The Higgs sector, SUSY breaking and Inflation in String Theory

Memoria de Tesis Doctoral realizada por

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A todo aquel que alguna vez pensó o escuchó:
...e'que yo quiego ap'ender...

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1
Introducción

La búsqueda de las leyes fundamentales que describen nuestro universo nos ha llevado a preguntarnos por el comportamiento de la naturaleza a escalas infinitamente pequeñas e infinitamente grandes. La Fisica de Partículas y la Cosmología son las ramas de la física que estudian el comportamiento de los procesos físicos a estas dos escalas tan diferentes.

La Física de Partículas busca entender cuáles son los constituyentes elementales que forman el universo y las interacciones que los gobiernan. El concepto de bloque fundamental e indivisible (apodado entonces como átomo) viene ya de la Antigua Grecia, pero no es hasta el siglo XIX de mano de la física experimental que se establece como base para modelizar el comportamiento de la materia a escalas más pequeñas. La tabla periódica es la primera clasificación de estos átomos o elementos químicos remarcando la existencia de un orden y periodicidad en su propiedades químicas. Desde entonces, gracias a poder realizar experimentos a escalas cada vez más pequeñas, hemos pasado de los átomos a los núcleos y electrones, de los núcleos a los protones y neutrones, y de estos últimos a los quarks. La "tabla periódica" del siglo XXI se conoce como el Modelo Estándar (SM) de Física de Particulas, y describe bajo el mismo formalismo matemático tanto las partículas que forman la materia como las responsables de las interacciones fundamentales. De la misma manera el equivalente a los "microscopios" son hoy los aceleradores de partículas, que con sus altas energías permiten separar la materia en sus constituyentes elementales y obtener información sobre la física a escalas cada vez más pequeñas. En concreto, el modelo estándar es una teoría cuántica de campos que describe las partículas elementales conocidas junto con la interacción electromagnética, nuclear fuerte y nuclear débil, es decir, todas las interacciones fundamentales salvo la gravedad. El origen del valor de las masas y acoplos de estas partículas es un problema abierto hoy en día; y la incorporación de la gravedad en una formulación cuántica consistente, la principal tarea pendiente de la Física Teórica de este siglo. Pero la motivación sigue siendo la misma, buscar esos constituyentes elementales que al combinarse de diferentes formas dan lugar a la materia que nos rodea. El principio que ha guiado y sigue guiando esta búsqueda es el concepto de unificación. La naturaleza nos ha enseñado que a medida que vamos a escalas más pequeñas, las leyes que dominan los procesos físicos son más sencillas y requieren de menos ingredientes. Unos pocos tipos de partículas y tan solo cuatro fuerzas elementales son capaces de explicar todos los fenómenos y estructuras observadas hoy en día.

Por otra parte, la Cosmología estudia la evolución y composición del universo, desde su inicio hasta nuestros días. A estas inmensas escalas la interacción que juega el papel dominante es precisamente la gravedad. Observar el universo nos da información sobre su pasado, sobre épocas en las que la energía media era suficientemente alta como para que toda la materia estuviese disociada en sus constituyentes elementales. Cuanto más atrás en el tiempo más alta era la energía media (o temperatura) y más elementales las partículas que regían el comportamiento del universo. Por ello se dice que el propio universo es el acelerador de partículas más grande que tenemos. El Modelo Estándar de Cosmología describe el universo desde unos pocos segundos después del Big Bang. Sin embargo, estos primeros segundos son claves para entender el origen del universo, y aunque poco en tiempo, en realidad equivalen a 16 órdenes de magnitud en energía. Es durante este primer segundo, cuando el universo era tan pequeño y la densidad de energía tan alta, que los efectos cuánticos de la gravedad pasan a ser apreciables. Por ello, para poder entender esos primeros instantes en los que todas las partículas estaban disociadas en aquellos elementos fundamentales que componen el universo, es imprescindible tener una teoría cuántica consistente de la gravedad.

Teoría de Cuerdas es hoy por hoy el mejor candidato para un teoría cuántica de la gravedad que unifica además la gravedad con el resto de las interacciones fundamentales de la naturaleza.

1.1. Fenomenología de Cuerdas

Teoría de Cuerdas (ST) sostiene que toda partícula es en realidad un objeto de dimensión uno (una cuerda sin grosor). La escala a la cual la dimensión extensa de la cuerda se hace apreciable se conoce como escala de la cuerda y es típicamente cercana a la escala de Planck (escala a la cual los efectos de gravedad cuántica son apreciables). Las cuerdas pueden ser abiertas o cerradas, y los diferentes tipos de partículas observadas en nuestro universo corresponden a diferentes modos de vibración de estas cuerdas. Una de las características más importantes es que el primer modo de vibración de una cuerda cerrada equivale a una partícula de spin 2 que juega el papel del gravitón. Las ecuaciones de relatividad general de Einstein aparecen a su vez como condición necesaria para la consistencia interna de la teoría (invariancia conforme en la worldsheet). Por tanto, Teoría de Cuerdas necesariamente predice la existencia de gravedad. Este es uno de los mayores logros que hizo que Teoría de Cuerdas empezase a considerarse como un serio candidato a una teórica cuántica de gravedad. Además ST ha demostrado ser suficientemente rica como para incluir también los elementos necesarios para formar teorías gauge no abelianas y fermiones quirales, pudiendo dar cabida también al SM de partículas. Por ello, ST se considera una “teoría del todo” que permite dar una descripción cuántica unificada y autocontenida de todas las partículas e interacciones existentes.

Por otra parte, cabe destacar que ST no tiene parámetros libres. Todos ellos se fijan dinámicamente dentro de la propia teoría, es decir, corresponden a valores esperados en el vacío de los propios campos escalares que aparecen en última instancia como vibraciones de las cuerdas. El ejemplo más típico es el propio acoplo de la cuerda, que no es más que el valor esperado de un campo escalar conocido como el dilatón. El único parámetro libre realmente es la escala de la cuerda e incluso se espera que este sea un artefacto de la descripción perturbativa de ST, pues hoy por hoy aún no tenemos una descripción completa a nivel no perturbativo de la teoría. La propia dimensión del espacio-tiempo tampoco es un parámetro que se especifique inicialmente sino que viene fijado por la propia autoconsistencia interna de la teoría. En el caso de teorías de supercuerdas (es decir, teorías de cuerdas supersimétricas para poder incluir la presencia de fermiones en el espacio-tiempo) esta dimensión es diez. Esto implica que a parte de las cuatro dimensiones espaciotemporales en las que vivimos, hay seis dimensiones espaciales extra. Estas dimensiones extra deben ser compactas y tener un tamaño suficientemente pequeño como para no ser observables a las escalas de energía en las que vivimos. Lamentablemente, toda la simplicidad y predictividad de la teoría se pierde en el proceso de compactificación de diez a cuatro dimensiones. Las innumerables maneras de compactificar la teoría dan lugar a los múltiples vacíos o soluciones que conforman el *landscape* de teoría de cuerdas. De la misma manera que diferentes soluciones a las ecuaciones de Maxwell describen diferentes configuraciones del campo electromagnético, o que diferentes soluciones a las ecuaciones de Einstein describen diferentes sistemas planetarios o galaxias, diferentes soluciones en Teoría de Cuerdas describen diferentes universos. Cada vacío, pues, corresponde a un posible universo consistente con ST. Las características de la compactificación (como el tamaño y forma de las dimensiones extras) están parametrizados también por los valores esperados de campos escalares (conocidos como moduli de la compactificación) que se fijan dinámicamente dando lugar a un número inmenso (pero finito) de posibles soluciones.

Fenomenología de Cuerdas trata de encontrar cuál es el vacío (o compactificación) que corresponde a nuestro universo. En ciertas teorías de supercuerdas (Tipo II) es posible definir sectores localizados de la teoría que se puede desacoplar del resto de propiedades de la compactificación. Esto permite atacar el problema a trozos, en un alarde del dicho "divide y vencerás". Ciertas propiedades de nuestro universo por tanto dependen de aspectos puramente locales y pueden ser modelizadas independientemente del resto de la compactificación, como es el caso del Modelo Estándar de Partículas. Esto permite atacar y responder preguntas tales como cuál es el origen del valor de las masas de la partículas, el porqué de tener tres familias de partículas o el origen microscópico de las interacciones gauge. Sin duda, en los últimos quince años se han realizado grandes progresos para embeber el Modelo Estándar de Física de Partículas en Teoría de Cuerdas, y mucho se ha conseguido gracias a la construcción de modelos locales. Por contra, otras propiedades de nuestro universo dependen de aspectos globales y no pueden responderse con exactitud sin un conocimiento pleno de la compactificación. El ejemplo típico es el valor de la constante cosmológica que da lugar a la expansión acelerada del universo. También los modelos inflacionarios pueden ser extremedamente sensibles a otras regiones de la compactificación en principio desacopladas. Aunque grandes progresos se han llevado a cabo en la resolución de los diferentes problemas por separado, mucho queda aún por hacer para encontrar una compactificación que lo tenga en cuenta todo a la vez. Por suerte, aún queda mucho por aprender.

A día de hoy Teoría de Cuerdas tiene una motivación puramente teórica de definir una teoría del todo que unifique y describa todos los elementos de nuestro universo. Su propia autoconsistencia interna ha arrojado luz sobre muchos aspectos de gravedad cuántica, a la vez que inspirado la formulación de dualidades entre gravedad e interacciones gauge cuyas aplicaciones a otras áreas de la Física como Materia Condensada están aún por sorprendernos. También ha sido la fuente de grandes avances en el propio campo de las Matemáticas. La comprobación experimental de la teoría en cambio parece estar aún lejos de llevarse a cabo. Si efectivamente la escala de la cuerda y de las dimensiones extras está cerca de la escala de Planck, la detección directa por parte de aceleradores de partículas es inviable. Aunque esta escala tan alta de energía para la escala de la cuerda no es necesario para la autoconsistencia interna de la teoría, lo es si queremos unificar todas las interacciones gauge como resultantes de una única interacción a alta energía. Observaciones cosmológicas podrían arrojar luz sobre estas escalas de energía tan altas y tal vez incluso detectar de manera indirecta remanentes de posibles modelos de Cuerdas, como cuerdas cósmicas o señales de *tunneling* de un vacío a otro de la teoría en etapas tempranas del universo. Cuanto más ambiciosas son las preguntas que queremos responder más lejos tenemos que irnos del rango de energías en el que vivimos y de lo que nos es confortable. Pero esto no hace que dejemos de preguntarnos sobre el inicio del universo y las leyes fundamentales que rigen la naturaleza. El tener una teoría consistente que sea capaz de dar respuesta a todas estas preguntales fundamentales sobre el mundo que nos rodea es un proyecto ambicioso a la vez que extraordinario. Y Teoría de Cuerdas es un paso hacia adelante en el camino de conseguirlo.

1.2. Resultados experimentales en altas energías

Estamos viviendo unos años de nuevos descubrimientos tanto en Física de Partículas como en Cosmología. El 4 de Julio de 2012 se anunció [1,2] el descubrimiento del tan buscado bosón de Higgs por parte del acelerador de partículas del CERN situado en Ginebra y apodado LHC (*Large Hadron Collider*). El bosón de Higgs, predicho hace nada menos que 50 años, era la pieza del rompecabezas que faltaba para completar el Modelo Estándar de Física de Partículas, que como hemos dicho describe las interacciones fundamentales y partículas conocidas de nuestro universo. La masa del Higgs era una incógnita dentro del SM y de hecho, plantea un problema teórico. En general, lo más natural es que las partículas escalares (como el Higgs) tengan una masa del orden del límite de validez (*cutoff*) de la teoría efectiva. Por tanto la masa es una indicación de que a partir de ahí debe aparecer nueva física (nuevas partículas) y la teoría debe ser completada teniendo en cuenta esos nuevos grados de libertad. El que una teoría sea efectiva no significa que sea errónea, sino que simplemente es incompleta. Todas las teorías físicas que tenemos y en las que nos basamos hoy en día son efectivas en el sentido de que son válidas solo en un rango de energías y deben ser completadas a escalas más altas. Física no relativista por relatividad especial de Lorentz, mecánica clásica por mecánica cuántica, etc. Este paradigma de teorías efectivas constituye nuestro entendimiento moderno del universo, por el cual a medida que vamos a longitudes menores (o escalas de energías mayores) accedemos a nuevos fenómenos físicos que necesitan de una nueva teoría. Además esta capacidad de poder definir una teoría efectiva desacoplada de la física a energías más altas es lo que nos permite estudiar por ejemplo la física de materiales sin tener que preocuparnos por la existencia de los quarks dentro de los núcleos. Volviendo a la masa del Higgs, el concepto de naturalidad se basa en asumir que las constantes físicas y parámetros libres de la teoría deben ser de orden uno. La masa de un escalar es natural solo si es del orden de la escala donde aparece nueva física. Si en cambio no aparece nueva física hasta muchos órdenes de magnitud después, significa que esta masa está *finetuned*, es decir, que hace falta un ajuste de los parámetros de la teoría con una precisión extrema para dar lugar a ese valor no natural de la masa. Este concepto de naturalidad en el Higgs ha inspirado la mayor parte de toda la investigación en física más allá del Modelo Estándar hoy en día, toda ella basada en el supuesto de que nueva física debe aparecer a escalas de energías no muy lejanas a la masa del Higgs y por tanto accesibles en el LHC. Sin embargo, el LHC aún no ha detectado señal alguna de nueva física más allá del SM. Aún es pronto para concluir nada, pero si en el próximo *run* del LHC a 14 TeV no hay signo aún de nueva física quizá estamos asistiendo a la primera evidencia clara de fine-tuning en la naturaleza (tal vez junto con el valor de la constante cosmológica), dando lugar a un cambio de paradigma en el entendimiento del universo.

Por otra parte, este último año ha sido también un año de gran revuelo en la comunidad cosmológica. En Marzo de 2014, el experimento BICEP2 [3] anunció la observación de ondas gravitacionales primordiales, constituyendo una fuerte evidencia indirecta de la teoría de Inflación. Como veremos durante la tesis, Inflación postula la existencia de un periodo de expansión acelerada al inicio del universo (tan solo 10^{-34} segundos después del Big Bang), motivada de nuevo por un concepto de naturalidad en las condiciones iniciales del universo. De ser confirmado el experimento, constituiría la primera evidencia de física más allá del SM a una escala de energía altísima, tan solo dos órdenes de magnitud por debajo de la escala de Planck y a nada menos que trece por encima del rango accesible por el LHC. Por desgracia, recientes estudios muestran que toda la señal de BICEP2 podría ser debida a ruido (polvo) proveniente del fondo galáctico. Los próximos años prometen sin duda arrojar luz sobre este asunto gracias a experimentos como Planck y Keck. Mientras tanto, el estudio de inflación a estas escalas de energía tan altas es interesante por sí mismo. Su extrema sensibilidad a física en torno a la masa de Planck, ha motivado una estrecha relación con Teoría de Cuerdas y puede convertirse en una gran fuerte de información en la búsqueda de ese vacío dentro de Teoría de Cuerdas que corresponde a nuestro universo.

Gran parte de esta tesis ha sido motivada por los hallazgos experimentales recién comentados: la masa del Higgs, la falta de evidencia de nueva física a baja energía $\sim 10^3$ GeV, y la posible detección de nueva física relacionada con inflación a alta energía implicando la existencia de una partícula escalar (el inflatón) a $\sim 10^{13}$ GeV. Estudiaremos una clase de compactificaciones concreta de Teoría de Cuerdas en la que el SM vive en un sistema de D-branas y supersimetría (SUSY) es rota espontáneamente por flujos de cuerda cerrada. Veremos que ello motiva una escala de supersimetría alta, entre $10^{10} - 10^{13}$ GeV, consistente con la masa del Higgs pero implicando efectivamente la ausencia de nueva física a baja energía. Identificaremos además esta escala con la escala de inflación, uniendo ambos mecanismos y llevando a una identificación del Higgs con el inflatón. De esta manera conectaremos Física de Partículas y Cosmología, todo ello siempre dentro del marco teórico consistente de Teoría de Cuerdas.

1.3. Esquema de la tesis

La tesis está estructurada de la siguiente forma. En el capítulo 3 introduciremos los ingredientes principales del tipo de compactificaciones que estudiaremos de Teoría de Cuerdas. En concreto, compactificaciones de Tipo IIB (y sus extensiones a F-theory) en variedades Calabi-Yau (CY) con orientifolds y flujos, en las que el SM se encuentra en un sector localizado de D7-branas. Estudiaremos en detalle la acción efectiva de una D7-brana pues será el punto de partida tanto para estudiar el proceso de ruptura de SUSY en el MSSM como la generación del potencial inflacionario. El capítulo 4 está dedicado a la conexión de Teoría de Cuerdas con Física de Partículas. Estudiaremos las implicaciones de romper supersimetría con flujos de cuerda cerrada sobre las partículas del MSSM. En concreto, en 4.1 estudiaremos la estructura de los términos de ruptura de SUSY (*soft terms*) inducidos y en 4.2 la presencia de no universalidades en los soft terms originada por densidades de flujos no constantes. En 4.3 nos centraremos en el tamaño de estos soft terms, motivando una escala alta de supersimetría que como veremos es consistente con la masa del Higgs medida experimentalmente. También estudiaremos la relación de esta escala con unificación gauge dentro de F-theory, y propondremos soluciones a posibles problemas fenomenológicos como el decaimiento del protón o un candidato para materia oscura. En el capítulo 5 pasaremos a estudiar la conexión entre Teoría de Cuerdas y Cosmología, en concreto, la construcción de modelos de Inflación dentro de Teoría de Cuerdas. En 5.1 introduciremos los conceptos básicos de Inflación y el origen de su extrema sensibilidad a la física de alta energía. Nos centraremos en el estudio de modelos de *large field inflation* que requieren de un correcto tratamiento dentro de Teoría de Cuerdas. En 5.2 propondremos un nuevo modelo inflacionario apodado Higgsotic inflation, en el que el inflaton es un escalar responsable a la vez de ruptura espontánea de una simetría gauge no abeliana y de inflación, siendo el obvio candidato para ello el bosón de Higgs. Embeberemos el modelo en un sistema de D7-branas y calcularemos el potencial efectivo y los observables cosmológicos resultantes. Finalmente el capítulo 6 contiene las conclusiones de la tesis, seguido de dos apéndices.

2
Introduction

The search for the fundamental laws that describe our universe has led us to wonder about the behaviour of nature at infinitely small and infinitely large scales. Particle Physics and Cosmology are the branches of physics which study the physical phenomena at these two vastly differing scales.

Particle Physics attempts to understand the nature of the elementary constituents which are the basis of the matter and interactions of our universe. The concept of a fundamental and indivisible block (dubbed *atom*) comes from Ancient Greece, but it is not until the 19th century that, motivated by experimental physics, it is really used to explain the behaviour of matter at very small distances. The periodic table is the first classification of the different chemical elements (fundamental blocks of that time), highlighting the existence of an order and periodicity in their chemical properties. Since then, our understanding of the fundamental constituents has evolved thanks to experiments at ever smaller distances, going from atoms to nuclei and electrons, from nuclei to protons and neutrons, and from these to quarks. The current 21^{st} century "periodic table" is known as the Standard Model (SM) of Particle Physics, which describes within the same mathematical framework the elementary particles leading both to matter and interactions. Similarly the new "microscopes" are the particle colliders, which are able to dissociate matter into its elemental constituents at very high energies, allowing us to obtain information about physics on ever decreasing scales. More specifically, the SM is a quantum field theory which provides a unified description of matter particles and electromagnetic, strong and weak interactions, i.e. all fundamental interactions except for gravity. The origin of the value of the masses and couplings of these particles is today an open problem; and the formulation of a consistent quantum description of gravity, the main task ahead of Theoretical Physics this century. But the motivation is still the same, to look for those elemental constituents which, combined in different ways, give rise to the world around us. The principle that has guided and continues to guide this search is the concept of unification. Nature has taught us that as we go to smaller scales, the laws describing the physical processes are simpler and require fewer ingredients. Just a few kinds of particles and only four elementary forces are enough to explain all the phenomena and structures observed today.

Cosmology, on the other hand, studies the evolution and composition of the universe, from its beginnings up to the present day. At these huge scales the interaction playing the dominant role is precisely gravity. The observation of the universe gives us information about its past, about periods when the average energy was sufficiently high that all matter was dissociated into its elementary constituents. The further back in time we go, the higher the average energy (or temperature) was, and the more elemental the particles governing the behavior of the universe were. For this reason, the universe itself is said to be the largest particle collider. The Standard Model of Cosmology describes the universe from just a few seconds after the Big Bang. However, these first few seconds are crucial to the understanding of the origin of the universe. And while it is very small in terms of time, it actually corresponds to 16 orders of magnitude in terms of energy. It is indeed during this first second, when the universe is so small and the average energy so high, that the quantum effects of gravity are expected to be significant. That is why, having a consistent quantum theory of gravity is essential to understand those first moments of the universe, when all particles were dissociated into their fundamental units.

String Theory is currently the best candidate for a consistent quantum theory of gravity, which in addition unifies gravity with the other fundamental interactions of nature.

2.1. String Phenomenology

String Theory (ST) states that every particle is actually a one-dimensional object (a zero-thickness string). The scale at which the extended dimension of the string becomes appreciable is known as string scale and it is typically close to the Planck scale (that at which the quantum effects of gravity become important). Strings can be open or closed, and the different types of particles observed in our universe arise from different excitation modes of these strings. One of the most important features is that the first excitation mode of a closed string leads to a particle of spin 2 which plays the role of the graviton. The Einstein equations of general relativity arise, in turn, as necessary conditions for the internal consistency of the theory (conformal invariance in the worldsheet). Therefore string theory necessarily predicts the existence of gravity. This is one of the biggest achievements that caused string theory to begin to be considered as a serious candidate for a quantum theory of gravity. In addition, ST has demonstrated to be rich enough to accommodate non-abelian gauge symmetries and chiral fermions, allowing for the embedding of the SM. Therefore, ST is considered as a “theory of everything” which provides a unified and self-contained quantum description of all particles and interactions of nature.

Moreover, it should be pointed out that ST has no free parameters. All the free parameters within ST are fixed dynamically by the theory itself, that is, they correspond to vacuum expectation values of scalar fields which arise from the excitation string modes. The most typical example is the string coupling itself, which is simply the vacuum expectation value of a scalar known as the dilaton. The only free parameter is the string scale, and even this is expected to be an artifact of the perturbative description of ST. For the time being we do not have yet a complete nonperturbative description of string theory. Interestingly, the space-time dimension is not a parameter imposed by hand but it is also fixed by the internal self-consistency of the theory. In the case of superstring theories (ie, supersymmetric string theories to account for the presence of fermions) the space-time has to be ten dimensional. This implies that there are six extra spatial dimensions in addition to the usual four dimensional space-time in which we live. These extra dimensions must be compact and sufficiently small to be undetectable at the scales currently accessible by experiments. Unfortunately, all the simplicity and predictability of the theory is lost in the process of compactification from ten to four dimensions. The countless ways to compactify the theory lead to the multiple vacua or solutions that form the *string landscape*. In the same way that different solutions of Maxwell equations describe different configurations of the electromagnetic field, or that different solutions of Einstein equations describe different planetary systems or galaxies, then different solutions of string theory describe different universes. Each vacuum thus corresponds to a possible universe consistent with ST. The compactification properties (such as size and shape of the extra dimensions) are also parameterised by the vacuum expectation values of scalar fields (known as moduli of the compactification) which are fixed dynamically leading to a huge (but finite) number of possible solutions.

String Phenomenology attempts to find what is the vacuum (or compactification) corresponding to our universe. In certain superstring theories (Type II) one can define localised sectors which can be decoupled from the rest of properties of the compactification. This allows us to attack by breaking down the problem into pieces, as the saying goes, "divide and rule". Certain properties of our universe, such as many properties of the Standard Model of Particles, depend on purely local aspects and can be decoupled from the rest of the compactification. This allows us to address questions such as the origin of the value of the masses of the particles, why there are three families of particles or what is the microscopic origin of the gauge interactions. Certainly in the last fifteen years there have been big improvements in embedding the SM in string theory using local models. In contrast, other properties of our universe depend on global aspects and can not be addressed without a full understanding of the compactification. The typical example is the value of the cosmological constant responsible for the accelerated expansion of the universe. Inflationary models can also be extremely sensitive to other regions of the compactification a priori decoupled. While big achievements have been made in the resolution of the different problems separately, much work remains to be done to find a complete compactification which takes all of them into account at once. Fortunately, there is still a lot to learn.

Currently, string theory is mostly motivated by the theoretical challenge of finding a theory of everything which provides a unified description of all the elements in our universe. The internal self-consistency of the theory has shed light on many aspects of quantum gravity, inspiring the formulation of dualities between gravity and gauge interactions whose applications to other areas of physics as condensed matter are very promising. It has also been a source of major advances in the field of pure mathematics. The experimental verification of the theory though seems to be still far from being achieved. Cosmological observations might shed light on the physics at very high energy scales and perhaps even indirectly detect possible remnants of string models, like cosmic strings or signs of tunneling between different vacua in the early stages of the universe. However, if indeed the string and compactification scales are close to the Planck scale, the direct detection by the colliders will be unfeasible. While not required by the internal consistency of the theory, such a high value for the string scale is necessary to achieve successful unification of all gauge interactions. The more ambitious are the questions we want to answer, the further we have to go beyond the range of energies in which we live and beyond what it is confortable to us. But that does not make us stop wondering about the beginning of the universe and the fundamental laws describing the world around us. Having a consistent theory capable of providing an answer to all fundamental questions about our universe is an ambitious and extraordinary project. And string theory is a step forward on the way to achieve it.

2.2. New experimental results and future expectations

We are living an exciting time of new discoveries in both particle physics and cosmology. On July 4th 2012 it was announced [1, 2] the discovery of the long-sought Higgs boson by the LHC (Large Hadron Collider) at CERN and located in Geneva. The Higgs boson, predicted 50 years ago, was the missing piece of the puzzle to complete the standard model of particle physics which, as commented above, describes the fundamental interactions and particles known in our universe. The Higgs mass was a free parameter in the SM and in fact it poses a theoretical problem. In general, it is *natural* that the scalar particles (such as the Higgs) have a mass of the order of the cutoff of the effective theory, indicating that new physics (new particles) should appear at that scale and the theory has to be completed taking into account those new degrees of freedom. The fact that a theory is effective does not make it wrong, but simply incomplete. Indeed all the physical theories we have and on which we have built our knowledge of the universe are effective in the sense that they are valid only in a range of energies and must be completed at scales beyond that. For instance, non-relativitic mechanics must be completed by special relativity, classical by quantum mechanics, etc. The notion of effective theories is the basis of our modern understanding of nature. By going to shorter lengths (or higher energies) we are able to observe new physical phenomena that will require the formulation of a new theory. Furthermore, the fact that one can define an effective theory decoupled from high energy physics allows us to study, for instance, material science without concerning about the existence of quarks within the nuclei. Returning to the Higgs mass, the concept of naturalness is based on the assumption that the physical constants and free parameters of the theory should be of order one. The mass of a scalar is natural only if it is of the order of the scale of new physics. But if new physics does not appear until many orders of magnitude later, it implies that the mass is *finetuned*, ie, one needs a fine adjustment of the parameters with extreme precision to give rise to such an unnatural value of the mass. This concept of naturalness in the Higgs (leading to the hierarchy problem) has inspired most of the research beyond the standard model (BSM), everything based on the assumption that new physics must appear at scales not very far from the Higgs mass and therefore attainable in the LHC. However, there is no sign of new BSM physics yet at the LHC. While it is too early to make conclusions, if there is still no sign of new physics in the next run of the LHC at 14 TeV, we might be witnessing the first clear evidence of fine-tuning in nature (perhaps together with the value of the cosmological constant), leading to a change in our current understanding of nature.

Furthermore, this last year has also been very important for the cosmological community. In March 2014, the BICEP2 experiment [3] announced the observation of primordial gravitational waves, providing strong indirect evidence for the theory of inflation. As we will see during this thesis, inflation postulates the existence of a period of accelerated expansion at the beginning of the universe (just 10^{-34} seconds after the Big Bang). Inflation was also formulated to allow for naturalness in the initial conditions of the universe. If this experiment is confirmed, it would be the first evidence for physics beyond the SM at very high energies, at only two orders of magnitude below the Planck scale and thirteen above the range attainable by the LHC. Unfortunately, recent studies show that the BICEP2 signal could be due to dust from the galactic background. The next few years promise to be decisive in clarifying this issue thanks to experiments like Planck and Keck. Meanwhile, the study of inflation at these high energy scales is interesting for its own sake. Its extreme sensitivity to Planck scale physics has promoted a close relationship with string theory. Inflation might become an essential source of information in finding the specific vacuum of string theory corresponding to our universe.

Most of this thesis has been motivated by the recent experimental findings detailed above: the Higgs mass, the absence of new physics at low energy $\sim 10^3$ GeV, and the possible detection of new physics related to inflation implying the existence of a scalar particle (the inflaton) at $\sim 10^{13}$ GeV. We will study a particular class of string theory compactifications in which the SM lives in a system of D-branes and supersymmetry (SUSY) is spontaneously broken by closed string fluxes. We will see that this motivates a high scale for supersymmetry, $10^{10} - 10^{13}$ GeV, consistent with the Higgs mass and indeed implying the absence of new physics at low energy. We also identify this scale with the scale of inflation, unifying both mechanisms and leading to the identification of the Higgs with the inflaton. In this way we will connect Particle Physics and Cosmology, all within the theoretical framework of string theory.

2.3. Plan of the thesis

The outline of this thesis is as follows. In chapter 3 we introduce the main ingredients of the string compactifications we will be dealing with during the thesis. In particular, type IIB compactifications (and their F-theory extensions) on Calabi-Yau (CY) manifolds with orientifolds and fluxes, in which the SM lives in a localised system of D7-branes. We will describe in detail the effective action of a D7-brane as it will be the starting point for studying both the SUSY breaking in the MSSM and the generation of the inflationary potential. Chapter 4 is dedicated to the connection between string theory and particle physics. We study the implications of flux-induced supersymmetry breaking on the MSSM fields. In particular, in 4.1 we study the structure of soft SUSY breaking terms and in 4.2 the presence of non-universalities in the soft terms due to non-constant flux densities. In 4.3 we will focus on the size of the soft terms, motivating a high scale of supersymmetry breaking which we will show to be consistent with the experimental Higgs mass. We will also study the relation between this scale and gauge coupling unification within F-theory, and propose solutions to possible phenomenological problems such as proton decay or a candidate for dark matter. In chapter 5 we turn to studying the connection between string theory and cosmology, in particular the construction of inflationary models within string theory. In 5.1 we review the basic concepts of inflation and discuss its extreme sensitivity to ultraviolet (UV) physics. We will focus on the study of large field inflation models that require a proper treatment within string theory. In 5.2 we propose a new model dubbed Higgsotic inflation, in which the inflaton is a scalar responsable for both spontaneous non-abelian gauge symmetry breaking and large field inflation, with the Higgs boson being the obvious candidate. We will embed the model in a system of D7-branes and compute the effective potential and the resulting cosmological observables. Finally chapter 6 contains the conclusions of the thesis, followed by two appendices.

3

String Theory ingredients

There are five different superstring theories according to the concrete way by which the anomalies are cancelled in ten dimensions: Type IIA, Type IIB, Type I, $E_8 \times E_8$ Heterotic and $SO(32)$ Heterotic. However all these string theories are related by a rich web of dualities indicating that all of them might be perturbative limits of a more fundamental theory, dubbed M-theory, whose microscopic formulation is still a mistery. In addition, these dualities provide relations between different phenomenologically interesting vacua of string theory. In this thesis we will work mostly in the context of Type IIB and its non-perturbative F-theory extension. As we will see, such compactifications allow many appealing features from the point of view of string model building. For an overview of string phenomenology and model building see e.g. [4] and references therein.

In this chapter we introduce the basic ingredients of these compactifications which will appear continously throughout the thesis. In particular, in 3.1 we review the basic tools of Type IIB orientifold compactifications to four dimensions with special interest in the closed string 3-form fluxes. The D-brane open string sector and its effective action is considered in 3.2 while in 3.3 we describe a particular string embedding of the SM in a local SU(5) GUT model of F-theory.

