LANDAU-POMERANCHUK-MIGDAL EFFECT IN QCD

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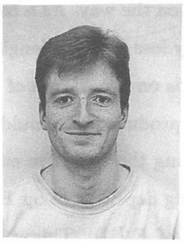

The radiative energy loss per unit length $-dE/dz$ of a high energy parton crossing a QCD medium is derived by generalizing the Landau-Pomeranchuk-Migdal effect to QCD. As a consequence, $-dE/dz \propto \alpha_s\sqrt{E}$.

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1. Introduction

Amongst the numerous Quark Gluon Plasma (QGP) signals which have been proposed, jet production could be an interesting probe of the dense or hot medium created in ultra-relativistic heavy ion collisions. In heavy ion collisions, due to hard interactions, partons of high transverse energy E are produced at an early time ~ 0.1 fm/c. It is assumed that for ultrarelativistic energies a hot deconfined medium might be created at a time ~ 1 fm/c , with a transverse size ~ 10 fm. Therefore, the energetic partons would have to cross this medium on a distance ~ 10 fm, loosing energy on the way. As a consequence, the jet production cross section depends directly on the energy loss per unit length of the energetic partons $-\frac{dE}{dz}$. If the behaviour of this quantity depends on the nature of the medium (cold hadronic matter or hot plasma), the characteristics of produced jets could signal the QGP.

Our study is devoted to calculating the radiative energy loss, which appears to be the dominant contribution: we get a strong dependence on E, $-\left.\frac{dE}{dz}\right)_{rad} \propto \alpha_s\sqrt{E}$ (whereas the collisional energy loss is independent of E [1]). This is found by generalizing the Landau-Pomeranchuk-Migdal (LPM) effect known in QED [2,3] to QCD. The important jet quenching which is obtained could be a crucial signal for QGP.

The LPM effect in QED has been discovered in the 1950's. Consider an electron which scatters off fixed nuclei and radiates one photon of energy ω. When its energy E is small, the radiation spectrum induced by the crossing of a distance L of the medium is additive in the number of scatterings :

$$\left.\omega\frac{dI}{d\omega}\right)_L = \frac{L}{\lambda}\left.\omega\frac{dI}{d\omega}\right)_{BH} \quad . \tag{1}$$

λ is the electron mean free path between two successive scatterings and $\left.\omega\frac{dI}{d\omega}\right)_{BH}$ is the radiation spectrum induced by one scattering, independent of ω, given by the Bethe-Heitler law :

$$\left.\omega\frac{dI}{d\omega}\right)_{BH} \propto \alpha \quad . \tag{2}$$

When E increases, the electron and the radiated photon become collinear. The photon formation time (time of separation between electron and photon in the transverse direction) becomes very large in front of λ :

$$t_f = \frac{2}{\omega\,\theta^2} \gg \lambda \quad , \tag{3}$$

where θ is the angle between electron and photon. Then destructive interferences appear between emission amplitudes induced by scattering centres separated by a distance $\sim t_f$.

This leads to a suppression of the spectrum for small ω, known as the LPM effect [2,3] :

$$\frac{1}{L}\,\omega\frac{dI}{d\omega}\Big)_L \equiv \omega\frac{dI}{d\omega dz} \propto \alpha\sqrt{\omega} \quad . \tag{4}$$

This effect has been measured with a good quantitative agreement only recently in a SLAC experiment [4].

To study the importance of coherence effects in QCD, we go through the following steps [5] :

- We first rederive the LPM effect in QED in an amplitude approach.
- We generalize the derivation to QCD using a formal analogy with QED.

The result reads :

$$\omega\frac{dI}{d\omega dz}\Big)_{QCD} \propto \alpha_s \frac{1}{\sqrt{\omega}} \Rightarrow -\frac{dE}{dz}\Big)_{QGP} \propto \alpha_s \sqrt{E} \quad , \tag{5}$$

where here E is the energy of the incoming parton, and ω the energy of the radiated gluon.

