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# Cluster-daughter overlap as a new probe of alpha-cluster formation in medium-mass and heavy even-even nuclei

Dong Bai<sup>a</sup>, Zhongzhou Ren<sup>b,\*</sup>

<sup>a</sup> School of Physics, Nanjing University, Nanjing, 210093, China
 <sup>b</sup> School of Physics Science and Engineering, Tongji University, Shanghai, 200092, China

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### ABSTRACT

We study the possibility to use the cluster-daughter overlap as a new probe of alpha-cluster formation in medium-mass and heavy even-even nuclei. We introduce a dimensionless parameter  $\mathfrak{O}$ , which is the ratio between the root-mean-square (rms) intercluster separation and the sum of rms point radii of the daughter nucleus and the alpha particle, to measure the degree of the cluster-daughter overlap quantitatively. By using this parameter, a large (small) cluster-daughter overlap between the alpha cluster and daughter nucleus corresponds to a small (large)  $\mathfrak O$  value. The alpha-cluster formation is shown explicitly, in the framework of the quartetting wave function approach, to be suppressed when the  ${\mathfrak O}$ parameter is small, and be favored when the  $\mathfrak{O}$  parameter is large. We then use this  $\mathfrak{O}$  parameter to explore systematically the landscape of alpha-cluster formation probabilities in medium-mass and heavy even-even nuclei, with  $\mathfrak O$  being calculated from experimentally measured charge radii. The trends of alpha-cluster formation probabilities are found to be generally consistent with previous studies. The effects of various shell closures on the alpha-cluster formation are identified, along with some hints on a possible subshell structure at N = 106 along the Hg and Pb isotopic chains. The study here could be a useful complement to the traditional route to probe alpha-cluster formation in medium-mass and heavy even-even nuclei using alpha-decay data. Especially, it would be helpful in the cases where the target nucleus is stable against alpha decay or alpha-decay data are currently not available.

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### 1. Introduction

Alpha-cluster formation plays a key role in understanding structures and reactions of light, medium-mass, and heavy/superheavy nuclei across the nuclide chart [1–6]. For light nuclei, the existence of alpha-cluster structures was suggested in the late 1930s [7,8]. This idea is then explored in detail by generations of nuclear physicists, and inspires many important theoretical developments, such as resonating group method [9], generator coordinate method [10], antisymmetrized molecular dynamics [12], orthogonality condition method [11], THSR (Tohsaki–Horiuchi–Schuck–Röpke) wave function [13], etc. It is found that there could be alpha-cluster structures in ground states of light nuclei such as <sup>8</sup>Be =  $\alpha + \alpha$ , <sup>20</sup>Ne =  $\alpha + {}^{16}$ O, <sup>44</sup>Ti =  $\alpha + {}^{40}$ Ca [4], as well as in the famous Hoyle and Hoyle-like excited states of self-conjugate nuclei near alphaparticle disintegration thresholds [13–17]. The study of alphacluster formation in heavy/superheavy nuclei could date back to

E-mail addresses: dbai@itp.ac.cn (D. Bai), zren@tongji.edu.cn (Z. Ren).

Rutherford's discovery of alpha decay more than one hundred years ago [18]. Till today, alpha decay is still an important direction being continuously developed [6,19–23], from which we gain lots of information on alpha-cluster formation in heavy/superheavy nuclei [20,24–37]. Especially, it is found that alpha-cluster formation probabilities change significantly at magic numbers N = 126 and Z = 82. Alpha-cluster formation could also exist in the mediummass nuclei. Possible candidates include  $^{46,54}$ Cr =  $\alpha + ^{42,50}$ Ti [38, 39],  $^{90}$ Sr =  $\alpha + ^{86}$ Kr [40],  $^{92}$ Zr =  $\alpha + ^{88}$ Sr [40],  $^{94}$ Mo =  $\alpha + ^{90}$ Zr [40–43],  $^{96}$ Ru =  $\alpha + ^{92}$ Mo [40],  $^{98}$ Pd =  $\alpha + ^{94}$ Ru [40],  $^{136}$ Te =  $\alpha + ^{132}$ Sn [44], etc. Recently, there are also systematic studies on the landscape of alpha-cluster formation probabilities in mediummass nuclei using alpha-decay data [45].