3.1. Fluxes on Type IIB orientifolds

Let us consider 4d compactifications of Type IIB theories on Calabi-Yau (CY) manifolds. They lead to $N = 2$ theories in four dimensions, so they are not directly suitable for particle physics model building since that amount of supersymmetry does not allow for chirality. One can then further reduce the number of supercharges by introducing an orientifold action which truncates the supersymmetry to 4d $N = 1$ (see [4,5] for reviews). To cancel the RR tadpoles induced by these orientifold planes one needs to include as well another kind of higher dimensional objects called D-branes. These objects may support non-abelian guage interactions and chiral matter, so they will be crucial in the realisation of the SM of particle physics in string theory. In this section we will focus on the closed string sector of the compactification leaving the open string sector tied to the D-branes for the next section.

The bosonic content of Type IIB in 10d is given by the NSNS sector, ie. the graviton G_{MN}, the antisymmetric tensor B_{MN} and the dilaton ϕ, and the RR sector containing the antisymmetric p-forms C_0, C_2 and C_4. The 10d supergravity effective action is given by

$$S_{IIB} = \frac{1}{2k_{10}^2}\int d^{10}x\sqrt{-g}\left(R - \frac{\partial_M\bar{\tau}\partial^M\tau}{2(Im\tau)^2} - \frac{1}{2}|F_1|^2 - \frac{|G_3|^2}{2Im\tau} - \frac{1}{4}|\tilde{F}_5|^2\right) + \frac{1}{2k_{10}^2}\int C_4\wedge\frac{G_3\wedge\bar{G}_3}{4iIm\tau} + S_{local} \quad (3.1)$$

where

$$\tau = C_0 + ie^{-\phi}\ , \quad G_3 = F_3 - \tau H_3\ , \quad F_3 = dC_2\ , \quad H_3 = dB_2 \quad (3.2)$$

and

$$\tilde{F}_5 = F_5 - \frac{1}{2}C_2\wedge H_3 + \frac{1}{2}B_2\wedge F_3\ , \quad *_{10}\tilde{F}_5 = \tilde{F}_5 \quad (3.3)$$

The term S_{local} includes the contribution from localised sources of the compactification, like D-branes or O-planes. The complete 4d massless spectrum of Type IIB CY compactifications on X_6 can be obtained by dimensional Kaluza-Klein (KK) reduction of the 10d bosonic fields and further completion to 4d $N=2$ supermultiplets to account for the fermionic partners. They lead to the gravity multiplet, $h_{1,1}$ vector multiplets and $h_{2,1}+1$ neutral hypermultiplets of $N=2$. The integers $h_{p,q}$ are topological invariants known as Hodge numbers which give the dimension of the corresponding cohomology group $H^{p,q}(X_6)$.

As we mentioned above, Type IIB orientifolds are obtained by considering IIB on a CY and modding out by an orientifold action given by $\Omega\mathcal{R}$. Ω is the action on the worldsheet while $\mathcal{R}$ is a geometric symmetry acting holomorphically on the complex coordinates of X_6. Here we will focus on models with O7/D7 planes, implying an orientifold action of the form $\Omega R_i(-1)^{F_L}$ where R_i locally acts on a single complex coordinate as $z_i\to -z_i$, while keeping the other two coordinates invariant. F_L is the left-moving fermion number, required for the orientifold action to square to the identity. This introduces O7-planes at the fixed points of R_i wrapping a 4-cycle parametrized by the other two invariant complex coordinates, and whose RR charges must be cancelled by D7-branes. These models are of particular interest because they correspond to the weak coupling limit of the F-theory constructions briefly discussed in section 3.3.

The spectrum is obtained by truncating the massless spectrum of type IIB to states invariant under the orientifold action. This projects out a subset of the original spectrum leaving the $N=1$ gravity multiplet, $h_{1,1}+1$ chiral multiplets coming from the $N=2$ hypermultiplets, $h_{2,1}^+$ vector multiplets and $h_{2,1}^-$ chiral multiplets (both two coming from the original $N=2$ vector multiplets). $h_{2,1}^\pm$ refer to number of (2,1)-forms which are even or odd with respect to the geometrical action. In the following we present the microscopic description of these multiplets in terms of the 10d fields.

Let us introduce for this purpose a basis of harmonic p-forms which split in turn into even and odd harmonic forms according to their behaviour under the orientifold action. The elements of the cohomology basis satisfy the following relations,

$$\int_{X_6}\omega_a\wedge\tilde{\omega}^b = \delta_a^b\ , \quad \int_{X_6}\omega_\alpha\wedge\tilde{\omega}^\beta = \delta_\alpha^\beta\ , \quad \int_{X_6}\alpha_K\wedge\beta^L = \delta_K^L \quad (3.4)$$

where ω_a (ω_α) denotes an odd (even) (1,1)-form and $\tilde{\omega}_a$ ($\tilde{\omega}_\alpha$) denotes its dual odd (even) (2,2)-form. Analogously, α_k and its dual β^K form a basis of 3-forms. The massless 4d fields correspond to harmonic forms on the internal manifold X_6, so that we have to expand the

10d fields into the different non-trivial cohomology basis. The fields B_2 and C_2 are odd while C_4 is even under the orientifold action, leading to the expansion

$$B_2 = b^a(x)\omega_a \ , \quad C_2 = c^a(x)\omega_a \quad , \quad a = 1, \dots, h^-_{1,1} \tag{3.5}$$

$$C_4 = A^k_1 \wedge \alpha_k + C_\alpha(x)\tilde{\omega}^\alpha \quad , \quad k = 1, \dots, h^+_{2,1} \ , \ \alpha = 1, \dots, h^+_{1,1} \tag{3.6}$$

where b^a, c^a and C_α are 4d scalars and A^k_1 are gauge bosons. The geometric action also acts as $J \to J$ and $\Omega_3 \to -\Omega_3$, being J the Kahler 2-form and Ω_3 the holomorphic 3-form of the CY manifold X_6. Therefore we can expand the Kahler form as

$$J = v^\alpha \omega_\alpha \quad , \quad \alpha = 1, \dots, h^+_{1,1} \tag{3.7}$$

obtaining $h^+_{1,1}$ real kahler moduli. In addition, we have $h^-_{2,1}$ complex structure moduli surviving the orientifold projection. The moduli space of CY orientifolds is then given by $h^+_{1,1}$ chiral multiplets T_α containing (v^α, C_α), $h^-_{1,1}$ chiral multiplets G_a containing (b^a, c^a), $h^-_{2,1}$ chiral multiplets U_i containing the complex structure moduli and one additional chiral multiplet S combining the even universal axion C_0 and the dilaton ϕ. We will come back to this classification when discussing the candidates for inflation in Type IIB compactifications in section 5.1.

The above scalars are known as moduli of the compactification. The vevs of these scalars parametrize among other things the properties of the compactification (like size or shape of the internal dimensions) and in the absence of further ingredients they are massless [1]. This presents a problem because they interact with the matter fields and they would give rise to fifth forces which have not been observed experimentally. Therefore any realistic attempt to embed our universe in string theory must sooner or later deal with the problem of moduli stabilization, ie. must generate a scalar potential which fixes the vevs of these scalars and provide them a large enough mass. Scalar potentials for these fluxes can be induced by compactifying the theory on a non-trivial background [7–11]. Concretely, flux compactifications with non-trivial fluxes for the field strength tensor of the diverse 10d fields are essential in the construction of realistic string models of particle physics and cosmology. Let us go back then to the effective action (3.1) and consider a IIB compactification on a 6d manifold X_6 with CY topology but in the presence of non-trivial NSNS and RR 3-form fluxes H_3, F_3. Due to the backreaction of the fluxes with gravity, the actual 10d spacetime is not the product of 4d Minkowski spacetime times a CY, but must account for a non-trivial warp factor [7]. The ansatz for the metric so that 4d Poincaré invariance is maintained, reads

$$ds^2_{10} = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}\tilde{g}_{mn}dy^m dy^n \tag{3.8}$$

where $\tilde{g}_{mn}$ is the underlying CY metric, x^μ the coordinates in Minkowski and y^m the coordinates on the compact manifold. The ansatz for the five-form field strength consistent with its Bianchi identity

$$d\tilde{F}_5 = H_3 \wedge F_3 + 2k^2_{10}\mu_3\rho^{loc}_3 \tag{3.9}$$

and Poincare invariance, is given by

$$\tilde{F}_5 = (1 + *_{10})(d\alpha \wedge dx^1 \wedge dx^2 \wedge dx^3) \tag{3.10}$$

[1] Some of the moduli could be estabilized by the presence of D-branes, but not all of them so additional ingredients are still required, see e.g. [6]

with α a function on the compact space and ρ_3^{loc} the D3 charge density from localized sources. Inserting (3.10) in (3.9) we get a laplace equation for the potential α, which combined with the Einstein equation of motion for the Ricci tensor (also in the form of a laplace equation for the warp factor) gives rise to the following condition

$$\tilde{\nabla}^2(e^{4A}-\alpha) = \frac{e^{2A}}{6Im\tau}|iG_3 - *_6G_3|^2 + e^{-6A}|\partial(e^{4A}-\alpha)|^2 + 2k_{10}^2 e^{2A}(\frac{1}{4}(T_m^m - T_\mu^\mu)^{loc} - T_3\rho_3^{loc}) \tag{3.11}$$

The last term above refers to the contribution from the localised sources, being T_{MN} the stress tensor and T_p the tension of the object. The LHS integrates to zero on a compact manifold while the RHS is positive definite (as long as we are considering O3/O7-planes and D-(anti)branes). Therefore the condition holds if all terms in the RHS vanish, implying that the 3-form fluxes must be imaginary self-dual (ISD)

$$*_{6d}\, G_3 = iG_3 \tag{3.12}$$

the warp factor and the four-form potential are related such that

$$\alpha = e^{4A} \tag{3.13}$$

and the localised sources satisfy $\frac{1}{4}(T_m^m - T_\mu^\mu)^{loc} = T_3\rho_3^{loc}$ which allows the presence of objects with D3 but not anti-D3 charge. It is interesting that the supergravity solution becomes analogous to that of BPS D3-branes. Indeed, the integrated Bianchi identity (3.9) states that the total D3 charge from supergravity backgrounds and localized sources vanishes. The three-form fluxes must satisfy the Bianchi identities

$$dF_3 = 0\ , \quad dH_3 = 0 \tag{3.14}$$

and obey a Dirac quantization condition

$$\frac{1}{(2\pi)^2\alpha'}\int_\gamma F_3 = m_\gamma \in \mathbb{Z}\ , \quad \frac{1}{(2\pi)^2\alpha'}\int_\gamma H_3 = n_\gamma \in \mathbb{Z} \tag{3.15}$$

where γ denote a 3-cycle on X_6 and m_γ, n_γ are the flux quanta of the corresponding cycle. This implies that the 3-form fluxes scale as $G_3 \sim f\alpha'/R^3$, where f is related to the flux quanta and R measures the size of the overall CY.

In the large volume limit the fluxes are diluted and the 4d effective theory will be given by adding a superpotential to the efecctive action of the flux-less CY compactification [12]. This can be described in terms of the $N=1$ Kahler potential

$$K_{IIB} = -log(-i\int \Omega_3 \wedge \bar{\Omega}_3) - log(S+S^*) - 2log(e^{-3\phi/2}\int J\wedge J\wedge J) \tag{3.16}$$

at leading order in α'. The flux induced $N=1$ superpotential, also known as Gukov-Vafa-Witten superpotential [13], takes the form

$$W = \int_{X_6} G_3 \wedge \Omega_3 = \int_{X_6}(F_3 - iSH_3)\wedge\Omega_3 \tag{3.17}$$

and leads to a scalar potential for the complex axio-dilaton and the complex structure moduli. The scalar potential is given by

$$V = e^K(g^{a\bar{b}}D_aWD_{\bar{b}}\bar{W} - 3|W|^2) \tag{3.18}$$

where we sum over all moduli and $D_aW = \partial_a W + K_a W$. In the large volume regime the last term on the Kahler potential (regarding the Kahler moduli) becomes

$$K(T) = -3\log(T + T^*) \tag{3.19}$$

where we have only considered the overall Kahler modulus T. Combining the fact that the superpotential (neglecting non-perturbative effects) does not depend on T, the effective theory is said to have a no-scale structure. There is a cancellation between the T contribution and the $-3|W|^2$ so that the scalar potential becomes

$$V = e^K g^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} \tag{3.20}$$

where the sum is now only over the dilaton and complex structure moduli. This scalar potential leads to Minkowski vacua for configurations with $D_iW = 0$. If in addition $D_TW = 0$ these vacua are supersymmetric. We can translate the supersymmetric conditions into conditions on the components of the 3-form fluxes. The auxiliary fields for the dilaton and complex structure moduli are given by $\bar{F}^i = -e^{K/2}K^{\bar{i}j}D_jW$ with

$$D_SW = -\frac{1}{S+S^*}\int_{X_6} \bar{G}_3 \wedge \Omega \ , \quad D_{U_i}W = \int_{X_6} \bar{G}_3 \wedge \chi_i \tag{3.21}$$

where χ_i is a complete basis of (2,1)-forms. These auxiliary fields vanish if

$$G_{(3,0)} = 0 \ , \quad G_{(2,1)} = 0 \tag{3.22}$$

This is equivalent in a CY manifold to impose that G_3 must be ISD[2]. The vacua will be supersymmetric if the auxiliary field for the Kahler modulus

$$\bar{F}^T = (S+S^*)^{-1/2}(T+T^*)^{-1/2}\int_{X_6} G_3 \wedge \Omega \tag{3.23}$$

also vanishes, implying

$$G_{(0,3)} = 0 \tag{3.24}$$

Therefore supersymmetric Minkowski vacua are obtained only for (2,1) fluxes. Otherwise, supersymmetry will be spontaneously broken upon turning on some other component of the flux. The typical scenario to break supersymmetry comes from simply adding ISD (0,3) fluxes, so that the supergravity equations of motion are still satisfied and the theory preserves $N = 1$ supersymmetry which is in turn spontaneously broken in the vacua of the theory. This situation corresponds to modulus dominated SUSY breaking, which corresponds to have a non-vanishing auxiliary field for the Kahler modulus.

Notice that the 3-form fluxes are able to stabilize the dilaton and the complex structure moduli, but not the kahler moduli. The KKLT [8] and large volume scenarios [14–17] of moduli stabilization consider in addition non-perturbative effects to the superpotential and α' corrections to the Kahler potential (this latter in the LVS scenario) to stabilize the Kahler moduli. These models are able to stabilize all the moduli in an AdS vacuum, so it is required an extra contribution to uplift the vacuum energy to a dS vacuum. This uplifting is source of debate and controversy nowadays.

[2] Recall that there are not non-primitive components in a CY compactification, so (3.22) implies the vanishing of the IASD fluxes.

Let us remark the relation just found between the 3-form fluxes and the 4d $N = 1$ supergravity variables. Closed string 3-form fluxes correspond to non-vanishing vevs for the auxiliary fields of the chiral fields T, S, U. We have seen that these fluxes are generic in a realistic Type IIB compactification, since they are necessary to stabilize at least part of the moduli of the compactification. Therefore they typically provide an important source of SUSY breaking. In this section we have studied the effect of these fluxes over the closed string moduli sector. However, they also affect the D-brane open string sector and may generate SUSY breaking soft terms on the SM fields, as we will discuss in section 4.1.

3.2. Dp-brane effective action

The second superstring revolution was triggered by the renewed interest on D-branes, which allowed for a better understanding of string theory at strong coupling, beyond the perturbative regime in which the theory had been defined by then. It turned out that String Theory is not a theory of strings after all. It contains extended objects, like D-branes, which may become arbitrarily light at strong coupling dominating the low energy physics at that regime. This gave rise to the introduction of the weak/strong coupling dualities relating different superstring theories [18] and suggesting the existence of a unique underlying theory which was dubbed M-theory. The different string theories correspond to perturbative corners of this more fundamental theory, whose nature and microscopic description is still unknown.

Dp-branes can be understood as extended (p+1)-dim surfaces at which open strings endpoints are linked to. These objects are indeed dynamical and can be described at weak coupling in terms of the massless spectrum of the open string sector, leading to an effective supersymmetric Yang Mills (SYM) theory in the worldvolume of the brane. These D-branes have to be interpreted as non-perturbative states of the same Type II theory, in analogy with the solitons in quantum field theory. This implies that it should be possible to construct these states as collective excitations of the spacetime fields. This interaction with the closed strings leads to a non-trivial background which can also be described as a solution to the supergravity equations of motion. These two equivalent descriptions lead Maldacena [19] to introduce the AdS/CFT correspondence or gauge/gravity duality.

The interest on D-branes is also justified by their enormous possibilities in model building [4]. D-branes became an essential ingredient in the construction of Type II orientifold compactifications and opened a new avenue to embed the SM of Particle Physics in ST. They provide a beautiful geometrical picture of the 4d properties: non-abelian gauge groups arise from coincident D-branes and matter fields live at the brane intersections.

The massless spectrum of the open string sector ending on a Dp-brane is depicted in table 3.1. The massless states fill an U(1) vector supermultiplet with respect to the 16 supercharges in (p+1)-dimensions propagating in the (p+1)-dimensional worldvolume of the Dp-brane. In the case of a set of N coincident Dp-branes, the gauge group is enhanced to U(N) and all the fields (gauge bosons, scalars and fermions) are promoted to be on the adjoint representation of the gauge group. In this thesis we will construct our models around a system of D7-branes.

As mentioned above, the dynamics of the brane is described by the open string

Sector	**SO(p-1)**	**(p+1)-dim field**
NS	vector	gauge boson A_μ
NS	scalar	9-p real scalars Φ^i
R	spinor	fermions λ_α

Table 3.1: Massless spectrum of a Dp-brane.

modes. For instance, the transverse scalars play the role of the goldstone bosons coming from the translational symmetries broken by the presence of the brane. Therefore a vev to these scalars parametrizes the D-brane position in the transverse space, and a non-trivial profile describes the fluctuations of the brane worldvolume on spacetime. The effective action for the massless open string modes is thus the worldvolume action describing the dynamics of the brane. The bosonic action contains two pieces, known as the Dirac-Born-Infeld (DBI) action and the Chern-Simons (CS) action.

The DBI action is given by

$$S_{DBI} = -\mu_p \int_{W_{p+1}} d^{p+1}\xi \, e^{-\phi} \sqrt{-\det\left(P[E_{ab}] + 2\pi\alpha' F_{ab}\right)} \tag{3.25}$$

where the coefficent $\mu_p = (2\pi)^{-p}\alpha'^{-(p+1)/2}$ is related to the Dp-brane tension (and charge). The embedding of the Dp-brane into the 10d space-time induces a world-volume metric which is described by the pullback of the space-time tensor $E_{ab} = G_{ab} - B_{ab}$ to the brane worldvolume. The field-strength of the worldvolume gauge field is denoted by $F_{\mu\nu}$. Notice that this action describes the interaction of the worldvolume fields with the NSNS closed string background determined by the dilaton ϕ, the metric $G_{\mu\nu}$ and the antisymmetric tensor $B_{\mu\nu}$.

The derivation of this action is quite technical and involves the computation of the vacuum cylinder diagram describing the interaction between two identical branes by interchanging closed strings. The contribution from the NSNS and RR fields cancel each other, so the amplitude vanishes indicating that a D-brane is a BPS state. However the study of the two different pieces give us information about the DBI and CS action independently. It was obtained that the effective action describing the interaction with the NSNS fields coincides with the Born-Infeld action which was introduced 40 years before in a different context, as the action of an electromagnetic field which is invariant under general transformations of coordinates. Even if the full derivation of the action is beyond the scope of this section, we can give an intuitive reasoning to explain the functional dependence of the action on the different 10d fields. Notice that the piece of the action depending on the metric is no more than the Nambu-Goto action of an extended p-dimensional object with tension μ_p/g_s

$$S_{NG} = \frac{\mu_p}{g_s} \int d^{p+1}\xi (-g)^{1/2} \tag{3.26}$$

This action is the generalization to higher dimensions of the usual relativistic action of a particle described by the total path length swept out in spacetime. The worldline in the case of a particle is generalized to the wordsheet for strings or the worldvolume for branes. The explicit appearance of the field strength $F_{\mu\nu}$ can be guessed by T-duality from the simple situation of a D1-brane expanding a spatial dimension tilted at an angle θ with respect to ξ^1. The Nambu-Goto action of the D1-brane is simply $\int d\xi^1 \sqrt{1+(\partial_1\xi^2)^2}$. By T-dualizing along ξ^2 we get a D2-brane extended along ξ^1 and ξ^2 but with a non-trivial magnetic flux F_{12}. Since this flux is related to the angle by $tan\theta = 2\pi\alpha' F_{12}$ the action for the D2-brane becomes $\int d\xi^1 d\xi^2 \sqrt{1+(2\pi\alpha' F_{12})^2}$, matching with the result anticipated in (3.25). In fact, if we expand the DBI action in terms of $F_{\mu\nu}$ (or in α') the first non-trivial term on $F_{\mu\nu}$ corresponds to the usual Yang-Mills action for the worldvolume field strength,

$$S_{YM} = \frac{1}{g_{YM}^2} \int d^{p+1} (-g)^{1/2} tr F_{\mu\nu} F^{\mu\nu} \tag{3.27}$$

with $g_{YM}^2 = g_s \alpha'^{(p-3)/2} (2\pi)^{p-2}$. Finally the appearance of B through the combination $B - 2\pi\alpha' F$ is required to respect the gauge invariance of the B-field. Besides the appearance of B in equal footing with the metric is consistent with the fact that both fields arise microscopically from the same string states. Finally the dilaton background parametrizes the string coupling as $g_s = e^{\phi}$, so the dependence of the brane tension on the string coupling $T_p \sim 1/g_s$ reflects the non-perturbative nature of these states.

The second piece of the action, known as CS action, describes the coupling of the open string modes with the RR closed string background and is given by

$$S_{CS} = \mu_p \int_{W_{p+1}} P[\sum_q C_q] \wedge e^{2\pi\alpha' F_2 - B_2} \tag{3.28}$$

where we have neglected the effects of spacetime curvature. As any BPS state, a D-brane must have conserved charges which correspond to antysimmetric RR charges in the case of D-branes. The topological nature of the above actions refers to the fact that it describes indeed the RR charges of the brane. If we expand the action on α'

$$S_{CS} = \mu_p \left(\int_{W_{p+1}} C_{p+1} + 2\pi\alpha' \int_{W_{p+1}} C_{p-1} \wedge tr F + \dots \right) \tag{3.29}$$

the interpretation of each term is more clear. The first term describes the coupling of the Dp-brane with the (p+1)-form potential C_{p+1} as expected. Besides, the presence of worldvolume gauge field backgrounds induce lower dimensional RR charges on the brane, giving rise to the additional terms above.

The generalization of the effective action to a stack of N branes requires to account for the non-abelian character of the worldvolume fields. We have to promote the partial derivatives appearing in the pullback of tensor space time fields to covariant derivatives of the full non-abelian theory, and keep trace of non-trivial commutators such as $[A, \phi]$ or $[\phi, \phi]$. Notice that both gauge fields and scalars are now $N \times N$ matrices in the adjoint representation of the gauge group. One also have to introduce a symmetrized trace over gauge indices 'STr' to ignore all commutators between the non-abelian expressions such as the field strength, the covariant derivatives or the commutators above themselves. This prescription for the trace is controversial and it can lead to incomplete results in general configurations for sixth or higher orders in the commutators. However, it describes properly the physics for supersymmetric configurations. The leading behaviour of the non-abelian action has been confirmed with the computation of string scattering amplitudes, and the full form

$$\begin{aligned} S = -\mu_p \mathrm{STr} \int_{W_{p+1}} d^{p+1}\xi \, e^{-\phi} \sqrt{-\det\left(P[E_{ab} + E_{ai}(Q^{-1} - \delta)^{ij} E_{jb}] + 2\pi\alpha' F_{ab}\right) det(Q^i_j)} \\ + \mu_p \mathrm{STr} \int_{W_{p+1}} P[\sum_q C_q] \wedge e^{2\pi\alpha' F_2 - B_2} \end{aligned} \tag{3.30}$$

has been shown to be consistent with T-duality [20], where $Q^i_j = \delta^i_j + i2\pi\alpha'[\Phi^i, \Phi^k]E_{kj}$. Recall that a, b denote the dimensions extended by the brane and i, j the transverse directions.

We have seen that a stack of N branes lead to a SYM U(N) gauge theory with bosons, scalars and fermions in the adjoint representation. To construct realistic embeddings of the SM we need to consider several stacks of D-branes in non-trivial open string backgrounds (namely, intersections of branes and magnetic worldvolume fluxes) to allow for the additional presence of chiral matter fields in the bifundamental representation of the gauge group. In the next section we describe a particular local model of D7-branes in Type IIB which can also be understood as arising at the weak coupling limit of an F-theory configuration. We will use this local model in the forthcoming chapters.

3.3. F-theory local model building

F-theory [21–23] may be considered as a non-perturbative extension of Type IIB orientifold compactifications with 7-branes. This class of compactifications have two main phenomenological virtues compared to other string constructions (see [24–29] for reviews). First, in Type IIB compactifications it is well understood how moduli could be fixed in the presence of closed string fluxes and non-perturbative effects. Secondly, particularly within F-theory, GUT symmetries like $SU(5)$ appear allowing for a correct structure of fermion masses (in particular a sizeable top quark mass). Here we just review a few concepts which are required for the understanding of the forthcoming sections (see [30–33] for model building in F-theory). Our general discussion mostly applies both to perturbative Type IIB and their F-theory extensions but we will refer to them as F-theory constructions for generality.

In Type IIB orientifold/F-theory unified models the $G_{GUT} = SU(5)$ symmetry arises from the worldvolume fields of five 7-branes with their extra 4 dimensions wrapping a 4-cycle S_{GUT} inside a six dimensional compact manifold B_3, see fig.3.1. The matter fields transforming in 10-plets and 5-plets have their wave functions in extra dimensions localized on complex curves, the so called matter curves. These matter curves, which have two real dimensions, may be understood as intersections of the $SU(5)$ 7-branes with extra $U(1)$ 7-branes wrapping other 4-cycles in B_3. In other words, new degrees of freedom appear at these intersections and the gauge symmetry is enhanced to a larger group like SU(6) or SO(10). The breaking of these groups and the decomposition of the adjoint representation is performed as follows,

$$\begin{array}{ll} SU(6) \to SU(5) \times U(1) & SO(10) \to SU(5) \times U(1)' \\ 35 \to 24_0 + 1_0 + 5_{-1} + \bar{5}_1 & 45 \to 24_0 + 1_0 + 10_4 + \bar{10}_{-4} \end{array} \tag{3.31}$$

which gives matter in the 5, $\bar{5}$ and 10 of SU(5), as required to accommodate all the SM particles. Fig.3.1 shows a pictorial scheme of a SU(5) F-theory GUT model. The gauge bosons live in the bulk of S_{GUT} while the matter (quarks, leptons and Higgses) are localized in the matter curves. In order to get chirality we have to add a non-trivial background worldvolume flux $\langle F \rangle$ along the extra U(1) 7-branes, so that the gauge symmetry SU(5) remains unbroken but the matter curves become charged under the magnetic flux. If that is the case we will have a 4d chiral spectrum arising from the corresponding matter curves. The magnetic flux also sets the number of local families in such curves. Finally

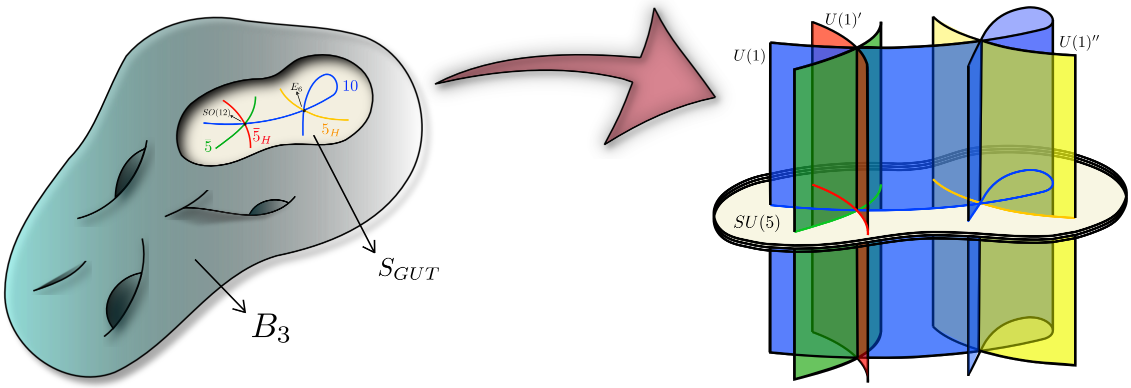


Figure 3.1: Scheme of an F-theory $SU(5)$ GUT. The six extra dimensions are compactified on B_3 whereas the $SU(5)$ degrees of freedom are localized on a 4-cycle submanifold S_{GUT}. The gauge bosons live on the bulk of S_{GUT} but the chiral multiplets are localized on complex matter curves. At the intersection of two matter curves with a Higgs curve a Yukawa coupling develops.

in order to break the SU(5) GUT to the SM gauge group we have to add a second piece of worldvolume flux along the hypercharge generator, as we will see later. Hypercharge fluxes have an additional use in typical F-theory GUT's. Indeed, by appropriately choosing these open string fluxes one can get doublet-triplet splitting in the $SU(5)$ Higgs 5-plet, see refs. [24–33] for details. However, as we will remark later, this is only required if SUSY is broken at low energies.

Furthermore, two matter curves may intersect at a point $p \in S$ where the gauge group is enhanced to G_p giving rise to Yukawa couplings between the chiral multiplets of the GUT matter fields. G_p contains the gauge group G_{σ_i} of each matter curve involved (and thereby also G_{GUT}). Taking $G_{GUT} = SU(5)$ one can end up in the context of F-theory compactifications with different enhanced groups at p such that $SO(12), E_6, E_7$ or E_8.As we have commented, the existence of these enhanced groups is a very attractive feature of F-theory, not realizable in perturbative Type IIB compactifications where the gauge groups are limited to be $SU(n+1)$, $SO(2n)$ or $Sp(n)$. Indeed, the top Yukawa coupling can be described only in the presence of exceptional groups which are not realizable in Type IIB.

Yukawa couplings appear then at triple intersection points in S_{GUT} at which two matter curves involving 10-plets and 5-plets cross with a matter curve containing the Higgs 5-plets, see fig.3.1. The Yukawa couplings may be computed as in standard Kaluza-Klein compactifications from triple overlap integrals of the form

$$\mathcal{Y}^{ij}_{D,L} = \int_S \Psi^i_{10}\Psi^j_{\bar{5}}\Phi_{H_D} \qquad \mathcal{Y}^{ij}_{U} = \int_S \Psi^i_{10}\Psi^j_{10}\Phi_{H_U} \,. \tag{3.32}$$

where i, j are family indices. The wave functions have a Gaussian profile so that one only needs local information about these wave functions around the intersection points in order to compute the Yukawa couplings. This local information may be extracted from the local equations of motion which give quite explicit expressions for the wave functions (see [34–43] for details). We will use these local wave functions to compute the SUSY breaking soft terms for chiral bifundamental fields in section 4.1.

Let us remark the importance of going to an F-theory construction to provide a realistic embedding of the SM. As commented, the top Yukawa coupling is forbidden at perturbative level in Type IIB due to the selection rules of the overall U(1) of the GUT stack of branes, and can only be generated by non-perturbative effects. Instead F-theory provides a strong coupling description of Type IIB allowing for the presence of exceptional groups and a sizeable top Yukawa coupling. In F-theory the axio-dilaton of Type IIB is promoted to be the complex structure parameter of an elliptic fibration over the ten dimensional space-time. This allows to deal with the backreation of the 7-branes in a well globally defined way and translates fields and properties of Type IIB into geometric aspects of the elliptically fibered CY four-fold. For instance, 7-branes appear as singularities in the elliptic fiber. The divisor S_{GUT} wrapped by the GUT branes corresponds to a codimension one singularity such that it yields a SU(5) gauge group. In the same way, matter curves and Yukawa points correspond to codimension two and three singularities respectively. On the other hand, both 3-form closed string fluxes and worldvolume fluxes on the branes have the same origin. They all come from G_4-fluxes in F-theory. Since we will mostly work at the local level we will not need to use the algebraic geometry machinery associated to F-theory. We will keep the language inherited from perturbative Type IIB, but keeping in mind that most of our discussion regarding the embedding of the SM actually refers to the local model of F-theory discussed here.