2. LPM effect in QED

To describe multiple scatterings we use a model where the electron scatters off screened Coulomb potentials created by static centres located at $\vec{x}_i$ $(i = 1, -, N)$:

$$V_i(\vec{q}_i) = \frac{e}{\vec{q}_i^{\,2} + \mu^2} e^{-i\vec{q}_i.\vec{x}_i} \quad . \tag{6}$$

μ is the Debye mass in the medium and $\vec{q}_i$ the momentum transferred to the electron during scattering on centre $\vec{x}_i$. We use :

$$\lambda \gg \mu^{-1} \quad . \tag{7}$$

As μ^{-1} is the range of the Coulomb potentials, this means that successive elastic scatterings are independent. Then we consider soft photon emission :

$$\omega \ll E \quad , \tag{8}$$

and the approximation :

$$\frac{\mu}{E} \ll 1 \quad . \tag{9}$$

Using time-ordered perturbation theory leads to the radiation spectrum induced by N scatterings :

$$\omega\frac{dI}{d\omega}\Big)_N \propto \int d\Omega \left|\sum_{i=1}^{N} M_i\, e^{ik.x_i}\right|^2 \quad , \tag{10}$$

where M_i is the effective radiation amplitude induced by the transfer $\vec{q}_i$, corresponding to photon emission before and after centre i :

$$M_i \propto \left(\frac{\varepsilon \cdot p_{i+1}}{k \cdot p_{i+1}} - \frac{\varepsilon \cdot p_i}{k \cdot p_i}\right) \quad . \tag{11}$$

k and ε are the photon momentum and polarization, p_i and p_{i+1} the electron momenta before and after centre i. Dividing by $N\lambda$ to get the radiation spectrum per unit length ($N \gg 1$) :

$$\omega \frac{dI}{d\omega dz} \propto \int d\Omega \; 2Re \sum_{j=i+1}^{\infty} M_i \cdot M_j^* \left(e^{ik\cdot(x_j - x_i)} - 1\right) \quad . \tag{12}$$

This may be evaluated by averaging over the electron trajectory [5].

We give here the following heuristic derivation to get the qualitative behaviour of the energy spectrum. After summing over the photon polarizations, we get :

$$M_i \cdot M_j^* \propto \left(\frac{\vec{u}_{i+1}}{u_{i+1}^2} - \frac{\vec{u}_i}{u_i^2}\right)\left(\frac{\vec{u}_{j+1}}{u_{j+1}^2} - \frac{\vec{u}_j}{u_j^2}\right) \quad , \tag{13}$$

where $\vec{u}_j$ is the "two-dimensional angle" $\vec{u}_j = \vec{k}_\perp/\omega - \sum_{l=1}^{j-1} \vec{q}_{l\perp}/E$. The essential feature is the phase which is expressed in terms of u_j^2 :

$$k \cdot (x_j - x_i) \underset{j-i \gg 1}{\simeq} \omega \; (j-i) \; \lambda \; u_j^2/2 \quad . \tag{14}$$

In the approximation of independent successive elastic scatterings, the electron undergoes a random walk between i and j, i.e. $u_j^2 \propto (j-i)(\mu/E)^2$:

$$k \cdot (x_j - x_i) \underset{j-i \gg 1}{\simeq} \frac{\lambda\mu^2}{2} \frac{\omega}{E^2} (j-i)^2 \equiv \frac{(j-i)^2}{\nu^2}$$

$$\nu \equiv \sqrt{\frac{2}{\lambda\mu^2} \frac{E^2}{\omega}} \quad . \tag{15}$$

ν is the coherence number and depends on ω; $\nu \gg 1$ for $E \gg \lambda\mu^2$. For $|j-i| < \nu$, the phase difference between M_i and M_j is smaller than 1, which means that M_i and M_j are coherent. Therefore, as far as the emission of a photon of energy ω is concerned, the effective independent centres are groups of ν centres. The spectrum is obtained by dividing by ν the Bethe-Heitler spectrum induced by one scattering :

$$\omega \frac{dI}{d\omega dz} = \frac{1}{\nu\lambda} \left.\omega \frac{dI}{d\omega}\right)_{BH} \propto \alpha\sqrt{\omega} \quad . \tag{16}$$

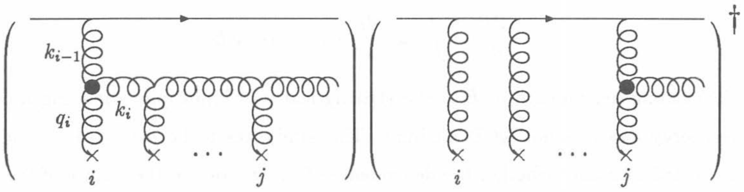
This behaviour is confirmed by analytic calculation [5].

