General Relativity and Gravitation

Bulk dominated fermion emission on a Schwarzschild background

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Abstract

Using the WKBJ approximation we obtain semi-analytic expressions of the absorption probability for Dirac fermions on a higher dimensional Schwarzschild background. We then relate the absorption probability to the absorption cross-section, and then use these results to plot the emission rates. Our results lead to the interesting conclusion that for $d > 5$ bulk fermion emission dominates brane localised emission.

Introduction

Large extra-dimensional scenarios [1] have led to the somewhat striking prediction that black holes (BHs) may be observed at particle accelerators such as the LHC [2]. However, in order to suppress a rapid proton decay quarks and leptons need to be physically separated in the higher dimension(s). Such models are generically called split fermion models [3, 4]. Note that in supersymmetric versions of this idea the localizing scalars and bulk gauge fields will also have fermionic bulk superpartners.

In a previous work we applied conformal methods, which allowed us to separate the Dirac equation on a higher dimensional spherically symmetric background, to discuss the quasinormal modes (QNMs) for Schwarzschild BHs [5]. In this work we used this same method to calculate the greybody factors [6] and emission rates for Dirac perturbations on a d-dimensional Schwarzschild background by writing the background as:

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega_{d-2}^2 \ , \tag{1}$$

with $f(r) = 1 - \left(\frac{r_H}{r}\right)^{d-3}$ and where the horizon is at $r = r_H$.

After the conformal transformation [5], the Dirac equation separates into a time-radial part and a $(d-2)$-sphere. Moreover, the radial part reduces to a Schrödinger-like equation in the tortoise coordinate r_*:

$$\left(-\frac{d^2}{dr_*^2} + V_1\right) G = E^2 G \ , \tag{2}$$

where $dr = f(r)dr_*$, and the potential is given by $V_1(r) = \kappa^2 \frac{f}{r^2} + \kappa f \frac{d}{dr}\left[\frac{\sqrt{f}}{r}\right]$, with $\kappa = \ell + \frac{d-2}{2}$. Note that the above potential reduces to the brane-localized results when we set $\kappa = \ell + 1$, and therefore provides an alternate derivation of the brane localized potential.

Absorption probabilities via the WKBJ approximation

In a recent work by two of the authors [7] we applied the intermediate WKBJ approximation (up to first order) to evaluate the absorption probability of a graviton to a static BH. The WKBJ approximation can, however, be applied at all energies (including low energy) as has been discussed in reference [8].

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Low Energy WKBJ

In terms of the WKBJ approximation, in general, it will be convenient to make a change of variables to $x = Er$ [7]. Writing $E^2 Q(x_*) = E^2 - V_1$ the Schrödinger equation, equation (2), takes the form:

$$\left(\frac{d^2}{dx_*^2} + Q\right) G = 0 \ . \tag{3}$$

The low energy absorption probability corresponds to the probability for a particle to tunnel through the potential barrier. The result to first order in the low energy WKBJ approximation is given by:

$$|\mathcal{A}_\kappa(E)|^2 = \exp\left[-2\int_{x_1}^{x_2} \frac{dx'}{f(x')}\sqrt{-Q(x')}\right] \ , \tag{4}$$

where x_1 and x_2 are the turning points, $Q(x_{1,2}) = 0$. This approximation is valid for $V_1 \gtrsim E^2$ and as long as we can solve for the turning points in $V_1(x) = E^2$. Note that we can numerically integrate equation (4) for each energy E to obtain the absorption probability as a function of E.

Intermediate Energy: 3rd Order WKBJ

An adapted form of the WKBJ method can be employed to find the QNMs, or the absorption probability (which we are primarily interested in here), when the scattering takes place near the top of the potential barrier. In the following we shall use the same notation as reference [9], where we have confirmed their results to fourth order. However, for the purposes of this work, we shall consider only up to and including third order, in which case we express the absorption probability as:

$$|\mathcal{A}_\kappa(E)|^2 = \frac{1}{1 + e^{2S(E)}} \ , \tag{5}$$

where $S(E)$ has been defined in reference [5].

It should be noted that as we go to higher orders the approximation becomes valid for lower energies. However, as can be seen from figure 1, even orders in the intermediate WKBJ method drop back down to zero for large energy. For this reason we shall work to third order in our calculations, as odd orders have the nice property that $|A|^2 \to 1$ for large energy. Note also that the WKBJ approximation, in general, is accurate for larger angular momentum channels, whereas the Unruh approach [10] is valid for only the lowest angular momentum channels and $\varepsilon \ll 1$ (namely small BHs). However, it is interesting to note that although the low energy WKBJ result does not agree exactly with the Unruh result, they both tend to zero for $\varepsilon \to 0$. On the other hand, unlike the Unruh result, the low energy WKBJ is valid for energies up to $\varepsilon \sim \mathcal{O}(1)$, where it matches onto the intermediate WKBJ. For a more in depth discussion of this method see references [5].

High Energy

For high energies the absorption probability tends to unity, and the cross-section reduces to that of the classical cross-section, see reference [7]. However, as discussed in reference [8], there will always be small corrections to the large energy limit. A high energy WKBJ approach can be applied in this limit, but for the purposes of this current study it will be sufficient to use $|\mathcal{A}_\kappa(E)|^2 = 1$.

