

# Anomalies in (semi)-leptonic B decays $B^\pm \rightarrow \tau^\pm \nu$ , $B^\pm \rightarrow D\tau^\pm \nu$ and $B^\pm \rightarrow D^*\tau^\pm \nu$ , and possible resolution with sterile neutrino\*

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**Abstract:** The universality of the weak interactions can be tested in semileptonic  $b \rightarrow c$  transitions, and in particular in the ratios  $R(D^{(*)}) \equiv \Gamma(B \rightarrow D^{(*)}\tau\nu)/\Gamma(B \rightarrow D^{(*)}l\nu)$  (where  $l = \mu$  or  $e$ ). Due to the recent differences between the experimental measurements of these observables by BaBar, Belle and LHCb on the one hand and the Standard Model predicted values on the other hand, we study the predicted ratios  $R(D^{(*)}) = \Gamma(B \rightarrow D^{(*)}\tau + \text{“missing”})/\Gamma(B \rightarrow D^{(*)}l\nu)$  in scenarios with an additional sterile heavy neutrino of mass  $\sim 1$  GeV. Further, we evaluate the newly defined ratio  $R(0) \equiv \Gamma(B \rightarrow \tau + \text{“missing”})/\Gamma(B \rightarrow \mu\nu)$  in such scenarios, in view of the future possibilities of measuring the quantity at Belle-II.

**Keywords:** sterile neutrino, lepton universality, semileptonic B decays

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## 1 Introduction

The lepton universality of weak gauge theory can be tested in exclusive semileptonic B decays, possibly through the existence of new charged currents, as well as the quark flavor mixing structure [1] of the Standard Model (SM). To achieve these goals, it is usually necessary to calculate accurately the corresponding hadronic matrix elements. However, in ratios like

$$R(D^{(*)}) \equiv \frac{\Gamma(B \rightarrow D^{(*)}\tau\nu)}{\Gamma(B \rightarrow D^{(*)}l\nu)}, \quad (1)$$

with  $l = e$  or  $\mu$ , most of the hadronic uncertainties cancel,

making such ratios particularly relevant for testing the universality of weak interactions. In Table 1 we show the SM predictions which we will use. The shown (hadronic) uncertainties originate, respectively, from lattice calculations [2] and from estimated higher order corrections in HQET to the ratio  $A_0/A_1$  of form factors [3]. Recently, new improved experimental results from the Belle [4] and LHCb [5] Collaborations have appeared, in addition to the older BaBar results [6]. As a result, the world average values reported by the HFAG group [7, 8] are larger than the SM predicted values for  $R(D)$  by  $1.9\sigma$ , and for  $R(D^*)$  by  $3.3\sigma$  (cf. Table 1).

Table 1. Experimental results [4–6] and SM predictions [2] of  $R(D)$  and  $R(D^*)$ . The SM prediction with the scalar form factor (SFF) variations [9] is also given. The first and second experimental errors are statistical and systematic, respectively.

	$R(D)$	$R(D^*)$
BaBar	$0.440 \pm 0.058 \pm 0.042$	$0.332 \pm 0.024 \pm 0.018$
Belle	$0.375 \pm 0.064 \pm 0.026$	$0.302 \pm 0.030 \pm 0.011$
LHCb	–	$0.336 \pm 0.027 \pm 0.030$
experimental average [7, 8]	$0.397 \pm 0.040 \pm 0.028$	$0.316 \pm 0.016 \pm 0.010$
SM Prediction [2]	$0.300 \pm 0.008$	$0.252 \pm 0.003$
SM Prediction (SFF Variation) [9]	0.335	–

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Many theoretical explanations have been proposed to explain these indications of possible lepton universality violation, including charged scalar exchanges [10], vector resonances [11] or a  $W'$  boson [12–14], and leptoquarks (or, equivalently,  $R$ -parity violating supersymmetry) [12, 14–16]. The effects of exchange of on-shell sterile neutrinos have also been evaluated, cf. [17, 18]. Explanation of the anomalies within an effective field theory approach with dimension-6 scalar, vector and tensor operators appeared in Refs. [16, 19]. It was shown in Ref. [20] that the perturbative QCD (pQCD) combined with lattice results reduces the difference from the experimental values. The electroweak effects in B-anomalies were investigated in Ref. [21].

In Ref. [9], the authors checked how robust the SM predictions are for the  $R(D^{(*)})$  ratios. In contrast to the predictions of the vector form factors for  $B \rightarrow D l \nu$  decays, which have been determined well in measurements of the branching ratios and  $q^2$ -distributions in light lepton channels, the scalar form factor (SFF) is measurable only in decays with  $\tau$  leptons. Even small deviations of the SFF from the lattice values can bring the SM prediction closer to the current measured values of  $R(D)$ , as shown in Table 1.

As closely related to the decays,  $B \rightarrow D^{(*)} \tau \nu$ , the branching fraction (BF) of  $B^+ \rightarrow \tau^+ \nu$  decay<sup>1)</sup> was measured by Belle and BaBar [22], as shown in Table II [7, 8, 23]. The SM prediction for the decay BF is given by [24]

$$\mathcal{B}_{\text{SM}}(B^+ \rightarrow \tau^+ \nu) = (0.848_{-0.055}^{+0.036}) \times 10^{-4}. \quad (2)$$

This implies that if the measurement is improved in future B-factory experiments such as Belle-II [25], the comparison can clarify whether new physics scenarios are needed.

Table 2. Branching fractions of  $B^+ \rightarrow e^+ \nu$ ,  $\mu^+ \nu$ ,  $\tau^+ \nu$  in units of  $10^{-6}$  [7, 8, 23].

$B^+ \rightarrow$	$e^+ \nu$	$\mu^+ \nu$	$\tau^+ \nu$
BABAR	$< 1.9$	$< 1.0$	$179 \pm 48$
Belle	$< 0.98$	$< 1.7$	$91 \pm 19 \pm 11$
experimental average	$< 0.98$	$< 1.0$	$106 \pm 19$

As a probe of new physics beyond the SM, the leptonic decays  $B^+ \rightarrow l^+ \nu$  are very interesting. This is because these decay rates can be evaluated very precisely, and even at the tree-level new physics effects may appear, e.g., contributions of charged Higgs [26] in two-Higgs doublet models [27]. In the SM, the leptonic decay rates of  $B^+ \rightarrow l^+ \nu$  are proportional to the square of the charged lepton mass,  $m_l^2$ . Thus, the decays of  $B^\pm$  to  $e^\pm \nu$  and  $\mu^\pm \nu$  are strongly suppressed in comparison with the

decays to  $\tau^\pm \nu$ . Here we define new ratio  $R(0)$  [26] as

$$R(0) \equiv \frac{\Gamma(B \rightarrow \tau \nu)}{\Gamma(B \rightarrow \mu \nu)}, \quad (3)$$

which is one of the most interesting to test the universality of weak interactions, since all the hadronic uncertainties cancel in the ratio, and the ratio is a function of  $M_\tau^2/M_B^2$  (and  $M_\mu^2/M_B^2$ ).

Heavy sterile neutral particles (a.k.a. “heavy neutrinos”) have suppressed mixing with SM neutrinos and appear in various new physics scenarios, among them the original seesaw [28] with very heavy neutrinos, seesaw with neutrinos with mass  $\sim 0.1$ –1 TeV [29], or with mass  $\sim 1$  GeV [30]. For some studies of the production of very heavy neutrinos with mass  $\sim 100$  GeV at the LHC we refer to Ref. [31]. We will include in our considerations the reactions  $B^\pm \rightarrow \tau^\pm N$ ,  $B^\pm \rightarrow D \tau^\pm N$ ,  $B^\pm \rightarrow D^* \tau^\pm N$ , where  $N$  is any heavy sterile neutrino of the Dirac or Majorana type, and interpret the measured branching fractions in the new physics scenario. For example, even if  $N$  is invisible in the detector, we can still distinguish  $B^\pm \rightarrow l^\pm N$  signals from  $B^+ \rightarrow l^+ \nu$  for  $l = e$  or  $\mu$ , because these are two-body decays and therefore the momentum of the charged lepton in the B meson rest frame is fixed by the mass of  $N$ . However, in the case of decays  $B^\pm \rightarrow \tau^\pm N$ , the produced  $\tau^\pm$  particle decays fast and hence there is more than one neutrino in the final state, and the decay signature of  $B^\pm \rightarrow \tau^\pm N$  cannot be distinguished from the ordinary  $B^\pm \rightarrow \tau^\pm \nu$ . Therefore, the experimentally observed signal of  $B^\pm \rightarrow \tau^\pm \nu$  may include contributions from  $B^\pm \rightarrow \tau^\pm N$ , and this signal we will denote as  $B^\pm \rightarrow \tau^\pm +$  “missing”.

Massive neutrinos  $N$  in general mix with the standard flavor neutrinos, e.g. as in a seesaw type new physics scenario. We denote as  $U_{lN}$  the mixing coefficient for the heavy mass eigenstate  $N$  with the standard flavor neutrino  $\nu_l$  ( $l = e, \mu, \tau$ )<sup>2)</sup>. The standard sub-eV neutrino  $\nu_1$  ( $l = e, \mu, \tau$ ) can then be represented as

$$\nu_1 = \sum_{k=1}^3 U_{l\nu_k} \nu_k + U_{lN} N, \quad (4)$$

where  $\nu_k$  ( $k = 1, 2, 3$ ) are the light mass eigenstates. The  $3 \times 3$  matrix  $U_{l\nu_k}$  is the usual PMNS matrix [34]. In the relations (4) we assume the existence of only one additional massive sterile neutrino  $N$ , however, it can be extended with any number of  $N$ . Then the extended (unitary) PMNS matrix  $U$  would be in this case a  $4 \times 4$  matrix, implying the relations

$$\sum_{k=1}^3 |U_{l\nu_k}|^2 = 1 - |U_{lN}|^2. \quad (5)$$

1) Throughout this paper, formulas can also be applied to charge-conjugate modes.

