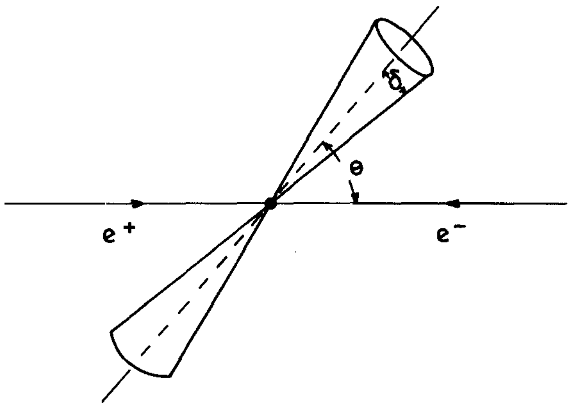

Fig.3

Fig.3. The two opposing cones of half-angle δ used in the derivation of the Sterman-Weinberg cross-section.

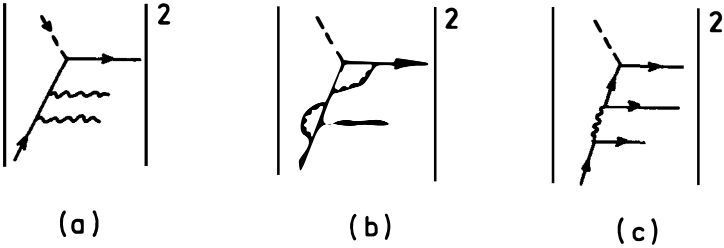

Fig. 4

Fig. 4. A sample set of diagrams which contribute to the deep inelastic structure functions of a quark.

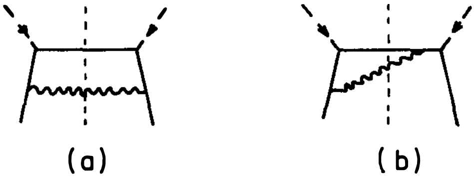

Fig. 5

Fig. 5. Some lowest-order diagrams contributing to the non flavor singlet components of the deep inelastic structure of a quark.

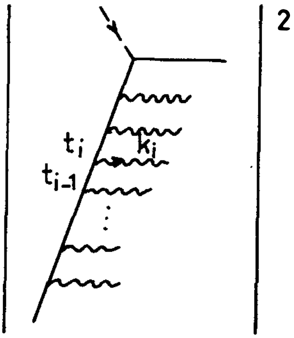

Fig. 6

Fig. 6. Contribution of gluon radiation by the incoming quark to the hard cross section quark + $\gamma \to$ anything.

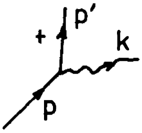

Fig. 7

Fig. 7. Kinematical variables in the vertex for gluon radiation.

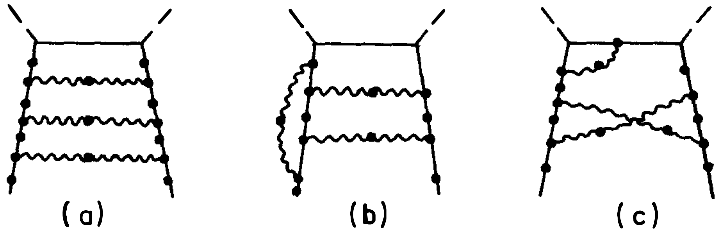

Fig. 8

Fig. 8. A sample set of diagrams which contribute to forward Compton scattering. Dotted lines and vertices represent dressed propators and vertices.

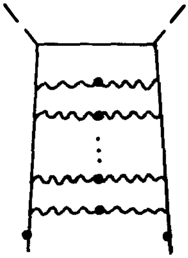

Fig. 9

Fig. 9. Diagrams which contribute to the quark structure function in the leading log approximation. Dotted lines represent dressed propagators.

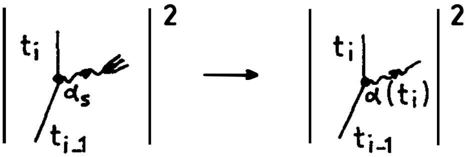

Fig.10

Fig. 10. Schematic representation of the effect of dressing the gluon propagator.

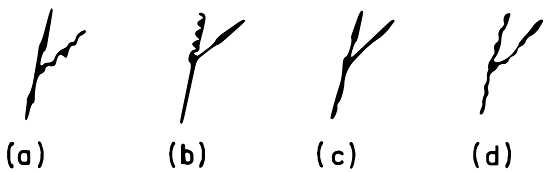

Fig.11

Fig. 11. The four basic processes associated with the Altarelli-Parisi probabilities Pab(x).