Another important issue is the relation between the different energy scales arising in Type IIB/F-theory compactifications. The string scale $M_s = \alpha'^{-1/2}$ is related to the Planck scale M_p by[3]

$$M_p^2 \;=\; \frac{8M_s^8 V_6}{(2\pi)^6 g_s^2} \tag{3.33}$$

where V_6 is the volume of the six dimensional internal manifold B_3 and g_s in the string coupling constant. The above relation can be obtained by KK dimensionally reducing the IIB effective action (3.1) to four dimensions. Note that one can lower M_s by having a large volume V_6 (or decreasing g_s), so that the string scale is in principle a free parameter.

The volume V_4 of the 4-fold S_{GUT} which is wrapped by the 7-branes is usually independent of the overall volume of B_3. This volume V_4 is indeed related to the inverse GUT coupling constant α_G, which can be obtained by using (3.27) and performing a KK compactification to 4d. In particular one has at tree level

$$\frac{1}{\alpha_G} \;=\; 4\pi Ref_{SU(5)} \;=\; \frac{1}{8\pi^4 g_s}\left(\frac{V_4}{\alpha'^2}\right) \tag{3.34}$$

with $f_{SU(5)}$ the gauge kinetic function. Parametrizing $V_4 = (2\pi R_c)^4$ one then obtains

$$M_c \;=\; M_s\left(\frac{2\alpha_G}{g_s}\right)^{1/4} \tag{3.35}$$

where we have defined the compactification scale as $M_c = 1/R_c$. This is slightly below the string scale (i.e. for $g_s = 1/2$ and $\alpha_G = 1/24$ one has $M_c \simeq 0.6M_s$). This scale M_c can be identified with the GUT scale at which $SU(5)$ is broken down to the SM. Indeed

[3]The perturbative string scale M_s is not well defined in purely F-theoretical terms. Instead one can use the mass scale of the supergravity action M_* which also involves the g_s dependence, so that $M_p^2 \sim M_*^8 V_6$. The conclusions are unchanged.

in F-theory GUTs (with the 4-cycle being a del Pezzo surface) there are no adjoint Higgs multiplets nor discrete Wilson lines available and it is a hypercharge flux background $< F_Y > \neq 0$ which does the job [30–33]. These fluxes go through holomorphic curves Σ inside S_{GUT} and they are quantized, $\int_\Sigma F_Y =$ integer. Thus on dimensional grounds one has $< F_Y > \simeq 1/R_c^2 = M_c^2$ and indeed one can identify the compactification scale M_c with the GUT scale.

There is an important feature to remark from these models. The strengths of gravitational and gauge interactions are independent since they arise from different sectors of the theory, the closed string sector living in the bulk and the localised D-brane open string sector respectively. This independence is fully related to the possible independence of the overall volume V_6 and the smaller volume V_4 wrapped by the 7-branes in CY manifolds. This allows for the construction of local models of particle physics, in which gravity can be decoupled from the SM physics. This is an attractive feature which is not realizable in heterotic compactifications, where gravity and gauge interactions have the same origin. Concretely, this decoupling limit exists in F-theory compactifications if the 4-cycle wrapped by the 7-branes is a del Pezzo surface (see e.g. [44]). Then gravity can be succesfully decoupled sending $M_p \to \infty$ while keeping the gauge interactions on, or in geometrical terms, the size of the del Pezzo surface can be shrunk to zero while keeping the overall volume finite. In that case the 4d effective theory can be expanded in a power series of

$$\varrho^3 \equiv \frac{\mathrm{Vol}(S)^{3/4}}{\mathrm{Vol}(B_3)^{1/2}} = \frac{\sqrt{2}}{\alpha_{\mathrm{GUT}}} \frac{M_{\mathrm{GUT}}}{M_{\mathrm{Pl}}} \ll 1 \tag{3.36}$$

Many phenomenological properties depend only on the local configuration which motivates the *bottom-up approach* in String Theory. In this approach one looks for local configurations of Dp-branes (or local models in F-theory) resembling as much as possible the SM, without worrying about the global aspects of the theory. As a second step one then tries to embed the local model in a consistent global compactification to study the constraints that the global theory may impose over the effective local models. We will follow this approach in chapter 4 to study the flux-induced SUSY breaking soft terms for the fields living in a system of 7-branes. However, the same phylosophy that works so well in particle physics, fails to study most of the cosmological open issues in string theory. Many cosmological observables depend on global properties of the compactification and the interaction with gravity is essential. However, we will see in chapter 5 that the D-brane effective action can still play a relevant role in the construction of inflationary models in which a careful controll over all quantum corrections is mandatory.

4

From String Theory to Particle Physics

In the previous chapter we have seen how closed string 3-form fluxes may lead to spontaneous supersymmetry breaking of the initial 4d $N = 1$ supersymmetry on Type IIB orientifold compactifications, and how this induces a scalar potential fixing the dilaton and complex structure moduli. In this chapter we study the implication of flux-induced SUSY breaking in the D-brane open string sector of the theory, and how the 3-form fluxes may induce as well SUSY breaking soft terms for the fields of the Standard Model via gravity mediation. The structure of soft terms for adjoint and bifundamental fields arising from a system of D7-branes is studied in section 4.1, while in section 4.2 we will point out that non-constant fluxes may give rise to sustantial flavor-non universalities in the soft terms. In section 4.3 we will focus on the scale of these fluxes motivating an Intermediate SUSY breaking scale $M_{SS} \simeq 10^{10} - 10^{12}$ GeV consistent with the Higgs mass and gauge coupling unification.

4.1. Flux-induced SUSY breaking soft terms

A crucial ingredient to make contact with low-energy physics is the structure of the SUSY breaking soft terms. In trying to study those, two complementary paths have been followed:

- *Bottom-up local approach.* In this case one studies the physics of a local set of D7-branes (or D3-branes), without a full knowledge of the complete compact space. SUSY-breaking is felt by the D-branes as induced by the closed string backgrounds in the vicinity of the branes. These backgrounds include RR and NS 3-form fluxes as well as a 5-form flux, dilaton and metric backgrounds. They parametrize our ignorance of the full compactification details. The SUSY-breaking soft terms may be obtained by expanding the DBI+CS 7-brane action around its location including closed string background insertions.

- $\mathcal{N} = 1$ *supergravity effective action.* Here one starts from the effective supergavity action in terms of the Kähler potential of the moduli fields and the Kähler metric of the matter fields. The superpotential includes a Gukov-Vafa-Witten moduli-dependent piece fixing the complex dilaton and the complex structure moduli as well as non-perturbative superpotentials included to fix the Kähler moduli.

Both approaches have advantages and shortcomings. The first gives us a microscopic description of the origin of the soft terms but no information on the global structure of the compactification, including how closed string moduli are fixed. On the other hand the effective supergravity approach requires a detailed knowledge of the Kähler potential of the moduli as well as the matter metrics and the allowed non-perturbative effects. Having a full control of these latter aspects in specific compactifications is a challenge. In this chapter we will follow the first bottom-up strategy to study SUSY-breaking soft terms induced by closed and open string backgrounds on localised sets of bulk and/or intersecting 7-branes.

We are working in a local limit where the volume of the 4-cycle submanifold S_{GUT} (wrapped by the 7-branes) is much smaller than the overall volume of the base B_3, so that the gravitational effects can be consistently decoupled from the gauge dynamics. Notice that whereas closed string fields are non-dynamical in that limit, the 4d soft terms induced by their backgrounds survive. From the supergravity point of view they are defined in the limit $M_p \to \infty$ keeping the gravitino mass $m_{3/2}$ finite (ie, $Vol(B_3) \to \infty$ but $Vol(S)$ finite). Thus in order to obtain the soft terms we need only local information of the closed string background around the position of the branes, justifying the bottom-up approach.

In Ref. [45] (see also [46]) soft terms induced by closed string 3-form G_3 fluxes on bulk D7-branes were obtained by starting with the DBI+CS action and inserting closed string backgrounds. The matter fields transform in the adjoint representation, so that the results are not of direct phenomenological interest. In this thesis we generalize those computations to the phenomenologically more relevant case of chiral bi-fundamental fields laying at 7-brane intersections. We apply the results to the study of soft SUSY breaking terms on the local SU(5) F-theory GUT model described in 3.3, combining for first time local wavefunctions for chiral matter fields and closed string fluxes giving rise to SUSY breaking.

4.1.1. Bulk matter fields

In this section we review and extend the local computation of flux-induced SUSY-breaking soft-terms that was performed in Ref. [45] for 7-brane scalars transforming in the adjoint represention of the gauge group. However, we consider slightly more general configurations than in [45], allowing for the simultaneous presence of imaginary self-dual (ISD) and imaginary anti self-dual (IASD) 3-form fluxes as well as for magnetization on the 7-branes. Even though only ISD fluxes provide for solutions to the 10D classical equations of motion, complete compactifications addressing moduli fixing typically include additional non-perturbative ingredients that generically induce IASD fluxes and other closed string backgrounds. That is why it is interesting to keep trace also of those.

More precisely, we consider closed string backgrounds of the general form

$$\begin{aligned} ds^2 &= Z(x^m)^{-1/2}\eta_{\mu\nu}d\hat{x}^\mu d\hat{x}^\nu + Z(x^m)^{1/2}ds^2_{\text{CY}} \\ \tau &= \tau(x^m) \\ G_3 &= \frac{1}{3!}G_{lmn}(x^m)dx^l \wedge dx^m \wedge dx^n \\ \chi_4 &= \chi(x^m)d\hat{x}^0 \wedge d\hat{x}^1 \wedge d\hat{x}^2 \wedge d\hat{x}^3 \\ F_5 &= d\chi_4 + *_{10}d\chi_4 \end{aligned} \tag{4.1}$$

with $\tau = C_0 + ie^{-\phi}$ the complex axio-dilaton, $G_3 = F_3 - \tau H_3$ (with F_3 and H_3 the RR and NSNS flux respectively) and ds^2_{CY} the Ricci-flat metric of the underlying Calabi-Yau.

Hatted coordinates are along the non-compact directions.

At any point in the internal space the background can be decomposed according to the SU(3)-structure preserved by the compactification. In general the relation between local and global parameters of the background is however highly non-trivial, except for simple cases like toroidal compactifications where the local SU(3)-structure can be straightforwardly extended into a global one.

From the viewpoint of the local SU(3)-structure the antisymmetric flux G_3 transforms as a $\mathbf{20} = \overline{\mathbf{10}} + \mathbf{10}$, with the $\overline{\mathbf{10}}$ and $\mathbf{10}$ representations corresponding respectively to the ISD G_3^+ and IASD G_3^- components of the 3-form flux, defined as

$$G_3^\pm = \frac{1}{2}(G_3 \mp i *_6 G_3) \ , \qquad *_6 G_3^\pm = \pm i G_3^\pm \tag{4.2}$$

These components can be further decomposed into irreducible representations of SU(3). Thus, IASD fluxes in the $\mathbf{10}$ are decomposed according to $\mathbf{10} = \mathbf{6} + \mathbf{3} + \mathbf{1}$, where the $\mathbf{6}$ and $\mathbf{3}$ representations read [47]

$$\begin{aligned} S_{ij} &= \frac{1}{2}(\epsilon_{ikl} G_{j\bar{k}\bar{l}} + \epsilon_{jkl} G_{i\bar{k}\bar{l}}) \\ A_{ij} &= \frac{1}{2}(\epsilon_{\bar{i}\bar{k}\bar{l}} G_{kl\bar{j}} - \epsilon_{\bar{j}\bar{k}\bar{l}} G_{kl\bar{i}}) \end{aligned} \tag{4.3}$$

respectively, whereas the singlet is given by the G_{123} component of the flux, proportional to the holomorphic 3-form Ω of the internal space. Local coordinates are complexified according to the local complex structure as $z^m = \frac{1}{\sqrt{2}}(x^{2m+2} + i x^{2m+3})$, $m = 1, 2, 3$. Similar definitions apply in the decomposition of ISD fluxes into $G_{\bar{1}\bar{2}\bar{3}}$, $S_{\bar{i}\bar{j}}$ and A_{ij}. For simplicity and to avoid cumbersome expressions, in this thesis we take $S_{12} = A_{12} = S_{\bar{1}\bar{2}} = A_{\bar{1}\bar{2}} = 0$. The dependence on these components can be obtained by requiring $SO(4) \times SO(2)$ convariance in our expressions [45]. Furthermore, the tensors A_{ij} and $A_{\bar{i}\bar{j}}$ correspond respectively to (1,2) and (2,1) non-primitive components of the flux, that are incompatible with the cohomology of a Calabi-Yau (although a local component in principle could be allowed). In addition we set $S_{3i} = S_{\bar{3}\bar{i}} = 0$, where z^3 is the complex direction transverse to the D7-branes, since those flux components generically lead to Freed-Witten (FW) anomalies in the worldvolume of D7-branes, as discussed in [45].

Being defined in the $M_{\mathrm{Pl.}} \to \infty$ limit, soft-terms in the effective 8d theory of a stack of 7-branes can be understood from the background in a local transverse patch around the stack of 7-branes. Such local background receives in general contributions from globally non-trivial fluxes as well as from the backreaction of distant sources, as we discuss in section 4.2.4. Thus, we expand the background (4.1) around the stack of 7-branes as

$$\begin{aligned} Z^{-1/2} &= 1 + \frac{1}{2} K_{mn} y^m y^n + \ldots \\ \tau &= \tau_0 + \frac{1}{2} \tau_{mn} y^m y^n + \ldots \\ \chi &= \text{const.} + \frac{1}{2} \chi_{mn} y^m y^n + \ldots \\ G_{lmn}(x^r) &= G_{lmn} + \ldots \end{aligned} \tag{4.4}$$

where we have denoted by y^m the two coordinates that are transverse to the stack of 7-branes and which for the sake of concreteness in what follows we take to be x^8 and

x^9. Dots in the rhs of eqs. (4.4) represent higher order terms in the expansion, and will only contribute to non-renormalizable couplings in the 4d effective action. In the next subsections we make use of this local expansion to compute the flux-induced soft-breaking terms for the adjoint fields of a stack of 7-branes.

4.1.1.1. Unmagnetized bulk D7-brane fields

We first address the case of unmagnetized 7-branes, leaving the case of magnetized branes for the next subsection. We closely follow the procedure developed in [45]. Thus, we expand the DBI+CS action of D7-branes in transverse coordinates in presence of the local background (4.1) and (4.4), and make use of the identification

$$z^3 = 2\pi\alpha'\Phi \tag{4.5}$$

to derive an 8d effective action that contains flux-induced SUSY-breaking soft terms. Dimensional reduction then leads to a soft-breaking Lagrangian in the 4d effective theory.

The relevant piece of the D7-brane DBI+CS action for the computation of flux-induced soft terms is given by

$$S = -\mu_7 \,\mathrm{STr}\left[\int d^8\xi\, e^{-\phi}\sqrt{-\det\left(P[E_{ab}] + \sigma F_{ab}\right)} - g_s \int P\left[-C_6 \wedge \mathcal{F}_2 + C_8\right]\right] \tag{4.6}$$

where

$$E_{ab} = e^{\phi/2}G_{ab} - B_{ab}\ , \qquad \sigma = 2\pi\alpha', \qquad \mu_7 = (2\pi)^{-3}\sigma^{-4}g_s^{-1}\ , \qquad \mathcal{F}_2 \equiv B_2 - \sigma F_2 \tag{4.7}$$

'STr' denotes the symmetrized trace over gauge indices and $P[\,\cdot\,]$ is the pull-back to the 7-brane worldvolume. Our conventions are such that the metric has signature diag$(-++\dots)$ whereas $dz^1 \wedge d\bar{z}^1 \wedge dz^2 \wedge d\bar{z}^2$ has negative signature.

The terms contributing to the determinant in the DBI piece of the action are given by[1]

$$\det(P[E_{ab}]) = e^{4\phi}\det\left(g_{ab} + 2\sigma^2 D_{(a}\Phi D_{b)}\bar{\Phi} - e^{-\phi/2}\mathcal{F}_{ab}\right) \tag{4.8}$$

Expanding the determinant as well as the square root in the DBI piece then leads to the following 8d Lagrangian[2]

$$\mathcal{L}_{8\mathrm{d}} = \mu_7 e^{\phi}\,\mathrm{STr}\left(-1 - \sigma^2 D_a\Phi D_a\bar{\Phi} - \frac{g_s^{-1}}{4}\mathcal{F}_{ab}\mathcal{F}_{ab} + C_8 - C_6 \wedge \mathcal{F}_2\right)\ . \tag{4.9}$$

In order to proceed further we should relate the dilaton ϕ, the B-field and the RR-fields that appear in this expression to fluctuations of the 8d field Φ in the limit $M_{\mathrm{Pl}} \to \infty$. Let us first address the case of the axio-dilaton. Complexifying the second equation in (4.4) and making use of eq. (4.5) we write

$$\tau = i g_s^{-1}\left(1 + \frac{\sigma^2\tau_{33}}{2}\Phi^2 + \frac{\sigma^2\tau_{\bar{3}\bar{3}}}{2}\bar{\Phi}^2 + \sigma^2\tau_{3\bar{3}}|\Phi|^2 + \dots\right) \tag{4.10}$$

[1] See appendix A for more details.

[2] The non-conventional sign for the kinetic term of Φ is due to the particular signature that we have taken of the 8d metric.

where for simplicity we have fixed $\langle\tau\rangle = ig_s^{-1}$. The 10d supergravity equations of motion then put restrictions on the parameters of this expansion. More precisely, from the equation

$$\nabla^2\tau = \frac{1}{i\,\mathrm{Im}\,\tau}\nabla^M\tau\,\nabla_M\tau + \frac{1}{12i}G_{mnp}G^{mnp} - \frac{4\kappa_{10}^2(\mathrm{Im}\,\tau)^2}{\sqrt{-g}}\frac{\delta S_7}{\delta\bar\tau} \tag{4.11}$$

we get the constraint

$$\tau_{3\bar 3} = \frac{1}{2i}\left(G_{123}G_{\bar 1\bar 2\bar 3} + \frac{1}{4}\sum_{k=1}^{3}S_{kk}S_{\bar k\bar k}\right) \tag{4.12}$$

and therefore in the presence of both ISD and IASD 3-form fluxes the dilaton is generically non-constant. In this expression we have assumed that localised distant 7-brane sources do not contribute to the soft terms, and thus have ignored last term in eq. (4.11). This is the case if there are no anti-D7-brane charges present in the compactification, as we assume in what follows.

Similarly, from the equation

$$dB_2 = -\frac{\mathrm{Im}\,G_3}{\mathrm{Im}\,\tau} \tag{4.13}$$

we obtain for the B-field components

$$\begin{aligned} B_{12} &= \frac{g_s\sigma}{2i}\left[(G_{\bar 1\bar 2\bar 3})^*\Phi - \frac{1}{2}S_{\bar 3\bar 3}\bar\Phi - G_{123}\Phi + \frac{1}{2}(S_{33})^*\bar\Phi\right] \\ B_{1\bar 2} &= \frac{g_s\sigma}{4i}\left[-S_{\bar 2\bar 2}\Phi + (S_{\bar 1\bar 1})^*\bar\Phi - S_{11}\bar\Phi + (S_{22})^*\Phi\right] \end{aligned} \tag{4.14}$$

And from the equations

$$\begin{aligned} dC_6 &= H_3\wedge C_4 - *_{10}\,\mathrm{Re}\,G_3 \\ dC_8 &= H_3\wedge C_6 - *_{10}\,\mathrm{Re}\,\tau \end{aligned} \tag{4.15}$$

we get respectively for the RR 6-form and 8-form potentials

$$\begin{aligned} C_{\hat 0\hat 1\hat 2\hat 3 12} &= \frac{\sigma}{2i}\left[-(G_{\bar 1\bar 2\bar 3})^*\Phi - G_{123}\Phi + \frac{1}{2}S_{\bar 3\bar 3}\bar\Phi + \frac{1}{2}(S_{33})^*\bar\Phi\right] \\ C_{\hat 0\hat 1\hat 2\hat 3 1\bar 2} &= \frac{\sigma}{4i}\left[S_{\bar 2\bar 2}\Phi + (S_{22})^*\Phi - (S_{\bar 1\bar 1})^*\bar\Phi - S_{11}\bar\Phi\right] \end{aligned} \tag{4.16}$$

and

$$\begin{aligned} C_{\hat 0\hat 1\hat 2\hat 3 1\bar 1 2\bar 2} = &-\frac{g_s\sigma^2}{16}\left[(-2G_{123}S_{33} + (S_{11}S_{22})^* - S_{\bar 1\bar 1}S_{\bar 2\bar 2} + 2(G_{\bar 1\bar 2\bar 3}S_{\bar 3\bar 3})^*)\,\Phi^2 \;+\; \text{c.c.}\right. \\ &\left.+\left(|S_{33}|^2 - |S_{\bar 3\bar 3}|^2 - 4|G_{\bar 1\bar 2\bar 3}|^2 + |S_{\bar 1\bar 1}|^2 + |S_{\bar 2\bar 2}|^2 + 4|G_{123}|^2 - |S_{22}|^2 - |S_{11}|^2\right)|\Phi|^2\right] \\ &+\frac{ig_s\sigma^2}{4}\left[\tau_{33} + (\tau_{\bar 3\bar 3})^*\right]\Phi^2 \;+\; \text{c.c.} \end{aligned} \tag{4.17}$$

In particular the non-constant contribution (4.12) of the axio-dilaton is crucial for dC_8 to be locally integrable when ISD and IASD 3-form fluxes are simultaneously present.

Plugging eqs. (4.12), (4.14), (4.16) and (4.17) into the 8d Lagrangian (4.9) and rescaling the fields in order to have canonically normalized 8d kinetic terms, we get the following Lagrangian for the worldvolume bosons of 7-branes

$$\mathcal{L}_{\rm B} = {\rm Tr}\left(D_a\Phi D_a\bar{\Phi} - \frac{1}{4g_8^2}F_{ab}F_{ab} - \frac{g_s}{4}\left[2|G_{\bar{1}\bar{2}\bar{3}}|^2 + \frac{1}{2}|S_{\bar{3}\bar{3}}|^2 + \frac{1}{2}|S_{11}|^2 + \frac{1}{2}|S_{22}|^2\right]|\Phi|^2 \right.$$
$$+\frac{g_s}{4}\left[(S_{\bar{3}\bar{3}})^*((G_{\bar{1}\bar{2}\bar{3}})^* - G_{123}) + \frac{1}{2}(S_{11})^*((S_{22})^* - S_{\bar{2}\bar{2}}) - (G_{\bar{1}\bar{2}\bar{3}})^*S_{33} - \frac{1}{2}(S_{22})^*S_{\bar{1}\bar{1}} - 2i\tau_{33}\right]\Phi^2$$
$$+{\rm h.c.} - \frac{ig_s^{1/2}}{2}\Phi\left[(S_{22})^*\left(\partial_{\bar{1}}A^{\bar{2}} - \partial_2 A^1 + g_8[A^1, A^{\bar{2}}]\right) + (S_{11})^*\left(\partial_1 A^2 - \partial_{\bar{2}}A^{\bar{1}} + g_8[A^{\bar{1}}, A^2]\right)\right.$$
$$\left.\left. +(S_{\bar{3}\bar{3}})^*\left(\partial_1 A^{\bar{2}} - \partial_2 A^{\bar{1}} + g_8[A^{\bar{1}}, A^{\bar{2}}]\right) + 2(G_{\bar{1}\bar{2}\bar{3}})^*\left(\partial_{\bar{1}}A^2 - \partial_{\bar{2}}A^1 + g_8[A^1, A^2]\right)\right] + {\rm h.c.}\right) \tag{4.18}$$

where the 8d gauge coupling constant g_8 is given by

$$g_8 = g_s^{1/2}(2\pi)^{5/2}\alpha' \ . \tag{4.19}$$

The closed string background therefore may induce scalar masses as well as trilinear couplings for the fields in the worldvolume of the 7-branes. Apart from the terms that were derived in [45], sourced by purely ISD or IASD 3-form fluxes, there are extra contributions to the B-term coming from the simultaneous presence of ISD and IASD 3-form fluxes as well as from the non-constant complex axion-dilaton. These contributions can arise in non Calabi-Yau compactifications, but also may result from the backreaction of non-perturbative effects in more conventional compactifications [48–50]. Observe also the presence of quadratic derivative couplings induced by the 3-form fluxes. These couplings were already noticed in [51], where it was shown that represent a mixing between massive modes due to Majorana mass terms induced by 3-form fluxes. Such mixing however does not affect the lightest mode of each KK tower and those derivative couplings therefore can be safely neglected for the purposes of this thesis.[3]

Flux-induced masses for the 8d fermions localized in the worldvolume of 7-branes can be computed similarly, starting in this case with the DBI+CS fermionic action. Following closely the procedure described in [45], we obtain

$$\mathcal{L}_{\rm F} = \frac{g_s^{1/2}}{2\sqrt{2}}{\rm Tr}[(G_{\bar{1}\bar{2}\bar{3}})^*\lambda\lambda + \frac{1}{2}(S_{\bar{3}\bar{3}})^*\Psi^3\Psi^3 + \frac{1}{2}S_{11}\Psi^1\Psi^1 + \frac{1}{2}S_{22}\Psi^2\Psi^2] + {\rm h.c.} \tag{4.20}$$

where λ is the 8d gaugino and Ψ^i, $i = 1, 2, 3$, the three additional complex fermions that live in the worldvolume of the 7-branes, and that in flat space form the fermionic content of an $\mathcal{N} = 4$ vector supermultiplet.

Having the bosonic and fermionic 8d Lagrangians for the lightest fields of 7-branes, we can obtain the 4d soft SUSY-breaking Lagrangian by dimensional reduction. For the case of fields transforming in the adjoint representation of the gauge group dimensional reduction is straightforward, since their internal wavefunctions are constant over the 4-cycle. In the same notation for the 4d soft-term Lagrangian of Ref. [52]

$$\mathcal{L}_{\rm soft} = -(m^2)_{ij}\phi^i\bar{\phi}^j - \left(\frac{1}{3!}A_{ijk}\phi^i\phi^j\phi^k + \frac{1}{2}B_{ij}\phi^i\phi^j - \frac{1}{2}M\lambda\lambda + \frac{1}{2}\mu_{ij}\psi^i\psi^j \right.$$
$$\left. - \frac{1}{2}C_{ijk}\bar{\phi}^i\bar{\phi}^j\phi^k + {\rm h.c.}\right) \tag{4.21}$$

[3]Note that even if the mixing were involving the lightest modes, it could be still safely neglected based on the large separation between the SUSY-breaking and Planck scales.

and for the case of 7-branes wrapping a T^4, we obtain the 4d soft-terms

$$
\begin{aligned}
m^2_{1\bar{1}} &= m^2_{2\bar{2}} = 0 \ ; \ \ B_{ij} \ = \ 0 \ \ , \ i,j \neq 3 \\
m^2_{3\bar{3}} &= \frac{g_s}{2}\left(|G_{\bar{1}\bar{2}\bar{3}}|^2 + \frac{1}{4}|S_{\bar{3}\bar{3}}|^2 + \frac{1}{4}|S_{11}|^2 + \frac{1}{4}|S_{22}|^2\right) \\
B_{33} &= \frac{g_s}{2}\left(-(G_{\bar{1}\bar{2}\bar{3}}S_{\bar{3}\bar{3}})^* - \frac{1}{2}(S_{22}S_{11})^* + (S_{\bar{3}\bar{3}})^* G_{123}\right. \\
&\qquad \left. + \frac{1}{2}(S_{11})^* S_{\bar{2}\bar{2}} + (G_{\bar{1}\bar{2}\bar{3}})^* S_{33} + \frac{1}{2}(S_{22})^* S_{\bar{1}\bar{1}} + 2i\tau_{33}\right) \\
A^{ijk} &= -h^{ijk}\frac{g_s^{1/2}}{\sqrt{2}}\,(G_{\bar{1}\bar{2}\bar{3}})^* \\
C^{ijk} &= -\frac{g_s^{1/2}}{2\sqrt{2}}\left[h^{jk1}S_{11} + h^{jk2}S_{22} - h^{jk3}(S_{\bar{3}\bar{3}})^*\right] \\
M^a &= \frac{g_s^{1/2}}{\sqrt{2}}\,(G_{\bar{1}\bar{2}\bar{3}})^* \\
\mu_{33} &= -\frac{g_s^{1/2}}{2\sqrt{2}}(S_{\bar{3}\bar{3}})^* \\
\mu_{ii} &= -\frac{g_s^{1/2}}{2\sqrt{2}}S_{ii} \ , \qquad i = 1,2 \ \ ,
\end{aligned}
$$

with

$$h_{ijk} = 2\epsilon_{ijk}\sqrt{2}g_{\mathrm{YM}} \tag{4.22}$$

the Yukawa coupling and $g_{\mathrm{YM}} = g_8/\sqrt{\mathrm{Vol}(T^4)}$ the 4d gauge coupling constant. Note that only the geometric field Φ gets a mass at this level, whereas the $A_i, A_{\bar{i}}$ remain massless. This is expected from 8d gauge invariance. This will be relevant in our generalisation to the matter curve case below.

Although for concreteness here we have reduced the 8d theory in a 4-torus, we could have equally performed dimensional reduction in a different type of 4-cycle, obtaining analogous expressions for the soft terms of a stack of 7-branes that wraps such 4-cycle. For that aim, note that no knowledge of the metric of the 4-cycle is required, but only its topological features. Whereas the 7-brane field content will change according to the homology of the 4-cycle, we expect expressions for the soft terms not far from those obtained here in the toroidal case. This will be even more certain in the case of soft terms for bifundamental matter fields discussed in the next section, since the wave-functions of those fields are localized also along some of the directions of the 4-cycle.

4.1.1.2. Magnetized bulk D7-brane fields

We now consider the addition of magnetic fluxes on the world-volume of D7-branes. Namely, we consider the presence of a local magnetic background

$$\langle F_2 \rangle = i(F_+ + F_-)\, dz^1 \wedge d\bar{z}^1 + i(F_+ - F_-)\, dz^2 \wedge d\bar{z}^2 \tag{4.23}$$

where the D-term equations will in general require the vanishing of the self-dual component F_+. The magnetic flux $F_\mp$ induces a charge of $D3(\overline{D3})$-brane in the worldvolume of D7-branes, and therefore it is expected the presence of flux-induced D3-brane soft-terms proportional to the magnetic background, apart from the soft-terms described in the previous subsection for unmagnetized D7-branes.

In more precise terms, the effect of magnetic fluxes on the D7-branes can be understood in terms of two different mechanisms. On one side the magnetic flux sources new renormalizable couplings in the 4d Lagrangian that originate from higher order couplings on which two or more of the gauge field-strengths are taken to be background. On the other side, the magnetic flux deforms the internal wavefunctions of charged fields and induces the mixing of massive modes in order to minimize the additional source of potential energy introduced by the flux. In this subsection we address the first of these effects. This is the only relevant one for the soft masses and B-terms of geometric moduli Φ in magnetized non-intersecting D7-branes. In this subsection we compute those contributions to soft masses. Then, in the next section we address the more interesting case of chiral-matter bifundamental fields, where the effect of the magnetization in the internal wavefunctions turns out to be the leading effect.