2) Other notations for  $U_{lN}$  exist in the literature, among others  $V_{l4}$  in Ref. [32] and  $B_{lN}$  in Ref. [33].

One of our scenarios will be with this unitarity assumption. This will modify the decay width, due to the existence of a massive neutrino  $N$ , by the amount  $\Gamma(B^+ \rightarrow \tau^+ N) - \Gamma(B^+ \rightarrow \tau^+ N)|_{M_N=0}$  and  $\Gamma(B^+ \rightarrow D^{(*)} \tau^+ N) - \Gamma(B^+ \rightarrow D^{(*)} \tau^+ N)|_{M_N=0}$ , where the minus terms are due to the unitarity of  $U$ .

In the other scenario, the  $3 \times 3$  PMNS mixing matrix is unitary, and  $N$  will be regarded as a neutral fermion which does not mix with the SM flavor neutrinos  $\nu_l$ , but couples with charged leptons such as  $\tau$  in the same form as in the first scenario, for example,

$$\Delta\mathcal{L} \sim \tilde{g} \bar{\tau} \gamma^\mu \tilde{W}_\mu^- N + \text{h.c.} \Rightarrow \delta\mathcal{L} = \left( -\frac{g}{\sqrt{2}} \right) U_{\tau N} \bar{\tau} W_{X\mu}^- N + \text{h.c.}, \quad (6)$$

via mediation of a new physics charged gauge boson  $\tilde{W}^\pm$ , where subscript  $X$  denotes either L or R (left or right-handed projection). Here  $\tilde{W}^\pm$  has the light SM gauge boson  $W^\pm$  component, and this may lead to the couplings in Eq. (6), where the suppression effects of such (or similar) scenarios are parameterized in the parameter  $U_{\tau N}$ . These couplings have the same form as in the previous scenario, but now we have no condition of unitarity (5). Such a violation of the unitarity indicates unknown new physics beyond the SM.

For our analysis, we want to keep in a most generic form both the scenarios which lead to Eqs. (4)–(5) and those which lead to Eq. (6). Nonetheless, we wish to mention, as a representative example for the mechanism of Eq. (6), the LR-models [35–37] with the gauge group  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ . Such models have in the scalar sector a  $2 \times 2$  LR-doublet  $\phi$  and triplets  $\Delta_L, \Delta_R$ . The vacuum expectation value (VEV)  $u_R$  of  $\Delta_R$  is much larger than the other VEVs,  $|u_R| \gg |v|, |w| \gg |u_L|$  (where  $v, w \sim 10^2$  GeV are VEVs in  $\phi$ ), leading to a hierarchical mixing of the charged flavor bosons  $\tilde{W}_L^\pm$  and  $\tilde{W}_R^\pm$ . The flavor boson  $\tilde{W}_R^\pm$  has, as a result, a small component ( $\sim |v^2/u_R^2|$ ) of the SM mass eigenstate boson  $W^\pm$ . More specifically,

$$\tilde{W}_R^\pm = -e^{\mp i\lambda} \sin\xi W^\pm + \cos\xi W_2^\pm, \quad (7)$$

where  $W_2^\pm$  is the heavy mass eigenstate with mass  $M_2^2 \approx g^2 |u_R|^2 / 4 \gg M_W^2$ ,  $\lambda$  is a real phase, and the mixing angle  $\xi$  is

$$\xi \approx \frac{2|vw|}{|v|^2 + |w|^2} \left( \frac{M_W}{M_2} \right)^2, \quad (8)$$

where  $(|v|^2 + |w|^2) = 1/(\sqrt{2}G_F)$ . In such models we can have a heavy neutrino  $N_R$  which forms with  $\tau_R$  an

$SU(2)_R$ -doublet  $(N_R, \tau_R)$ , where  $N_R$  is a flavor and mass eigenstate, i.e., it does not mix with other flavor neutrinos such as  $(\nu_{l,L})^c$  ( $l = e, \mu, \tau$ ) and  $\nu_{l',R}$  ( $l' = e, \mu$ ). Then the gauge boson mixing (7) gives us in the terms which couple  $\tilde{W}_R^\pm$  with leptons the following contributions (we take  $g_R = g_L = g$ ):

$$\begin{aligned} & -\frac{g}{\sqrt{2}} \bar{\tau} \gamma^\mu \left( \frac{1+\gamma_5}{2} \right) N \tilde{W}_{R\mu}^- + \text{h.c.} \\ & = -\frac{g}{\sqrt{2}} \bar{\tau} \gamma^\mu \left( \frac{1+\gamma_5}{2} \right) N \left( -e^{i\lambda} \sin\xi W_\mu^- + \cos\xi W_{2\mu}^- \right) + \text{h.c.} \end{aligned} \quad (9)$$

$$= -\frac{g}{\sqrt{2}} U_{\tau N} \bar{\tau} \gamma^\mu \left( \frac{1+\gamma_5}{2} \right) N W_\mu^- + \text{h.c.} + \dots, \quad (10)$$

where the ellipsis stands for the couplings with the heavy  $W_2^\pm$  boson. Here we see that in such a case the heavy-light mixing parameter is  $|U_{\tau N}|^2 = \sin^2\xi \approx \xi^2$ , which, according to Eq. (8) is then

$$|U_{\tau N}|^2 \approx \left( \frac{M_W}{M_2} \right)^4. \quad (11)$$

According to the analysis of general LR-models of Ref. [37], there is a lot of freedom in  $V_R$ , the right-handed quark mixing matrix, resulting in a relatively generous bound  $M_2 > 300$  GeV coming predominantly from the  $K_L$ - $K_S$  mass difference  $\Delta m_K$  and by  $B_d \bar{B}_d$  mixing (and assuming  $g_R = g_L$ ). This implies the upper bound  $|U_{\tau N}|^2 < 5 \times 10^{-3}$  in such LR-scenarios<sup>1)</sup>.

We further point out that the right-handedness of the coupling  $\tau$ - $W$ - $N$ , Eq. (10), does not affect the formulas for the decay widths  $\Gamma(B \rightarrow (D^{(*)}) \tau \nu)$  that we use in the present paper: we checked that these formulas turn out to be the same as in the case of the left-handedness of the  $\tau$ - $W$ - $N$  coupling; we recall that the couplings of quarks to  $W$  are, of course, always left-handed.

In our numerical analysis, we will derive the results for two scenarios: either the  $(4 \times 4)$  matrix  $U$  is unitary, or  $U$  without the unitarity assumption, *i.e.* analyses with and without the unitarity assumption. Our formulas, to be derived in the following sections, will be applicable also to cases with more than one additional massive neutral fermion  $N$  where the second fermion has a mass  $M_{N'} \gtrsim 10$  GeV, such particles being too heavy to be produced on-shell in the considered decays.

At present, the upper bounds for the mixing parameters  $|U_{\tau N}|^2$  are available from the measurements and analyses of the CHARM [40] and DELPHI [41] Collaborations. These were dedicated direct measurements and

1) The LR-models are restricted to the minimal (symmetric) versions, where  $V_R$  is closely related with  $V_L (= V_{CKM})$ , the resulting bound is more restrictive,  $M_2 > 2.3$  TeV [38]. Signals of  $W_R$  were searched for at the LHC (CMS Collaboration) [39], in specific minimal LR-model scenarios where  $W_R^\pm$  would decay in the  $l=e, \mu$  channels to  $lN_{l,R}$ ; mass exclusion regions  $M_2 > 3.3$  TeV were found [39] in such scenarios, but only if  $M_{N_l} > 200$  GeV.

analyses for decays producing heavy neutrinos  $N^1$ ). On the other hand, there are indirect indications that the heavy-light mixing parameters  $|U_{\tau N}|^2$  have more restrictive upper bounds, coming from the  $\tau$  lepton decays where the analyses were made under the assumption of the SM scenario of (practically) massless neutrinos and unitary  $3 \times 3$  PMNS matrix  $U_{\text{PMNS}}$ , and there the lepton universality of the electroweak coupling  $g$  was shown to a large precision [8] (Sec. 9.2 there). However, in these latter analyses, unlike in Refs. [40, 41], it was assumed that heavy neutrinos do not exist. In view of the lack of any new updated dedicated measurements and analyses of the  $\tau$  decays with heavy neutrinos, we will usually present here the upper bounds on the mixing parameters  $|U_{\tau N}|^2$  as those from Refs. [40, 41]. Nonetheless, in the next section we will present an analysis of the lepton universality results [8] in the scenario of one additional heavy neutrino  $N$ , and in the subsequent analyses in this work we will keep in mind the restrictions on the heavy-light mixing from such an analysis.

In this paper, we will consider the recent experimental anomalies,  $R(D)$  and  $R(D^*)$ , with the theoretical assumption of one not very heavy neutrino  $N$ :  $M_N \sim 1$  GeV. We will also predict the newly defined  $R(0)$ , which can be measured at Belle-II, as a function of the unknown parameters,  $M_N$  and  $U_{\tau N}$ .