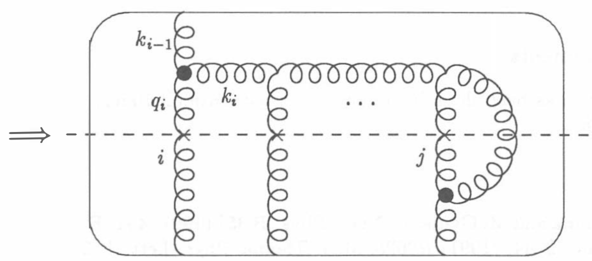
3. Generalization to QCD

To generalize to QCD, we have to consider interferences between gluon radiation amplitudes induced by distant centres. Now the effective radiation amplitude induced by the momentum transfer $\vec{q}_i$ is the sum of three terms: radiation of the incident parton (let us choose a quark) before and after centre i, and radiation from the exchanged gluon. In the soft gluon limit, this takes the same form as in QED, up to the replacement of the photon angle by the gluon transverse momentum :

$$M_i \propto \left[\frac{\vec{k}_{i\perp}}{k_{i\perp}^2} - \frac{(\vec{k}_{i\perp} - \vec{q}_{i\perp})}{(\vec{k}_{i\perp} - \vec{q}_{i\perp})^2} \right] [T^b, T^{a_i}] \quad , \tag{17}$$

where colour factors are included. Drawing interference terms analogous to those appearing in (12) in a connected form, we get the following planar diagrams, which give the dominant contribution to the spectrum (with $\vec{k}_{i\perp} \equiv k_i = k_{i-1} + q_i$) :





Interference terms involving quark rescattering [6] lead to non-planar diagrams, suppressed by colour factors, which give a subdominant contribution [5]. As the relevant variable is now $\vec{k}_\perp$, we write :

$$k \cdot (x_j - x_i) \underset{j-i\gg 1}{\simeq} \omega\,(j-i)\,\lambda\, u_j^2/2 \;=\; \frac{\lambda}{\omega}\,(j-i)\,k_j^2/2 \quad . \tag{18}$$

With $k_j^2 \propto (j-i)\mu^2$:

$$k \cdot (x_j - x_i) \underset{j-i\gg 1}{\simeq} \frac{\lambda\mu^2}{2}\,\frac{1}{\omega}\,(j-i)^2 \;\equiv\; \frac{(j-i)^2}{\nu_{QCD}^2}$$

$$\nu_{QCD} \equiv \sqrt{\frac{2}{\lambda\mu^2}\,\omega} \quad . \tag{19}$$

The relevant coherence number is obtained from (15) by the replacement $\omega/E^2 \to 1/\omega$. In a similar way we get the energy spectrum :

$$\left.\omega\frac{dI}{d\omega dz}\right)_{QCD} = \frac{1}{\nu_{QCD}\lambda}\left.\omega\frac{dI}{d\omega}\right)_{BH} \propto \alpha_s\,\frac{1}{\sqrt{\omega}} \quad . \tag{20}$$

This yields the energy loss per unit length of the incoming quark :

$$-\frac{dE}{dz} = \int_0^E \omega\frac{dI}{d\omega dz}d\omega \;\propto\; \alpha_s\,\sqrt{E} \quad . \tag{21}$$

This strong dependence in E is the striking feature of multiple scattering induced radiative energy loss in a hot QCD medium. This study has to be extended to cold nuclear matter, in order to infer whether the dependence of energy loss on the nature of the medium could signal the QGP. In any case, it would be quite motivating to check experimentally the $\sqrt{E}$ behaviour we obtain.

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References

1. M.H. Thoma and M. Gyulassy, Nucl. Phys. B 351 (1991) 491; E. Braaten and M.H. Thoma, Phys. Rev. D 44, (1991) R2625; M.H. Thoma, Phys. Lett. B 273 (1991) 128.
2. L.D. Landau and I.Ya. Pomeranchuk, Dokl. Akad. Nauk SSSR 92 (1953) 535, 735.
3. A.B. Migdal, Phys. Rev. 103 (1956) 1811, and references therein.
4. S. Klein et al, SLAC-PUB-6378 (Nov. 1993) (T/E); R. Becker-Szendy et al, SLAC-PUB-6400 (Dec. 1993) (E).
5. R. Baier, Yu.L. Dokshitzer, S. Peigné and D. Schiff, Phys. Lett. B 345 (1995) 277.
6. M. Gyulassy and X.-N. Wang, Nucl. Phys. B 420 (1994) 583.