The microscopic computation of soft masses for magnetized bulk D7-brane fields follows the same steps than in the previous subsection. We work to quadratic order in the magnetization. The relevant piece of the DBI+CS action is again given by eq. (4.6) with the addition of the CS coupling to the RR 4-form, that becomes also relevant in presence of magnetization,

$$S = -\mu_7 \int d^8\xi \, \mathrm{STr}\left[e^{-\phi}\sqrt{-\det\left(P[E_{\mu\nu}] + \sigma F_{\mu\nu}\right)}\right] + \mu_7 g_s \int \mathrm{STr}\left(P\left[\frac{1}{2}C_4 \wedge \mathcal{F}_2 \wedge \mathcal{F}_2 - C_6 \wedge \mathcal{F}_2 + C_8\right]\right) \; . \quad (4.24)$$

It is convenient to factorize the determinant that appears in the DBI piece of the action in Minkowski and 4-cycle pieces as

$$\det(P[E_{\mu\nu}]) = g_s^4 \det\left(\eta_{\mu\nu} + 2Z\sigma^2\partial_\mu\Phi\partial_\nu\bar{\Phi} + Z^{1/2}g_s^{-1/2}\sigma F_{\mu\nu}\right)\det\left(g_{ab} - Z^{-1/2}g_s^{-1/2}\mathcal{F}_{ab}\right) \; , \quad (4.25)$$

from which we get

$$\det(P[E_{\mu\nu}]) = -g_s^4 - g_s^3 Z\sigma^2\left(1 + \frac{Z^{-1}g_s^{-1}}{2}\mathcal{F}_{ab}\mathcal{F}_{ab}\right)\left(2g_s\partial_\mu\Phi\partial_\mu\bar{\Phi} - \frac{1}{2}F_{\mu\nu}F_{\mu\nu}\right) - \frac{1}{2}g_s^3 Z^{-1}\mathcal{F}_{ab}\mathcal{F}_{ab} + \frac{g_s^2}{4}Z^{-2}\mathcal{F}_{ab}\mathcal{F}_{bc}\mathcal{F}_{cd}\mathcal{F}_{da} - \frac{g_s^2}{8}Z^{-2}\left(\mathcal{F}_{ab}\mathcal{F}_{ab}\right)^2 \; . \quad (4.26)$$

Plugging this expression into eq. (4.24) and expanding the square root that appears in DBI part of the action, we find

$$\mathcal{L}_{8\mathrm{d}} = \mu_7 \int_{\Sigma_4} e^{\phi}\, \mathrm{STr}\left[\left(-1 - Z\sigma^2\hat{\theta}\partial_\mu\Phi\partial_\mu\bar{\Phi} - \frac{g_s^{-1}}{4}\sigma^2 Z\hat{\theta}F_{\mu\nu}F_{\mu\nu} - \frac{g_s^{-1}}{4}Z^{-1}\mathcal{F}_{ab}\mathcal{F}_{ab} + \frac{g_s^{-2}}{8}Z^{-2}\mathcal{F}_{ab}\mathcal{F}_{bc}\mathcal{F}_{cd}\mathcal{F}_{da} - \frac{1}{32}g_s^{-2}Z^{-2}\left[\mathcal{F}_{ab}\mathcal{F}_{ab}\right]^2\right)dx^4 + C_8 - C_6\wedge\mathcal{F}_2 + \frac{1}{2}\chi\,\mathcal{F}_2\wedge\mathcal{F}_2\right] \; , \quad (4.27)$$

where we have defined

$$\hat{\theta} \equiv 1 + \frac{Z^{-1}g_s^{-1}}{4}\mathcal{F}_{ab}\mathcal{F}_{ab} = 1 + Z^{-1}g_s^{-1}\sigma^2(F_+^2 + F_-^2) \; . \quad (4.28)$$

The contribution of magnetic fluxes to the soft masses of the 4d fields that descend from Φ can be read from this expression. The relevant terms in this equation are

$$\delta\mathcal{L}_{\Phi^2} = \mu_7\sigma^2 \int_{\Sigma_4} dx^4\, e^{\phi}\, \mathrm{STr}\left[-Z\hat{\theta}\partial_\mu\Phi\partial_\mu\bar{\Phi} - g_s^{-1}Z^{-1}(F_+^2+F_-^2) - \chi\,(F_+^2-F_-^2)\right.$$
$$\left. +\frac{g_s^{-2}}{2}Z^{-2}\left(F_{ab}F_{bc}B_{cd}B_{da} + \frac{1}{2}F_{ab}B_{bc}F_{cd}B_{da} - \frac{1}{8}B_{ab}B_{ab}F_{cd}F_{cd} - \frac{1}{4}F_{ab}B_{ab}F_{cd}B_{cd}\right)\right] \,. \tag{4.29}$$

Expanding $e^{\phi} = (\mathrm{Im}\,\tau)^{-1}$ as in eq. (4.10), Z and χ as

$$Z^{-1/2} = Z_0^{-1/2} + \frac{\sigma^2}{2}\left(K_{33}\Phi^2 + (K_{33})^*\bar{\Phi}^2 + 2K_{3\bar{3}}|\Phi|^2\right) + \ldots \tag{4.30}$$
$$\chi = \chi_0 + \frac{\sigma^2}{2}\left(\chi_{33}\Phi + (\chi_{33})^*\bar{\Phi} + 2\chi_{3\bar{3}}|\Phi|^2\right) + \ldots$$

making use of the identities (4.14)-(4.17),[4] dimensionally reducing over a T^4 and rescaling the fields to have canonically normalized 4d kinetic terms, we obtain the following additional contributions to the soft-masses and B-term (4.22) induced by the magnetization in the worldvolume of D7-branes

$$\begin{aligned}
\delta m^2_{3\bar{3}} = &-\sigma^2\left(\frac{1}{8}[4|G_{\bar{1}\bar{2}\bar{3}}|^2 + |S_{\bar{3}\bar{3}}|^2 + |S_{\bar{1}\bar{1}}|^2 + |S_{\bar{2}\bar{2}}|^2 + 2|S_{11}|^2 + 2|S_{22}|^2]\right.\\
&\left.-\frac{1}{4}\mathrm{Re}\,(S_{11}S_{\bar{1}\bar{1}} + S_{22}S_{\bar{2}\bar{2}}) - 2g_s^{-1}K_{3\bar{3}} - \chi_{3\bar{3}} - (\mathrm{Im}\,\tau)_{3\bar{3}}\right)F_+^2\\
&-\sigma^2\left(\frac{1}{8}[|S_{11}|^2 + |S_{22}|^2 + 4|G_{123}|^2 + |S_{33}|^2 + 2|S_{\bar{3}\bar{3}}|^2 + 8|G_{\bar{1}\bar{2}\bar{3}}|^2]\right.\\
&\left.-\mathrm{Re}\left(G_{123}G_{\bar{1}\bar{2}\bar{3}} + \frac{1}{4}S_{33}S_{\bar{3}\bar{3}}\right) - 2g_s^{-1}K_{3\bar{3}} + \chi_{3\bar{3}} - (\mathrm{Im}\,\tau)_{3\bar{3}}\right)F_-^2\\
\delta B_{33} = &\,\sigma^2\left(\frac{1}{4}[2(S_{\bar{3}\bar{3}}G_{\bar{1}\bar{2}\bar{3}})^* + S_{\bar{1}\bar{1}}S_{\bar{2}\bar{2}} + 2(S_{11}S_{22})^* - 2S_{\bar{2}\bar{2}}(S_{11})^* - 2S_{\bar{1}\bar{1}}(S_{22})^*\right.\\
&\left.-2G_{123}(S_{\bar{3}\bar{3}})^* - 2S_{33}(G_{\bar{1}\bar{2}\bar{3}})^*] + 2g_s^{-1}K_{33} + \chi_{33} + (\mathrm{Im}\,\tau)_{33}\right)F_+^2\\
&+\sigma^2\left(\frac{1}{4}[(S_{11}S_{22})^* + 2G_{123}S_{33} + 4(S_{\bar{3}\bar{3}}G_{\bar{1}\bar{2}\bar{3}})^* - 4S_{33}(G_{\bar{1}\bar{2}\bar{3}})^* - 4G_{123}(S_{\bar{3}\bar{3}})^*\right.\\
&\left.-S_{\bar{2}\bar{2}}(S_{11})^* - S_{\bar{1}\bar{1}}(S_{22})^*\,] + 2g_s^{-1}K_{33} - \chi_{33} + (\mathrm{Im}\,\tau)_{33}\right)F_-^2\,.
\end{aligned} \tag{4.31}$$

where we have defined

$$(\mathrm{Im}\,\tau)_{33} \;\equiv\; \frac{\tau_{33} - (\tau_{\bar{3}\bar{3}})^*}{2i}\,, \qquad (\mathrm{Im}\,\tau)_{3\bar{3}} \;\equiv\; \frac{\tau_{3\bar{3}} - (\tau_{3\bar{3}})^*}{2i} \tag{4.32}$$

In particular, note that among the contributions of antiself-dual magnetic fluxes to soft-masses there are terms proportional to $(2g_s^{-1}K_{3\bar{3}} - \chi_{3\bar{3}} + (\mathrm{Im}\,\tau)_{3\bar{3}})$, in agreement with the expressions for soft-masses in the worldvolume of D3-branes that were obtained in Ref. [52]. Similarly among the contributions of self-dual magnetic fluxes we identify terms that are proportional to $(2g_s^{-1}K_{3\bar{3}} + \chi_{3\bar{3}} + (\mathrm{Im}\,\tau)_{3\bar{3}})$, identified with the expressions for soft-masses in the worldvolume of anti D3-branes.

[4] The identities (4.15)-(4.17) in general receive additional contributions from the magnetization, however one may check that those contributions turn into sub-leading corrections to the soft-masses of Φ.

We can also compute the leading corrections of magnetic fluxes to trilinear couplings of the form $\Phi \times A \times A$. The starting point is again eq. (4.27). The relevant terms in that equation are now

$$\mathcal{L}_{\Phi AA,F} = \mu_7 \sigma^2 \int_{\Sigma_4} dx^4 \mathrm{STr} \Bigg(-Z\hat{\theta} g_s \partial_\mu \Phi \partial_\mu \bar{\Phi} - \frac{\hat{\theta}}{4} F_{\mu a} F_{\mu a} - \\ - \frac{1}{2} Z^{-2} g_s^{-1} \sigma B_{ab} F_{bc} F_{cd} F_{da} + \frac{1}{8} Z^{-2} g_s^{-1} \sigma B_{ab} F_{ab} F_{cd} F_{cd} \Bigg) \quad (4.33)$$

Some little algebra shows that

$$- \frac{1}{2} B_{ab} F_{bc} F_{cd} F_{da} + \frac{1}{8} B_{ab} F_{ab} F_{cd} F_{cd} = \\ = \frac{\sigma}{2i} \left(F_-^2 \left[\Phi A_{[\bar{1}} A_{\bar{2}]} \left(-2(G_{\bar{1}\bar{2}\bar{3}})^* + 2G_{123} \right) + \bar{\Phi} A_{[\bar{1}} A_{\bar{2}]} \left(S_{\bar{3}\bar{3}} - (S_{33})^* \right) \right] + \\ + F_+^2 \left[\Phi A_{[\bar{1}} A_{2]} \left(S_{\bar{2}\bar{2}} - (S_{22})^* \right) + \bar{\Phi} A_{[\bar{1}} A_{2]} \left(-(S_{\bar{1}\bar{1}})^* + S_{11} \right) \right] \right) + \text{h.c.} \quad (4.34)$$

where we are keeping only terms that contribute to soft trilinear couplings. Plugging this expression into eq. (4.33), dimensionally reducing over a T^4 and rescaling the fields to have canonically normalized 4d kinetic terms, we get the following corrections to trilinear soft couplings from magnetization in the worldvolume of D7-branes

$$\delta A^{ijk} = -h^{ijk} \frac{\sigma^2}{\sqrt{2 g_s}} \left[F_-^2 \left(G_{123} - 2(G_{\bar{1}\bar{2}\bar{3}})^* \right) - F_+^2 (G_{\bar{1}\bar{2}\bar{3}})^* \right] \quad (4.35)$$

$$\delta C^{ijk} = -\frac{\sigma^2}{2\sqrt{2 g_s}} \Big\{ h^{jk1} \left[F_+^2 \left((S_{\bar{1}\bar{1}})^* - 2S_{11} \right) - F_-^2 S_{11} \right] + h^{jk2} \left[F_+^2 \left((S_{\bar{2}\bar{2}})^* - 2S_{22} \right) - F_-^2 S_{22} \right] \\ + h^{jk3} \left[F_-^2 \left(2(S_{\bar{3}\bar{3}})^* - S_{33} \right) + F_+^2 (S_{\bar{3}\bar{3}})^* \right] \Big\}$$

These corrections are quadratic in the magnetic fluxes, as expected.

Dimensional reduction of eq. (4.24) also includes corrections to the gauge coupling constants upon replacing $\langle F^2 \rangle$ by its vev. In the context of F-theory SU(5) unification, corrections from the hypercharge flux F_Y are particularly relevant, since they generically induce non-universal thresholds for the three SM gauge coupling constants (see [32,33,53]), which may have interesting phenomenological implications, as we will discuss in section 4.3.3. Let us also remark that the SM gauginos also become slightly non-universal once the corresponding gaugino fields are normalized to one. We will not consider these gaugino mass corrections in what follows, since they are expected to be generically small if gauge coupling unification is to be maintained.

4.1.2. Chiral matter bifundamental fields

In the previous section we have considered soft-breaking terms for 4d fields that descend from geometric moduli Φ in non-intersecting magnetized branes. We have done this in two steps. First, a 8d field theory with the relevant operators induced by the closed string background has been derived in the limit $M_{\mathrm{Pl}} \to \infty$. Next, we have dimensionally reduced that 8d theory to obtain the soft-breaking Lagrangian in 4d. For the case of bulk fields, e.g. adjoints in non-intersecting magnetized D7-branes, this last step is straightforward. However, this general procedure can in principle be equally applied in more involved settings, such as intersecting magnetized D7-branes with 3-form fluxes.

In this section we compute 4d soft-breaking terms for chiral matter bifundamental fields localised at D7-brane intersections (or *matter curves*). Although the procedure described above is in principle feasible (see e.g. [51]), in practice it quickly becomes technically too involved as the background gets more general. Thus, we instead exploit a short-cut by making use of the general ideas behind Higgssing in 4d supersymmetric theories and the 4d soft-breaking Lagrangians for bulk fields obtained in the previous section.

4.1.2.1. Fields at matter curves

When computing the 4d effective theory of a stack of magnetized/intersecting D7-branes one dimensionally reduces an 8d supersymmetric gauge theory, as we have described in the previous section. In the case of locally vanishing closed string fluxes this 8d theory is simply given by topologically twisted 8d $\mathcal{N}=1$ SYM [30, 32].[5] The bosonic part reads

$$\mathcal{L}_{\text{SYM}} = \text{Tr}\left(D_a\Phi D_a\bar{\Phi} - \frac{1}{2}([\Phi,\bar{\Phi}])^2 - \frac{1}{4}F_{ab}F_{ab}\right) \tag{4.36}$$

where $D_a\Phi = \partial_a\Phi + i[A_a,\Phi]$ and $F_2 = dA + A\wedge A$. To linear order in the fluctuations, the corresponding equations of motion are

$$D_aD^a\Phi = 0\ , \qquad D_aF^{ab} = 0 \tag{4.37}$$

We take here for simplicity an underlying $U(N)$ gauge symmetry group, although the results may be extended easily to $SO(N)$ and E_n groups, as we will see later. $U(N)$ is broken to some product of smaller groups by the magnetization/intersections. The latter are parametrized in terms of backgrounds for F_2 and Φ

$$\begin{aligned}\langle F_2\rangle &= \left[i(F_+^\alpha + F_-^\alpha)\, dz_1\wedge d\bar{z}_1 + i(F_+^\alpha - F_-^\alpha)\, dz_2\wedge d\bar{z}_2\right]\, Q_\alpha \\ \langle\Phi\rangle &= m_i^\alpha z_i Q_\alpha\ ,\end{aligned} \tag{4.38}$$

where Q_α are the generators of the Cartan subalgebra of $U(N)$. Dimensionally reducing eq. (4.37) to 4d amounts to solving the following system of second-order differential equations for the internal wavefunction Ψ of a 4d scalar with mass m and $U(1)_\alpha\subset U(N)$ charges q_α (see e.g. [54])

$$\begin{aligned}(\mathbb{D}^+\mathbb{D}^- + q_\alpha F_+^\alpha\mathbb{I})\Psi &= m^2\Psi \\ (\mathbb{D}^-\mathbb{D}^+ - q_\alpha F_+^\alpha\mathbb{I})\bar{\Psi} &= m^2\bar{\Psi}\end{aligned} \tag{4.39}$$

with

$$\mathbb{D}^\pm \equiv \begin{pmatrix} 0 & D_1^\pm & D_2^\pm & D_3^\pm \\ -D_1^\pm & 0 & -D_3^\mp & D_2^\mp \\ -D_2^\pm & D_3^\mp & 0 & -D_1^\mp \\ -D_3^\pm & -D_2^\mp & D_1^\mp & 0\end{pmatrix}\ , \qquad \Psi = \begin{pmatrix} 0 \\ a^1 \\ a^2 \\ \phi\end{pmatrix}\ , \qquad \bar{\Psi} = \begin{pmatrix} 0 \\ a^{\bar{1}} \\ a^{\bar{2}} \\ \bar{\phi}\end{pmatrix} \tag{4.40}$$

[5] For simplicity we take the normal bundle of the magnetized/intersecting D7-branes to be trivial, so that we can ignore the effect of the twist. In the more general case, this can be however easily implemented by shifting the magnetization along the canonical bundle [40].

and

$$\begin{aligned} D_1^- &= \partial_1 - \frac{q_\alpha}{2}(F_+^\alpha + F_-^\alpha)\bar{z}_1 & D_1^+ &= \bar{\partial}_{\bar{1}} + \frac{q_\alpha}{2}(F_+^\alpha + F_-^\alpha)z_1 \\ D_2^- &= \partial_2 - \frac{q_\alpha}{2}(F_+^\alpha - F_-^\alpha)\bar{z}_2 & D_2^+ &= \bar{\partial}_{\bar{2}} + \frac{q_\alpha}{2}(F_+^\alpha - F_-^\alpha)z_2 \\ D_3^- &= -q_\alpha[(m_1^\alpha)^*\bar{z}_1 + (m_2^\alpha)^*\bar{z}_2] & D_3^+ &= q_\alpha(m_1^\alpha z_1 + m_2^\alpha z_2) \end{aligned} \tag{4.41}$$

In these expressions $a^{1,2}$ and ϕ are respectively the components of the internal wavefunction A along the Wilson lines and the geometric scalar Φ, and $[Q^\alpha, \Psi] = -q^\alpha\Psi$. Besides this local diffeo-algebraic equation, wavefunctions must also satisfy the global periodicity conditions of the 4-cycle S.

In order to solve eq. (4.39) note that $\mathbb{D}^+\mathbb{D}^-$ can be expressed as

$$\mathbb{D}^\pm\mathbb{D}^\mp = -\mathbb{I}\sum_{i=1}^{3} D_i^\pm D_i^\mp + \mathbb{B}^\pm \tag{4.42}$$

with

$$\mathbb{B}^\pm = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & [D_2^\pm, D_2^\mp] & [D_2^\mp, D_1^\pm] & [D_3^\mp, D_1^\pm] \\ 0 & [D_1^\mp, D_2^\pm] & [D_1^\pm, D_1^\mp] & [D_3^\mp, D_2^\pm] \\ 0 & [D_1^\mp, D_3^\pm] & [D_2^\mp, D_3^\pm] & [D_2^\pm, D_2^\mp] + [D_1^\pm, D_1^\mp] \end{pmatrix} \tag{4.43}$$

Diagonalising this matrix

$$\mathbb{J}^\dagger \cdot \mathbb{B}^+ \cdot \mathbb{J} = \mathrm{diag}(0, \lambda_1, \lambda_2, \lambda_3) \tag{4.44}$$

we get

$$\tilde{D}_p^- = \sum_j \xi_{p,j} D_j^- \ , \qquad \tilde{D}_p^+ = \sum_j \xi_{p,j}^* D_j^+ \tag{4.45}$$

where ξ_p is the p-th eigenvector of $\mathbb{B}$. These operators span the algebra of three quantum harmonic oscillators, namely

$$[\tilde{D}_p^+, \tilde{D}_p^-] \ = \ -\lambda_p \ , \qquad p = 1, 2, 3 \tag{4.46}$$

leading to three KK towers of 4d scalars. The matrix $\mathbb{B}$ has a single negative eigenvalue that, without loss of generality, we take here to be λ_1. Making use of eqs. (4.42), (4.44) and (4.46) we can explicitly solve eqs. (4.39). The wavefunction for the lightest mode of each tower is given by

$$\Psi_p = \xi_p \varphi_p \tag{4.47}$$

with φ_p a function on the 4-cycle S satisfying locally

$$\tilde{D}_p^- \varphi_p = \tilde{D}_q^+ \varphi_p = 0 \ , \qquad p, q = 1, 2, 3 \ , \quad q \neq p \tag{4.48}$$

The mass of the lowest mode for each tower of scalars is given by

$$m_{\Psi_p}^2 = \lambda_p - \lambda_1 + q_\alpha F_+^\alpha \tag{4.49}$$

And similarly for the complex conjugate degrees of freedom.

To give a concrete example, consider a stack of three D7-branes with gauge group $U(3)$ (see [34, 35, 40]), wrapping a 4-torus parametrized by the holomorphic condition

$$z_3 = 0 \tag{4.50}$$

In section 4.1.4 we will consider the phenomenologically most interesting case of $SO(12)$ or E_6 gauge groups, relevant for SU(5) F-theory unification. However, this simpler $U(3)$ model suffices to illustrate the main ideas of this section.

Let us tilt one of the D7-branes of the stack an angle so that instead of (4.50) it wraps a 4-torus parametrized by the condition

$$z_3 - m_a z_1 = 0 \tag{4.51}$$

with m_a a constant of the order of the string scale that determines the number of intersections in the complex 2-torus spanned by z_1. For future reference, we denote this matter curve as $\Sigma_a = \{z_1 = 0\}$.

The original $U(3)$ gauge group is broken as

$$\begin{array}{ccccc} U(3) & \to & U(2)\times U(1) & \to & SU(2)\times U(1) \\ \mathbf{8} & \to & \mathbf{3}^0 + \mathbf{1}^0 + \bar{\mathbf{2}}^+ + \mathbf{2}^- & & \end{array} \tag{4.52}$$

where the diagonal $U(1) \subset U(2)$ becomes massive due to the presence of Stückelberg couplings. From the point of view of the 8d $U(3)$ SYM theory the breaking (4.52) is encoded in a background for the geometric modulus

$$\langle \Phi \rangle = \frac{1}{\sqrt{6}} m_a z_1 (Q_1 + Q_2 - 2Q_3) \tag{4.53}$$

where Q_α, $\alpha = 1, 2, 3$, are the Cartan generators of $U(3)$. The 8d fields Φ and A can be decomposed according to (4.52) as

$$\Phi = \left(\begin{array}{c|c} \Phi_{\mathbf{3}^0} & \Phi_{a^+} \\ \hline \Phi_{a^-} & \Phi_{\mathbf{1}^0} \end{array} \right) + \langle \Phi \rangle \qquad A = \left(\begin{array}{c|c} A_{\mathbf{3}^0} & A_{a^+} \\ \hline A_{a^-} & A_{\mathbf{1}^0} \end{array} \right) \tag{4.54}$$

with the 4d scalars in the bifundamental representation arising from the $U(3)$ off-diagonal fluctuations.

One may easily check that eq. (4.43) gives rise in this case to

$$\mathbb{B}^+ = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_a \\ 0 & 0 & 0 & 0 \\ 0 & m_a & 0 & 0 \end{pmatrix} , \tag{4.55}$$

with eigenvalues 0 and $\pm|m_a|$. Thus, according to our discussion above, the internal wavefunctions for the lightest mode in each of the three KK towers of 4d scalars are given by

$$\Psi_{a_1^+} = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \varphi , \qquad \Psi_{a_2^+} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \varphi , \qquad \Psi_{a_3^+} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \varphi \tag{4.56}$$

where φ is a real function of the coordinates of the 4-cycle S, locally given by

$$\varphi = f(z_2) \exp\left[-\frac{|m_a|}{2}|z_1|^2\right] \ , \tag{4.57}$$

and $f(z_2)$ are holomorphic functions specified by the global properties of the 4-cycle S such that the wavefunctions (4.56) are orthonormalized. The exponential factor in (4.57) shows in particular the localization of the energy density along the matter curve Σ_a. The resulting 4d masses for the modes (4.56) are respectively

$$m^2_{a_1^+} = 0 \ , \qquad m^2_{a_2^+} = 2|m_a|^2 \ , \qquad m^2_{a_3^+} = |m_a|^2 \tag{4.58}$$

Wavefunctions and 4d masses for the charge conjugated sector a^- follow exactly the same expressions (4.56)-(4.58), with the role of $\Psi_{a_1^-}$ and $\Psi_{a_2^-}$ exchanged with respect to eq. (4.56). Thus, in total we obtain a massless vector-like pair of 4d charged fields localized in Σ_a and transforming in the $\bar{\mathbf{2}}^+ + \mathbf{2}^-$ representation of the gauge group, as expected.

This simple setting can be extended in several ways. First, one may consider magnetization in the worldvolume of D7-branes. The effect of magnetization is to modify the wavefunctions (4.56) and (4.57) and to lift one of the two chiral components of the above vector-like pair of 4d zero modes. Thus, turning on a magnetic flux in the above $U(3)$ D7-brane setting of the form

$$\langle F_2 \rangle \ = \ \frac{iF^a_-}{\sqrt{6}}\,(dz_1\wedge d\bar{z}_1 - dz_2\wedge d\bar{z}_2)\,(Q_1+Q_2-2Q_3) \tag{4.59}$$

leads to the modified wavefunctions

$$\Psi_{a_1^+} = \frac{1}{\sqrt{2\lambda_a(\lambda_a - F^a_-)}}\begin{pmatrix} F^a_- - \lambda_a \\ 0 \\ m_a\end{pmatrix}\varphi^- \ , \quad \Psi_{a_2^+} = \frac{1}{\sqrt{2\lambda(\lambda_a + F^a_-)}}\begin{pmatrix} F^a_- + \lambda_a \\ 0 \\ m_a\end{pmatrix}\varphi^+ \tag{4.60}$$

where $\lambda_a = \sqrt{(F^a_-)^2 + m_a^2}$ and

$$\varphi^{\pm} = f(z_2)\text{exp}\left[-\frac{\lambda_a}{2}|z_1|^2 \pm \frac{F^a_-}{2}|z_2|^2\right] \tag{4.61}$$

We have introduced subscript a to refer to quantities associated to curve Σ_a. Such notation is useful in later sections when several matter curves are present. Similar expressions to (4.60) again apply for $a^-_{1,2}$. Note that only one of the two wavefunctions (4.60) is normalizable in the presence of the magnetic flux and thus a chiral spectrum is indeed obtained, with local chirality determined by the sign of the magnetization. Besides those, there can be additional chiral fermions localized in other regions of the matter curve, with the total chirality determined by the integral of the magnetic flux along the matter curve (see also [55] for a discussion of local versus global chirality).

We now want to extend this simple setting to consider the effect of closed string 3-form fluxes in the neighbourhood of D7-branes. As we have discussed in the previous section, the effect of 3-form fluxes (and other closed string backgrounds) in the limit $M_{\text{Pl}} \to \infty$ is to deform the 8d theory (4.36) by adding new renormalizable couplings sourced by the closed string background. Hence, we should consider the more complicate 8d Lagrangian (4.18), which includes closed string fluxes, instead of (4.36). Dimensional reduction of this Lagrangian in presence of non-trivial magnetization and intersections becomes rather complicated. In particular, internal wavefunctions for chiral matter fields such as (4.56) and (4.60) receive also contributions from the closed string background.

In what follows, we pursue a simpler route to obtain the 4d soft-breaking Lagrangian for chiral matter fields. However before moving to the details, a comment regarding the consistency of the 3-form flux background in presence of intersecting D7-branes is in order. Note that the 3-form flux background has to satisfy some restrictions in order not to induce Freed-Witten anomalies in the worldvolume of the tilted D7-branes. Indeed, the condition for a NSNS 3-form flux not to induce a tadpole for the gauge field in the worldvolume of a stack of D7-branes is given by [56]

$$\int_{\Pi_a} P[H_3] = 0 \tag{4.62}$$

for any 3-cycle $\Pi_a \subset S$, as can be easily seen by integrating by parts the D7-brane CS coupling $\int_S B_2 \wedge F_2$. This condition puts constraints on the intersection parameters m_i^α in presence of non-trivial 3-form fluxes. For instance, in the above simple example of a tilted D7-brane wrapping the 4-torus parametrized by eq. (4.51) it leads to the local constraints

$$\begin{aligned} m_a[(S_{11})^* - S_{\bar{1}\bar{1}}] - m_a^*[S_{11} - (S_{\bar{1}\bar{1}})^*] = 0 \\ m_a[(S_{\bar{3}\bar{3}})^* - S_{33}] - m_a^*[S_{\bar{3}\bar{3}} - (S_{33})^*] = 0 \end{aligned} \tag{4.63}$$

and hence the phases of the intersection parameter m_a and those of the complexified 3-form fluxes must be suitably aligned. Note however that the constraint (4.62) is a global condition, and for generic 4-folds the toroidal constraints (4.63) need not apply locally. Thus, we do not impose them in what follows.

4.1.2.2. Soft terms for fields on matter curves

To compute the expression of soft terms for bifundamental fields localised on matter curves, we combine the information about 4d soft terms for bulk fields obtained in section 4.1.1 with our discussion on matter field wavefunctions of previous subsection. For simplicity we first consider the case with no magnetic fluxes and only pure ISD closed string fluxes, namely only the flux components $G_{\bar{i}\bar{j}\bar{k}}$ and $S_{\bar{3}\bar{3}}$ are non-vanishing. The effect of magnetization on the soft terms for bifundamental fields will be discussed in subsection 4.1.2.3. For simplicity we also assume that closed string fluxes are approximately constant over the 4-cycle S, so that they can be factored out when performing dimensional reduction. The case of locally varying closed and open string fluxes will be considered in section 4.2.

The reader may easily check that the soft scalar terms for D7-brane adjoints that we found in eqs. (4.22) can be rewritten in terms of a 4d scalar potential of the form

$$V_{\text{ISD}} \;=\; |M^*\Phi^* \;+\; F_\Phi|^2 \;+\; |F_{A_{\bar{1}}}|^2 \;+ |F_{A_{\bar{2}}}|^2 \tag{4.64}$$

where $M = g_s^{1/2}(G_{\bar{1}\bar{2}\bar{3}})^*/\sqrt{2}$ is the gaugino mass, and F_i are the auxiliary fields for the different 4d complex scalar fields,

$$F_\Phi = \;\partial_\Phi W \;, \qquad F_{A_{\bar{1}}} = \;\partial_{A_{\bar{1}}} W \;, \qquad F_{A_{\bar{2}}} = \;\partial_{A_{\bar{2}}} W \;. \tag{4.65}$$

In these expressions W is the physical superpotential of the 4d effective theory (with normalised fields) and it includes a μ-term for Φ, with $\mu = -g_s^{1/2}(S_{\bar{3}\bar{3}})^*/(2\sqrt{2})$, and a cubic term proportional to the Yukawa coupling, i.e.

$$W \;=\; \frac{\mu}{2}\,\Phi^2 \;+\; h_{123}\,A_{\bar{1}}A_{\bar{2}}\Phi \;+\; \ldots \tag{4.66}$$

The scalar potential (4.64) is positive definite. This is consistent with the fact that ISD fluxes locally preserve a no-scale structure [7]. In terms of the physical scalar fields we have

$$V_{\rm ISD} \;=\; \left(|M|^2+|\mu|^2\right)|\Phi|^2 + \; M\mu\,\Phi^2 \;+\; \mu\,h_{123}^*\,\Phi A^{\bar{1}}A^{\bar{2}} \;+\; Mh_{123}\,\Phi A^1A^2 \;+\; \text{h.c.} \tag{4.67}$$

Indeed, comparing with eq. (4.21) we read out the following pattern of soft terms,

$$m^2_{3\bar{3}} = |M|^2 \;+\; |\mu|^2 \quad ; \quad A_{ijk} = -Mh_{ijk} \tag{4.68}$$

$$B_{33} = 2M\mu \quad ; \quad C_{3jk} = -\mu h^*_{3jk} \tag{4.69}$$

This reproduces the result for non-magnetized and non-intersecting 7-branes obtained in eq. (4.22) when only ISD closed string fluxes are turned on. We will see in subsection 4.1.2.4 that this pattern corresponds to modulus dominance SUSY-breaking in an effective supergravity approach.