## 2 Restrictions on $|U_{\tau N}|^2$ from lepton universality tests in $\tau$ decays

The Heavy Flavor Averaging Group (HFAG) [8] obtained restrictions coming from lepton universality tests of the SM. They analyzed, among other things, the measured widths  $\Gamma(\tau \rightarrow e + \text{missing})$  and  $\Gamma(\mu \rightarrow e + \text{missing})$ , where “missing” stands for  $\nu_\tau \bar{\nu}_e(\gamma)$  and  $\nu_\mu \bar{\nu}_e(\gamma)$ . They thus obtained

$$\left(\frac{g_\tau}{g_\mu}\right) = 1.0010 \pm 0.0015, \quad (12)$$

where the above notation stands for

$$\begin{aligned} \left(\frac{g_\tau}{g_\mu}\right)^2 &= \left(\frac{M_\mu}{M_\tau}\right)^5 \frac{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e(\gamma))}{\Gamma(\mu^- \rightarrow \nu_\mu e^- \bar{\nu}_e(\gamma))} \\ &\times \left[ \frac{R_\gamma(\mu) R_W(\mu) f(M_e^2/M_\mu^2)}{R_\gamma(\tau) R_W(\tau) f(M_e^2/M_\tau^2)} \right] \\ &= 1 + (2 \pm 3) \times 10^{-3}. \end{aligned} \quad (13)$$

The above ratio in their analysis includes the QED [ $(\gamma)$ ] [42] and other corrections [ $\sim (M_\tau/M_W)^2$ ,  $\sim (M_e/M_\mu)^2$ ]. However, the net contribution of all these effects to the above quantity, represented as the correction factor in the brackets in Eq. (13), turns out to be  $\sim 10^{-4}$  and will

thus be ignored in the analysis here.

If, on the other hand, we have an additional heavy neutrino  $N$  which couples to  $\tau$  (but not to  $e$  or  $\mu$ ), either in a scenario in which the  $4 \times 4$  matrix  $U$  is nonunitary [e.g., scenarios as in Eq. (6)], or in a scenario where the  $4 \times 4$  mixing matrix  $U$  is unitary [cf. Eq. (5)], the above analysis changes significantly.

Namely, in the scenario where the formal  $4 \times 4$  matrix  $U$  is nonunitary (and the  $3 \times 3$  PMNS mixing matrix is unitary), accounting for the nonzero mass  $M_N$  changes the above analysis in the following way. The quantity  $\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)$  gets replaced by

$$\begin{aligned} \Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e) &\mapsto \Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e) + \Gamma(\tau^- \rightarrow N e^- \bar{\nu}_e) \quad (14) \\ &= \Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e) + |U_{\tau N}|^2 \bar{\Gamma}(\tau^- \rightarrow N e^- \bar{\nu}_e), \end{aligned} \quad (15)$$

where  $\bar{\Gamma}(\tau^- \rightarrow N l^- \bar{\nu}_l)$  stands for the decay width to a (massive) neutrino  $N$  and the coupling parameter  $|U_{\tau N}|^2 = 1$

$$\begin{aligned} \bar{\Gamma}(\tau^- \rightarrow N l^- \bar{\nu}_l) &= \frac{G_F^2}{96\pi^3} M_\tau^5 \int_{z_1}^{(1-\sqrt{z_N})^2} dz \lambda^{1/2}(1, z_N, z)(z - z_1) \\ &\times \left\{ (1 + z_N - z) \left(1 - \frac{z_1}{z}\right)^2 \right. \\ &\left. + \left[ (1 - z_N)^2 \frac{1}{z} - z \right] \left[ 1 + \frac{z_1}{z} - 2 \left(\frac{z_1}{z}\right)^2 \right] \right\}. \end{aligned} \quad (16)$$

Here,  $z_N = (M_N/M_\tau)^2$ ,  $z_1 = (M_l/M_\tau)^2$  ( $l=e$  or  $l=\mu$ ), and  $z = p_W^2/M_\tau^2$ , where  $p_W^2$  is square of the invariant mass of  $W^{*-} = (l\bar{\nu}_l)$ . The first term on the right-hand side of Eq. (15) denotes the usual SM contribution

$$\begin{aligned} \Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e) &= \sum_{k=1}^3 |U_{\tau\nu_k}|^2 \bar{\Gamma}(\tau^- \rightarrow \nu_k e^- \bar{\nu}_e) \\ &= \bar{\Gamma}(\tau^- \rightarrow \nu e^- \bar{\nu}_e)|_{M_\nu=0}, \end{aligned} \quad (17)$$

where the above identities hold because of the (practical) masslessness of the SM neutrinos  $\nu_k$  ( $k=1,2,3$ ) and the unitarity of the  $3 \times 3$  PMNS matrix.

The following ratio is crucial in the present analysis:

$$G_1(M_N) = \frac{\Gamma(\tau^- \rightarrow N l^- \bar{\nu}_l)}{\Gamma(\tau^- \rightarrow \nu_\tau l^- \bar{\nu}_l)} = |U_{\tau N}|^2 \bar{G}_1(M_N), \quad (18)$$

where  $l=e$  or  $l=\mu$ , and

$$\bar{G}_1(M_N) = \frac{\bar{\Gamma}(\tau^- \rightarrow N l^- \bar{\nu}_l)}{\bar{\Gamma}(\tau^- \rightarrow \nu l^- \bar{\nu}_l)|_{M_\nu=0}} \quad (19)$$

is the corresponding canonical ratio.

The values (13), together with Eq. (15) and the notations (18)-(19), then lead in the mentioned scenario to

1) CHARM limits [40] were obtained for  $M_N < 300$  MeV, based on the absence of signals  $N \rightarrow \nu_\tau Z^0 \rightarrow \nu_\tau e^+ e^-$  in a neutrino beam dump experiment, where  $N$  is produced from decays of  $D_s$  mesons. DELPHI limits [41] were obtained for  $M_N > 250$  MeV, based on the absence of signals  $e^+ e^- \rightarrow Z^0 \rightarrow \nu_l N$  for long-lived and short-lived  $N$ , and they apply to all three parameters  $|U_{lN}|^2$  ( $l=e, \mu, \tau$ ).

the following predictions for  $|U_{\tau N}|^2$ :

$$G_e(M_N) = (2 \pm 3) \times 10^{-3} \Rightarrow |U_{\tau N}|^2 = \frac{(2 \pm 3) \times 10^{-3}}{\overline{G}_e(M_N)}. \quad (20)$$

We recall that  $N$  here is massive, on-shell, couples only to  $\tau$  (and not to  $\mu$  or  $e$ ), and the  $3 \times 3$  PMNS matrix is as in the SM, i.e., unitary. The ratio function  $G_1(M_N)$  is presented in Fig. 1 for the cases  $|U_{\tau N}|^2 = 1$  and  $|U_{\tau N}|^2 = 10^{-3}$ .

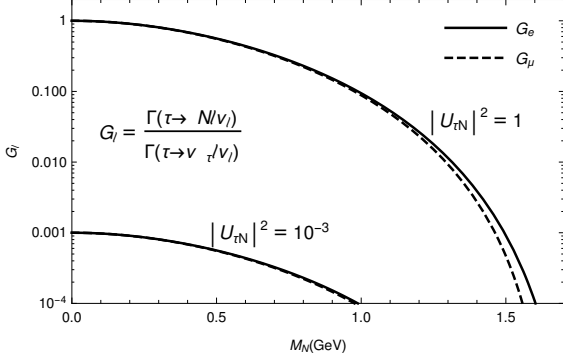


Fig. 1. The ratios  $G_e$  and  $G_\mu$ , Eq. (18), as a function of  $M_N$ , for two choices of the heavy-light mixing:  $|U_{\tau N}|^2 = 1$  (then  $G_1 = \overline{G}_1$ ), and  $|U_{\tau N}|^2 = 10^{-3}$ .

If, on the other hand, the  $4 \times 4$  heavy-light mixing matrix  $U$  is unitary, i.e., Eq. (5), an analogous analysis leads to the conclusion that  $|U_{\tau N}|^2$  has a very restrictive upper bound. Namely, in such a case the ratio on the left-hand side of Eq. (13) must be below the value of unity, and the most generous upper bound for  $|U_{\tau N}|^2$  is then obtained if the right-hand side of Eq. (13) is  $1 - 1 \times 10^{-3}$

$$|U_{\tau N}|^2 < \frac{(-1) \times 10^{-3}}{(\overline{G}_e(M_N) - 1)} = \frac{(+1) \times 10^{-3}}{|\overline{G}_e(M_N) - 1|}. \quad (21)$$

The restrictions (20) and (21) are presented in Fig. 2.

From Fig. 2(a) we can see that in the scenario of the  $3 \times 3$  unitary PMNS matrix (the  $4 \times 4$  matrix  $U$  is then nonunitary), the bounds (13) imply for the heavy-light mixing parameter  $|U_{\tau N}|^2$  stringent upper bounds  $|U_{\tau N}|^2 \lesssim 0.5 \times 10^{-2}$  if the mass of  $N$  is low ( $M_N < 0.4$  GeV), and for higher masses the bounds are not stringent:  $|U_{\tau N}|^2 \lesssim 10^{-1}$  for  $M_N > 1$  GeV.

On the other hand, if the  $4 \times 4$  matrix  $U$  is considered unitary, Fig. 2(b) implies that the upper bounds for  $|U_{\tau N}|^2$  are quite stringent ( $\lesssim 10^{-3}$ ) for higher masses  $M_N > 0.6$  GeV, while for light masses less stringent upper bounds apply. The results of Fig. 2(b) are consistent with the results of the analysis [43] of the experimental constraints for the well-measured decay ratios  $\mathcal{B}(\tau \rightarrow l \nu \nu')$  where  $l = e, \mu$ , and  $\nu$  and  $\nu'$  are light neutrinos or the heavy neutrino  $N$ , in the scenario which corresponds here to the unitary  $4 \times 4$   $U$  matrix. The authors of Ref. [43] obtain the upper bound  $|U_{\tau N}|^2 < 0.01$  for  $0.3 \text{ GeV} < M_N < 1 \text{ GeV}$ .

We will keep in mind these restrictions which come indirectly from the lepton universality tests Eq. (13), i.e., from the decays  $\tau \rightarrow e + \text{missing}$  and  $\mu \rightarrow e + \text{missing}$ . Nonetheless, in the graphs we will account for the bounds on  $|U_{\tau N}|^2$  coming from the dedicated direct measurements of the CHARM [40] and DELPHI [41] Collaborations mentioned in the Introduction.