Unlike alpha clustering in light nuclei which could be studied by using microscopic methods, probing alpha clustering in medium-mass and heavy/superheavy nuclei is more challenging thanks to intrinsic difficulties in solving quantum systems with a large number of nucleons. In literature, alpha-cluster formation in medium-mass and heavy/superheavy nuclei is usually investigated systematically by exploiting experimental data on alpha de-

Corresponding author

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cay. However, neither all the nuclei with alpha-cluster structures permit alpha decays, nor all the nuclei permitting alpha decays have their decay widths be measured. Therefore, new probes of alpha-cluster formation besides those based on the alpha decay are necessary. In this note, we would like to study the possibility to adopt the cluster-daughter overlap as a new probe of alpha-cluster formation in medium-mass and heavy nuclei, motivated by the observation that alpha-cluster formation is suppressed when the cluster-daughter overlap is large and favored when the cluster-daughter overlap is small. In Section 2, we analyze the relation be-tween the cluster-daughter overlap and the alpha-cluster formation quantitatively within the framework of the quartetting wave func-tion approach [46-48], which is a microscopic method proposed recently to estimate alpha-cluster formation probabilities. The in-clusion of shell-model properties in the quartetting wave function approach is also discussed recently in Ref. [49]. We introduce a dimensionless parameter  $\mathfrak{O}$  to measure quantitatively the degree of the cluster-daughter overlap. In Section 3, the  $\mathfrak{O}$  parameter is then adopted to explore the landscape of alpha-cluster formation probabilities in medium-mass and heavy even-even nuclei, with D being calculated directly from experimentally measured charge radii. Special attentions are paid to the closed-(sub)shell effects. In Section 4, we give the conclusions. 

## 2. Cluster-daughter overlap versus alpha-cluster formation in the light of quartetting wave function approach

We first study the relation between the cluster-daughter overlap and the alpha-cluster formation probability within the framework of the quartetting wave function approach. The quartetting wave function approach is inspired by the THSR wave function for light self-conjugate nuclei, and has been applied successfully in studying alpha-cluster formation in heavy/superheavy nuclei. Within this framework, we divide the wave function of the four valence nucleons uniquely as the product of the center-of-mass (COM) part and the intrinsic part. Following Refs. [46–49], by adopting the local-density approximation and ignoring the derivative terms of the intrinsic wave function, the Schrödinger equation for the COM wave function could be given by

$$-\frac{\hbar^2}{2M_{\alpha}}\nabla_{\mathbf{r}}^2\Phi(\mathbf{r}) + W(\mathbf{r})\Phi(\mathbf{r}) = E\Phi(\mathbf{r}).$$
(1)

 $W(\mathbf{r})$  is the effective potential felt by the COM motion of the alpha cluster. A detailed derivation of Eq. (1) could be found in Ref. [46]. The alpha-cluster formation probability  $P_{\alpha}$  could be then obtained as follows:

$$P_{\alpha} = \int d^{3}\mathbf{r} |\Phi(\mathbf{r})|^{2} \Theta[\rho_{B}^{\text{Mott}} - \rho_{B}(\mathbf{r})].$$
<sup>(2)</sup>

Here,  $\rho_B^{\text{Mott}} = 0.02917 \text{ fm}^{-3}$  is the Mott density higher than which the alpha cluster is believed to dissolve and merge with the daughter nucleus. The root-mean-square (rms) intercluster separation is given by

$$R_{i} \equiv \langle R^{2} \rangle_{i}^{1/2} = \left[ \int_{0}^{\infty} \mathrm{d}r \, r^{2} \, |\phi(r)|^{2} \right]^{1/2}, \tag{3}$$

where  $\phi(r)$  is the normalized radial wave function of  $\Phi(\mathbf{r})$ . To measure the degree of the cluster-daughter overlap quantitatively, we introduce a dimensionless parameter  $\mathfrak{D}$  to measure the relative size between the rms intercluster separation  $R_i$  and the sum of the rms point radii of the alpha particle  $R_{\alpha} \equiv \langle R^2 \rangle_{\alpha}^{1/2}$  and the daughter nucleus  $R_d \equiv \langle R^2 \rangle_d^{1/2}$  (see Fig. 1 for a schematic representation),