Let us now turn to the case of bifundamental fields living on intersecting 7-branes. To simplify the discussion we consider the above simple $U(3)$ example with no magnetization, although the results are valid for more realistic (e.g. SU(5), see section 4.1.4) group theory structures. We slightly generalize the setting by considering the three D7-branes in the original stack to be tilted an arbitrary angle, so that the gauge group is fully broken to $U(1)^3$. As before, 4d bifundamental scalars arise from $U(3)$ off-diagonal fluctuations of the adjoint fields

$$\Phi = \begin{pmatrix} \Phi_{\mathbf{1}^0} & \Phi_{a^+} & \Phi_{c^-} \\ \Phi_{a^-} & \Phi'_{\mathbf{1}^0} & \Phi_{b^+} \\ \Phi_{c^+} & \Phi_{b^-} & \Phi''_{\mathbf{1}^0} \end{pmatrix} \quad , \qquad A = \begin{pmatrix} A_{\mathbf{1}^0} & A_{a^+} & A_{c^-} \\ A_{a^-} & A'_{\mathbf{1}^0} & A_{b^+} \\ A_{c^+} & A_{b^-} & A''_{\mathbf{1}^0} \end{pmatrix} \tag{4.70}$$

In absence of magnetic fluxes the three sectors a, b and c are vector-like and contain massless chiral matter fields $a^{\pm}$, $b^{\pm}$, $c^{\pm}$ that are described by wavefunctions of the form (4.56). For concreteness we take the curves Σ_a, Σ_b and Σ_c to be given by

$$\Sigma_a \;=\; \{z_1 = 0\}\;, \qquad \Sigma_b \;=\; \{z_2 = 0\}\;, \qquad \Sigma_c \;=\; \{z_1 = z_2\} \tag{4.71}$$

as in the $U(3)$ model presented in section 2.3 of [35].

One important effect of turning on a background for the transverse scalar Φ is that the eigenstates (4.56) that solve the equations of motion in the internal space are generically a combination of A^1, A^2 and Φ. Since the rotation induced in the space of internal wavefunctions commutes with dimensional reduction, we can think of the following three-step procedure to obtain the 4d Lagrangian of bifundamental fields. We first dimensionally reduce the 8d Lagrangian (4.18) to obtain a 4d Lagrangian for bulk fields, as we have already done in section 4.1.1. Next, we trace over the gauge indices in order to express this Lagrangian in terms of bifundamental fields. Last, we rotate the 4d fields to a new basis that diagonalizes eqs. (4.39) and decouple massive modes that are at the string scale. Note that the rotation is different for each of the sectors of the theory, a, b and c, involved in a Yukawa coupling.

For instance, for the matter fields localized in curve $\Sigma_a = \{z_1 = 0\}$ the rotation in the space of wavefunctions is given by

$$\begin{pmatrix} \varphi_{a_1^+} \\ \varphi_{a_2^+} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} A^1_{a^+} \\ \Phi_{a^+} \end{pmatrix} \quad ; \quad \varphi_{a_3^+} = A^2_{a^+} \tag{4.72}$$

Neglecting the effect of closed string fluxes on the internal wavefunctions (based on the assumed large hierarchy of scales $M_{ss} \ll M_s$), the fields φ_{a_i} correspond to mass eigenstates with $m^2_{\varphi_{a_1^+}} = 0$, $m^2_{\varphi_{a_2^+}} = 2|m_a|^2$ and $m^2_{\varphi_{a_3^+}} = |m_a|^2$, as we saw in the previous subsection. For the sector a^- the rotation is equivalent but the role of the fields φ_{a_1} and φ_{a_2} is interchanged. Moreover, by supersymmetry the same rotation also acts on the auxiliary fields, namely

$$\begin{pmatrix} F_{A_{a^+}} \\ F_{\Phi_{a^+}} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} F_{\varphi_{a_1^+}} \\ F_{\varphi_{a_2^+}} \end{pmatrix} . \tag{4.73}$$

Since the fields $\varphi_{a_2^+}$, $\varphi_{a_1^-}$ and $\varphi_{a_3^\pm}$ are very heavy (with masses of order the string scale), correct decoupling in the effective theory dictates that in the effective 4d Lagrangian we should set

$$F_{\varphi_{a_2^+}} \;=\; F_{\varphi_{a_1^-}} \;=\; F_{\varphi_{a_3^\pm}} \;=\; 0 \tag{4.74}$$

along with

$$\varphi_{a_2^+} \;=\; \varphi_{a_1^-} \;=\; \varphi_{a_3^\pm} \;=\; 0 \tag{4.75}$$

Thus, we can make use of the following replacements in the effective action (4.64),

$$F_{A_{a^+}} \;=\; -\frac{F_{\varphi_{a_1^+}}}{\sqrt{2}} \;, \quad F_{\Phi_{a^+}} \;=\; \frac{F_{\varphi_{a_1^+}}}{\sqrt{2}} \;, \quad \Phi_{a^+} \;=\; \frac{\varphi_{a_1^+}}{\sqrt{2}} \tag{4.76}$$

and the analogous ones for a_2^- and for the sectors b and c. This leads to a scalar potential of the form

$$V \;=\; \sum_{\alpha=a,b,c} \left(\frac{1}{2}|M^* \varphi^*_{\alpha_1^+} + F_{\varphi_{\alpha_1^+}}|^2 \;+\; \frac{1}{2}|F_{\varphi_{\alpha_1^+}}|^2 \;+\; (\alpha_1^+ \leftrightarrow \alpha_2^-) \right) \tag{4.77}$$

where the first term in this expression originates from F_Φ whereas the second term comes from F_A. To see explicitly how the soft terms for matter fields arise from this expression we expand the squared sum that appears in the above potential,

$$V \;=\; \sum_{\alpha=a,b,c} \left(\frac{1}{2}|M|^2|\varphi_{\alpha_1^+}|^2 \;+\; |F_{\varphi_{\alpha_1^+}}|^2 \;+\; \frac{1}{2} M F_{\varphi_{\alpha_1^+}} \varphi_{\alpha_1^+} \;+\; \text{h.c.} \;+\; (\alpha_1^+ \leftrightarrow \alpha_2^-) \right) . \tag{4.78}$$

The first term corresponds to a soft mass for the scalar fields $\varphi_{\alpha_1^+}$, which is a factor 1/2 smaller than the one that we had for adjoint fields. Moreover, in absence of magnetization the superpotential contains μ-terms proportional to $\varphi_{\alpha_1^+}\varphi_{\alpha_2^-}$ and hence we can express the auxiliary field of $\varphi_{a_1^+}$ as

$$F_{\varphi_{a_1^+}} \;=\; \frac{1}{\sqrt{2}} \left(F_{\Phi_{a^+}} \;+\; F_{A_{a^+}} \right) = \frac{\mu \Phi_{a^-}}{\sqrt{2}} \;+\; \ldots \;=\; \frac{\mu}{2} \varphi_{a_2^-} \;+\; \ldots \;=\; \mu^a_{\text{bif}} \varphi_{a_2^-} \;+\; \ldots \tag{4.79}$$

where the dots represent higher-order superpotential terms such as Yukawa couplings. Therefore, $\mu^a_{\rm bif} = \mu/2$ and similarly for the other two matter curves, if they host vector-like states. The second term in eq. (4.78) hence gives rise to supersymmetric masses for the scalar fields, given by $|\mu^\alpha_{\rm bif}|^2$, and to the usual supersymmetric trilinear coupling, that can be written as a product of $\mu^\alpha_{\rm bif}$ and the effective Yukawa coupling. Finally, the last term in eq. (4.78) gives rise to B-terms and SUSY-breaking trilinear couplings A. Comparing with the case of adjoint fields, they are also suppressed by a factor 1/2.

Summing over the three curves a, b and c we therefore obtain soft masses of the form $|M|^2/2$ for each of the 4d scalars. In addition, we get μ-terms, B-terms and a supersymmetric trilinear coupling for each non-chiral curve. Recall that for the soft SUSY-breaking trilinear coupling we get the same result three times (one for each curve), leading to an extra multiplicative factor 3.

Summarizing, we have obtained the following set of soft terms for bifundamental fields in a system of intersecting non-magnetized D7-branes with ISD 3-form fluxes,

$$\begin{aligned} m^2_{\rm bif,\alpha} &= \frac{|M|^2}{2} + \frac{|\mu|^2}{4} = \frac{g_s}{4}\left(|G_{\bar{1}\bar{2}\bar{3}}|^2 + \frac{1}{8}|S_{\bar{3}\bar{3}}|^2\right) \\ (B_{\rm bif}\,\mu_{\rm bif})^\alpha &= \frac{1}{2}B_\Phi\,\mu^\alpha_{\rm bif} = M\,\mu^\alpha_{\rm bif} = -\frac{g_s}{8}(G_{\bar{1}\bar{2}\bar{3}})^*(S_{\bar{3}\bar{3}})^* \\ A^{ijk}_{\rm bif} &= \frac{3A_\Phi}{2} = -\frac{3}{2}Mh^{ijk} = -\frac{3g_s^{1/2}}{2\sqrt{2}}(G_{\bar{1}\bar{2}\bar{3}})^*h^{ijk}. \end{aligned} \tag{4.80}$$

where $\alpha = a, b, c$ and we have now factored out in this expression explicitly the $\mu_{\rm bif}$ factor from the definition of the B-parameter. Gaugino masses remain unaltered since they are not localized by the non-trivial background of Φ. Hence, for the fermonic masses we have

$$\begin{aligned} M &= \frac{g_s^{1/2}}{\sqrt{2}}(G_{\bar{1}\bar{2}\bar{3}})^* \\ \mu^\alpha_{\rm bif} &= \frac{\mu}{2} = -\frac{g_s^{1/2}}{4\sqrt{2}}(S_{\bar{3}\bar{3}})^* \end{aligned} \tag{4.81}$$

We can also guess the contribution to soft scalar masses coming from the IASD fluxes S_{ii}, $i = 1, 2$. Indeed, looking at the results for the bulk D7-brane fields in eq. (4.22), we expect an additional dependence on IASD fluxes through the replacement

$$|S_{\bar{3}\bar{3}}|^2 \;\rightarrow\; |S_{\bar{3}\bar{3}}|^2 + |S_{11}|^2 + |S_{22}|^2 \tag{4.82}$$

in the mass squared and

$$G_{\bar{1}\bar{2}\bar{3}}S_{\bar{3}\bar{3}} \;\rightarrow\; G_{\bar{1}\bar{2}\bar{3}}S_{\bar{3}\bar{3}} + \frac{1}{2}(S_{11}S_{22})^* \;. \tag{4.83}$$

for the B-term. This is suggested by symmetry arguments similar to those used in section 3.1 of [45]. However, no contribution to trilinear couplings or fermion masses is expected from S_{ii} IASD fluxes, since there are no holomorphic gauge components $A_{1,2}$ present in the chiral matter fields.

We now turn to discuss how magnetization modifies this pattern of soft terms for matter fields.

4.1.2.3. Effect of magnetic fluxes on soft terms for fields at matter curves

Magnetization leads to 4d chiral spectra, as reviewed in section 4.1.2.1, with total chirality determined by the integral of $\langle F_2 \rangle$ over the various matter curves of the theory. For concreteness, let us assume that the curves Σ_a and Σ_b in our $U(3) \to U(1)^3$ toy model above are now charged under the flux, such that only the modes a^+ and b^+ survive in the 4d spectrum. We take the matter curve Σ_c however to be neutral under the flux, and so the spectrum arising from this curve is unaffected, containing the vector-like pair given by c^+ and c^-. In this toy model we may think of the non-chiral sector localized in Σ_c as the Higgs sector, whereas the chiral sectors localized in the curves Σ_a and Σ_b can be thought as MSSM chiral sectors. A more realistic example is given in section 4.1.4, where we apply the results of this section to study the hypercharge dependence of soft terms in a local F-theory SU(5) GUT model.

As we have already mentioned, magnetic fluxes affect the 4d soft SUSY-breaking Lagrangian in two ways. On one side, the presence of a non-trivial background for F_2 leads to new renormalizable couplings in 4d which can be traced back to higher-dimensional couplings in the 8d theory where some of the fields-strengths present in the coupling are replaced by the background flux. These corrections were computed in subsection 4.1.1.2 for the case of bulk D7-brane fields. They are quadratic in the magnetic flux density and from the point of view of the 4d effective supergravity correspond to renormalizable thresholds to the Kähler potential and/or the gauge kinetic function of the 4d effective theory. The other effect of magnetic fluxes, relevant for matter fields, is to modify the profile of the internal wavefunctions, as it has been described in subsection 4.1.2.1, and therefore also the rotation in the space of internal wavefunctions. For instance, in our $U(3)$ example above the internal wavefunctions for 4d charged fields in presence of magnetic fluxes were given in eq. (4.60). The rotation eq. (4.72) in the space of fields is thus modified such that

$$\begin{pmatrix} \varphi_{a_1^+} \\ \varphi_{a_2^+} \end{pmatrix} = \begin{pmatrix} \frac{F_-^a - \lambda_a}{\sqrt{2\lambda_a(\lambda_a - F_-^a)}} & \frac{m_a}{\sqrt{2\lambda_a(\lambda_a - F_-^a)}} \\ \frac{F_-^a + \lambda_a}{\sqrt{2\lambda_a(\lambda_a + F_-^a)}} & \frac{m_a}{\sqrt{2\lambda_a(\lambda_a + F_-^a)}} \end{pmatrix} \begin{pmatrix} A^1_{a^+} \\ \Phi_{a^+} \end{pmatrix} \quad ; \quad \varphi_{a_3^+} = A^2_{a^+} \tag{4.84}$$

Mass eigenstates therefore still originate from a mixture between Wilson lines and transverse scalars, but this mixture now depends on the magnetic flux in the curve Σ_a. Only in the case without magnetic flux, Φ and A contribute equally to the mass eigenstates. Moreover, this correction begins at linear order on the magnetic flux, and therefore for bifundamental fields the quadratic corrections described in section 4.1.1.2 are sub-leading. We thus ignore those and just consider the leading effect coming from the modification of the $\Phi - A$ mixing induced by magnetic fluxes.

In order to compute the soft terms of matter fields localized in the curves Σ_a, Σ_b and Σ_c, we follow the same procedure described in the previous subsection. The rotation matrix for the fields localized in the matter curve Σ_a is now given by eq. (4.84). A similar rotation also applies to the fields localized in curve Σ_b, after interchanging $A^1 \leftrightarrow A^2$. The rotation for auxiliary fields is modified in the same way than for the scalar fields. Therefore, for the fields localized in the matter curves $\Sigma_{a,b}$ we can make the replacements

$$F_{A_{\alpha^+}} = -\sqrt{\frac{\lambda_\alpha - F_-^\alpha}{2\lambda_\alpha}} F_{\varphi_{\alpha_1^+}} \, , \qquad F_{\Phi_{\alpha^+}} = \sqrt{\frac{\lambda_\alpha - F_-^\alpha}{2\lambda_\alpha}} \frac{\lambda_\alpha + F_-^\alpha}{m_\alpha} F_{\varphi_{\alpha_1^+}} \, , \qquad \alpha = a, b \tag{4.85}$$

where we have already set the auxiliary fields of massive modes to zero. In these expressions $\lambda_\alpha = \sqrt{(F_-^\alpha)^2 + m_\alpha^2}$. Note that only the fields coming from the sector a^+ and b^+ are normalizable and therefore those coming from the sectors a^- and b^- are not present in the low energy spectrum of the theory. This implies that μ- and B-terms are absent in the matter curves Σ_a and Σ_b. Curve Σ_c on the other hand is not affected by magnetic fluxes, and therefore the same expression (4.73) for the rotation of auxiliary fields in absence of magnetic fluxes still applies for Σ_c.

Making all these substitutions in the potential (4.64) we get

$$\begin{aligned} V \;=\; & \sum_{\alpha=a,b} \left(\frac{\lambda_\alpha - F_-^\alpha}{2\lambda_\alpha} \frac{(\lambda_\alpha + F_-^\alpha)^2}{m_\alpha^2} |M^* \varphi^*_{\alpha_1^+} + F_{\varphi_{\alpha_1^+}}|^2 \;+\; \frac{\lambda_\alpha - F_-^\alpha}{2\lambda_\alpha} |F_{\varphi_{\alpha_1^+}}|^2 \right) + \\ & + \frac{1}{2}|M^* \varphi^*_{c_1^+} + F_{\varphi_{c_1^+}}|^2 \;+\; \frac{1}{2}|F_{\varphi_{c_1^+}}|^2 \;+\; (c_1^+ \leftrightarrow c_2^-) \end{aligned} \tag{4.86}$$

and expanding perturbatively in powers of the ratio F_-^α/m_α between the magnetization and the intersection parameter, the contribution to the scalar potential that comes from the sectors $\alpha = a, b$ becomes

$$\begin{aligned} V_\alpha \;=\; & \frac{1}{2}\left[1 - \left|\frac{F_-^\alpha}{m_\alpha}\right| + \mathcal{O}\left(\left|\frac{F_-^\alpha}{m_\alpha}\right|^3\right)\right] |M|^2 |\varphi_{\alpha_1^+}|^2 + \; |F_{\varphi_{\alpha_1^+}}|^2 + \\ & + \frac{1}{2}\left[1 - \left|\frac{F_-^\alpha}{m_\alpha}\right| + \mathcal{O}\left(\left|\frac{F_-^\alpha}{m_\alpha}\right|^3\right)\right] M F_{\varphi_{\alpha_1^+}} \varphi_{\alpha_1^+} \end{aligned} \tag{4.87}$$

The μ-term $\mu^c_{\rm bif}$ is not modified to linear order on the fluxes, since the curve Σ_c is neutral under the magnetic flux. Hence, to leading order we still have $\mu_{\rm bif} = \mu/2$, as in the case with no magnetization discussed in the previous subsection.

Summarizing, from eqs. (4.86) and (4.87) we have therefore derived the following set of flux-induced soft terms for intersecting magnetized 7-branes in the above $U(3)$ toy model, for fields localized in each of the matter curves Σ_α,

$$\begin{aligned} m^2_{{\rm bif},\alpha} \;&=\; \frac{|M|^2}{2}\left(1 - \left|\frac{F_-^\alpha}{m_\alpha}\right|\right) \;=\; \frac{g_s}{4}|G_{\bar{1}\bar{2}\bar{3}}|^2 \left(1 - \left|\frac{F_-^\alpha}{m_\alpha}\right|\right) \qquad \alpha = a,b \\ m^2_{{\rm bif},c} \;&=\; \frac{|M|^2}{2} + \frac{|\mu|^2}{4} \;=\; \frac{g_s}{4}\left(|G_{\bar{1}\bar{2}\bar{3}}|^2 + \frac{1}{8}|S_{\bar{3}\bar{3}}|^2\right) \\ (B_{\rm bif}\,\mu_{\rm bif})^c \;&=\; \frac{1}{2} B_\Phi\, \mu^c_{\rm bif} \;=\; M \mu^c_{\rm bif} \;=\; -\frac{g_s}{8}(G_{\bar{1}\bar{2}\bar{3}})^* (S_{\bar{3}\bar{3}})^* \\ A^{ijk}_{\rm bif} \;&=\; -\frac{M}{2}\left(3 - \left|\frac{F_-^a}{m_a}\right| - \left|\frac{F_-^b}{m_b}\right|\right) h^{ijk} \;=\; -\frac{g_s^{1/2} h^{ijk}}{2\sqrt{2}} (G_{\bar{1}\bar{2}\bar{3}})^* \left(3 - \left|\frac{F_-^a}{m_a}\right| - \left|\frac{F_-^b}{m_b}\right|\right) \end{aligned} \tag{4.88}$$

Note in particular that μ- and B-terms do not receive corrections linear in the magnetic fluxes, as we have already mentioned. This is because in this particular case there are no magnetic fluxes along the curve c, which is the one hosting the Higgs fields. On the other hand there would appear corrections quadratic in the magnetic fields, analogous to those appearing for adjoint fields in the previous chapter.

Let us mention for completeness that there is also a third possibility for brane distributions with consistent Yukawa couplings, even in compactifications with a rigid S

divisor. Indeed, we can have a coupling of the form (I-I-A) involving two fields coming from the intersection of D7-branes and one field coming from the reduction of the gauge field A living in the 7-brane worldvolume. If this is the case, it is natural to assume that the Higgs field arises from the worldvolume of the D7-branes while the MSSM chiral matter arise from intersections (labelled by a, b). Hence the soft mass and the B-term for the Higgs are forbidden by gauge invariance, whereas the soft masses for the chiral fields take the form described above. We can summarize this structure in the following scalar potential,

$$V = \sum_{\alpha=a,b} V_\alpha + |F_A|^2 \tag{4.89}$$

where V_α is described in eq. (4.87) and F_A is the auxiliary field for the A field. Recall that it does not include a $\mu-$term by gauge invariance. Thus the trilinear coupling will have only two contributions coming from $V_{a,b}$. To sum up, this brane distribution leads to the following flux-induced soft SUSY-breaking terms,

$$\begin{aligned} m_H^2 &= 0\,, \qquad B_H \;=\; 0 \\ m^2_{\mathrm{bif},\,a,b} &= \frac{|M|^2}{2}\left(1-\left|\frac{F_-^{a,b}}{m_a}\right|\right) \\ A^{Hij} &= -\frac{M}{2}\left(2-\left|\frac{F_-^{a}}{m_a}\right|-\left|\frac{F_-^{b}}{m_b}\right|\right)h^{Hij}\,. \end{aligned} \tag{4.90}$$

4.1.2.4. Comparison with effective $\mathcal{N}=1$ supergravity

As emphasized in refs. [45, 52], the pattern of flux-induced soft terms that arise in the worldvolume of D3/D7-branes for ISD 3-form fluxes can be also understood in terms of effective $\mathcal{N}=1$ supergravity. For the case of adjoint fields with no magnetization, discussed in section 4.1.1.1, soft terms agree with those obtained from a simple no-scale Kähler potential for a single Kähler modulus T and a gauge kinetic function of the form

$$K = -3\log(T+T^*)\,, \qquad f_a = T\,, \tag{4.91}$$

as well as a Kähler metric for matter fields

$$K_{ij} = \frac{\delta_{ij}}{(T+T^*)^{\xi_i}} \tag{4.92}$$

with ξ_i the so-called *modular weight* of the scalar field ϕ_i. This structure is more than a toy model. Indeed, one obtains such a simple structure in isotropic toroidal orientifolds in which T is the overall Kähler modulus with $T=T_1=T_2=T_3$, and a stack of D7-branes wraps a 4-torus $T^2\times T^2$ within the T^6. The modular weight of 4d adjoint fields that descend from Φ and A is given respectively by $\xi=0$ and $\xi=1$. We ignore the dependence of these expressions on the complex axion-dilaton, the complex structure moduli and the other Kähler moduli present in the theory, since those are not relevant for the computation of soft terms below.

Assuming that the F-term auxiliary field F_T of the modulus T is non-vanishing (*modulus dominance*), the standard $\mathcal{N}=1$ supergravity formulae (see e.g. [57]) yield

$$\begin{aligned} m^2_{\phi_i} &= |M|^2(1-\xi_i) \\ A_{ijk} &= -Mh_{ijk}\sum_{\alpha=i,j,k}(1-\xi_\alpha) \\ B^i &= M\sum_{\alpha=\phi^i_u,\phi^i_d}(1-\xi_\alpha) \end{aligned} \tag{4.93}$$

where $\phi^i_{u,d}$ represent possible vector-like states allowing for a supersymmetric μ-term. In particular, for adjoint fields that descend from Φ and A we get

$$m^2_{\Phi_i} = |M|^2 , \qquad A_{ijk} = -Mh_{ijk} , \qquad B_\Phi = 2M \tag{4.94}$$

and $B_A = m^2_A = 0$. This is consistent with the more general result shown in eq. (4.22) particularized to case in which only ISD fluxes $G_{\bar{1}\bar{2}\bar{3}}$ and $S_{\bar{3}\bar{3}}$ are present. In the case on which magnetic fluxes are also present, the Kähler metric (4.92) is suitably corrected by the magnetization as [58, 59]

$$K_{ij} = \frac{\delta_{ij}}{t^{\xi_i}}\left(1 + c_\xi t^{\xi-1}\right) \tag{4.95}$$

with $t = T + T^*$ and c_ξ some flux-dependent constant whose value will depend on the modular weight and the flux quanta. The corrections to the soft terms that arise from this Kähler metric are in agreement with those found in eqs. (4.31) and (4.35) particularized to the case of ISD 3-form fluxes and anti self-dual magnetic flux F_- once we identify the flux correction of (4.95) with our microscopic description of the flux density, $\rho \equiv \frac{c_\Phi}{t} = g_s^{-1}\sigma^2 F_-^2$. One also finds that for A fields, which have $\xi = 1$, one has $c_A = 0$ and the fields that descend from A remain massless even after the addition of magnetic fluxes.

Similarly, the soft terms for matter fields in intersecting D7-branes given in eqs. (4.80) and (4.81) can be also reproduced by the above $\mathcal{N}=1$ supergravity formulae. Indeed, the modular weight of chiral fields localized at intersecting D7-branes is given by $\xi = 1/2$. In absence of magnetic fluxes, standard supergravity formulae then leads to

$$m^2_{\text{bif},i} = \frac{|M|^2}{2} , \qquad A_{ijk} = -\frac{3M}{2}h_{ijk} , \qquad B^i_{\text{bif}} = M \tag{4.96}$$

in agreement with eqs. (4.80). The corrections from magnetic fluxes arising from (4.95) to the different soft terms are parametrized for the case of fields with modular weight $\xi = 1/2$ by $\rho_{\text{bif}} \equiv \frac{c_{\text{bif}}}{t^{1/2}}$. Note that in the large t limit (corresponding to the flux diluted regime) these corrections are dominant since $\rho_{\text{bif}} > \rho$. This is consistent with the linear (instead of quadratic) dependence on the fluxes found in (4.88).

Finally we can also derive the structure of soft terms in a (I-I-A)-type configuration using the Kähler metric above. In this case we have two matter fields coming from D7-brane intersections with modular weight $\xi = 1/2$ and one adjoint field that descends from A with modular weight $\xi = 1$. The standard $\mathcal{N}=1$ supergravity formulae yield

$$m^2_f = \frac{|M|^2}{2} , \qquad A_{ijk} = -Mh_{ijk} , \qquad B_H = m^2_H = 0 \tag{4.97}$$

in agreement with eq.(4.90) as expected. The flux correction for the matter fields will be also parametrized by $\rho_f = \frac{c_f}{t^{1/2}}$ consistent with the linear dependence found in (4.90).

The above structure of soft terms does not only arises in toroidal settings but also in *swiss-cheese* compactifications [14–17] in which a stack of 7-branes containing the SM fields wraps a small cycle of size $t_s = \mathrm{Re}(T_s)$ inside a large-volume CY manifold with overall volume modulus T_b. This is also the type of configurations that one expects in local F-theory GUT models, where T_s would correspond to the local Kähler modulus associated to the local divisor S.

In the simplest type IIB swiss-cheese examples the Kähler potential for the moduli T_s and T_b is given by [60]

$$K \;=\; -2\log\big(t_b^{3/2} \;-\; t_s^{3/2}\big)\,, \tag{4.98}$$

with $t_b = \mathrm{Re}(T_b) \gg t_s$, whereas to leading order the gauge kinetic function is given by $f = T_s$. The Kähler metric for the matter fields reads

$$K_\alpha \;=\; \frac{t_s^{(1-\xi_\alpha)}}{t_b}\,, \tag{4.99}$$

with ξ_α the corresponding modular weights. Expanding the action in powers of t_s/t_b and assuming $F_{t_b,t_s} \neq 0$ we obtain the same patterns of soft-terms as in the above toroidal case, where now $M = F_{t_s}/t_s$ [58].

Note however that the microscopic derivation of soft-terms in section 4.1.1 and in this section go beyond these $\mathcal{N}=1$ supergravity results in various respects. In particular they do not assume any form for the $\mathcal{N}=1$ Kähler potential but give explicit expressions for the soft-terms in terms of the underlying general closed string background. In this regard, they are expected to be valid in more complicated non-toroidal settings and may also include the effect of IASD sources. Obtaining the closed string background around the D7-branes in a general compactification is usually a too complicated task, but once the closed string background is known, the techniques developed in the above sections allow to obtain the soft-breaking patterns for the fields in the worldvolume of D7-branes. This approach might be particularly useful for fields localized at D7-brane intersections, since their Kähler metrics are only fully known in the case of toroidal compactifications, whereas for the case of local systems like the swiss-cheese kind of setting discussed above the structure of the Kähler metrics for matter fields can at present only be guessed in terms of scaling arguments [58, 60].

4.1.3. Effect of distant branes on the local soft terms

When building phenomenological type IIB orientifold compactifications the degrees of freedom of the SM typically are located in the worldvolume of D7-branes and/or D3-branes subject to closed string and open string fluxes. The type of settings that are typically considered is shown in figure 4.1. Apart from the branes of the SM sector, there may also be additional localised sources at other regions of the compact space. For instance, there could be distant D7-branes giving rise to gaugino condensation and stabilizing some of the Kähler moduli of the compactification. There might also be anti-D3-branes, as in the KKLT setting [8], required to uplift the vacuum from AdS to dS. Alternatively, this role might also be played by distant D7-branes with self-dual magnetic fluxes in their worldvolume [61]. The effect of distant localised sources on the SM branes may be discussed in terms of their backreaction near the SM branes, as discussed e.g. in [41, 49] for the particular case of gaugino condensation on D7-branes. In this section we discuss the effect of distant localised sources on the pattern of soft breaking terms by computing the backreaction of localised sources on the local geometry.

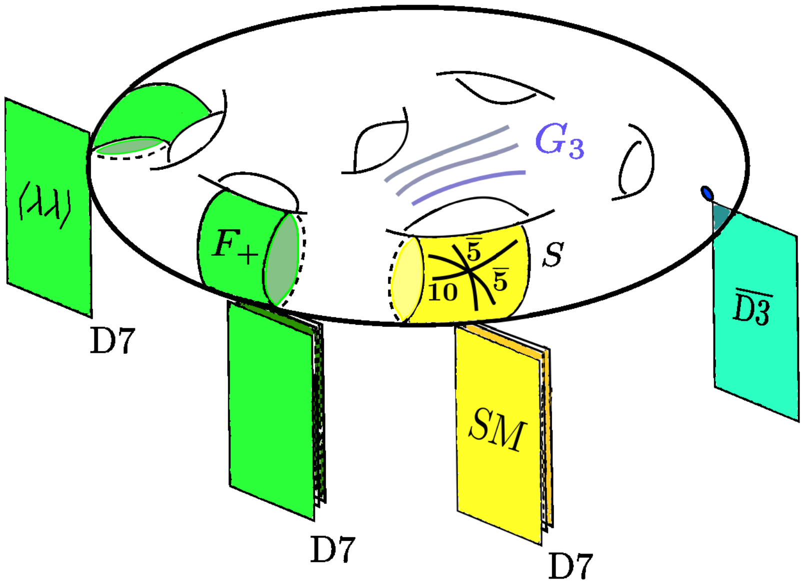


Figure 4.1: Summary of the type of sources that are present in a standard phenomenological IIB orientifold compactification. The SM is located in a stack of intersecting D7-branes with a higher dimensional SU(5) GUT structure. Apart from topologically non-trivial closed string 3-form fluxes G_3 , there are distant localized sources that may also contribute to SUSY-breaking and/or moduli stablization. These include gaugino condensation in the worldvolume of D7-branes, self-dual magnetic fluxes also in the worldvolume of D7-branes and/or anti-D3-branes. The effect of distant sources in the effective theory on the worldvolume of the SM D7-branes can be studied in terms of their backreaction in the local patch.