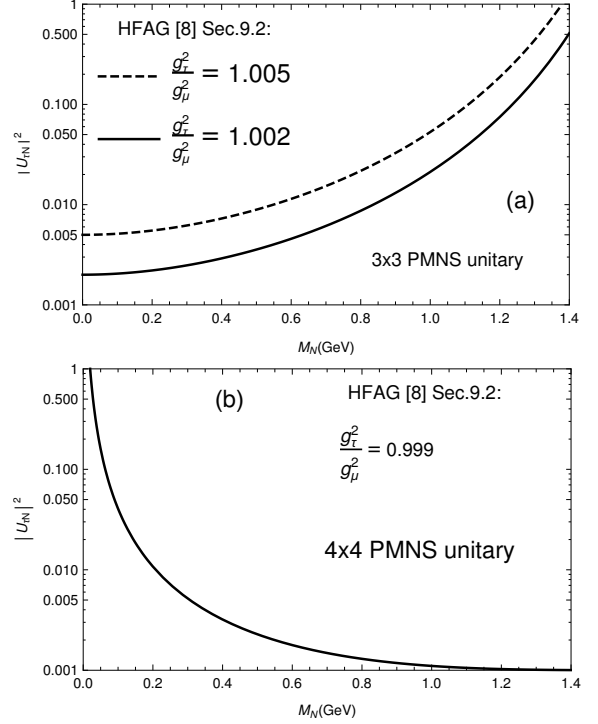


Fig. 2. The upper bounds as obtained by lepton universality tests, Eq. (13): (a) in the case when the  $3 \times 3$  PMNS mixing matrix is unitary (and the  $4 \times 4$  matrix  $U$  is nonunitary) - the solid line is the upper bound if the central value ( $1 + 2 \times 10^{-3}$ ) is taken in the values of Eq. (13), the dashed line when the largest value ( $1 + 5 \times 10^{-3}$ ) is taken there; (b) in the case when the  $4 \times 4$  mixing matrix  $U$  is unitary, and in that case the lowest value ( $1 - 1 \times 10^{-3}$ ) is taken in Eq. (13).

### 3 Theoretical details of $R(D)$ , $R(D^*)$ and $R(0)$ with Light Sterile Neutrino

#### 3.1 $R(D)$ and decays of $B \rightarrow D N$

In the SM the amplitude for hadronic transition  $B \rightarrow D$  is given in terms of vector and scalar form factors,  $F_1(q^2)$  and  $F_0(q^2)$ , defined as

$$\begin{aligned} & \langle D(p_D) | \bar{c} \gamma^\mu b | B^-(p_B) \rangle \\ &= \left[ (2p_D + q)^\mu - \frac{(M_B^2 - M_D^2)}{q^2} q^\mu \right] F_1(q^2) \\ &+ \frac{(M_B^2 - M_D^2)}{q^2} q^\mu F_0(q^2), \end{aligned} \quad (22)$$

where  $q = p_B - p_D$  is the momentum of the virtual  $W^-$ . Ref. [18] as  
 For the process  $B \rightarrow D l N$ , the decay width is given in

$$\Gamma(B \rightarrow D l N) = |U_{lN}|^2 \bar{\Gamma}(B \rightarrow D l N), \quad (23)$$

$$\begin{aligned} \bar{\Gamma}(B \rightarrow D l N) &= \frac{1}{384\pi^3} G_F^2 |V_{cb}|^2 \frac{1}{M_B} \int_{(M_N+M_l)^2}^{(M_B-M_D)^2} dq^2 \frac{1}{(q^2)^2} \lambda^{1/2} \left( 1, \frac{q^2}{M_B^2}, \frac{M_D^2}{M_B^2} \right) \lambda^{1/2} \left( 1, \frac{M_l^2}{q^2}, \frac{M_N^2}{q^2} \right) \\ &\times \left\{ F_1(q^2)^2 [2(q^2)^2 - q^2 M_N^2 + M_l^2 (2M_N^2 - q^2) - M_N^4 - M_l^4] [(q^2 - M_D^2)^2 - 2M_B^2 (q^2 + M_D^2) + M_B^4] \right. \\ &\left. + F_0(q^2)^2 3(M_B^2 - M_D^2)^2 [q^2 M_N^2 + M_l^2 (2M_N^2 + q^2) - M_N^4 - M_l^4] \right\}, \quad (24) \end{aligned}$$

where the kinematically allowed values of  $q^2$  are  $(M_N + M_l)^2 \leq q^2 \leq (M_B - M_D)^2$ . Notice that if N is replaced by  $\nu_1$  (i.e.,  $M_N \approx 0$ ),  $\bar{\Gamma}(B \rightarrow D l \nu_1)$  in Eq. (24) becomes the SM decay width of  $B \rightarrow D l \nu_1$ , i.e.,  $\Gamma_{SM}(B \rightarrow D l \nu_1)$ .

The form factor  $F_1(q^2)$  is well known [44]<sup>1)</sup>. It can be expressed in terms of the variable  $w$

$$w = \frac{(M_B^2 + M_D^2 - q^2)}{2M_B M_D}, \quad (25)$$

$$z(w) = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}, \quad (26)$$

in the following approximate form [44]:

$$\begin{aligned} F_1(q^2) &= F_1(w=1) (1 - 8\rho^2 z(w) + (51\rho^2 - 10)z(w)^2 \\ &\quad - (252\rho^2 - 84)z(w)^3), \quad (27) \end{aligned}$$

where the free parameters  $\rho^2$  and  $F_1(w=1)$  have been recently determined with high precision by the Belle Col-

laboration, see Ref. [46]:

$$\rho^2 = 1.09 \pm 0.05, \quad (28)$$

$$|V_{cb}| F_1(w=1) = (48.14 \pm 1.56) \times 10^{-3}. \quad (29)$$

The value (29) was deduced from their value of  $\eta_{EW} \mathcal{G}(1) |V_{cb}| = \eta_{EW} F_1(w=1) \sqrt{4r/(1+r)} = (42.29 \pm 1.37) \times 10^{-3}$ , where  $r = M_D/M_B$  and  $\eta_{EW} = 1.0066 \approx 1$  [47]. In our numerical evaluations, we will use the central values  $\rho^2 = 1.09$  and  $|V_{cb}| F_1(w=1) = 48.14 \times 10^{-3}$ . A recent study [9] has shown that the mostly unknown scalar form factor  $F_0(q^2)$ , expressed as

$$F_0(q^2) = (1 + \alpha q^2 + \beta q^4) F_1(q^2), \quad (30)$$

can enhance the value of  $R(D)$  up to 0.335 within the SM, as shown in Table 1. We will use this scaling relation for  $F_0(q^2)$  with  $\alpha = +0.16 \text{ GeV}^{-2}$  and  $\beta = -0.003 \text{ GeV}^{-2}$  as used in Ref. [9].

Taking the contribution from  $B \rightarrow D \tau N$  decays into account, and assuming the unitarity (5) of the matrix  $U$ , the ratio of branching fractions  $R(D)$  is

$$R(D) \equiv \frac{\Gamma(B \rightarrow D \tau + \text{"missing"})}{\Gamma(B \rightarrow D l \nu)} = \frac{\{\bar{\Gamma}(B \rightarrow D \tau \nu) + |U_{\tau N}|^2 [\bar{\Gamma}(B \rightarrow D \tau N) - \bar{\Gamma}(B \rightarrow D \tau \nu)]\}}{\bar{\Gamma}(B \rightarrow D l \nu)} \quad (l=e, \mu), \quad (31)$$

where  $\bar{\Gamma}(B \rightarrow D \tau \nu)$  is the expression (24) for zero neutrino mass  $M_\nu = 0$  and  $M_l = M_\tau$ . If unitarity is not assumed,  $R(D)$  becomes

$$R(D) = \frac{\bar{\Gamma}(B \rightarrow D \tau \nu) + |U_{\tau N}|^2 \bar{\Gamma}(B \rightarrow D \tau N)}{\bar{\Gamma}(B \rightarrow D l \nu)} \quad (l=e, \mu). \quad (32)$$

### 3.2 $R(D^*)$ and decays of $B \rightarrow D^* l N$

The matrix elements for the  $B \rightarrow D^*$  transition are more complicated than those for the  $B \rightarrow D$  transition, because of the vector character of  $D^*$ , including four form factors:

$$\begin{aligned} H_\eta^\mu &= i2\eta \frac{\epsilon^{\mu\nu\alpha\beta}}{(M_B + M_{D^*})} \epsilon_\nu^*(p_D)_\alpha (p_B)_\beta V(q^2) - \left[ (M_B + M_{D^*}) \epsilon^{*\mu} A_1(q^2) - \frac{\epsilon^* \cdot q}{(M_B + M_{D^*})} (p_B + p_D)^\mu A_2(q^2) \right] \\ &\quad + 2M_{D^*} \frac{\epsilon^* \cdot q}{q^2} q^\mu (A_3(q^2) - A_0(q^2)), \quad (33) \end{aligned}$$

1) For early attempts to account for the flavor symmetry breaking in form factors of heavy pseudoscalars, cf. Ref. [45].

where

$$H_{(\eta=-1)}^\mu \equiv \langle D^{*-}(p_D) | \bar{c}(1-\gamma_5)\gamma^\mu b | B^0(p_B) \rangle = \langle \bar{D}^{*0}(p_D) | \bar{c}(1-\gamma_5)\gamma^\mu b | B^+(p_B) \rangle \quad (34)$$

$$H_{(\eta=+1)}^\mu \equiv \langle D^{*+}(p_D) | \bar{b}(1-\gamma_5)\gamma^\mu c | \bar{B}^0(p_B) \rangle = \langle D^{*0}(p_D) | \bar{b}(1-\gamma_5)\gamma^\mu c | B^-(p_B) \rangle, \quad (35)$$

The four form factors are

$$A_1(q^2) = \frac{1}{2} R_*(w+1) F_*(1) [1 - 8\rho_*^2 z(w) + (53\rho_*^2 - 15)z(w)^2 - (231\rho_*^2 - 91)z(w)^3], \quad (36)$$

$$V(q^2) = A_1(q^2) \frac{2}{R_*^2(w+1)} [R_1(1) - 0.12(w-1) + 0.05(w-1)^2], \quad (37)$$

$$A_2(q^2) = A_1(q^2) \frac{2}{R_*^2(w+1)} [R_2(1) + 0.11(w-1) - 0.06(w-1)^2], \quad (38)$$

$$A_3(q^2) = \frac{(M_B + M_{D^*})}{2M_{D^*}} A_1(q^2) - \frac{(M_B - M_{D^*})}{2M_{D^*}} A_2(q^2). \quad (39)$$

Here,  $R_* = 2\sqrt{M_B M_{D^*}} / (M_B + M_{D^*})$ , the variables  $w$  and  $z(w)$  are given by Eq. (25), and the values of the free parameters determined in Ref. [48] are

$$\rho_*^2 = 1.214(\pm 0.035), \quad 10^3 F_*(1) |V_{cb}| = 34.6(\pm 1.0), \quad (40)$$

$$R_1(1) = 1.401(\pm 0.038), \quad R_2(1) = 0.864(\pm 0.025). \quad (41)$$

We will use the central values of these parameters.