**Fig. 1.** A schematic representation of the cluster-daughter system. In this work, we assume that the centers of mass of the alpha particle and the daughter nucleus coincide with their centers of charge.  $R_{\alpha}$  and  $R_d$  are the rms point radii of the alpha particle and the daughter nucleus.  $R_i$  is the rms intercluster separation. *O* is the center of mass of the parent nucleus.  $\mathbf{R}_1$  ( $\mathbf{R}_2$ ) is the displacement vector between *O* and the center of mass of the daughter nucleus (alpha particle).  $\mathbf{r}_i$  ( $\mathbf{r}_j$ ) is the displacement vector between the center of mass of the daughter nucleus (alpha particle) to a representative nucleon inside the daughter nucleus (alpha particle).

$$\mathfrak{O} = \frac{R_i}{R_\alpha + R_d}.$$
(4)

As a result, a large (small) overlap between the alpha cluster and the daughter nucleus corresponds to a small (large) value of the  $\mathfrak{O}$  parameter.

In the rest part of this section, we would like to adopt the quartetting wave function approach to study the relation between the  $\mathfrak{O}$  parameter and the alpha-cluster formation probability. We take  $^{212}$ Po =  $^{208}$ Pb +  $\alpha$  as an example, which could be viewed as an alpha cluster moving on the top of the doubly magic nucleus  $^{208}$ Pb and has been investigated intensively by previous studies [41,42, 50–53]. For the density distributions for the daughter nucleus, we adopt [47,54]

$$\rho_{\pi}(r) = \frac{0.0628948}{1 + \exp[(r - 6.68 \text{ fm})/0.447 \text{ fm}]} \text{ fm}^{-3}, \tag{5}$$

$$\rho_{\nu}(r) = \frac{0.0937763}{1 + \exp[(r - 6.70 \text{ fm})/0.550 \text{ fm}]} \text{ fm}^{-3}.$$
 (6) <sup>108</sup>  
<sup>109</sup>

as the proton and neutron distributions, respectively. The critical radius that marks the dissolution of the alpha cluster is then given by  $r_{\text{cluster}} = 7.43825$  fm. In other words, the alpha cluster persists only for  $r > r_{cluster}$  and dissolves once  $r < r_{cluster}$ . When  $r > r_{cluster}$ , the effective potential  $W(\mathbf{r}) = W^{\text{ext}}(\mathbf{r}) + W^{\text{intr}}(\mathbf{r})$ , with  $W^{\text{ext}}(\mathbf{r})$  be-ing the external potential inside which the alpha cluster moves and  $W^{\text{intr}}(\mathbf{r})$  being the intrinsic energy of the alpha cluster inside the nuclear medium. The external potential  $W^{\text{ext}}(\mathbf{r})$  could be approxi-mated by  $W^{\text{ext}}(\mathbf{r}) = V_{\text{M3Y}}(\mathbf{r}) + V_{\text{C}}(\mathbf{r})$  following the double-folding procedure. The microscopic M3Y nucleon-nucleon interaction takes the form  $V_{\rm NN}(s) = c \exp(-4s)/(4s) - d \exp(-2.5s)/(2.5s)$ . For the alpha emitter  $^{212}$ Po, the two free parameters *c* and *d* could be determined by fitting the emission energy  $Q_{\alpha}$  and the alpha-decay half-life  $T_{1/2}$ . The intrinsic potential  $W^{intr}(\mathbf{r})$ , on the other hand, is given by  $W^{\text{intr}}(\mathbf{r}) = W^{\text{Pauli}}(\mathbf{r}) + E_{\alpha}^{(0)}$ , with  $W^{\text{Pauli}}(\mathbf{r}) = 4515.9\rho_B(\mathbf{r}) - 100935\rho_B^2(\mathbf{r}) + 1202538\rho_B^3(\mathbf{r})$  [46] and  $E_{\alpha}^{(0)} = -28.3$  MeV being the energy of the alpha particle in the vacuum. When  $r < r_{cluster}$ , the cluster state of the four valence nu-cleons merges with the shell-model state of the daughter nucleus. In the local density approximation, we have  $W(\mathbf{r}) = \mu_4$ , with  $\mu_4$ 

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Fig. 2. A study on <sup>212</sup>Po within the quartetting wave function approach by varying values of  $\mu_4$ . Figure (a) shows the behavior of the effective potential W(r). Figure (b) shows the normalized radial wave function  $\phi(r)$ . Figure (c) shows the relation between the dimensionless parameter  $\mathfrak{O}$  and the alpha-cluster formation probability  $P_{\alpha}$ . The critical radius  $r_{\text{cluster}} = 7.43825$  fm is displayed explicitly by the vertical line in (a) and (b).

being a constant related to the Fermi energy of the four valence nucleons.