For concreteness we focus on the case of distant anti-D3-branes. In general these backreact the metric and the RR 5-form field-strength through the equations of motion. We have seen in previous sections that in absence of magnetization soft-terms for fields on D7-branes (both bulk and on intersections) do not depend on the metric nor on the RR 5-form and thus the presence of distant D3- or anti-D3-branes does not modify D7-brane soft terms within this approximation. This is expected, since unmagnetised D7-branes have no net D3-brane charge. However, once anti self-dual or self-dual magnetic fluxes are switched on in the worldvolume of D7-branes, some D3- or anti-D3-brane charge is respectively induced in their worldvolume. This implies that distant anti-D3-branes (or D3-branes, respectively) are now expected to give rise to corrections for the soft-terms in the worldvolume of magnetized D7-branes. Indeed, we saw in section 4.1.1.2 that magnetization leads to corrections to D7-brane soft terms that depend on the background for the metric and the RR 5-form. Although these corrections are quadratic in the magnetic fluxes, they can lead to relevant physical effects if 3-form fluxes are suppressed or in the context of fine-tuned scalar potentials, in which minute effects become important.

We begin this section by reviewing the computation of soft scalar masses induced on the worldvolume of D3-branes by distant anti-D3-branes in flat space [52]. We then move to the same computation for magnetised D7-branes in flat space. Finally, we consider compactification effects in these computations.

4.1.3.1. Scalar masses for D3-branes in the presence of distant anti-D3-branes

We first consider the case of a probe D3-brane located in (non-compact) locally flat space and a distant stack of N anti-D3-branes, and compute the induced soft scalar masses in the worldvolume of the D3-brane. This computation was addressed in Ref. [52], but we revisit it here with the aim of extending it to other settings in the next subsections.

In general, anti-D3-branes backreact the metric and the RR 4-form potential through the following type IIB supergravity equations of motion

$$-\tilde{\nabla}^2 Z = \frac{g_s}{12} G_{mnp}\bar{G}^{\widetilde{mnp}} + (2\pi\sigma)^2\tilde{\rho}_3^{\rm loc} + Z^{-1}\left[\partial_m Z\tilde{\partial}^m Z - (Z\chi)^4\partial_m\chi^{-1}\tilde{\partial}^m\chi^{-1}\right] \quad (4.100)$$
$$-\tilde{\nabla}^2\chi^{-1} = \frac{ig_s(Z\chi)^2}{12} G_{mnp} *_6 \bar{G}^{\widetilde{mnp}} + (2\pi\sigma)^2\tilde{Q}_3^{\rm loc} + 2\left[Z^{-1}\partial_m\chi^{-1}\tilde{\partial}^m Z - \chi\partial_m\chi^{-1}\tilde{\partial}^m\chi^{-1}\right]$$

where tilded quantities are taken with respect to the unwarped metric and $\tilde{\rho}_3^{\rm loc}(z)$ and $\tilde{Q}_3^{\rm loc}(z)$ are the energy density and D3/$\overline{\rm D3}$-brane charge density associated to localized sources. These equations are easily solved for backgrounds that only involve same-sign D3-brane charges (recall that we are taking the D3-branes as probes). For the particular case of a stack of N anti-D3-branes and vanishing 3-form fluxes

$$\tilde{Q}_3^{\rm loc}(z) = -\tilde{\rho}_3^{\rm loc} = -N\frac{\delta(\vec{z}_0 - \vec{z})}{\sqrt{\tilde{g}}}\ , \qquad Z = -\chi^{-1} \quad (4.101)$$

where $\vec{z}_0$ denotes the position of the stack of anti-D3-branes in the internal space Eqs. (4.100) are then proportional to each other and reduce to a standard Poisson equation in the internal space

$$-\tilde{\nabla}^2 Z = (2\pi\sigma)^2\tilde{\rho}_3^{\rm loc} \quad (4.102)$$

When the internal space is non-compact flat space this leads to the standard supergravity solution for anti-D3-branes in asymptotically flat space, namely

$$Z = 1 - \frac{g_s N\sigma^2}{\pi\,|\vec{z} - \vec{z}_0|^4} \quad (4.103)$$

Soft terms in the worldvolume of the probe D3-branes are fully determined in terms of the local backreaction around the D3-branes. Concretely, for the soft scalar masses [52]

$$m^2_{i\bar{j}} = 2K_{i\bar{j}} - \chi_{i\bar{j}} + g_s({\rm Im}\,\tau)_{i\bar{j}} \quad (4.104)$$
$$B_{ij} = 2K_{ij} - \chi_{ij} + g_s({\rm Im}\,\tau)_{ij}\ .$$

where K, χ and τ were defined in eq.(4.4). For concreteness we take the probe D3-branes to be located at the origin of coordinates. Expanding eq. (4.103) around the origin leads in real coordinates to

$$Z \ = \ 1 - \frac{g_s N\sigma^2}{\pi\, r_0^6}\left[r_0^2 + 4x_0^m x^m + 2\left(\frac{6x_0^m x_0^n}{r_0^2} - \delta^{mn}\right)x^m x^n + \ldots\right]. \quad (4.105)$$

where $r_0^2 = \sum_n (x_0^n)^2$. The linear term shows the expected instability due to the attraction between branes and anti-branes. We assume in what follows that such a term is absent, leading to a static configuration. That may originate in a variety of ways like e.g. an orbifold projection, a particularly symmetric configuration or the D3-branes being fractional and stuck at a singularity.

Comparing to eqs. (4.4) and making use of eqs. (4.101) and (4.104) we then obtain the following scalar masses and B-term in the worldvolume of the probe D3-brane

$$m^2_{i\bar{j}} = \text{const.}\left(\frac{6}{r_0^2} z_0^i \bar{z}_0{}^j - \delta^{ij}\right) , \qquad B_{ij} = \text{const.}\,\frac{6}{r_0^2} z_0^i z_0^j \tag{4.106}$$

where the proportionality constant is $8g_s N\sigma^2/(\pi r_0^6 - g_s N\sigma^2 r_0^2)$. We would have obtained the same result if we have instead considered the reverse situation, namely soft terms induced on a anti-D3-brane by the presence of a distant D3-brane.

These mass terms by themselves may easily trigger instabilities for the scalars on the D3-brane, since they may be tachyonic. For instance, if along the i-th complex plane $|z_0^i| \ll |z_0^j|$ with $i \neq j$, the second piece in the first equation (4.106) dominates and the D3-brane scalar field Φ^i becomes tachyonic. In the isotropic case, where $z_0^1 = z_0^2 = z_0^3$ and thus $\frac{z_0^i}{r_0} = \frac{1}{\sqrt{6}}$, one gets

$$m^2_{i\bar{j}} \;=\; \text{const.}\,(1-\delta^{ij}) , \qquad B_{ij} \;=\; \text{const.} \tag{4.107}$$

Hence, in that case diagonal masses vanish and off-diagonal ones are equal to the B-term. Still, there are tachyonic mass eigenstates, since the mass matrix is traceless. Note that, as emphasized in [52] this source of SUSY breaking *by itself* would lead in addition to no gaugino masses nor μ-terms and would therefore not be phenomenologycally viable for MSSM soft terms without the addition of further ingredients.

4.1.3.2. Scalar masses for magnetized D7-branes in the presence of distant anti-D3-branes

We can perform the same analysis as above for the case of magnetised D7-branes in the presence of distant anti-D3-branes in asymptotically flat space. To simplify the presentation let us consider only a non-vanishing anti self-dual magnetic flux F_- in the worldvolume of some probe D7-branes. From eq. (4.31) we get for the scalar bilinears

$$m^2_{3\bar{3}} = \sigma^2\left(2g_s^{-1}K_{3\bar{3}} - \chi_{3\bar{3}}\right)F_-^2 , \qquad B_{33} = \sigma^2\left(2g_s^{-1}K_{33} - \chi_{33}\right)F_-^2 . \tag{4.108}$$

Note that $K_{3\bar{3}}$, K_{33}, $\chi_{3\bar{3}}$ and χ_{33} obtained in the previous subsection depend on the coordinates of the D7-branes along the internal space, z^1 and z^2, and dimensional reduction to 4d is therefore non-trivial. However, if the wavefunctions of the 4d fields are strongly localized in the internal space, as occurs for instance for fields localized at Yukawa coupling enhancement points in F-theory GUTs, we can approximate wavefunctions by a delta function. Here we take for instance the case of a vector-like pair of scalars localized at the origin of coordinates. Then, making use of eqs. (4.101) and (4.105) in (4.108) we find for $|z_0^3| \gg |z_0^1|, |z_0^2|$

$$m^2_{3\bar{3}} \;=\; \text{const.}\, g_s^{-1}\sigma^2 F_-^2 , \qquad B_{33} \;=\; \frac{3}{4}\text{const.}\, g_s^{-1}\sigma^2 F_-^2 \tag{4.109}$$

where we have included an extra factor 1/2 with respect to eq. (4.108) to account for the fact that we are now considering a vector-like pair of bifundamental scalars, according to what we found in section 4.1.2.

4.1.3.3. Compactification effects

The situations discussed in this section so far are unrealistic in that they are non-compact. However, they served us to illustrate how the expressions that we found in sections 4.1.1 and 4.2.1 for the soft breaking terms can capture the contributions from distant localised sources that break supersymmetry. We would like now to consider a slightly more complete toy model on which the internal space is taken to be compact, in order to illustrate compactification effects on soft terms. Thus, we consider again the case of a stack of magnetized probe D7-branes and N distant anti-D3-branes, but we now solve eq. (4.102) in a 2-torus transverse to the D7-branes (and we smear the D3-brane charge along the remaining internal directions). Concretely, we take the magnetized D7-brane to be at the origin of coordinates and the anti-D3-branes exactly at the opposite point in the transverse T^2, as depicted in figure 4.2. In that case, linear terms automatically vanish due to the balance between the attraction forces on the two sides of the D7-branes.

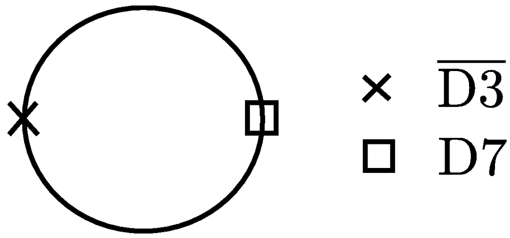


Figure 4.2: A stack of $\overline{\text{D3}}$-branes on the opposite side of the magnetized D7-branes.

Following [62] we can express the solution to eq. (4.102) in terms of the Green's function $G(x-y)$ on the transverse space to the D7-branes as

$$Z(z) = (2\pi\sigma)^2 N G(z_0^3 - z^3) \tag{4.110}$$

The Green function for a 2-torus with unit volume is

$$G(z) = \frac{1}{2\pi}\log\left|\frac{\vartheta_1(z;U)}{\vartheta_1'(0;U)}\right|^2 - \frac{(\text{Im } z)^2}{\text{Im } U} \tag{4.111}$$

where U is the complex structure of the torus and ϑ_i are the usual Jacobi theta functions. Expanding around $z_k = 1/2$ reads

$$G(z) = -\frac{1}{\pi}\left(\log\pi + \log|\vartheta_4(0;\,2U)|^2\right) - \frac{\left|z-\frac{1}{2}\right|^2}{2\,\text{Im } U} - \frac{\pi}{12}(\hat{E}_2(U) + \vartheta_3^4(0;\,U) + \vartheta_4^4(0;\,U))\left(z-\frac{1}{2}\right)^2$$
$$-\frac{\pi}{12}(\hat{E}_2(U) + \vartheta_3^4(0;\,U) + \vartheta_4^4(0;\,U))^*\left(\bar{z}-\frac{1}{2}\right)^2 + \ldots \tag{4.112}$$

where $\hat{E}_2$ is the modified second Eisenstein series defined as

$$\hat{E}_2(U) = E_2(U) - \frac{3}{\pi\text{Im } U} \tag{4.113}$$

and

$$E_2(U) = 1 - 24\sum_{n=1}^{\infty}\frac{nq^n}{1-q^n} \tag{4.114}$$

and $q = e^{2\pi i U}$. From eqs. (4.110) and (4.31) then we get in this case the following set of scalar bilinears in the worldvolume of magnetized D7-branes

$$\begin{aligned} m^2_{3\bar{3}} &= -\frac{g_s^{-1}\sigma^4\pi^2 F_-^2 N}{4\,\mathrm{Vol}(T^6)\,\mathrm{Im}\,U}\ , \\ B_{33} &= \frac{g_s^{-1}\sigma^4\pi^2 F_-^2 N}{4\,\mathrm{Vol}(T^6)\,\mathrm{Im}\,U}\left[1 - \frac{3\,\mathrm{Im}\,U}{\pi}(E_2(U) + \vartheta_3^4(0;\,U) + \vartheta_4^4(0;\,U))\right] \end{aligned} \tag{4.115}$$

where we have introduced the volume of the internal space back in these expressions. Note that soft-terms in particular now depend on the complex structure of the transverse 2-torus. It is also interesting to recall the interpretation of the different terms in these expressions from an effective field theory point of view. Indeed, scalar masses and the first contribution in the B-term are tree-level contributions similar to those computed in the previous subsections. However, in the present compact case the B-term receives in addition loop threshold corrections that are exponential in the complex structure of the 2-torus. Those come from integrating out heavy modes that propagate in the transverse T^2 and stretch between the D7-branes and the anti-D3-branes.

This example, as it stands, is a toy model with no direct phenomenological interest. In particular, scalar masses are tachyonic, showing the instability of D7-branes under small fluctuations. The tachyonic instability in this setting was expected, since once the anti-D3-branes move a bit from their original position, the attractive forces on the two sides of the anti-D3-branes are no-longer balanced and they quickly decay towards the magnetized D7-branes. In this regard, it might be interesting to extend this example by including closed string fluxes and see whether it is possible to make it stable.

4.1.4. Hypercharge dependence of soft terms in F-theory

In this section we study the effect of fluxes on soft terms for fields on local F-theory SU(5) models with enhanced $SO(12)$ and E_6 symmetries. We are particularly interested in the dependence on the hypercharge flux, required for the breaking from SU(5) to the SM gauge group.

4.1.4.1. Hypercharge dependence at SO(12) point

In previous sections we have considered corrections of open string magnetic fluxes to the soft terms of 7-brane fields, including also fields localized at intersections. To simplify the discussion, we considered a toy model with an underlying $U(3)$ gauge symmetry. The generalization to gauge symmetries of phenomenological interest is however straightforward. Indeed, we now apply the results of the above sections to SU(5) unification in the context of type IIB/F-theory compactifications. More precisely, in this subsection we concentrate in the case of local F-theory SU(5) GUTs with gauge symmetry enlarged to $SO(12)$ at complex co-dimension 3 singularities. This is the gauge symmetry enhancement that is relevant for the presence of local Yukawa couplings of the form $\bar{\mathbf{5}}\times\mathbf{10}\times\bar{\mathbf{5}}_{\mathbf{H}}$ that lead to masses for charged leptons and D-type quarks. In particular, we identify the possible (e.g. hypercharge) magnetic flux dependence of the scalar soft masses, as it might be of phenomenological relevance.

We consider the same local $SO(12)$ F-theory structure as introduced in Ref. [42] (see also [63]). To avoid expressions with two many indices, throughout this section we use the alternative notation $x \equiv z_1$ and $y \equiv z_2$ to denote the two local complex coordinates in the 4-cycle S. The vev for the transverse 7-brane position field is given by

$$\langle \Phi_{xy} \rangle \ = \ m^2(xQ_x \ + \ yQ_y) \tag{4.116}$$

where m is related to the intersection slope of the 7-branes, as we have already discussed, and it is generically of order the string scale. Q_x and Q_y are $SO(12)$ Cartan generators breaking the symmetry respectively down to $SU(6) \times U(1)$ and $SO(10) \times U(1)$. As in the $U(3)$ toy model of previous sections, we have matter curves Σ_a, Σ_b and Σ_c at $x = 0$, $y = 0$ and $x = y$ respectively. Matter curves Σ_a and Σ_b host respectively 5-plets and 10-plets associated to quarks and leptons, while Σ_c hosts 5_H-plets that include the Higgs multiplets. In order to get chiral matter and family replication we must add magnetic fluxes to this setting. We follow Ref. [42] and consider the above local system of matter curves subject to approximately constant magnetic fields, that break the gauge symmetry down to that of the SM and give rise to chirality. The magnetic flux background comes in three pieces,

$$\langle F_2 \rangle \ = \ \langle F_{(1)} \rangle \ + \ \langle F_{(2)} \rangle \ + \ \langle F_Y \rangle \tag{4.117}$$

with

$$\begin{aligned} \langle F_{(1)} \rangle \quad &= \quad i\,(M_x\,dx \wedge d\bar{x} + M_y\,dy \wedge d\bar{y})\;Q_F \\ \langle F_{(2)} \rangle \quad &= \quad i\,(dx \wedge d\bar{y} + dy \wedge d\bar{x})\,(N_a Q_x + N_b Q_y) \\ \langle F_Y \rangle \quad &= \quad i\left[(dx \wedge d\bar{y} + dy \wedge d\bar{x})\,N_Y + (dy \wedge d\bar{y} - dx \wedge d\bar{x})\,\tilde{N}_Y\right] Q_Y \ . \end{aligned} \tag{4.118}$$

and $Q_F = -Q_x - Q_y$. The first piece leads to chirality (and matter replication) for fields that are localized in the matter curves $\Sigma_{a,b}$. The second piece leads to chirality for the Higgs fields, localized in the matter curve Σ_c. This is interesting in order to obtain doublet-triplet splitting and a suppressed μ-term. Finally, the third piece corresponds to a magnetic flux along the hypercharge direction, that breaks SU(5) down to the SM gauge group and for the particular choice $N_Y = 3(N_a - N_b)$ is consistent with doublet-triplet splitting. We refer to [42] for further details on this configuration.

Thus, putting all pieces together the complete flux may be written as

$$\langle F_2 \rangle = i(dy \wedge d\bar{y} - dx \wedge d\bar{x})Q_P + i(dx \wedge d\bar{y} + dy \wedge d\bar{x})Q_S + i(dy \wedge d\bar{y} + dx \wedge d\bar{x})M_{xy}Q_F \tag{4.119}$$

where

$$Q_P = \tilde{M}Q_F + \tilde{N}_Y Q_Y \quad , \quad Q_S = N_a Q_x + N_b Q_y + N_Y Q_Y \tag{4.120}$$

and

$$\tilde{M} = \frac{1}{2}(M_y - M_x) \ , \qquad M_{xy} = \frac{1}{2}(M_y + M_x) \tag{4.121}$$

The local D-term SUSY condition would imply $M_{xy} = 0$.

The local internal wavefunctions of the fields must satisfy the system of differential equations that we have described in section 4.2.1 and were solved in [42] for this particular case. The zero modes for each sector are given (in the *holomorphic gauge*) by

$$\Psi_\rho = \begin{pmatrix} -\frac{i\lambda_x}{m^2} \\ \frac{i\lambda_y}{m^2} \\ 1 \end{pmatrix} \chi^i_\rho E_\rho \ , \qquad \chi^i_\rho = e^{-q_\Phi(\lambda_x\bar{x} - \lambda_y\bar{y})} f_i(\lambda_x y + \lambda_y x) \qquad \rho = a^\pm, b^\pm, c^\pm \tag{4.122}$$

where E_ρ are the corresponding $SO(12)$ step generators for fields in $\Sigma_{a,b,c}$. The values of the parameters q_Φ and $\lambda_{x,y}$ are given in table 4.1 for each of the fields in the curves $\Sigma_{a,b,c}$. The physical, normalizable, wavefunctions in the *real gauge* can be obtained from

$$\chi_\rho^{\text{real}} \ = \ e^{i\Omega}\chi_\rho^{\text{hol}} \tag{4.123}$$

where

$$\Omega \ = \ \tfrac{i}{2}\left[\left(|y|^2-|x|^2\right)Q_P + (x\bar{y}+y\bar{x})\,Q_S + M_{xy}\left(|y|^2+|x|^2\right)Q_F\right] \ . \tag{4.124}$$

The constants $\lambda_\pm$ that appear in table 4.1 are defined as the lowest eigenvalue for the

ρ	q_Φ	λ_x	λ_y	SU(5) rep.
a_p^+	$-x$	λ_+	$-q_s\frac{\lambda_+}{\lambda_+-q_p}$	$\bar{\mathbf{5}}$
a_p^-	x	λ_-	$q_s\frac{\lambda_-}{\lambda_-+q_p}$	$\mathbf{5}$
b_q^+	y	$-q_s\frac{\lambda_+}{\lambda_++q_p}$	λ_+	$\mathbf{10}$
b_q^-	$-y$	$q_s\frac{\lambda_-}{\lambda_--q_p}$	λ_-	$\overline{\mathbf{10}}$
c_r^+	$x-y$	$\frac{q_s\lambda_+-m^4}{\lambda_++q_p-q_s}$	$-\lambda_+-\frac{q_s\lambda_+-m^4}{\lambda_++q_p-q_s}$	$\bar{\mathbf{5}}$
c_r^-	$-(x-y)$	$-\frac{q_s\lambda_-+m^4}{\lambda_--q_p+q_s}$	$-\lambda_++\frac{q_s\lambda_-+m^4}{\lambda_--q_p+q_s}$	$\mathbf{5}$

Table 4.1: Wavefunction parameters.

sectors $a_p^\pm, b_q^\pm$ and $c_r^\pm$ and satisfy the following cubic equations [42]

$$\begin{aligned}(\lambda_i^a)^3-(m^4+(q_p^a)^2+(q_s^a)^2)\lambda_i^a+m^4q_p^a \ &= \ 0\\ (\lambda_i^b)^3-(m^4+(q_p^b)^2+(q_s^b)^2)\lambda_i^b-m^4q_p^b \ &= \ 0\\ (\lambda_i^c)^3-(2m^4+(q_p^c)^2+(q_s^c)^2)\lambda_i^c+2m^4q_s^c \ &= \ 0\end{aligned} \tag{4.125}$$

where for simplicity we have assumed the D-term condition $M_{xy}=0$. To first order in the fluxes the constants $\lambda_\pm$ are given by

$$\lambda_\pm^a=\mp m^2-\frac{1}{2}q_p^a+\ldots \ ; \ \lambda_\pm^b=\mp m^2+\frac{1}{2}q_p^b+\ldots \ \ ; \ \lambda_\pm^c=\mp\sqrt{2}m^2-\frac{1}{2}q_s^c+\ldots \tag{4.126}$$

In order to compute the physical soft masses we need to normalize these local wave functions. It is useful to factorize the normalization of the vector in eq. (4.122) from the normalization of the scalar function χ_ρ^i so that

$$\langle\Psi_\rho^i|\Psi_\rho^j\rangle = m_*^2\int_S \mathrm{Tr}(\Psi_\rho^i\Psi_\rho^j)\,\mathrm{dvol}_S = 2m_*^2||\vec{v}_\rho||^2\int_S\chi_\rho^i(\chi_\rho^j)^*\,\mathrm{dvol}_S=\delta_{ij} \tag{4.127}$$

where

$$\vec{v}_\rho=\begin{pmatrix}-\frac{i\lambda_x}{m^2}\\ \frac{i\lambda_y}{m^2}\\ 1\end{pmatrix}_\rho \tag{4.128}$$

and hence $||\vec{v}_\rho||^2 = 1 + \frac{\lambda_x^2}{m^4} + \frac{\lambda_y^2}{m^4}$. By using the definition of λ_x and λ_y in table 4.1 and eq. (4.126), we get

$$\begin{aligned} ||\vec{v}_{a^\pm}||^{-2} &\simeq \frac{1}{2}\left(1 \mp \frac{q_p^{a^\pm}}{2m^2} + \ldots\right) \\ ||\vec{v}_{b^\pm}||^{-2} &\simeq \frac{1}{2}\left(1 \pm \frac{q_p^{b^\pm}}{2m^2} + \ldots\right) \\ ||\vec{v}_{c^\pm}||^{-2} &\simeq \frac{1}{2}\left(1 \mp \frac{q_s^{c^\pm}}{2\sqrt{2}m^2} + \ldots\right) \end{aligned} \tag{4.129}$$

Having the normalized internal wavefunctions, we can compute the soft masses for these fields by making use of the results of previous sections. For simplicity we only consider soft scalar masses in the presence of an ISD (0,3)-form closed string background. We expand the non-Abelian DBI+CS action of the 7-brane in powers of the transverse adjoint $SO(12)$ scalar Φ in the presence of a non-trivial $G_{(0,3)}$ flux, as we did in section 4.1.1, obtaining

$$\mathcal{L}_{8\mathrm{d}} = \mathrm{Tr}\left(D_a\Phi D^a\bar{\Phi} - \frac{1}{4}F_{ab}F^{ab} - \frac{g_s}{2}|G|^2|\Phi|^2 + \ldots\right) \tag{4.130}$$

The local flux density induces a 8d mass term for the transverse scalar Φ. Upon dimensional reduction in the presence of non-trivial backgrounds $\langle\Phi\rangle$ and $\langle F_2\rangle$ this leads to 4d soft masses for the fields localized at the matter curves. The scalar Φ transforms in the adjoint representation of $SO(12)$ and can thus be decomposed as

$$\mathrm{Tr}(|\Phi|^2) = |\Phi_{a_p^+}|^2 + |\Phi_{a_p^-}|^2 + |\Phi_{b_q^+}|^2 + |\Phi_{b_q^-}|^2 + |\Phi_{c_r^+}|^2 + |\Phi_{c_r^-}|^2 + \ldots \tag{4.131}$$

where Φ_ρ corresponds to the third component of eq. (4.122), namely the internal wavefunction of the transverse scalar that solves the equation of motion in each sector. The induced 4d soft masses for the matter fields living in the sector a^+ therefore read

$$(m_{ij}^{a_p^+})^2 = \frac{g_s}{2\,\mathrm{Vol}(S)}\int_S \mathrm{dvol}_S\,|G|^2|\Phi_{a_p^+}|^2 = \frac{g_s}{2\mathrm{Vol}(S)||\vec{v}_{a_p}||^2}\int_S \mathrm{dvol}_S\,|G|^2\chi^i_{a_p^+}(\chi^j_{a_p^+})^* \tag{4.132}$$

Using the definition of λ_x and λ_y in table 4.1 we obtain

$$(m^{ij}_{a_p^+})^2 = \frac{g_s}{4\mathrm{Vol}(S)}\int_S \mathrm{dvol}_S\,|G|^2\left(1 - \frac{q_p^{a^+}}{2m^2}\right)\chi^i_{a_p^+}(\chi^j_{a_p^+})^*\,. \tag{4.133}$$

Note that in the presence of magnetic fluxes, scalar kinetic terms get also flux corrections. However, those start at quadratic order in the magnetic flux, so that they only give rise to subleading corrections to this expression for the soft masses. When the flux G is constant over the 4-fold we recover the results of eq. (4.88), extended to the SU(5) GUT case here considered. Analogously for the sector a^- we get,

$$(m^{ij}_{a_p^-})^2 = \frac{g_s}{4\mathrm{Vol}(S)}\int_S \mathrm{dvol}_S\,|G|^2\left(1 + \frac{q_p^{a^-}}{2m^2}\right)\chi^i_{a_p^-}(\chi^j_{a_p^-})^* \tag{4.134}$$

Taking into account that the zero mode in the sector a_p^+ (a_p^-) is normalizable only if $q_p^{a^+} > 0$ ($q_p^{a^-} < 0$), we can rewrite these expressions as

$$(m_{a_p}^{ij})^2 \;=\; \frac{g_s}{4\text{Vol}(S)} \int_S \text{dvol}_S\, |G|^2 \left(1 - \frac{|q_p^{a_p}|}{2m^2}\right) \chi_{a_p}^i (\chi_{a_p}^j)^* \tag{4.135}$$

assuming that only one of the two modes is actually present in the massless spectrum. The result for the sector $b_q^{\pm}$ reads

$$(m_{b_q}^{ij})^2 \;=\; \frac{g_s}{4\text{Vol}(S)} \int_S \text{dvol}_S\, |G|^2 \left(1 - \frac{|q_p^{b_q}|}{2m^2}\right) \chi_{b_q}^i (\chi_{b_q}^j)^* \tag{4.136}$$

where we have used that the zero mode in the sector b_q^+ (b_q^-) is normalizable only if $q_p^{b^+} < 0$ ($q_p^{b^-} > 0$). Finally for the sector $c_r^{\pm}$ we obtain

$$(m_{c_r}^{ij})^2 \;=\; \frac{g_s}{4\text{Vol}(S)} \int_S \text{dvol}_S\, |G|^2 \left(1 - \frac{|q_s^{c_r}|}{2\sqrt{2}m^2}\right) \chi_{c_r}^i (\chi_{c_r}^j)^* \;. \tag{4.137}$$

Thus we observe that only fields with different absolute value of hypercharge have different soft masses at the unification scale. In particular, both Higgs fields H_u and H_d have equal soft masses as long as they feel the same amount of hypercharge flux. This will be important in section 4.3.4.

Making use of the SO(12) chiral spectrum summarized in table 4.2, we can express the above results in a more compact form

$$(m^{ij})^2 \;=\; \frac{g_s}{4\text{Vol}(S)} \int_S \text{dvol}_S\, |G|^2 \left(1 - \frac{1}{2m^2}\left|\eta\tilde{M} - q_Y\tilde{N}_Y\right|\right) \chi_{c_r}^i (\chi_{c_r}^j)^* \tag{4.138}$$

where $\eta = +1, -1, 0$ respectively for matter fields in the $\bar{\mathbf{5}}$, $\mathbf{10}$ and $\bar{\mathbf{5}}_{\mathbf{H}}$ multiplets, and q_Y is the usual SM hypercharge (i.e.$Y(E_R) = 1$). Moreover, for the case of the Higgs doublets the replacement $\tilde{N}_Y \to \frac{5}{3\sqrt{2}}N_Y$ should be also made in this expression.

If the fluxes are approximately constant over the 4-cycle S we can perform the integral over the normalized wave functions, getting

$$(m^{ij})^2 \;=\; \frac{M^2\delta_{ij}}{2}\left(1 - \frac{1}{2m^2}\left|\eta\tilde{M} - q_Y\tilde{N}_Y\right|\right) \tag{4.139}$$

where we have expressed G in terms of the gaugino mass M, see eq. (4.22).

The possible phenomenological relevance of the magnetic flux contributions depend on the size of the fluxes. A naive estimate shows that these corrections are potentially important. Indeed, flux quantization imply $\int_{\Sigma_2}\langle F_2\rangle \simeq 2\pi$, so that we expect $\tilde{M} \simeq N_Y \simeq \tilde{N}_Y \simeq (2\pi)/\text{Vol}_S^{1/2}$. On the other hand we know that $\alpha_G \simeq 1/(M_s^4\text{Vol}_S) \simeq 1/24$, so that flux contributions are expected to be of order $\sim 0.2\, M_s^2$.

We can extract some additional information on the structure and size of the fluxes from other phenomenological considerations. Indeed, magnetic fluxes have also been shown to play an important role in the computation of Yukawa couplings in local F-theory models. In [42] it was found an expression relating ratios of second and third generation quark/lepton masses to local fluxes in an F-theory $SO(12)$ setting,

$$\frac{m_\mu/m_\tau}{m_s/m_b} = \left(\frac{(\tilde{M} + \frac{1}{2}\tilde{N}_Y)(\tilde{M} + \tilde{N}_Y)}{(\tilde{M} - \frac{1}{3}\tilde{N}_Y)(\tilde{M} + \frac{1}{6}\tilde{N}_Y)}\right)^{1/2} \tag{4.140}$$

Sector	Chiral mult.	$SU(3) \times SU(2)$	q_Y	q_p
a_1^+	D_R	$3(\bar{\mathbf{3}}, \mathbf{1})$	$\frac{1}{3}$	$\tilde{M} - \frac{1}{3}\tilde{N}_Y$
a_2^+	L	$3(\mathbf{1}, \mathbf{2})$	$-\frac{1}{2}$	$\tilde{M} + \frac{1}{2}\tilde{N}_Y$
b_1^+	U_R	$3(\bar{\mathbf{3}}, \mathbf{1})$	$-\frac{2}{3}$	$-\tilde{M} + \frac{2}{3}\tilde{N}_Y$
b_2^+	Q_L	$3(\mathbf{3}, \mathbf{2})$	$\frac{1}{6}$	$-\tilde{M} - \frac{1}{6}\tilde{N}_Y$
b_3^+	E_R	$3(\mathbf{1}, \mathbf{1})$	1	$-\tilde{M} - \tilde{N}_Y$
Sector	Chiral mult.	$SU(3) \times SU(2)$	q_Y	q_s
c_1^+	D_d	$(\bar{\mathbf{3}}, \mathbf{1})$	$\frac{1}{3}$	0
c_2^+	H_d	$(\mathbf{1}, \mathbf{2})$	$-\frac{1}{2}$	$\frac{5}{6}N_Y$

Table 4.2: SO(12) chiral spectrum.