For the decays  $B \rightarrow D^* \ell N$ , the width is given in Ref. [18] as

$$\Gamma(B \rightarrow D^* \ell N) = |U_{\ell N}|^2 \bar{\Gamma}(B \rightarrow D^* \ell N), \quad (42)$$

where the canonical decay width (i.e., without the heavy-light neutrino mixing) is

$$\begin{aligned} \bar{\Gamma}(B \rightarrow D^* \ell N) = & \frac{1}{64\pi^3} \frac{G_F^2 |V_{cb}|^2}{M_B^2} \int_{(M_N + M_\ell)^2}^{(M_B - M_{D^*})^2} dq^2 \bar{\lambda}^{1/2} |\vec{q}| q^2 \left\{ \left( 1 - \frac{(M_N^2 + M_\ell^2)}{q^2} - \frac{1}{3} \bar{\lambda} \right) \left[ 2(M_B + M_{D^*})^2 A_1(q^2)^2 \right. \right. \\ & + \left. \frac{8M_B^2 |\vec{q}|^2}{(M_B + M_{D^*})^2} V(q^2)^2 + \frac{M_B^4}{4M_{D^*}^2 q^2} \left( (M_B + M_{D^*}) \left( 1 - \frac{(q^2 + M_{D^*}^2)}{M_B^2} \right) A_1(q^2) - \frac{4|\vec{q}|^2}{(M_B + M_{D^*})} A_2(q^2) \right)^2 \right] \\ & + \left[ - \left( \frac{M_N^2 - M_\ell^2}{q^2} \right)^2 + \frac{(M_N^2 + M_\ell^2)}{q^2} \right] \frac{M_B^2 |\vec{q}|^2}{M_{D^*}^2 q^2} \left[ \frac{2M_{D^*} (M_B + M_{D^*})^2}{(2M_{D^*} (M_B + M_{D^*}) - q^2)} \right]^2 \\ & \left. \times \left[ 1 - \frac{(M_B - M_{D^*}) A_2(q^2)}{(M_B + M_{D^*}) A_1(q^2)} \right]^2 A_1(q^2)^2 \right\}, \quad (43) \end{aligned}$$

where  $\bar{\lambda} = \lambda \left( 1, \frac{M_N^2}{q^2}, \frac{M_\ell^2}{q^2} \right)$  and  $|\vec{q}| = \frac{1}{2} M_B \lambda^{1/2} \left( 1, \frac{q^2}{M_B^2}, \frac{M_{D^*}^2}{M_B^2} \right)$ . As in the case of  $B \rightarrow D \ell N$ ,  $\bar{\Gamma}(B \rightarrow D^* \ell N)$  expresses the SM decay width  $\Gamma_{\text{SM}}(B \rightarrow D^* \ell \nu_1)$ , when  $N$  is replaced by  $\nu_1$  (i.e.,  $M_N \approx 0$ ).

Analogous to the case of  $R(D)$ , including the possible contribution from  $B \rightarrow D^* \tau N$  decays and assuming unitarity (5) of the full  $U$  matrix, the ratio of branching fractions  $R(D^*)$  can be written as

$$R(D^*) \equiv \frac{\Gamma(B \rightarrow D^* \tau + \text{"missing"})}{\Gamma(B \rightarrow D^* \ell \nu)} = \frac{\{\bar{\Gamma}(B \rightarrow D^* \tau \nu) + |U_{\tau N}|^2 [\bar{\Gamma}(B \rightarrow D^* \tau N) - \bar{\Gamma}(B \rightarrow D^* \tau \nu)]\}}{\bar{\Gamma}(B \rightarrow D^* \ell \nu)} \quad (l=e, \mu), \quad (44)$$

where  $\bar{\Gamma}(B \rightarrow D^* \tau \nu)$  is the expression (43) for zero neutrino mass  $M_\nu = 0$  and  $M_\ell = M_\tau$ . When the matrix  $U$  is not assumed to be unitary,  $R(D^*)$  becomes

$$R(D^*) = \frac{\bar{\Gamma}(B \rightarrow D^* \tau \nu) + |U_{\tau N}|^2 \bar{\Gamma}(B \rightarrow D^* \tau N)}{\bar{\Gamma}(B \rightarrow D^* \ell \nu)} \quad (l=e, \mu). \quad (45)$$

### 3.3 $R(0)$ and decays of $B \rightarrow \ell N$

In the decay  $B^+ \rightarrow \ell^+ \nu_1$  ( $l=e, \mu, \tau$ ), within the SM with  $M_{\nu_1} \approx 0$ , the decay width is given by

$$\Gamma_{\text{SM}}(B^+ \rightarrow \ell^+ \nu_1) = \frac{1}{8\pi} G_F^2 f_B^2 |V_{ub}|^2 M_B^3 y_1 (1 - y_1)^2 \quad (46)$$

where  $y_1 = M_\ell^2 / M_B^2$ . Here,  $M_B$  and  $f_B$  are the  $B^+$  meson mass and the decay constant, respectively,  $|V_{ub}|$  is the corresponding CKM matrix element, and  $G_F =$

$1.1664 \times 10^{-5} \text{ GeV}^{-2}$  is the Fermi coupling constant.

The decay width for the process  $B^+ \rightarrow l^+ N$  ( $l=e, \mu, \tau$ ) is, as shown in Ref. [18]:

$$\Gamma(B^+ \rightarrow l^+ N) = |U_{lN}|^2 \bar{\Gamma}(B^+ \rightarrow l^+ N), \quad (47)$$

where the canonical width  $\bar{\Gamma}$  (i.e., without the heavy-light mixing factor  $|U_{lN}|^2$ ) is

$$\begin{aligned} \bar{\Gamma}(B^+ \rightarrow l^+ N) = & \frac{1}{8\pi} G_F^2 f_B^2 |V_{ub}|^2 M_B^3 \lambda^{1/2}(1, y_N, y_l) \\ & \times [(1-y_N)y_N + y_l(1+2y_N-y_l)], \end{aligned} \quad (48)$$

where  $y_N = M_N^2/M_B^2$  and the function  $\lambda^{1/2}$  is given by

$$\lambda^{1/2}(x, y, z) = (x^2 + y^2 + z^2 - 2xy - 2yz - 2zx)^{1/2}. \quad (49)$$

It is obvious that if  $N$  is replaced by  $\nu_l$  (i.e.,  $y_N \approx 0$ ),  $\bar{\Gamma}(B^+ \rightarrow l^+ N)$  in Eq. (48) becomes  $\Gamma_{\text{SM}}(B^+ \rightarrow l^+ \nu_l)$  in Eq. (46). The SM expectation for the newly defined  $R(0)$ , which is completely independent of hadronic un-

certainty, is given by

$$R(0)_{\text{SM}} \equiv \frac{y_\tau(1-y_\tau)^2}{y_\mu(1-y_\mu)^2} = (2.2255 \pm 0.0002) \times 10^2, \quad (50)$$

where the uncertainty comes almost entirely from the uncertainty in the  $\tau$  lepton mass [49].

For the decays  $B^+ \rightarrow \tau^+ +$  “missing momentum”, taking the contribution from the decays  $B^+ \rightarrow \tau^+ N$  into account, the widths can be obtained with the unitarity assumption (5) by

$$\begin{aligned} & \Gamma(B^+ \rightarrow \tau^+ + \text{“missing momentum”}) \\ & = \bar{\Gamma}(B^+ \rightarrow \tau^+ \nu) + |U_{\tau N}|^2 [\bar{\Gamma}(B^+ \rightarrow \tau^+ N) - \bar{\Gamma}(B^+ \rightarrow \tau^+ \nu)], \end{aligned} \quad (51)$$

where  $\bar{\Gamma}(B^+ \rightarrow \tau^+ \nu)$  is for zero neutrino mass  $M_\nu = 0$ , i.e., it is equal to the SM expression (46) with  $l = \tau$ . Considering the contribution from  $B^+ \rightarrow \tau^+ N$ , the ratio  $R(0)$  in the considered scenario with one heavy neutrino  $N$  and the unitarity (5) of the  $U$  matrix is

$$R(0) \equiv \frac{\Gamma(B \rightarrow \tau + \text{“missing”})}{\Gamma(B \rightarrow \mu \nu)} = \frac{\{\bar{\Gamma}(B^+ \rightarrow \tau^+ \nu) + |U_{\tau N}|^2 [\bar{\Gamma}(B^+ \rightarrow \tau^+ N) - \bar{\Gamma}(B^+ \rightarrow \tau^+ \nu)]\}}{\bar{\Gamma}(B^+ \rightarrow \mu^+ \nu)}. \quad (52)$$

Here,  $\bar{\Gamma}(B^+ \rightarrow l^+ N)$  is given in Eq. (48), and  $\bar{\Gamma}(B^+ \rightarrow l^+ \nu)$  is the same expression with zero neutrino mass  $M_\nu = 0$ , i.e., Eq. (46). On the other hand, without the unitarity assumption, we have instead of the relation in Eq. (51) the following relation:

$$\begin{aligned} & \Gamma(B^+ \rightarrow \tau^+ + \text{“missing momentum”}) \\ & = \bar{\Gamma}(B^+ \rightarrow \tau^+ \nu) + |U_{\tau N}|^2 \bar{\Gamma}(B^+ \rightarrow \tau^+ N); \end{aligned} \quad (53)$$

and the ratio  $R(0)$  becomes

$$R(0) = \frac{\bar{\Gamma}(B^+ \rightarrow \tau^+ \nu) + |U_{\tau N}|^2 \bar{\Gamma}(B^+ \rightarrow \tau^+ N)}{\bar{\Gamma}(B^+ \rightarrow \mu^+ \nu)}. \quad (54)$$

As we shall see later, the observation of  $R(0)$  can give very useful information on the values of  $|U_{\tau N}|$  and the mass of a sterile neutrino  $M_N$ .