Emission Rates

The emission rate for a massless fermion from a BH is related to the cross-section by a $d^{d-1}k$ dimensional momentum integral times a fermionic thermal temperature distribution:

$$\frac{d\mathcal{E}}{dt} = \sum_{\lambda,E} \sigma_{\lambda,E} \frac{E}{e^{\frac{E}{T_H}} + 1} \frac{d^{d-1}k}{(2\pi)^{d-1}} \ , \tag{6}$$

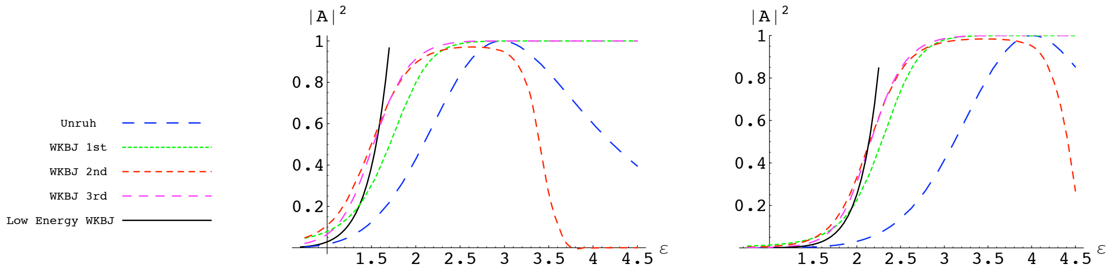

Figure 1: *Plots of the absorption probability, via various schemes, for $d = 7$ and the first two angular momentum channels: $\ell = 0$ (left) and $\ell = 1$ (right). Note the Unruh result is only valid for $\varepsilon \ll 1$.*

where T_H is the Hawking temperature, $\sigma_{\lambda,E}$ are the greybody factors and the sum is a generic sum over all angular momentum and momentum variables. We were able to relate the greybody factor to the absorbtion probability by considering the results of reference [11]:

$$\sigma_{\lambda,E} = \frac{1}{2\Omega_{d-2}} \left(\frac{2\pi}{E}\right)^{d-2} \sum_{\kappa} D_\kappa |\mathcal{A}_\kappa(E)|^2 \ . \tag{7}$$

In the above we have used D_κ as the degeneracy. Given that the angular integration over the momentum for a massless field ($|k| = E$) leads to the Jacobian $\int d^{d-1}k = \int \Omega_{d-2} E^{d-2} dE$, the fermion emission rate can be expressed solely in terms of the absorption probability. However, as the sum over κ is for $\kappa = \pm\left(\frac{d}{2} - 1\right), \pm\frac{d}{2}, \pm\left(\frac{d}{2} + 1\right)$ and since the integrand depends only on the absolute value of κ, we shall sum for $\kappa \geq 0$ and multiply by a factor of two. Therefore, after changing variables to $\varepsilon = E r_H$, and using the fact that the Hawking temperature is $T_H = (d-1)/(4\pi r_H)$, we obtain:

$$\frac{d^2\mathcal{E}}{dEdt} = \frac{1}{\pi r_H} \sum_{\kappa > 0} \frac{\varepsilon}{e^{\frac{4\pi\varepsilon}{d-1}} + 1} D_\kappa |\mathcal{A}_\kappa(\varepsilon)|^2 \ . \tag{8}$$

Results and Conclusions

We have calculated the total power by integrating over ε, see equation (8). The results are shown in Table 1. From these results we find that for $d > 5$ the emission is predominantly into the bulk. Note that in order to obtain convergence in equation (8) we must choose some value of $\kappa_{max} > \varepsilon$, and to ensure this we have taken $\kappa_{max} = 34 + \frac{d}{2}$.

Dimension d	5	6	7	8	9	10
$d\mathcal{E}_{\mathrm{Bulk}}/dt$	0.0579	0.1771	0.3380	1.4731	3.56403	18.2606
$d\mathcal{E}_{\mathrm{brane}}/dt$	0.0708	0.1172	0.204	0.3435	0.554892	0.860165
$\frac{d\mathcal{E}_{\mathrm{Bulk}}/dt}{d\mathcal{E}_{\mathrm{brane}}/dt}$	0.8181	1.5109	1.6587	4.2880	6.42292	21.9019

Table 1: *A comparison of the bulk and brane-localised power spectrum up to $d = 10$, where we have changed units from r_H to M and set $M = 1$.*

Our results for the Hawking emission rate of a massless Dirac field on a bulk d-dimensional Schwarzschild background, using the method we developed in reference [5], conclude that fermions are mainly emitted into the bulk for $d > 5$, as we have shown, see Table 1, which is in contrast to the scalar field case and for bulk to brane photons [12]. This is an example contrary to the conjecture that BHs radiate mainly on the brane [13]. Furthermore, bulk dominated fermion emission is also consistent with the original motivation for split-fermions, namely that of a suppression of a rapid proton decay [3]. Note that these results also agree qualitatively with our work for the QNMs on such a background [5], where the BH damping rate was found to increase with dimension.

We have also highlighted how semi-analytic results can be obtained by considering different versions of the semi-classical WKBJ approximation, where we also compared this to the low energy analytic results derived from the method first developed in reference [10], see figure 1. Note also that in a recent work these methods have been used to investigate the effect of brane tension on bulk fermion emission, with the result that QNMs were damped by the tension of the brane [14].

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