This expression is independent of the hierarchical (non-perturbative) origin of Yukawa couplings and is based on the fact that holomorphic Yukawas must respect the SU(5) gauge symmetry, even after flux-breaking to the SM gauge group. The difference in Yukawas of charged leptons τ, μ and b, s quarks originates exclusively from the different (hypercharge-dependent) fluxes present at the matter curves, which appear through wavefunction normalisation. Eq. (4.140) applies at the unification scale. Including the RG running and uncertainties one finds agreement with low-energy data for $\frac{m_\mu/m_\tau}{m_s/m_b} = 3.3 \pm 1$ at the GUT scale, therefore implying $\tilde{N}_Y/\tilde{M} = 1.2 - 2.4$ [42].

In order to see the implications of this relation on the structure of soft terms, let us demand without loss of generality that the local zero modes arise from the sectors a^+, b^+ and c^+. In terms of the local flux densities, that requires

$$-\tilde{M} - q_Y\tilde{N}_Y \;<\; 0 \;<\; \tilde{M} - q_Y\tilde{N}_Y \quad \text{and} \quad N_Y \;>\; 0 \tag{4.141}$$

for every possible value of the hypercharge q_Y. Eq. (4.139) then implies a hierarchy of soft scalar masses for each generation

$$m_E^2 \;<\; m_L^2 \;<\; m_Q^2 \;<\; m_D^2 \;<\; m_U^2 \;, \tag{4.142}$$

at the unification scale. This non-degenerate structure is different from those induced by the RG running or D-terms in the MSSM, and may have interesting phenomenological consequences. Moreover, the average scalar squared mass for fields in the 5-plet and 10-plet of each generation, m_0^2, is independent of the hypercharge flux,

$$m_0^2 \;=\; \frac{1}{5}\left(3m_D^2 + 2m_L^2\right) \;=\; \frac{1}{10}\left(6m_Q^2 + 3m_U^2 + m_E^2\right) \;=\; \frac{M^2}{2}\left(1 - \frac{\tilde{M}}{2}\right) \tag{4.143}$$

where fluxes are written in units of m^2. Thus, we can write soft masses for the 5-plet, the 10-plet and the Higgs H_d respectively as

$$\begin{aligned} m_{\mathbf{5}}^2 \;&=\; m_0^2 \;+\; \frac{q_Y}{4}\tilde{N}_Y M^2 \\ m_{\mathbf{10}}^2 \;&=\; m_0^2 \;-\; \frac{q_Y}{4}\tilde{N}_Y M^2 \\ m_{H_d}^2 \;&=\; m_0^2 \;+\; \frac{M^2}{2}\left(\frac{\tilde{M}}{2} \;-\; \frac{5}{6\sqrt{2}}|q_Y N_Y|\right) \end{aligned} \tag{4.144}$$

These equations neatly show the linear dependence of the soft masses on the hypercharge fluxes.

In [42] it was also shown that certain choices of the magnetic fluxes lead to h_b/h_τ Yukawa ratios that are consistent with the experimentally observed values, for example, $\tilde{M} \simeq 0.3$, $\tilde{N}_Y \simeq 0.4$ and $N_Y \simeq 0.6$ in units of m^2. With the above expressions, such values lead to the following pattern of soft masses at the unification scale

$$m^2(Q, U, D, L, E, H_d) \;=\; \frac{M^2}{2}(0.82,\, 0.98,\, 0.92,\, 0.75,\, 0.65,\, 0.82). \tag{4.145}$$

Thus, we observe that squark squared masses and slepton and Higgs squared masses become respectively $10-20\%$ and $25-35\%$ smaller than the hypercharge-uncorrected value. Note however that the precise results depend on the particular values for the fluxes, and there are other flux choices also leading to Yukawa couplings consistent with experimental constraints. It would we interesting to do a full scan over flux parameters giving consistent Yukawa results to see their impact on the obtained soft masses.

It is interesting to note how in this scheme the fermion mass spectrum gives information on the structure of sfermion masses, whereas in the standard context of supersymmetric field theory these would be independent quantities. We have not studied in detail the phenomenology of a MSSM model subject to a hierarchy of soft scalar masses of the form (4.142), but we note that a particularly interesting feature is that in such a scheme the stau has the smallest soft mass (after taking into account the running of the gauge and Yukawa couplings) and may easily be the next-to-lightest SUSY particle (NLSP). This in particular might be relevant for having the appropriate amount of neutralino dark matter through stau-neutralino coannihilation. It would be interesting to perform a RGE analysis and study the generation of EW radiative symmetry breaking in a model with this structure, including this new hypercharge degree of freedom. This would correspond to an extension of the work in [58, 64, 65].

We now turn to describe the effect of magnetic fluxes on the trilinear couplings, in the context of this local $SO(12)$ F-theory setting. As we discussed in section 4.1.2.3, the leading effect of the magnetization results from the modification of the $\Phi - A$ mixing. Since by supersymmetry this modification is the same for the scalar and auxiliary fields, we can factorize the correction induced by the fluxes in the scalar potential (see eq. (4.86)). Consequently, both scalar masses and trilinear couplings receive the same correction, that we have already derived in eqs. (4.135)-(4.137). After summing over the three matter curves that are involved in the coupling, the soft trilinear coupling takes the form

$$A = -\frac{M}{2}\left(3 - \frac{|q_p^{a_p}|}{2m^2} - \frac{|q_p^{b_q}|}{2m^2} - \frac{|q_s^{c_r}|}{2\sqrt{2}m^2}\right) \tag{4.146}$$

where the parameters q_p and q_s are given in table 4.2 for the relevant sectors of the theory (see also table 2 in [42] for the complete spectrum, including the non-normalizable modes). Requiring the flux densities to satisfy eq. (4.141) is equivalent to imposing $q_p^{a^+} > 0$, $q_p^{b^+} < 0$ and $q_s^{c^+} > 0$. Thus, making use of table 4.2 in eq. (4.146), we can recast the down- and lepton-type trilinear couplings as

$$
\begin{aligned}
A_d &= -\frac{M}{2}\left(3 - \tilde{M} + \frac{\tilde{N}_Y}{12} - \frac{5N_Y}{12\sqrt{2}}\right) \\
A_l &= -\frac{M}{2}\left(3 - \tilde{M} - \frac{3\tilde{N}_Y}{4} - \frac{5N_Y}{12\sqrt{2}}\right)
\end{aligned} \tag{4.147}
$$

where fluxes have been expressed in units of m^2.

A more complicate issue is that of the induced B-terms. Indeed, in these local F-theory $SO(12)$ and E_6 settings the Higgs fields H_u and H_d are chiral and live on different matter curves. A μ-term would have to be generated by some e.g. non-perturbative effect. The final physical μ-term is related to the integral of the two wavefunctions and is only non-vanishing if the H_u and H_d matter curves overlap. It would be interesting to study a local configuration in which, in addition, the two Higgs matter curves intersect at a point with $SU(7)$ enhancement, leading to an effective μ-term from the coupling to a singlet, as suggested e.g. in [31].

4.1.4.2. Soft terms at E_6 enhancement points

In the above subsection we have considered F-theory SU(5) unification with an underlying $SO(12)$ gauge symmetry enhancement at the point where the internal wavefunctions localize. Such configuration is incomplete in that up-type $\mathbf{10}\times\mathbf{10}\times\mathbf{5_H}$ Yukawa couplings are not generated, as those require an E_6 gauge symmetry enhancement [30]. In order to reproduce the desired rank-one structure of Yukawas, one must take into account non-trivial 7-brane monodromies, which may be conveniently described in terms of T-brane configurations [66]. From the point of view of the effective 8d theory, this amounts to considering non-Abelian profiles for the transverse scalar [37]. This approach was in particular used in [43] to perform the explicit computation of up-type Yukawa couplings in local F-theory SU(5) GUTs.

In this subsection we address the computation of soft masses for fields localized at a **10** matter curve near a point of E_6 gauge symmetry enhancement. The novel feature with respect to the $SO(12)$ case discussed above is that the profile of the transverse scalar $\langle\Phi\rangle$ does not necessarily commute with other elements of the background and, in particular, $[\langle\Phi\rangle, \langle\bar{\Phi}\rangle] \neq 0$. Thus, in order to satisfy the D-term condition,

$$
\omega \wedge F + \frac{1}{2}[\Phi, \bar{\Phi}] = 0 \tag{4.148}
$$

with ω the Kähler form, we must turn on a non-primitive background flux $\langle F_{\rm NP}\rangle$. This non-trivial background can be parametrized in terms of a real function f such that

$$
[\langle\Phi\rangle, \langle\bar{\Phi}\rangle] = \hat{m}^2(e^{2f} - \hat{m}^2|x|^2 e^{-2f})P\ , \qquad \langle F_{\rm NP}\rangle = -i\partial\bar{\partial} f P \tag{4.149}
$$

where P is some combination of the Cartan generators of E_6 and x a local coordinate of the 4-cycle S. At short distances the function f can be expanded as

$$
f(r) = \log c + c^2\hat{m}^2|x|^2 + \hat{m}^4|x|^4\left(\frac{c^4}{2} - \frac{1}{4c^2}\right) + \ldots \tag{4.150}
$$

Hence, we can parametrize the solution in terms of a real dimensionless constant c that encodes the details of the global embedding of the 7-brane local model.

Near the Yukawa point we can approximate $f(r) = \log c + c^2\hat{m}^2|x|^2 + \ldots$ such that the flux $F_{\rm NP}$ is constant and we can compute analytically the wavefunctions around that point. The two **10** matter curves, although coming from the same smooth curve $\Sigma_{\mathbf{10}}$, seem locally different. They have a different local zero mode associated to each curve, given by

$$\psi^j_{\mathbf{10}^+} = \frac{1}{||\vec{v}_{\mathbf{10}}||}\begin{pmatrix} \frac{i\lambda_{\mathbf{10}}}{\hat{m}^2} \\ -\frac{i\lambda_{\mathbf{10}}\xi_{\mathbf{10}}}{\hat{m}^2} \\ 0 \end{pmatrix} e^{f/2}\chi^j_{\mathbf{10}}\,, \qquad \psi^j_{\mathbf{10}^-} = \frac{1}{||\vec{v}_{\mathbf{10}}||}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} e^{-f/2}\chi^j_{\mathbf{10}} \tag{4.151}$$

where $||\vec{v}_{\mathbf{10}}||$ is the normalization factor of the wavefunction across the entire $\Sigma_{\mathbf{10}}$ matter curve and $\lambda_{\mathbf{10}}$ is the negative root of the equation

$$\hat{m}^4(\lambda_{\mathbf{10}} - q_p) \, + \, \lambda_{\mathbf{10}}c^2\left(c^2\hat{m}^2(q_p - \lambda_{\mathbf{10}}) - \lambda^2_{\mathbf{10}} + q_p^2 + q_s^2\right) \; = \; 0 \tag{4.152}$$

and $\xi_{\mathbf{10}} = -q_s/(\lambda_{\mathbf{10}} - q_p)$. The scalar wavefunction $\chi^j_{\mathbf{10}}$ takes the same form than in the $SO(12)$ model above. Indeed the only difference with respect to the above local model resides in the value of $\lambda_{\mathbf{10}}$ due to the presence of the parameter c in eq. (4.152). Solving that equation for small magnetic fluxes q_p, q_s we find that to first order in the fluxes $\lambda_{\mathbf{10}}$ is given by

$$\begin{aligned} \lambda_{\mathbf{10}} \; &= \; -\hat{m}^2 g_1(c) - g_2(c)\frac{q_p}{2} + \ldots \; = \; -\frac{\hat{m}^2}{2c}\left(c^3 + \sqrt{4+c^6}\right) - \frac{1}{2}\left(1 + \frac{c^3}{\sqrt{4+c^6}}\right) q_p + \ldots \\ &= \; -\hat{m}^2\left(\frac{1}{c} + \frac{c^2}{2} + \ldots\right) \; - \; \left(1 + \frac{c^3}{2} + \ldots\right)\frac{q_p}{2} \; + \; \ldots \end{aligned} \tag{4.153}$$

where in the last line we have expanded for small c. Thus, to linear order $\lambda_{\mathbf{10}} \simeq -\frac{\hat{m}^2}{c} - \frac{q_p}{2}$ and the wavefunctions (4.151) are well approximated by

$$\psi^j_{\mathbf{10}^+} \; \simeq \; \frac{1}{||\vec{v}_{\mathbf{10}}||}\begin{pmatrix} -i\left(1 + \frac{q_p c}{2\hat{m}^2}\right) \\ i\frac{q_s c}{\hat{m}^2} \\ 0 \end{pmatrix} \frac{1}{\sqrt{c}}\,\chi^j_{\mathbf{10}}\,, \qquad \psi^j_{\mathbf{10}^-} \; \simeq \; \frac{1}{||\vec{v}_{\mathbf{10}}||}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \frac{1}{\sqrt{c}}\,\chi^j_{\mathbf{10}} \tag{4.154}$$

To first order in the fluxes, the normalization factor reads

$$||\vec{v}_{10}||^{-2} = \frac{1}{2}\left(1 - \frac{q_p c}{2\hat{m}^2} + \ldots\right) \tag{4.155}$$

Soft masses for fields living in the **10** matter curve are (for constant fluxes) given by

$$m^2_{\mathbf{10}} \; = \; |M|^2 \int_S d\mathrm{vol}_S \, |\Phi_{\mathbf{10}}|^2 \; = \; |M|^2 \int_S d\mathrm{vol}_S \, \left(|\Phi_{\mathbf{10}^+}|^2 + |\Phi_{\mathbf{10}^-}|^2\right) \tag{4.156}$$

where M is the gaugino mass and $\Phi_{\mathbf{10}^\pm}$ is the lower entry of the vectors (4.154), including the normalization factor. Therefore, we obtain the following result for the soft masses

$$m^2_{\mathbf{10}} \; = \; |M|^2\frac{1}{1 + c^2 g_1^2}\left(1 - \frac{c^2 g_1 g_2}{1 + c^2 g_1^2}\frac{q_p}{\hat{m}^2} + \ldots\right) \; \simeq \; \frac{|M|^2}{2}\left(1 - \frac{q_p c}{2\hat{m}^2}\right). \tag{4.157}$$

where we have kept only the leading contribution of the primitive fluxes and taken the limit for small c in the last step. Note that the magnetic flux correction depends now on the parameter c, that parametrizes the non-primitive flux. Moreover, note that the limit $c \to 0$ does not correspond to the result that we obtained in the previous section for the curve Σ_b in the $SO(12)$ case. This is in fact something expected. Indeed, looking at the commutator in eq. (4.149)

$$[\langle\Phi\rangle, \langle\bar{\Phi}\rangle] \;=\; \hat{m}^2 \left(c^2 - \hat{m}^2|x|^2\frac{1}{c^2}\right) + \ldots \tag{4.158}$$

we observe that there is not a continuous way to make $[\langle\Phi\rangle, \langle\bar{\Phi}\rangle] \to 0$ by turning off c, as it diverges for $c \to 0$. Hence, this T-brane configuration gives rise to a new qualitative behaviour that is encoded in the non-trivial dependence of the soft masses on c. From a phenomenological point of view though this parameter can be seen just as a redefinition of the flux density that modifies the soft masses. In particular, the hierarchy between the masses for the fields living in the $\bar{\mathbf{5}}$ curve or the $\mathbf{10}$ curve depends on the value of c. Interestingly, extending the solution for $f(r)$ to all the real axis and requiring absence of poles leads to $c \sim 0.73$. If this is the case, there is only a small suppression on the flux correction and the scalars living in the $\mathbf{10}$ matter curve are only slightly heavier than those in the $\bar{\mathbf{5}}$ curve.

Let us conclude with the soft mass corresponding to H_u. In this setup the Higgs sector is chiral and both Higgses H_d and H_u live in different matter curves, $\bar{\mathbf{5}}$ and $\mathbf{5}$ respectively. In the previous section we studied the soft mass for H_d near a point of SO(12) gauge symmetry enhancement. However, in order to allow for an up-type Yukawa coupling we have seen that we need to go to a point of E_6 gauge symmetry enhancement. Fortunately, unlike the $\mathbf{10}$ curve, the $\mathbf{5}$ curve does not feel the presence of the non-primitive flux $\langle F_{NP}\rangle$ so the wavefunctions are the same than those obtained in the previous section for the $\bar{\mathbf{5}}$ curve but with the opposite chirality. We can borrow then the result for the soft mass obtaining

$$m^2_{H_u} \;=\; \frac{|M|^2}{2}\left(1 - \frac{|q_s|}{2\sqrt{2}m^2}\right) = \frac{|M|^2}{2}\left(1 - \frac{5|N_Y|}{12\sqrt{2}m^2}\right) \,. \tag{4.159}$$

We can see that the soft mass does not depend on the hypercharge sign, so in this setup the soft Higgs masses are universal whenever they feel the same amount of hypercharge flux density N_Y. This is a good approximation since both curves $\mathbf{5}$ and $\bar{\mathbf{5}}$ can not be very far away from each other in order to reproduce the known flavor structure and CKM matrix of the SM. It would be interesting though to apply these results to a more realistic F-theory compactification with E_7 or E_8 enhancement in which we could consider both Yukawa points and all the matter curves simultaneously.

4.2. String origin of Flavor violation

One of the most relevant aspects of low-energy supersymmetry is the potentially large contribution of supersymmetric particles to processes that involve Flavor Changing Neutral Currents (FCNC's). These include the $K^0 - \overline{K}^0$ oscillation and CP-violating parameters Δm_K and ϵ_K, as well as lepton number violating transitions like $\mu \to e\gamma$ and other hadronic and leptonic processes involving heavier generations. All these transitions may be induced by SUSY particles in the presence of flavor changing SUSY-breaking soft parameters like off-diagonal scalar masses $\tilde{m}_{ij}$, $i \neq j$ [67–71] . These flavor violating contributions may be too large and violate experimental bounds unless the squark masses of the first two generations are almost degenerated and off-diagonal masses are much suppressed. Otherwise the SUSY spectrum should be very heavy, effectively decoupling from these low-energy transitions.

The presence or not of flavor violating couplings depends, of course, on what the underlying source of SUSY-breaking is and on how it is transmitted to the visible sector. In the context of gravity mediation, off-diagonal flavor violating scalar masses may appear e.g. if the Kähler metric of the MSSM quark/lepton superfields and their derivatives on SUSY-breaking scalars do not align in flavor space, see e.g. [57] and references therein. Other schemes like gauge or anomaly mediation which transmit SUSY-breaking in a flavor universal manner were put forward in order to avoid the presence of too large flavor violating transitions.

The question that we want to address in this section is whether in the more fundamental setting of String Theory one can obtain information about the presence of flavor violating SUSY-breaking soft terms. This is a question which has been controversial and somewhat author-dependent in the last two decades (for some discussions on this issue see e.g. [63, 72–75].) The reason being that it is a general question which is difficult to answer without *both* a scheme giving rise to a MSSM compactification as well as a tractable source of SUSY-breaking. In this theis we have combined for the first time these two ingredients (closed string fluxes and local wavefunctions for *chiral* matter fields). Our results then give a microscopic derivation of the flux-induced soft terms which allow to deal with issues like flavor non-universalities.

Although we concentrate on the general setting of type IIB orientifold/F-theory local SU(5) compactifications, the idea is quite general and can be applied to large classes of type IIA, type IIB and heterotic compactifications; and more generally, to theories with extra dimensions in which chirality is induced by magnetic fluxes along the internal dimensions. In all such theories with the gauge and matter degrees of freedom localized within a compact $2n$-dimensional manifold S in the extra dimensions, SUSY-breaking masses arise from terms of the general form

$$\Phi_i(x)\Phi_j(x)^* \int_S d^n z\, d^n \bar{z}\; C(z,\bar{z})\; \phi_i^{(0)}(z,\bar{z})\phi_j^{(0)}(z,\bar{z})^* \tag{4.160}$$

where z denote collectively the complex coordinates of S. The indices $i,j = 1,2,3$ are SM generation indices and $\phi_i^{(0)}$ are zero mode wavefunctions corresponding to 4d SM matter fields Φ_i. The function $C(z,\bar{z})$ is a background factor, that in the case of String Theory depends on the internal geometry and the closed and open string fluxes. In general the internal wavefunctions of the three SM generations depend differently on the compact dimensions. Taking an orthonormal basis for the zero modes, the resulting 4d mass matrix is diagonal and degenerate when $C(z,\bar{z})$ is constant over S. On the other hand, in the most general situation on which $C(z,\bar{z})$ is not constant, the mass matrix is generation dependent and non-diagonal.

From now on we will focus on the local IIB/F-theory SU(5) models of section 3.3, with the aim of being more specific for the general expression (4.160) and estimate the bounds that FCNC currents impose over the size of the soft terms.

4.2.1. Flavor non-diagonal soft terms

Before turning to compute the flavor non-universailities coming from (4.160) let us introduce the mixing parameters which allow a direct comparison of our results with the experimental bounds coming from FCNC.

In the leptonic sector, the strongest flavor violating constraints come from limits for the branching ratio for $\mu \to e\gamma$. From the hadronic constraints, the strongest limits on flavor dependent soft terms come from the kaon system, in particular from $K^0 - \overline{K}^0$ oscillations and CP violation. The relevant quantities are the real and imaginary parts of the off-diagonal $\tilde{d} - \tilde{s}$ squark and $\tilde{e} - \tilde{\mu}$ slepton soft masses $\tilde{m}^2_{12}$. In both the leptonic and hadronic sectors, to directly compare with the experimental constraints it is customary to work in a fermion basis in which the fermion mass matrix is diagonal. In particular, for the kaon system one has

$$\tilde{m}^2_{ij} \;=\; \left(U_{\rm d}\, m^2_{\rm soft}\, U^\dagger_{\rm d} \right)_{ij} \;, \qquad i,j = 1,2,3 \tag{4.161}$$

where $m^2_{\rm soft}$ is the original squark mass matrix before quark diagonalisation and $U_{\rm d}$ is the unitary matrix that diagonalises the down-quark mass matrix. It is also customary to work in the mass insertion approximation [76] where we expand on the ratio of the off-diagonal terms over the averaged squark mass $\tilde{m}^2_{\tilde{q}}$,

$$\tilde{\delta}^{\rm d}_{ij} \;=\; \frac{\tilde{m}^2_{ij}}{\tilde{m}^2_{\tilde{q}}} \;. \tag{4.162}$$

There are in fact four 3×3 squark mass matrices (4.161), depending on the particular *chirality* of the squarks, namely $m^2_{\rm RR}$, $m^2_{\rm LL}$, $m^2_{\rm LR}$ and $m^2_{\rm RL}$. In the scheme for local SU(5) GUT's that we have described in the previous section, $m^2_{\rm LR}$ and $m^2_{\rm RL}$ are much suppressed for the two lightest generations, which are the ones relevant for the kaon system. This is because left- and right-handed squarks live in different matter curves and their mass terms are proportional to the Yukawa couplings, being those negligible for the first two generations. On the other hand $m^2_{\rm LL}$ and $m^2_{\rm RR}$ have both an analogous structure so that our results below apply to $m^2_{\rm RR}$ or $m^2_{\rm LL}$ irrespectively. Note that the down-quark mass matrix is in general not symmetric and it is actually diagonalized by a bi-unitary transformation involving matrices $U^{\rm R}_{\rm d}$ and $U^{\rm L}_{\rm d}$ simultaneously. Thus in eq. (4.161) we should actually take $U_{\rm d} = U^{\rm L}_{\rm d}$ for $m^2_{\rm LL}$ and $U_{\rm d} = U^{\rm R}_{\rm d}$ for $m^2_{\rm RR}$.

Focusing on the first two generations and taking $U_{\rm d}$ real for simplicity, we can parametrize $U_{\rm d}$ by an orthogonal 2×2 matrix

$$U_{\rm d} \;=\; \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{4.163}$$

From eq. (4.161) we obtain

$$\tilde{\delta}^{\rm d}_{12} \;=\; \frac{2m^2_{12}\cos 2\theta + (m^2_{22} - m^2_{11})\sin 2\theta}{2\tilde{m}^2_{\tilde{q}}} \;. \tag{4.164}$$

Barring accidental cancellations, the value of $\tilde{\delta}^d_{12}$ is therefore controlled by the ratios $\delta^{\rm d}_{12} \equiv m^2_{12}/\tilde{m}^2_{\tilde{q}}$ and $\rho^{\rm d}_{12} \equiv (m^2_{22} - m^2_{11})/2\tilde{m}^2_{\tilde{q}}$. Totally analogous formulae, with the obvious changes, may be written for the slepton mass matrices relevant for the $\mu \to e\gamma$ decay.

In the next two subsections we evaluate the size of $\tilde{\delta}_{12}$ in local SU(5) GUTs by making use of the expression (4.169) for the soft scalar masses that we have derived microscopically in the previous section. Note however that such expression is only valid at the unification scale M_{GUT}. To compare with the low-energy data we therefore have to take into account the renormalisation group (RG) running from the unification scale down to the electroweak scale. Since the Yukawa couplings of the first two generations are negligible, it is only the SUSY-gauge couplings that contribute to this running. Integrating the RG equations we find [77]

$$m^2(1^{\mathrm{st}}, 2^{\mathrm{nd}}\mathrm{gen.}) = m_0^2 + g(t) M_{1/2}^2 \tag{4.165}$$

where we are assuming universal gaugino and scalar masses, $M_{1/2}$ and m_0, at the scale M_{GUT}.[6] We have defined

$$g(t) = 2 \sum_{k=1}^{3} C_k b_k \left(1 - \frac{1}{(1+\beta_k t)^2}\right) \tag{4.166}$$

where C_k is the quadratic Casimir corresponding to the particular scalar field (namely, $C_k = \frac{N^2-1}{2N}$, $k = 2, 3$, for the fundamental of $\mathrm{SU}(N)_k$ and $C_1 = Y^2$ for $\mathrm{U}(1)_Y$) and the β-functions are

$$\beta_k = \frac{\alpha_k(0) b_k}{4\pi} \ . \tag{4.167}$$

For the evaluation of the β-functions we consider a MSSM spectrum, which yields $(b_1, b_2, b_3) = (11, 1, -3)$ with $\alpha_3(0) = \alpha_2(0) = \frac{5}{3}\alpha_1(0) = \alpha_{\mathrm{GUT}} \simeq \frac{1}{24}$ the gauge unification constants. Finally, $t = 2\log(M_{\mathrm{GUT}}/M_{\mathrm{SS}})$ where M_{SS} is the scale of supersymmetry-breaking.

In the case of squarks, the leading contribution to the running (4.165) comes from its SUSY-QCD part and thus the averaged squark mass $\tilde{m}_{\tilde{q}}$ has a substantial low-energy running. On the other hand, due to the universality of gauge couplings both $(m_{22}^2 - m_{11}^2)$ and m_{12}^2 have negligible low-energy running. This is important because it means that in going from M_{GUT} down to M_{SS}, the ratio $\tilde{\delta}_{12}$ is diluted by a RG factor which is typically of order

$$\tilde{\delta}_{12}^{\mathrm{d}} \ \rightarrow \ \frac{\tilde{\delta}_{12}^{\mathrm{d}}}{1 + g(t)\xi^2} \ , \qquad \xi \equiv \frac{M_{1/2}}{m_0} \ . \tag{4.168}$$

For instance, for the type of 3-form fluxes with $M_{1/2}^2 = 2m_0^2$ that we have discussed in the previous section, and taking $M_{\mathrm{GUT}} \simeq 10^{16}$ GeV and $M_{\mathrm{SS}} \simeq 2$ TeV, we obtain $g(t) \simeq 4.5$ and therefore a suppression of order $\simeq 1/10$ for the ratio $\tilde{\delta}_{11}^{\mathrm{d}}$ with respect to its value at the unification scale. This *RG dilution* turns out to be important in comparing with the low-energy data and, in particular, in evaluating the squark mass limits. On the other hand for the case of sleptons the dilution is weaker and for the above parameters $g(t) \simeq 0.5$ so the suppression is only of order $\simeq 1/2$. This different dilution will eventually make the slepton limits stronger than those coming from the kaon system.

In what follows we compute the flavor mixing induced by variations of closed and open string fluxes. We consider these two cases separately since, as it will become clear below, terms coming from the simultaneous variation of closed and open string fluxes are either suppressed or contribute to transitions between the first and third families rather than between the two lightest families.

[6]The effect of the non-universalities of the squark masses on the running can be safely neglected.

4.2.2. Flavor mixing from non-constant string fluxes

In the derivation of the soft masses for chiral matter curves in section 4.1, the closed string fluxes were assumed to be constant so that the integration over the internal dimensions was trivial. However if the fluxes are non-constant, we have to take a step back and recover the expression for the soft masses before the integration was done, which yields

$$m_{ij}^2 = \frac{g_s}{4\text{Vol}(S)} \int_S d^2z d^2\bar{z} \sqrt{g_4}\, |G|^2 \left(1 - \left|\frac{M}{m}\right|\right) \psi_i^+ (\psi_j^+)^* \tag{4.169}$$

where g_4 is the determinant of the metric in S and $\text{Vol}(S) = \int_S d^2z d^2\bar{z}\sqrt{g_4}$. Kinetic terms are diagonal and canonically normalized. We have used the internal wavefunctions for matter chiral fields computed in eq.(4.60) and that we repeat here for the convenience of the reader[7],

$$\Psi^{(0)}_{\bar{\mathbf{5}}_i}(z,\bar{z}) = \frac{1}{\sqrt{2\lambda(\lambda - M)}} \begin{pmatrix} M-\lambda \\ 0 \\ m \end{pmatrix} \psi_i^+(z,\bar{z}) \ , \qquad i = 1,\ldots,r \tag{4.170}$$

which are normalizable for $M < 0$ and where the functions ψ_i^+ are defined as

$$\psi_i^+(z,\bar{z}) \equiv N_i f_i(y) \exp\left[-\frac{q\lambda}{2}|x|^2 + \frac{qM}{2}|y|^2\right] \tag{4.171}$$

Here N_i is a normalization constant, $\lambda \equiv \sqrt{M^2+m^2}$, $q = \sqrt{3/5}$ is the U(1) charge normalization and $f_i(y)$ are holomorphic functions of y. We have also introduced an index i to label the number of families. The degeneracy r and the holomorphic functions $f_i(y)$ can only be determined in terms of the global topology of S and the gauge bundle. We take here $r = 3$, corresponding to the three families of the Standard Model. In that case we can take a basis such that the expansion of the holomorphic functions $f_i(y)$ around the origin reads

$$f_i(y) = y^{3-i} + \mathcal{O}\left(y^4\right) \ , \qquad i = 1,2,3 \tag{4.172}$$

In particular wavefunctions of different families localize in slightly different regions of the 4-cycle S due to their different holomorphic pieces $f_i(y)$.

Due to the Gaussian localization in both x and y the replacement $S \sim \mathbb{C}^2$ is a suitable approximation when Vol(S) is large in string units. Normalizing the wavefunctions in $\mathbb{C}^2$ we get

$$N_i = \frac{\sqrt{q\lambda}|qM|^{\frac{4-i}{2}}}{\pi\sqrt{(3-i)!}} \ . \tag{4.173}$$

For constant densities of magnetization $M(z,\bar{z}) = M_0$ and 3-form flux $G(z,\bar{z}) = G_0$ the integral in (4.169) becomes trivial and the standard supergravity formula for the soft scalar masses in intersecting magnetized 7-branes is recovered [58, 78, 79, 90]

$$m_{ij}^2 = \frac{g_s}{4}\delta_{ij}|G_0|^2 \left(1 - \left|\frac{M_0}{m}\right|\right) \ . \tag{4.174}$$

This mass matrix is diagonal and flavor universal, so that there are not new FCNC's induced beyond those of the Standard Model. However, in general there is no reason for the

[7] Notice the change of notation on the worldvolume flux, $F_- \equiv M$.

flux densities $M(z,\bar{z})$ and $G(z,\bar{z})$ to be constant over the GUT 4-cycle S. In that case non-universal soft masses are expected to arise from eq. (4.169). Morally, as different families of sfermions are localized in slightly different regions of S because of their holomorphic factors, each family feels a different density of 3-form flux and/or magnetization and therefore gets different soft masses, see figure 4.3.