## 4 Numerical analysis and discussion

### 4.1 $R(D)$ , $R(D^*)$ and $B \rightarrow D\tau N$ , $D^*\tau N$

In this numerical analysis, first we study the  $R(D)$  and  $R(D^*)$  anomalies in our scenarios: in one scenario the matrix  $U$  without the unitarity assumption; in the other scenario, the full  $4 \times 4$  matrix  $U$  is considered unitary. As mentioned earlier, the latter scenario is realized when seesaw-type mechanisms are used, and the former may appear when the heavy neutral fermion  $N$  originates

from a different, unknown, mechanism in a high energy framework beyond the SM. Also, we examine the possibility of finding certain “direct” bounds on magnitudes of the matrix elements  $|U_{lN}|$  from this analysis of  $R(D)$  as well as  $R(D^*)$ .

1) We first calculate  $R(D)$  including the effect of the process  $B \rightarrow D\tau N$  as given in Eqs. (32) and (31). By comparing the theoretical result with the experimental data shown in Table 1, the allowed parameter space for  $|U_{\tau N}|^2$  is found in terms of the mass of the sterile neutrino,  $M_N$ . The results are shown in Fig. 3 for the two scenarios.

In Fig. 3(a), the result is found for the scenario without the assumption of unitarity for the  $4 \times 4$  matrix  $U$ . It shows the allowed parameter space for  $|U_{\tau N}|^2$  and  $M_N$  obtained from the experimental average value of  $R(D)$ . The red (light grey) region is allowed by the experimental data at  $1\sigma$  level, i.e., when the deviation does not surpass  $1\sigma$ . The blue (dark grey) region is allowed by the data at  $2\sigma$  level, i.e., when the deviation does not surpass  $2\sigma$  (and is above  $1\sigma$ ). The white region can be regarded as excluded, as the deviation there surpasses  $2\sigma$ . For comparison, the known available upper bounds from the CHARM and DELPHI experiments [40, 41] for  $|U_{\tau N}|^2$  are also included, as black squares<sup>1)</sup>. We see that a certain range of values of  $|U_{\tau N}|^2$  and  $M_N$  can fit the experimental data of  $R(D)$ . At  $1\sigma$  level, there is a tendency that for a smaller value of  $M_N$ , a smaller  $|U_{\tau N}|^2$  can fit

1) These upper bounds were obtained from various physical processes searching for heavy sterile neutrino  $N$  and are given in Table 3, see footnote 2) and Refs. [32, 40, 41].

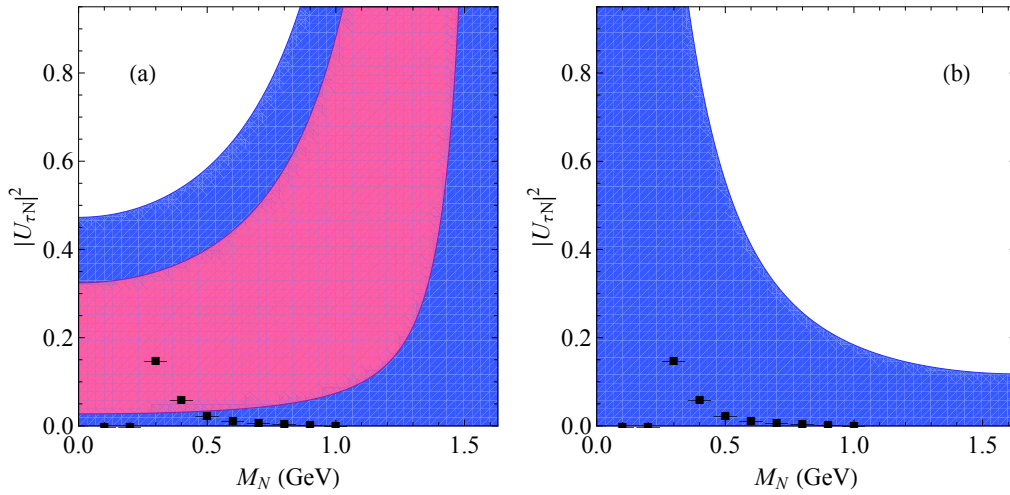


Fig. 3. (color online) The shaded regions represent the allowed parameter space obtained from the experimental average value of  $R(D)$  shown in Table 1. The red (light grey) region is allowed by experimental data for  $R(D)$  at  $1\sigma$  level, and the blue (dark grey) region at  $2\sigma$  level. The white region is excluded (more than  $2\sigma$  deviation): (a) when  $U$  is nonunitary, cf. Eq. (32); (b) when  $U$  is unitary, cf. Eqs. (5) and (31). In (a) and (b), the known present upper bounds from the CHARM and DELPHI experiments [40, 41] for  $|U_{\tau N}|^2$  are denoted by black squares. In the nonunitary case (a), if accounting for the lepton universality measurements, the three black squares at  $M_N=0.3, 0.4$  and  $0.5$  GeV decrease to the values  $\sim 10^{-2}$  as given in Fig. 2(a). In the unitary case (b), when accounting for the restrictions coming from  $\tau \rightarrow b\nu\nu'$  [43] and the lepton universality measurements Fig. 2(b), the black squares at  $M_N \geq 0.3$  GeV decrease to the values  $|U_{\tau N}|^2 \lesssim 0.01$ .

the data. For instance, for  $M_N=0.3$  GeV, the smallest value of  $|U_{\tau N}|^2$  allowed by the  $1\sigma$  data is  $2.8 \times 10^{-2}$ , while for  $M_N=1.0$  GeV, the smallest allowed value of  $|U_{\tau N}|^2$  is  $7.1 \times 10^{-2}$ .

In particular, for  $M_N=0.3$  GeV, the values of  $2.8 \times 10^{-2} \lesssim |U_{\tau N}|^2 \lesssim 3.5 \times 10^{-1}$  are allowed by the  $1\sigma$  data, in comparison with the known [40, 41] upper bound  $|U_{\tau N}|^2 = 1.5 \times 10^{-1}$ . Similarly, for  $M_N=0.4$  GeV, the  $1\sigma$  data allows the values of  $3.0 \times 10^{-2} \lesssim |U_{\tau N}|^2 \lesssim 3.7 \times 10^{-1}$ , compared to the DELPHI [41] upper bound of  $|U_{\tau N}|^2 = 6.0 \times 10^{-2}$ . We note that for  $M_N=0.3$  GeV and  $0.4$  GeV, the smallest allowed value of  $|U_{\tau N}|^2$  is smaller than its available [40, 41] upper bound. However, if  $M_N \geq 0.5$  GeV, the values of  $|U_{\tau N}|^2$  allowed by the  $1\sigma$  data are larger than the known DELPHI upper bounds. However, if using the indirect upper bounds coming from the lepton universality measurements, Fig. 2(a), all the allowed points move out of the  $1\sigma$  region in Fig. 3(a).

In Fig. 3 (b), the result corresponds to the case considering unitarity of the  $(4 \times 4)$  matrix  $U$ , cf. Eq. (5). In this scenario, there is no allowed parameter space for  $|U_{\tau N}|^2$  and  $M_N$  by the experimental data at  $1\sigma$  level. At  $2\sigma$  level, a certain region of the parameter space is allowed. For example, for  $M_N \lesssim 0.3$  GeV, the values of  $0 \lesssim |U_{\tau N}|^2 \lesssim 1$  are allowed, while for  $M_N=0.6$  GeV and  $1.0$  GeV,  $0 \lesssim |U_{\tau N}|^2 \lesssim 0.4$  and  $0 \lesssim |U_{\tau N}|^2 \lesssim 0.2$  are allowed, respectively.

2) In the case of  $R(D^*)$ , it is harder to fit the experimental data by including the contributions from

$B \rightarrow D^* \tau N$  together with those from the SM processes. Similarly to the analysis of  $R(D)$ , we compute  $R(D^*)$  including the effect of the decay  $B \rightarrow D^* \tau N$  as given in Eqs. (44) and (45), and again examine the two scenarios: (a) one considering the  $4 \times 4$  matrix  $U$  to be nonunitary, Eq. (45); (b) the other considering  $U$  to be unitary, Eqs. (5) and (44). It turns out that in the latter scenario (b), there are no allowed values of  $|U_{\tau N}|^2$  and  $M_N$ . Namely, Table 1 shows that the SM value of  $R(D^*)$  is below the central experimental value by more than  $3\sigma$ ; furthermore, the theoretical value in scenario (b) becomes even lower when  $U_{\tau N} \neq 0$ , cf. Eq. (44). In contrast, in the former scenario (a), certain values of  $|U_{\tau N}|^2$  and  $M_N$  are allowed at  $2\sigma$  level, but not at  $1\sigma$  level. The allowed parameter space is shown in Fig. 4 for the scenario (a). For instance, for  $M_N=0.3$  GeV and  $0.4$  GeV, the values of  $2.0 \times 10^{-1} \lesssim |U_{\tau N}|^2 \lesssim 3.6 \times 10^{-1}$  and  $2.1 \times 10^{-1} \lesssim |U_{\tau N}|^2 \lesssim 3.9 \times 10^{-1}$ , respectively, would be allowed at  $1\sigma$  level. However, as the black squares in Fig. 4 indicate, the present CHARM and DELPHI [40, 41] upper bounds on  $|U_{\tau N}|^2$  give compatibility of the experimental  $R(D^*)$  with the scenario (a) to at best  $2\sigma$  level, and this only if  $M_N \approx 0.3$  GeV. Further, if we include the indirect upper bounds coming from the lepton universality measurements, Fig. 2(a), the allowed points move out of the  $2\sigma$  region in Fig. 4.