In Fig. 2, we study the relation between the cluster-daughter overlap measured by the  $\mathfrak{O}$  parameter and the alpha-cluster for-mation probability  $P_{\alpha}$  by varying the values of  $\mu_4$  from 0.95  $Q_{\alpha}$ to  $1.05 Q_{\alpha}$ . Fig. 2(a) shows the behavior of the effective poten-tial W(r) with the critical radius  $r_{\text{cluster}} = 7.43825$  fm displayed by the vertical line. As  $\mu_4$  becomes larger, the horizontal platform of W(r) within the critical radius  $r_{cluster}$  becomes higher, and the energy pocket beyond the critical radius r<sub>cluster</sub> becomes deeper. Fig. 2(b) shows the normalized radial wave function  $\phi(r)$ . As  $\mu_4$ becomes larger, more and more parts of the radial wave function  $\phi(r)$  are pushed into the outer region  $r > r_{\text{cluster}}$ . According to Eq. (2), this indicates that the alpha-cluster formation probability becomes larger and larger. Fig. 2(c) displays the relation between the dimensionless parameter  $\mathfrak{O}$  and the alpha-cluster formation probability  $P_{\alpha}$ , hinting that there might exist approximately a simple positively-correlated linear relation between  $P_{\alpha}$  and  $\mathfrak{O}$ . Here, the rms point radii of the alpha particle, the daughter nuclei <sup>208</sup>Pb, and the alpha emitter <sup>212</sup>Po are compiled from the experimental values of their rms charge radii recorded in Ref. [55] by using  $R_{\text{point}} = \sqrt{R_{\text{ch}}^2 - R_o^2}$  with  $R_o = 0.88$  fm [56]. This linear relation allows us to probe the landscape of alpha-cluster formation probabilities by studying that of the cluster-daughter overlap.

### 3. The landscape of alpha-cluster formation probabilities probed by cluster-daughter overlap

We investigate the landscape of alpha-cluster formation probabilities in medium-mass and heavy even-even nuclei based on discussions in Section 2. Explicitly, we will study the landscape of the dimensionless parameter  $\mathfrak{O}$  along various isotopic and isotonic chains. Then, according to the linear relation between  $P_{\alpha}$  and D, the same landscape could also reveal important features of the landscape of alpha-cluster formation probabilities. To extract the D parameter from experimental nuclear ground-state charge radii, we make use of the relation [38,40,57-59],

$$R^{2}\rangle_{ch,P} = \frac{Z_{\alpha}}{Z_{\alpha} + Z_{d}} \left\langle R^{2} \right\rangle_{ch,\alpha} + \frac{Z_{d}}{Z_{\alpha} + Z_{d}} \left\langle R^{2} \right\rangle_{ch,d} + \frac{Z_{\alpha} A_{d}^{2} + Z_{d} A_{\alpha}^{2}}{(Z_{\alpha} + Z_{d})(A_{\alpha} + A_{d})^{2}} \left\langle R^{2} \right\rangle_{i}.$$
(7)

Here,  $\langle R^2 \rangle_{ch,P}^{1/2}$ ,  $\langle R^2 \rangle_{ch,\alpha}^{1/2}$ , and  $\langle R^2 \rangle_{ch,d}^{1/2}$  are the rms charge radii of the parent nucleus, the alpha particle, the daughter nucleus, while  $(R^2)^{1/2}_{i}$  is the rms separation between the centers of mass of alpha particle and the daughter nucleus. Eq. (7) assumes that the centers of mass of the alpha particle and the daughter nucleus coincide with their centers of charge, and follows directly from the definition of rms charge radius of the parent nucleus:

$$R^{2}\Big|_{\mathrm{ch},p} = \frac{\sum_{i=1}^{Z_{d}} \left\langle (\mathbf{R}_{1} + \mathbf{r}_{i})^{2} \right\rangle + \sum_{j=1}^{Z_{\alpha}} \left\langle (\mathbf{R}_{2} + \mathbf{r}_{j})^{2} \right\rangle}{Z_{\alpha} + Z_{d}}$$
$$= \frac{Z_{d}}{Z_{\alpha} + Z_{d}} \left\langle R^{2} \right\rangle_{\mathrm{ch},d} + \frac{Z_{\alpha}}{Z_{\alpha} + Z_{d}} \left\langle R^{2} \right\rangle_{\mathrm{ch},\alpha}$$
$$+ \frac{Z_{d} \left\langle R_{1}^{2} \right\rangle + Z_{\alpha} \left\langle R_{2}^{2} \right\rangle}{Z_{\alpha} + Z_{d}}.$$
(8)

An explanation of the variables here could be found in the caption  
of Fig. 1. Taking into account the fact that 
$$\langle R_1^2 \rangle^{1/2} = \frac{A_\alpha}{A_\alpha + A_d} \langle R^2 \rangle_i^{1/2}$$
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and  $\langle R_2^2 \rangle^{1/2} = \frac{A_d}{A_\alpha + A_d} \langle R^2 \rangle_i^{1/2}$ , we obtain Eq. (7). The experimental  
values of  $\langle R^2 \rangle_{ch,P}^{1/2}$ ,  $\langle R^2 \rangle_{ch,\alpha}^{1/2}$ , and  $\langle R^2 \rangle_{ch,d}^{1/2}$  could be found in Ref. [55],  
from which the value of  $R_i \approx \langle R^2 \rangle_i^{1/2}$  is obtained by solving Eq. (7).  
The  $\mathfrak{O}$  parameter could then be calculated according to its defini-  
tion in Eq. (4). In this work, we will study the landscape of the  $\mathfrak{O}$   
parameters of not only stable but also decaying nuclei which are  
unbound. For the latter case, the definition of the rms radius could  
be nontrivial. However, the lifetimes of these decaying nuclei are  
typically very long. For instance, the lifetime of <sup>210</sup>Po is about 140  
days, long enough to allow the electron scattering experiment to  
determine its charge radius practically. As a result, for our current  
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50 We start with the medium-mass nuclei Te. Xe. and Ba. whose 51 D-parameter landscapes are plotted in Fig. 3. It is found that, in 52 these three nuclei, the  $\mathfrak{O}$  parameter (the alpha-cluster formation 53 probability  $P_{\alpha}$ ) decreases first and reaches its minimum at around 54 the N = 82 shell closure. After crossing N = 82, the  $\mathfrak{O}$  parame-55 ter (the alpha-cluster formation probability  $P_{\alpha}$ ) gets suddenly a 56 large increase, manifesting clearly the effect of the N = 82 shell 57 closure on  $P_{\alpha}$ . Such a behavior is consistent with previous studies 58 on heavy nuclei in Refs. [28,48], which could be viewed as a non-59 trivial check of the assumption that the alpha-cluster formation 60 probability  $P_{\alpha}$  is approximately proportional to the dimensionless 61  $\ensuremath{\mathfrak{O}}$  parameter. Compared with Ref. [45], which studies the alpha-62 cluster formation probabilities in Te, Xe, and Ba isotopes near the 63 N = 50 shell based on limited alpha-decay data, the analysis here 64 allows one to get a complementary understanding on how  $P_{\alpha}$ 65 changes around the N = 82 shell, thanks to the availability of rich



**Fig. 4.** The landscape of the  $\mathfrak{O}$  parameter along the N = 64 and N = 66 isotonic chains around the Z = 50 shell. The vertical line denotes the Z = 50 shell closure.

experimental data on ground-state charge radii in this region. We also study the effects of the Z = 50 shell along the N = 64 and N = 66 isotonic chains in Fig. 4. As expected, a large increase of the  $\mathcal{D}$  parameter (the alpha-cluster formation probability  $P_{\alpha}$ ) is found from Z = 50 to Z = 52.