We comment on a feature of the results for  $R(D^{(*)})$  when the  $4 \times 4$   $U$  matrix is unitary. To be specific, let us consider the canonical decay widths  $\bar{\Gamma}$  for the decays

$B \rightarrow D^{(*)}\tau N$  given in Eqs. (24) and (43). We find that  $\bar{\Gamma}(B \rightarrow D^{(*)}\tau N) < \bar{\Gamma}(B \rightarrow D^{(*)}\tau\nu)$ , i.e.,  $\bar{\Gamma}(B \rightarrow D^{(*)}\tau N)$  is a monotonically decreasing function of  $M_N$ . Thus the  $|U_{\tau N}|^2$  term in  $R(D^{(*)})$ , i.e., in the numerator of Eqs. (31) and (44), becomes negative and the effect of this term is to reduce  $R(D^{(*)})$  with respect to its SM value. Only in the scenario where  $U$  is nonunitary, cf. Eqs. (32) and (45), does the presence of a heavy neutral fermion  $N$  increase the ratio  $R(D^{(*)})$ .

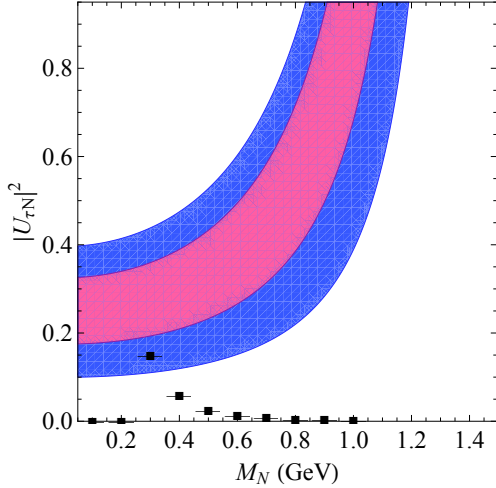


Fig. 4. (color online) The shaded regions represent the allowed parameter space obtained from the experimental average value of  $R(D^{*})$  shown in Table 1, in the case of nonunitarity of the full  $U$  matrix, cf. Eq. (45). The red (light grey) region is allowed by the experimental data at  $1\sigma$  level, and the blue (dark grey) region at  $2\sigma$  level. The known CHARM and DELPHI [40, 41] upper bounds for  $|U_{\tau N}|^2$  are denoted by black squares. If accounting for the lepton universality measurements, the three black squares at  $M_N = 0.3, 0.4$  and  $0.5$  GeV decrease to the values  $\sim 10^{-2}$  as given in Fig. 2(a).

#### 4.2 $R(0)$ and $B \rightarrow \tau N$

We now analyze the decay process  $B \rightarrow \tau +$  “missing momentum” by including the process  $B \rightarrow \tau N$ . We will first consider the BF of  $B \rightarrow \tau +$  “missing momentum”. Similarly to the previous cases of  $R(D)$  and  $R(D^{*})$ , two scenarios are examined: (a) one scenario considering the matrix  $U$  as nonunitary, and where  $\Gamma(B^+ \rightarrow \tau^+ + \text{“missing momentum”})$  is consequently given by Eq. (53); (b) the other scenario where the  $4 \times 4$  matrix  $U$  is considered to be unitary, Eq. (5), and where  $\Gamma(B^+ \rightarrow \tau^+ + \text{“missing momentum”})$  is consequently given by Eq. (51). The results of this analysis are presented in Figs. 5 (a) and (b). The allowed values of  $|U_{\tau N}|^2$  and  $M_N$  are in a wide range for both scenarios.

For future experiments, we make predictions for  $R(0)$  given in Eqs. (52) and (54). They are summarized in

Table 3. The known upper bounds for values of  $|U_{\tau N}|^2$ , for various specific values of  $M_N$ , are taken from Refs. [32, 40, 41]. Due to the smallness of these upper bounds for  $|U_{\tau N}|^2$ , the predicted values of  $R(0)$  for various values of  $M_N$  do not deviate much from the SM predicted value  $R(0)_{\text{SM}} = 2.2255 \times 10^2$ , cf. Eq. (50). Only for  $0.3 \text{ GeV} \leq M_N \leq 0.8 \text{ GeV}$  are sizable deviations from  $R(0)_{\text{SM}}$  expected in the scenario of nonunitarity of  $U$ . For example, for  $M_N = 0.3 \text{ GeV}$  and  $0.4 \text{ GeV}$ , the predicted values are  $R_0 = 2.5708 \times 10^2$  and  $2.3672 \times 10^2$ , respectively.

Table 3. Predicted values of  $R_0$ . The  $R_0$  values are calculated in two ways: (i) considering  $U$  to be nonunitary; (ii) considering  $U$  to be unitary. The results are given to four digits to facilitate comparison. The values of  $|U_{\tau N}|^2$  for various values of  $M_N$  are taken as equal to the known CHARM and DELPHI upper bounds [40, 41].

$M_N/\text{GeV}$	$ U_{\tau N} ^2$	$R_0$ [nonunitarity]	$R_0$ [unitarity]
0	0	$2.2255 \times 10^2$ [SM]	$2.2255 \times 10^2$ [SM]
0.1	$8.0 \times 10^{-4}$	$2.2273 \times 10^2$	$2.2255 \times 10^2$
0.2	$2.0 \times 10^{-4}$	$2.2259 \times 10^2$	$2.2255 \times 10^2$
0.3	$1.5 \times 10^{-1}$	$2.5708 \times 10^2$	$2.2370 \times 10^2$
0.4	$6.0 \times 10^{-2}$	$2.3672 \times 10^2$	$2.2336 \times 10^2$
0.5	$2.5 \times 10^{-2}$	$2.2864 \times 10^2$	$2.2307 \times 10^2$
0.6	$1.4 \times 10^{-2}$	$2.2608 \times 10^2$	$2.2297 \times 10^2$
0.7	$9.0 \times 10^{-3}$	$2.2491 \times 10^2$	$2.2291 \times 10^2$
0.8	$6.0 \times 10^{-3}$	$2.2420 \times 10^2$	$2.2286 \times 10^2$
0.9	$4.0 \times 10^{-3}$	$2.2370 \times 10^2$	$2.2281 \times 10^2$
1.0	$3.0 \times 10^{-3}$	$2.2345 \times 10^2$	$2.2278 \times 10^2$
2.0	$3.0 \times 10^{-4}$	$2.2268 \times 10^2$	$2.2262 \times 10^2$
3.0	$4.5 \times 10^{-5}$	$2.2257 \times 10^2$	$2.2256 \times 10^2$

More interesting predictions relevant to  $R(0)$  are depicted in Fig. 6(a) and (b), corresponding to the above two scenarios, respectively. The figures show the graphs of  $|U_{\tau N}|^2$  versus  $M_N$  for given values of  $R(0)$ . Provided that the value of  $R(0)$  is determined in future experiments (e.g., by measuring the BFs of  $B \rightarrow \mu\nu$  and  $B \rightarrow \tau +$  “missing” precisely), one can obtain useful information on  $|U_{\tau N}|^2$  and  $M_N$  from the figures. For example, if  $R(0)$  is determined to be  $2.250 \times 10^2$ , it corresponds to the case (ii) (i.e., thick red line) in Fig. 6(a) and (b), from which the value of  $|U_{\tau N}|^2$  can be extracted for a given  $M_N$ . However, to discriminate experimentally between cases (i) (SM) and (ii), the branching ratios for  $B \rightarrow \tau +$  “missing” and  $B \rightarrow \mu\nu$  will have to be measured with precision of 1% or better; it is possible that such a precision cannot be achieved at Belle-II. Further, if  $U$  is nonunitary and if, simultaneously, the  $\tau$ -N-W coupling comes from LR-model scenarios, then we have the upper bound  $|U_{\tau N}|^2 < 5 \times 10^{-3}$ , cf. discussion after Eq. (11). In such a case, we cannot expect to get values of  $R(0)$  over  $2.24 \times 10^2$  in such scenarios. Furthermore, if we take into account the indirect upper bounds on  $|U_{\tau N}|^2$  coming

from the lepton universality measurements, cf. Fig. 2(a), the three black squares at  $M_N=0.3, 0.4$  and  $0.5$  GeV in

Fig. 6(a) decrease to the values  $\approx 10^{-2}$ , and we cannot get values of  $R(0)$  over  $2.26 \times 10^2$ .