As mentioned before, lots of works have been devoted to study the alpha-cluster formation probabilities of heavy nuclei by exploiting alpha-decay data. Here, we would like to provide a complementary calculation by using experimental data of nuclear rms radii. The O-parameter landscapes of the heavy nuclei Hg, Pb, Po, Rn, and Ra around the N = 126 major shell could be found in Fig. 5. For the Po, Rn, and Ra isotopic chains in Fig. 5(c), (d) and (e), the  $\mathfrak{O}$  parameters decrease first and get a large increase across the N = 126 shell, consistent with previous studies based on alpha-decay data [28,48]. In Fig. 5(a), we plot the  $\mathfrak{O}$  parameters for the Hg isotopes in the region N = 102 - 122. It is interesting to see that the N = 106 subshell effect is made manifest in this plot, suggesting that the alpha-cluster formation probability gets a large increase when crossing the N = 106 subshell along the Hg isotopic chain. For the Pb isotopic chain shown in Fig. 5(b), it displays both the N = 106 subshell and the N = 126 shell structures, with the  $\mathfrak{O}$ parameters (the alpha-cluster formation probabilities  $P_{\alpha}$ ) getting a large increase at both N = 106 and N = 126. We study further the Z = 82 shell effects along the N = 114, 116, 118, and 120 isotonic chains in Fig. 6. Similar to the medium-mass nuclei, a large increase of the  $\mathfrak{O}$  parameter (the alpha-cluster formation probability  $P_{\alpha}$ ) is found from Z = 82 to Z = 84, consistent with Refs. [34, 48].

### 4. Conclusions

In this note, we study the possibilities to use the cluster-116 daughter overlap as a new probe of alpha-cluster formation in 117 medium-mass and heavy even-even nuclei. Quantitatively, the 118 cluster-daughter overlap is measured by a dimensionless  $\mathfrak{O}$  pa-119 rameter which is the ratio between the rms intercluster separation 120 121 and the sum of the rms point radii of the alpha particle and the 122 daughter nucleus. The larger (smaller) the cluster-daughter over-123 lap is, the smaller (larger) the  $\mathfrak{O}$  parameter will be. Within the 124 quartetting wave function approach, we show that there might ex-125 ist approximately a positively-correlated linear relation between 126 the alpha-cluster formation probability  $P_{\alpha}$  and the  $\mathfrak{O}$  parameter, which enables one to probe the landscape of alpha-cluster for-127 128 mation probabilities by studying instead the landscape of the  $\mathfrak{O}$ 129 parameter. By exploiting the experimental values of the groundstate charge radii of various medium-mass and heavy nuclei, we 130 D. Bai, Z. Ren / Physics Letters B ••• (••••) •••-•••



Fig. 5. The landscapes of the  $\mathfrak{O}$  parameter along the Hg, Pb, Po, Rn, and Ra isotopes around the N = 126 shell. The vertical line in (a) denotes the N = 106 subshell. The two vertical lines in (b) denote the N = 106 subshell and the N = 126 shell, respectively. The vertical lines in (c)-(e) denote the N = 126 major shell.



Fig. 6. The landscape of the  $\mathfrak{O}$  parameters along the N = 114, 116, 118, and 120 isotonic chains near the Z = 82 shell. The vertical line denotes the Z = 82 shell closure.

plot the landscape of the  $\mathfrak{O}$  parameters. The general trends of the alpha-cluster formation probabilities derived thereby are consistent with previous studies and natural theoretical expectations. The shell effects at N = 82, 126 and Z = 50, 82 are identified explicitly in terms of the cluster-daughter overlap, as well as interesting subshell effects at N = 106 along the Hg and Pb isotopic chains. Compared with previous studies based on alpha-decay data, the new probe could be used in cases where the target nucleus is stable against the alpha decay or the alpha-decay data are temporarily unavailable. The study here could be helpful for future theoretical and experimental studies on alpha-cluster formation in medium-mass and heavy regions, as well as providing extra moti-vations to carry out new experiments to measure charge radii in the medium-mass and heavy regions precisely. In this work, we assume both the alpha cluster and the daughter nucleus to be spherical. It is also important to consider further the impact of nuclear deformations, which according to previous studies such as

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Refs. [60,61] would enhance the alpha-cluster formation probability. Hopefully, this would be the topic of our future studies.

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