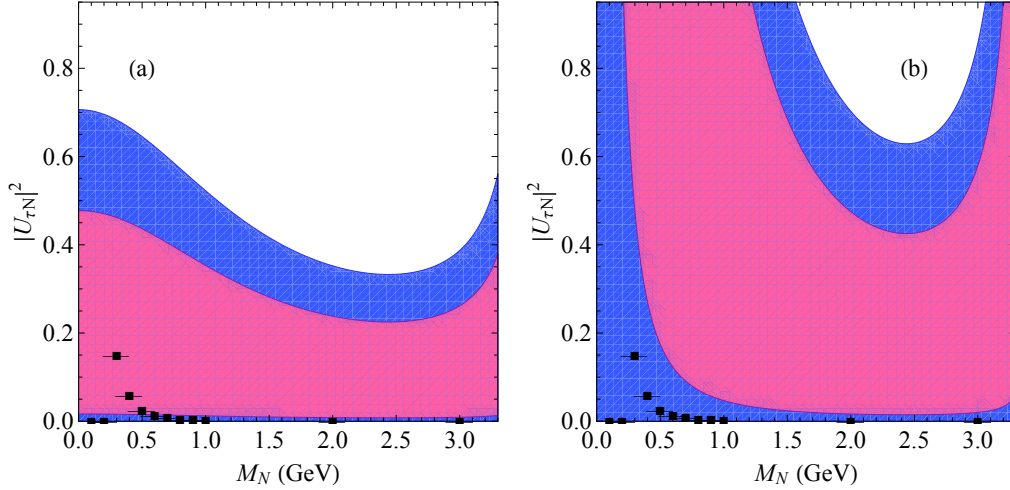


Fig. 5. (color online) The shaded regions represent the allowed parameter space obtained from the experimental average value of  $\text{BF}(B^+ \rightarrow \tau^+ \nu) = (1.06 \pm 0.19) \times 10^{-4}$  given in Table 2. The red (light grey) region is allowed by the experimental data at  $1\sigma$  level, and the blue (dark grey) region at  $2\sigma$  level. Plot (a) is obtained by considering  $U$  to be nonunitary, Eq. (53). Plot (b) is obtained by considering  $U$  to be unitary, Eq. (51). In (a) and (b), the known CHARM and DELPHI [40, 41] upper bounds for  $|U_{\tau N}|^2$  are denoted by black squares. If the nonunitary case is generated by a LR-model scenario, we have an additional upper bound  $|U_{\tau N}|^2 < 5 \times 10^{-3}$ . If accounting for the lepton universality measurements, the three black squares at  $M_N=0.3, 0.4$  and  $0.5$  GeV decrease to the values  $\sim 10^{-2}$  as suggested in Figs. 2(a),(b).

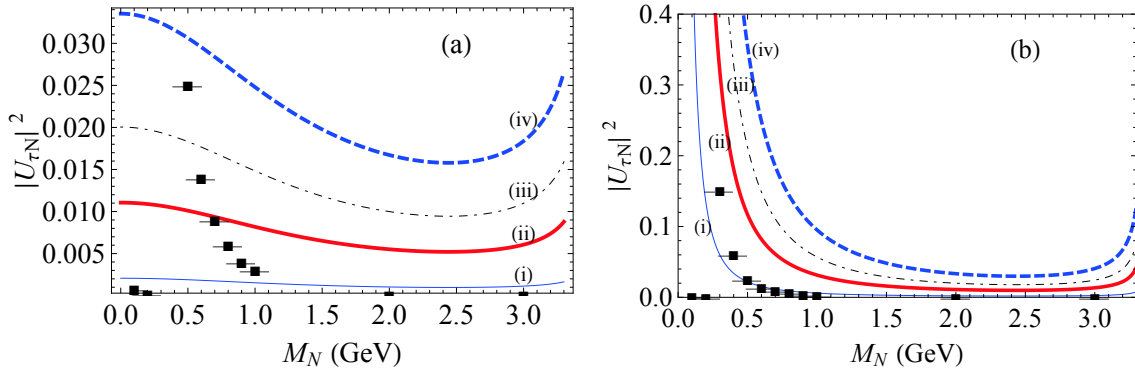


Fig. 6. (color online) The graphs of  $|U_{\tau N}|^2$  versus  $M_N$  when  $R_0$  is given. Plot (a) is obtained by considering  $U$  to be nonunitary, cf. Eq. (54). In plot (b) the  $4 \times 4$  matrix  $U$  is considered to be unitary, Eqs. (5) and (52). In (a) and (b), the cases (i), (ii), (iii), and (iv) correspond to the given value of  $R_0 = 2.23 \times 10^2, 2.25 \times 10^2, 2.27 \times 10^2$ , and  $2.30 \times 10^2$ , respectively. In (a) and (b), the known CHARM and DELPHI [40, 41] upper bounds for  $|U_{\tau N}|^2$  are denoted by black squares. If the nonunitary case (a) is generated by a LR-model scenario, we have an additional upper bound  $|U_{\tau N}|^2 < 5 \times 10^{-3}$ . If accounting for the lepton universality measurements, according to Fig. 2, the three black squares at  $M_N=0.3, 0.4$  and  $0.5$  GeV in (a) decrease to the values  $\approx 10^{-2}$ , and in (b) at  $M_N \geq 0.3$  GeV the black squares decrease to values  $\lesssim 10^{-2}$ .

## 5 Conclusions

In this work we studied the experimental anomalies of the ratios  $R(D)$  and  $R(D^*)$  related to the semileptonic B

decays  $B \rightarrow D \tau \nu$  and  $B \rightarrow D^* \tau \nu$ , and the newly suggested observable  $R(0)$  related to the purely leptonic B decays  $B \rightarrow \tau \nu$  and  $B \rightarrow \mu \nu$ , considering possible effects from the presence of a neutral fermion (sterile neutrino) N with

mass  $\sim 1$  GeV. In theoretical estimations of  $R(D)$ ,  $R(D^*)$ , and  $R(0)$ , the possible effects of the processes  $B \rightarrow D\tau N$ ,  $B \rightarrow D^*\tau N$ , and  $B \rightarrow \tau N$  were included. For generality we considered two possible scenarios: with the assumption of unitarity of the full  $4 \times 4$  mixing matrix  $U$  (extended PMNS matrix), and without the assumption of unitarity.

We analyzed each of  $R(D)$ ,  $R(D^*)$  and  $R(0)$  separately. Our findings are summarized as follows.

1) For the observable  $R(D)$ , assuming  $U$  to be nonunitary, the discrepancy between the experimental data and the theoretical prediction can be resolved at  $1\sigma$  level [cf. Fig. 3(a)] for  $M_N < 0.5$  GeV. The possible values of  $|U_{\tau N}|^2$  are found for various values of the mass  $M_N$ . Especially, we have found that the values of  $|U_{\tau N}|^2$  allowed by the  $1\sigma$  data are  $2.8 \times 10^{-2} \lesssim |U_{\tau N}|^2 \lesssim 3.5 \times 10^{-1}$  and  $3.0 \times 10^{-2} \lesssim |U_{\tau N}|^2 \lesssim 3.7 \times 10^{-1}$  for  $M_N = 0.3$  GeV and  $0.4$  GeV, which fall within these intervals, respectively. These values can be compared with the known CHARM and DELPHI [40, 41] upper bounds  $|U_{\tau N}|^2 = 1.5 \times 10^{-1}$  and  $6.0 \times 10^{-2}$  for  $M_N = 0.3$  GeV and  $0.4$  GeV, respectively. However, if the nonunitary  $U$  has its origin in general  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  models [37], these models would imply that  $|U_{\tau N}|^2 < 5 \times 10^{-3}$ . Further, if accounting for the indirect upper bounds on  $|U_{\tau N}|^2$  coming from the lepton universality measurements, Fig. 2(a), the otherwise generous CHARM and DELPHI upper bounds at  $M_N \leq 0.5$  GeV get decreased to  $|U_{\tau N}|^2 \lesssim 10^{-2}$ .

2) When the full  $4 \times 4$  matrix  $U$  is assumed to be unitary, in contrast to the above case, there is no resolution for the anomaly of  $R(D)$  within  $1\sigma$  level, as shown in Fig. 3(b).

3) We found it to be more difficult to resolve the

anomaly of  $R(D^*)$ , compared with  $R(D)$ . The discrepancy in  $R(D^*)$  can be resolved only when we assume that  $U$  is nonunitary, and at best only at  $2\sigma$  level and at  $M_N \approx 0.3$  GeV, cf. Fig. 4. When taking into account the indirect upper bounds on  $|U_{\tau N}|^2$  coming from the lepton universality measurements, then the discrepancy cannot be resolved even at  $2\sigma$  level.

4) We demonstrated that certain useful information on the parameters  $|U_{\tau N}|$  and  $M_N$  can be extracted from the purely leptonic B meson decays  $B \rightarrow \tau\nu$ ,  $B \rightarrow \tau N$  and  $B \rightarrow \mu\nu$ . If the observable  $R(0)$ , involving the rates of these decays, is measured in the future experiments, such as at Belle-II, the value of  $|U_{\tau N}|$  could be determined without any hadronic uncertainties, depending on  $M_N$ , cf. Fig. 6.

We assumed that the sterile heavy particle  $N$  is stable and invisible, hence does not decay inside the detector, and thus manifests itself as “missing momentum” in the measurements. However, depending on the values of  $U_{lN}$ ,  $M_N$  and the detector size, the produced sterile neutrino  $N$  can decay within or beyond the actual detector. When  $N$  is produced as  $B^+ \rightarrow \tau^+ N$  or  $B^+ \rightarrow D^{(*)}\tau^+ N$ , and if  $N$  also decays within the detector, the main signature of  $N$  will be  $N \rightarrow l^+ \pi^-$  (if  $N$  is Majorana) or  $N \rightarrow l^- \pi^+$  (if  $N$  is Dirac or Majorana), which will appear experimentally as a resonance in  $M(l^\pm \pi)$ . Then the experimental signatures would be  $B \rightarrow D^{(*)}l^\pm \pi$  and  $B \rightarrow l^\pm l^\pm \pi$  (if  $N$  is Majorana) or  $B \rightarrow D^{(*)}l^\pm l^\mp \pi$  and  $B \rightarrow l^\pm l^\mp \pi$  (if  $N$  is Dirac or Majorana). The details of such decays at Belle-II and LHCb have been discussed in Ref. [18], for sufficiently small values of  $|U_{lN}|$ , and  $M_N \lesssim 2$  GeV.

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