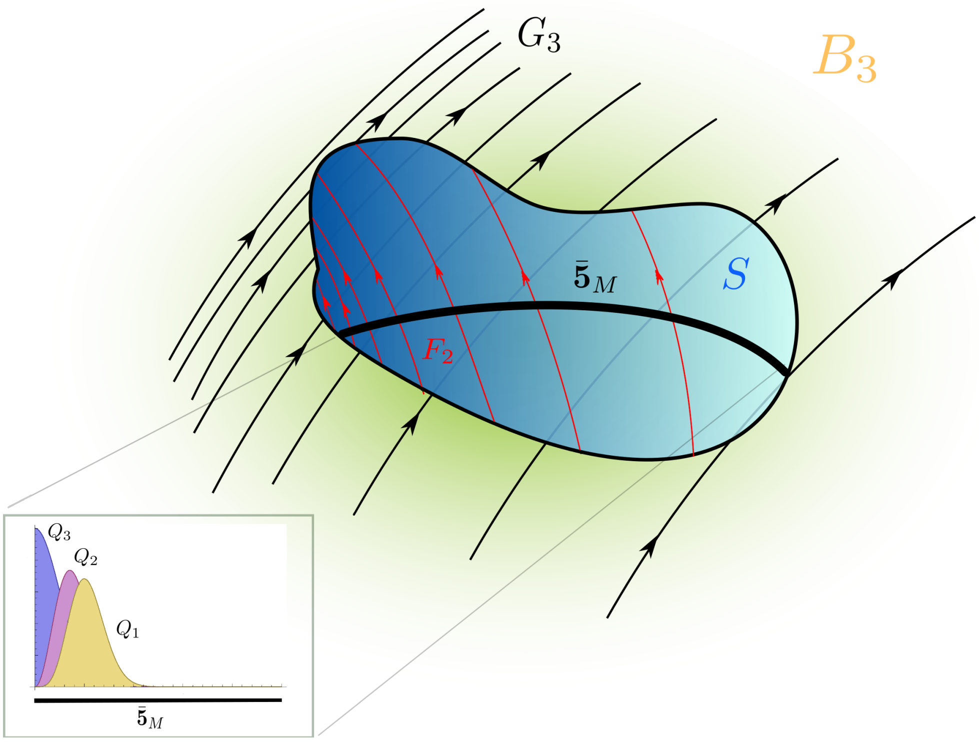


Figure 4.3: The 4-cycle S embedded in the ambient 3-fold B_3. The density of three-form flux G_3 (black arrows) and magnetization F_2 (red arrows) vary over S. Since wavefunctions of different families are localized in different regions of the $\bar{\mathbf{5}}_M$ matter curve, each family feels a different density of G_3 and F_2, leading to non-universal soft masses in the 4-dimensional effective theory.

As we have already commented, such (non-constant) flux densities can arise from the backreaction of distant sources or non-perturbative effects. The local background near the GUT D7-branes can be understood in that case as originating from massive open string modes that propagate between the distant sources and the stack of GUT branes, and eq. (4.169) computes the thresholds to the sfermion mass matrix that result from integrating out those heavy modes, in the same spirit than the supergravity computation of gauge thresholds [62, 80].

In this section we make use of eq. (4.169) to estimate the size of non-universal soft masses with non-constant magnetization and 3-form flux and compare the resulting FCNC transitions with the current experimental bounds.

4.2.2.1. Non-constant closed string fluxes

For concreteness we consider the $\bar{\mathbf{5}}$ matter curve $x = 0$ introduced above, with a non-constant density of closed string 3-form flux $G(z, \bar{z})$ and a constant density of magnetization M_0 along the 4-cycle S. The soft scalar masses for the right-handed down squarks and the left-handed sleptons that are contained in the $\bar{\mathbf{5}}$ matter curve can be computed microscopically by means of eq. (4.169). Due to the Gaussian localization of the wavefunctions ψ_i^+ around the point $x = y = 0$, the dominant contribution to the integral in (4.169) comes from the background near the localization point. It is therefore convenient to perform an expansion of the closed string flux density in powers of the local coordinates x, y of the 4-cycle

$$|G(z, \bar{z})|^2 \;=\; |G_0|^2 \left(1 \,+\, G_y^* \, y + G_y \, \bar{y} \,+\, G_{y\bar{y}} \, |y|^2 \,+\, \ldots\right) \tag{4.175}$$

where G_0, G_y, here defined, are complex constants and $G_{y\bar{y}}$ is real. We have only displayed terms of the expansion that contribute to the flavor dependence of the two lightest families. In particular, we have not shown the expansion in x since it has no consequences for the flavor dependence.

For sufficiently large sizes of S we can extend the domain of integration to infinity, so that (4.169) reads

$$m_{ij}^2 = \frac{g_s N_i N_j}{4} \int_{\mathbb{C}^2} d^2x d^2y \, \Big[|\hat{G}_0|^2 \left(1 \,+\, G_y^* y + G_y \bar{y} \,+\, G_{y\bar{y}} |y|^2 \,+\, \ldots\right) \\ y^{3-i} \bar{y}^{3-j} \, e^{-q\lambda|x|^2 - q|M_0||y|^2} \Big] \tag{4.176}$$

where we have defined

$$|\hat{G}_0|^2 \equiv |G_0|^2 \left(1 - \left|\frac{M_0}{m}\right|\right) \tag{4.177}$$

Let us compute first the diagonal terms $i = j$. Linear terms on x, y vanish upon integration, so that the only non-vanishing contributions are

$$m_{ii}^2 = \frac{g_s N_i N_j}{4} \int_0^\infty 2\pi x \, dx \int_0^\infty 2\pi y \, dy \, |\hat{G}_0|^2 \left(1 + G_{y\bar{y}} |y|^2\right) e^{-q\lambda|x|^2 - q|M_0||y|^2} |y|^{2(3-i)} \\ = \frac{g_s}{4} |\hat{G}_0|^2 \left(1 + G_{y\bar{y}} \frac{4-i}{q|M_0|}\right) \; . \tag{4.178}$$

Similarly, for the off-diagonal $\Delta F = 1$ soft masses we have

$$m_{ij}^2 = \frac{g_s N_i N_j}{4} \int_0^\infty 2\pi x \, dx \int_0^\infty 2\pi y \, dy \, |\hat{G}_0|^2 \left(G_y^* y + G_y \bar{y}\right) e^{-q\lambda|x|^2 - q|M_0||y|^2} y^{3-i} \bar{y}^{3-j} \\ = \frac{g_s k}{4} \frac{|\hat{G}_0|^2 G_y}{\sqrt{q|M_0|}} \, , \qquad \text{where} \qquad k \equiv \begin{cases} \sqrt{2} & \text{for} \quad i = 1, \; j = 2 \\ 1 & \text{for} \quad i = 2, \; j = 3 \end{cases} \tag{4.179}$$

Finally, the off-diagonal $\Delta F = 2$ mass term m_{13}^2 is proportional to higher derivatives of the 3-form flux and is therefore subleading with respect to m_{12}^2 and m_{23}^2.

Summing up, the structure of the soft mass matrix (4.176) is given by

$$\begin{pmatrix} m_{\tilde{q}}^2 + \delta m_1^2 & m_{12}^2 & m_{13}^2 \\ (m_{12}^2)^* & m_{\tilde{q}}^2 + \delta m_2^2 & m_{23}^2 \\ (m_{13}^2)^* & (m_{23}^2)^* & m_{\tilde{q}}^2 + \delta m_3^2 \end{pmatrix} \tag{4.180}$$

where $m^2_{\tilde{q}}$ is the universal soft mass for constant density of fluxes, eq. (4.174), and the flavor violating terms have the following hierarchical structure

$$m^2_{\tilde{q}} \;>\; m^2_{12},\, m^2_{23} \;>\; \delta m^2_i,\, m^2_{13} \tag{4.181}$$

Making use of eqs. (4.178) and (4.179) we can estimate the size of the non-universalities for the first two families in a generic model. Indeed, flux quantization gives us an estimate of the dependence of the flux densities on the volumes of S and B_3

$$\begin{aligned}\int_{\Sigma_i} F_2 = 2\pi n_i \quad &\Rightarrow \quad M_0 \sim \frac{2\pi n}{\mathrm{Vol}(S)^{1/2}} \\ \frac{1}{2\pi\alpha'}\int_{\gamma_j} G_3 = 2\pi f_j \quad &\Rightarrow \quad G_0 \sim \frac{4\pi^2\alpha' f}{\mathrm{Vol}(B_3)^{1/2}}\end{aligned} \tag{4.182}$$

where $\Sigma_i \in H_2(S)$ and $\gamma_j \in H_3(B_3)$ denote the 2-cycles and 3-cycles that support the open and closed string fluxes, n_i and f_j are integer numbers, and the parameter n (f) denote complex combinations of the various n_i (f_j) and the complex structure moduli of B_3. In the same vein, the derivatives of $G(z,\bar{z})$ scale as

$$G_y \sim \frac{2\, c_{y,G}}{\mathrm{Vol}(B_3)^{1/6}}\ , \qquad G_{y\bar{y}} \sim \frac{4\, c_{y\bar{y},G}}{\mathrm{Vol}(B_3)^{1/3}}\ , \tag{4.183}$$

where $c_{y,G}$ $(c_{y\bar{y},G})$ is an adimensional complex (real) constant.

A comment on the scale of SUSY-breaking is in order here. Defining the scale of SUSY-breaking as $M_{\mathrm{SS}} \simeq m_{\tilde{q}}$, from the above expressions we have

$$M_{\mathrm{SS}} \sim 2\pi^2\alpha'|f|\left(\frac{g_s}{\mathrm{Vol}(B_3)}\right)^{1/2} \sim \frac{|f|\, M^2_{\mathrm{GUT}}}{2\pi M_{\mathrm{Pl}}\sqrt{\alpha_{\mathrm{GUT}}}} \tag{4.184}$$

Hence, for $|f| \simeq 1$ and standard unification values $M_{\mathrm{GUT}} \simeq 10^{16}$ GeV and $\alpha^{-1}_{\mathrm{GUT}} \simeq 24$ we would obtain $M_{\mathrm{SS}} \simeq 6.5 \times 10^{12}$ GeV. However, it should be noted that the parameter f in general receives contributions from a large number of 3-cycles (c.f. eq. (4.182)), so that large cancellations can take place that lead to $|f| \ll 1$ and lower the scale of SUSY-breaking. This is similar to what occurs for the small superpotential parameter W_0 in KKLT vacua [8] and more generally for the cosmological constant in flux compactifications [81–84]. In this work we therefore take $|f|$ (and thus M_{SS}) to be a tunable parameter, perhaps selected on anthropic grounds (see section 4.3). Nevertheless, the reader may note that the dependence on f of the flavor mixing parameters δ_{ij} and ρ_{ij} cancels, and thus the limits that we obtain below for the sfermion masses do not actually depend on the tuning of f.

Making use of the scalings (4.182) and (4.183) we can estimate the dependence of the non-universal soft scalar masses on the local expansion parameter ϱ defined in eq. (3.36). Indeed, from eqs. (4.178) and (4.179) we observe that $(\delta m^2_{22} - \delta m^2_{11})$ is of order $\mathcal{O}\left(\varrho^2\right)$ while the off-diagonal mass m^2_{12} is of order $\mathcal{O}\left(\varrho\right)$ and is thus dominant. For the two lightest families we have

$$\tilde{\delta}^{\mathrm{d}}_{12} = \frac{\tilde{m}^2_{12}}{\tilde{m}^2_{\tilde{q}}} \;=\; \frac{\sqrt{2}G_y}{\sqrt{q|M_0|}} \;\sim\; \frac{2\, c_{y,G}}{\sqrt{\pi|n|}}\left(\frac{5}{3}\right)^{1/4}\varrho \tag{4.185}$$

In particular, when $\varrho \ll 1$ the closed string flux varies very little over S and the flavor dependence is suppressed, as expected. In terms of physical scales

$$\tilde{\delta}^{\mathrm{d}}_{12} = \frac{\tilde{m}^2_{12}}{\tilde{m}^2_{\tilde{q}}} \;\sim\; 1.4 \times \frac{c_{y,G}}{\sqrt{|n|}}\left(\frac{M_{\mathrm{GUT}}}{M_{\mathrm{Pl}}\,\alpha_{\mathrm{GUT}}}\right)^{1/3}\ . \tag{4.186}$$

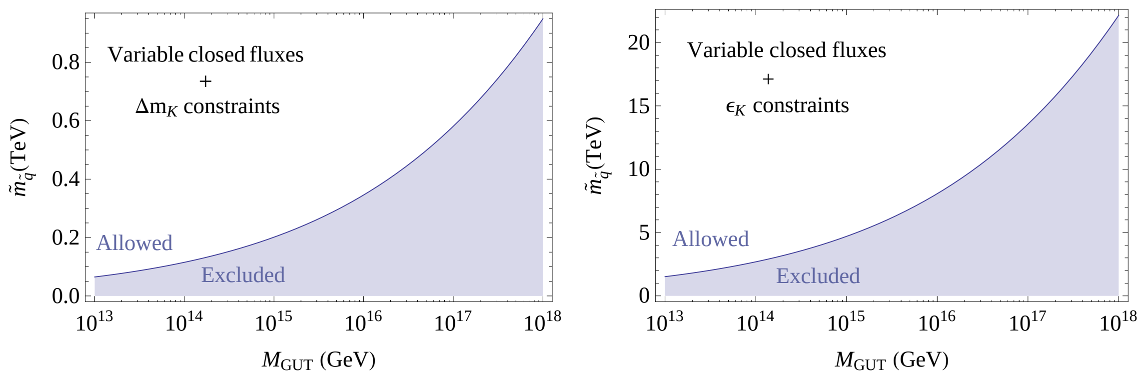


Figure 4.4: Left: Constraints on squark masses $\tilde{m}_{\tilde{q}}$ vs the unification scale $M_{\rm GUT}$ coming from the kaon mass difference Δm_K induced by non-constant densities of closed string fluxes along the GUT 4-cycle. Right: Analogous constraints coming from the CP violation parameter ϵ_K.

The same estimate applies also to the off-diagonal flavor transitions for sleptons, parametrized by $\tilde{\delta}^{\rm l}_{12}$. Note however that the effective local magnetization parameter n is expected to differ for sleptons and squarks, as they are differently charged under the hypercharge flux that breaks SU(5) down to the SM gauge group [31–33]. This effect is also responsible for the breaking of the degeneracy between down-quark and lepton Yukawa couplings [35]. Making use of the numerical estimates of [35] for the local density of hypercharge flux one may check that the effect of the latter on the off-diagonal soft masses is small and, within the model-dependent uncertainties of the above estimation, we can take $\tilde{\delta}^{\rm d}_{12} \sim \tilde{\delta}^{\rm l}_{12}$ at the unification scale. Thus, the difference in the limits below for sleptons and squarks comes mainly from their different running under the RG. We will further comment on these differences in section 4.2.3.

Note from eq. (4.186) that off-diagonal soft masses turn out to be parametrically suppressed by the ratio between the unification and the Planck scales. In a standard unification scheme with $M_{\rm GUT} \simeq 10^{16}$ GeV this suppression is only of order ~ 0.1 and the actual value at $M_{\rm GUT}$ therefore depends substantially on the particular details of the magnetization parameter n and the closed string flux variation parameter $c_{y,G}$, allowing for relatively large amounts of non-universal soft masses at the unification scale. On the other hand, as we have discussed in the beginning of this section, at low-energies there is an extra suppression factor $(1 + g(t)\xi^2)^{-1}$ that comes from the RG running between the unification scale and the SUSY-breaking scale $M_{\rm SS}$ and that dilutes the non-universal terms substantially in the case of squarks and more weakly in the case of sleptons.

Measurements of the kaon mass difference Δm_K put constraints on the real part of the mixing $\tilde{\delta}^{\rm d}_{12}$ at the low-energy scale. The current experimental bounds require [85]

$$\left|\text{Re } \tilde{\delta}^{\rm d}_{12}\right| \;=\; \frac{\tilde{m}^2_{12}}{\tilde{m}^2_{\tilde{q}}} \;<\; 4.2 \times 10^{-2}\, \frac{\tilde{m}_{\tilde{q}}}{350 \text{ GeV}} \tag{4.187}$$

where $\tilde{m}_{\tilde{q}}$ is the averaged squark mass at the scale $M_{\rm SS}$. From eq. (4.186) we then get a lower bound on the averaged squark mass

$$\tilde{m}_{\tilde{q}} \gtrsim \frac{|\text{Re } c_{y,G}|}{(1+g(t)\xi^2)\sqrt{|n|}} \times \left(\frac{M_{\text{GUT}}}{M_{\text{Pl}}}\right)^{1/3} \times 35 \text{ TeV} \tag{4.188}$$

where we have taken $\alpha_{\text{GUT}}^{-1} = 24$. Note that for typical values $M_{\text{GUT}} \simeq 10^{16}$ GeV, $n \simeq c_{y,G} \simeq 1$ and $\xi^2 \simeq 2$, this expression leads to $\tilde{m}_{\tilde{q}} \gtrsim 350$ GeV, which is in the range already excluded by direct LHC searches. Thus Δm_K does not provide strong bounds on the non-universalities induced by non-constant densities of closed string fluxes. This is also shown in figure 4.4 (left) where we have represented the low-energy bound on the averaged squark mass as a function of the unification scale. The bound becomes weaker as the unification scale is lowered, due to the decreasing of the flux variation over S.

The situation becomes much tighter if we consider the experimental constraints that come from the measured CP violation parameter ϵ_K. These yield [85]

$$\left|\text{Im } \tilde{\delta}_{12}^{\text{d}}\right| < 1.8 \times 10^{-3} \frac{\tilde{m}_{\tilde{q}}}{350 \text{ GeV}} . \tag{4.189}$$

The local density of closed string flux $G(z,\bar{z})$ is complex and so is the parameter $c_{y,G}$. We therefore expect the real and imaginary parts of $\tilde{m}_{12}$ to be generically of the same order. In that case, the strong constraints coming from ϵ_K translate into a more stringent limit for the averaged squark mass

$$\tilde{m}_{\tilde{q}} \gtrsim \frac{|\text{Im } c_{y,G}|}{(1+g(t)\xi^2)\sqrt{|n|}} \times \left(\frac{M_{\text{GUT}}}{M_{\text{Pl}}}\right)^{1/3} \times 8.1 \times 10^2 \text{ TeV} \tag{4.190}$$

A standard unification scale $M_{\text{GUT}} \simeq 10^{16}$ GeV and typical values $n \simeq c_{y,G} \simeq 1$ and $\xi^2 = 2$ imply an averaged squark mass of at least $\tilde{m}_{\tilde{q}} \gtrsim 8$ TeV, well above the current bounds coming from direct searches at the LHC. Thus, while we have seen that the constraint on the real part of $\tilde{m}_{12}^2$ is not very relevant in this context, the constraint on the imaginary part turns out to be much stronger. We have depicted the bound on the averaged squark mass versus the unification scale in figure 4.4 (right).

The limits coming from the (yet unmeasured) $\mu \to e\gamma$ decay turn out to be quite strong. Using results from [86] we can obtain an estimate based on recent experimental data [87]

$$|\tilde{\delta}_{12}^{\text{l}}| < 4 \times 10^{-4} \left(\frac{\tilde{m}_{\tilde{l}}}{500 \text{ GeV}}\right)^2 . \tag{4.191}$$

From the leptonic analogue of eq. (4.186) we then obtain

$$\tilde{m}_{\tilde{l}} \gtrsim \sqrt{\frac{|c_{y,G}|}{(1+g(t)\xi^2)|n|^{1/2}}} \times \left(\frac{M_{\text{GUT}}}{M_{\text{Pl}}}\right)^{1/6} \times 51 \text{ TeV} . \tag{4.192}$$

If $M_{\text{GUT}} \simeq 10^{16}$ GeV, $\xi^2 = 2$ and $c_{y,G} \simeq n \simeq 1$ then $\tilde{m}_{\tilde{l}} \gtrsim 11$ TeV. This is a remarkably strong bound which, of course, may be substantially weakened by playing with the model-dependent parameters, but it suggests that large masses for sleptons are generically required.

4.2.2.2. Non-constant open string fluxes

We now study the non-universalities that arise from non-constant local densities of open string fluxes. For that aim, we consider the same $\bar{\mathbf{5}}$ matter curve of the previous sections, in this case with a constant density of closed string 3-form flux G_0 and a non-constant density of magnetization $M(z,\bar{z})$ along the 4-cycle S. In order to evaluate (4.169) we proceed in the same way as we did for non-constant densities of closed string fluxes. Thus, we expand the density of magnetization around the point $x = y = 0$ where the wavefunctions (4.56) localize

$$M(z,\bar{z}) = M_0\left(1 + M_y^* y + M_y\bar{y} + M_{y\bar{y}}|y|^2 + \ldots\right) \tag{4.193}$$

We have not displayed the expansion on x since it plays no role in the generation of non-universalities, as it will become clear below.

In order to obtain the soft scalar mass matrix we must evaluate (4.169) on this background, namely

$$m_{ij}^2 \quad = \quad \frac{g_s}{4}\int_{\mathbb{C}^2} d^2x d^2y |G_0|^2 \left[1 - \left|\frac{M_0}{m}\right|\left(1 + M_y^* y + M_y\bar{y} + M_{y\bar{y}}|y|^2 + \ldots\right)\right]\psi_i^+(\psi_j^+)^* \tag{4.194}$$

Note that the local Gaussian wavefunctions introduced in section 4.1.2 were actually derived for constant open string fluxes and are in principle not directly applicable to this case. However, it was shown in [34] that the wavefunctions for non-constant open string fluxes have the same Gaussian structure (4.61) multiplied by a polynomial expansion on the local variables x, $\bar{x}$, y and $\bar{y}$. Thus, to first order in the coordinate expansion the effect of this factor is to contribute a further term, linear on y, on the integrand. This in practice amounts to a redefinition of the coefficient M_y in (4.193). Therefore, in what follows M_y represents an effective coefficient that includes not only the effect from the varying flux density $M(z,\bar{z})$ but also the correction from the modified wavefunction.

The integral (4.194) with the wavefunctions (4.171) is formally equivalent to (4.176), so that we can borrow the results of the previous subsection to obtain

$$\begin{aligned} m_{\tilde{q}}^2 &= \frac{g_s}{4}|G_0|^2\left(1 - \left|\frac{M_0}{m}\right|\right) , & \delta m_i^2 &= \frac{g_s}{4}|G_0|^2\frac{M_{y\bar{y}}}{q|m|}(4-i) \\ m_{12}^2 &= \frac{g_s}{4}|G_0|^2\frac{M_y}{|m|}\sqrt{\frac{2|M_0|}{q}} , & m_{23}^2 &= \frac{g_s}{4}|G_0|^2\frac{M_y}{|m|}\sqrt{\frac{|M_0|}{q}} \end{aligned} \tag{4.195}$$

where we have organized the soft scalar mass matrix accordingly to eq. (4.180). To estimate the size of the non-universalities we note in addition to eqs. (4.182) that the coefficients of the magnetization expansion scale with the volumes as

$$M_y \sim \frac{2\,c_{y,F}}{\mathrm{Vol}(S)^{1/4}} , \qquad M_{y\bar{y}} \sim \frac{4\,c_{y\bar{y},F}}{\mathrm{Vol}(S)^{1/2}} , \tag{4.196}$$

where $c_{y,F}$ and $c_{y\bar{y},F}$ are adimensional complex constants. Moreover, the parameter m in the background for Φ, eq. (4.53), scales as (see e.g. [54])

$$m \simeq \frac{\eta}{2\pi\alpha'} \tag{4.197}$$

where η is a complex adimensional parameter related to the angle of the intersection between the GUT branes and the extra U(1) D7-brane. Plugging these scalings into eqs. (4.195) we obtain

$$\begin{aligned}\delta^{\mathrm{d}}_{12} &= \frac{m^2_{12}}{m^2_{\tilde{q}}} \sim \frac{8\pi^{3/2}\,c_{y,F}\,|n|^{1/2}}{\eta}\left(\frac{5}{3}\right)^{1/4}\frac{\alpha'}{\mathrm{Vol}(S)^{1/2}} \\ \rho^{\mathrm{d}}_{12} &= \frac{\delta m^2_2-\delta m^2_1}{2m^2_{\tilde{q}}} \sim -\frac{4\pi\,c_{y\bar{y},F}}{\eta}\left(\frac{5}{3}\right)^{1/2}\frac{\alpha'}{\mathrm{Vol}(S)^{1/2}}\end{aligned} \tag{4.198}$$

Thus, unlike the case of varying closed string fluxes, for a non-constant density of open string flux along S, both δ^{d}_{12} and ρ^{d}_{12} are parametrically of the same order. Making use of eq. (4.164) and expressing the result in terms of the physical scales we have that in the basis where the quarks mass matrix is diagonal, $\tilde{\delta}^{\mathrm{d}}_{12}$ reads

$$\tilde{\delta}^{\mathrm{d}}_{12} \sim \frac{1}{\eta}\left(\frac{M_{\mathrm{GUT}}}{M_{\mathrm{st}}}\right)^2\left(1.28\cdot c_{y,F}\sqrt{|n|}\cos 2\theta - 0.41\cdot c_{y\bar{y},F}\sin 2\theta\right) \tag{4.199}$$

where $M_{\mathrm{st}} = \alpha'^{-1/2}$ is the string scale.

These results show that there are potentially sizeable off-diagonal transitions, which are parametrically suppressed by $M_{\mathrm{GUT}}/M_{\mathrm{st}} = (2\alpha_{\mathrm{GUT}}/g_s)^{1/4}$. Barring fine-tunings, for generic θ the second contribution in (4.199) is somewhat smaller than the first for $n \geq 1$, so that we use the first term for our estimation of squark limits. From the constraints on Δm_K discussed in the previous subsection we obtain that the averaged squark mass is bounded from below as

$$\tilde{m}_{\tilde{q}} \gtrsim \frac{1}{1+g(t)\xi^2}\sqrt{\frac{|n|}{g_s}\frac{c_{y,F}}{\eta}} \times 3.1\ \mathrm{TeV}\ . \tag{4.200}$$

In particular, for $g_s \simeq n \simeq c_{y,F} \simeq \eta \simeq 1$ and $\xi^2 = 2$, we get $\tilde{m}_{\tilde{q}} \gtrsim 330$ GeV, and as in the closed string flux case, the constraints are weaker than the direct limits from LHC.

The bound for the imaginary part of $\tilde{\delta}^{\mathrm{d}}_{12}$ coming from the CP violation parameter ϵ_K is stronger and gives

$$\tilde{m}_{\tilde{q}} \gtrsim \frac{1}{1+g(t)\xi^2}\sqrt{\frac{|n|}{g_s}\frac{c_{y,F}}{\eta}} \times 72\ \mathrm{TeV}\ . \tag{4.201}$$

so that for $g_s \simeq n \simeq c_{y,F} \simeq \eta \simeq 1$ and $\xi^2 = 2$ we get a lower bound $\tilde{m}_{\tilde{q}} \gtrsim 7.6$ TeV for the squarks, similar to the bound that arises for non-constant closed string flux densities.

We show in figure 4.5 the lower averaged squark mass bound from closed and open string fluxes as a function of the magnetization parameter n, coming from both Δm_K and ϵ_K. Combining closed and open string fluxes, squarks should be heavier than $\tilde{m}_{\tilde{q}} \gtrsim 8$ TeV to suppress sufficiently the contribution to CP violation in the kaon system.

It is also illuminating to look at the dependence of these bounds on the unification scale. Note in particular that the ratio $M_{\mathrm{GUT}}/M_{\mathrm{st}}$ in eq. (4.199) can be fixed in terms of g_s and α_{GUT}. Thus, for fixed g_s and α_{GUT} the non-universal corrections from the magnetization do not have a direct dependence on the value of the unification scale. Despite of this, an indirect dependence appears due to the renormalization of squark

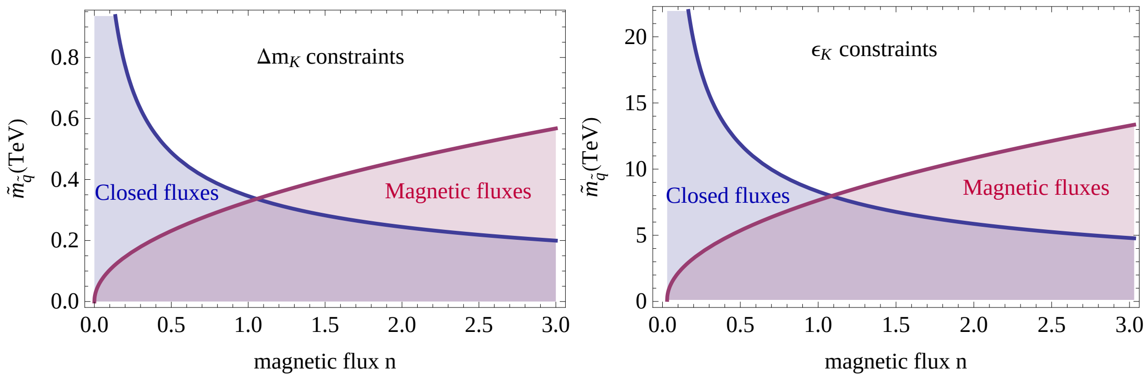


FIGURE 4.5: Bounds on the averaged squark mass due to non-constant closed and open flux densities along the 4-cycle S if $M_{\rm GUT} \simeq 10^{16}$ GeV. Left: bound coming from kaon mass mixing parameter Δm_K. Right: bound coming from the kaon CP violation parameter ϵ_K.

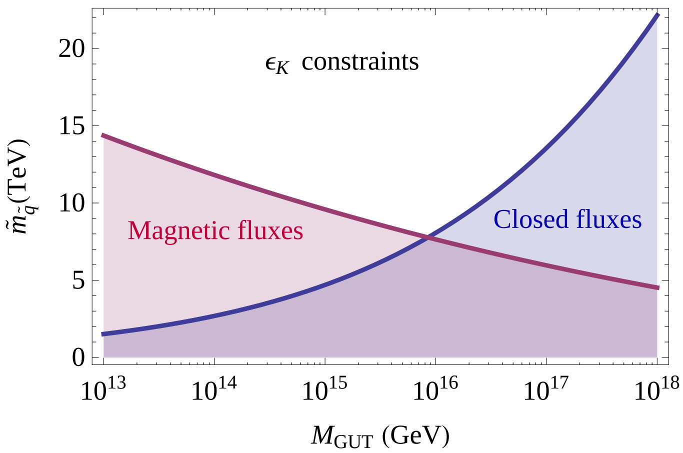


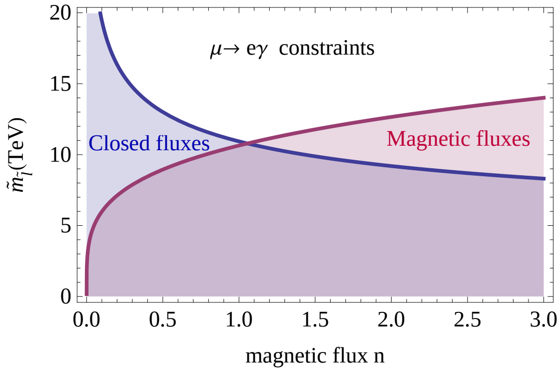
FIGURE 4.6: Lower bound on the squark mass from CP violation as a function of the unification scale $M_{\rm GUT}$. The contribution of both closed and open string fluxes is shown.

masses from $M_{\rm GUT}$ down to low-energies. The higher the value of $M_{\rm GUT}$, the larger is the effect of the running and the induced non-universalities are further diluted. This indirect dependence on $M_{\rm GUT}$ is of course also present for the non-universal terms induced by closed string fluxes, but in that case the direct dependence on $(M_{\rm GUT}/M_{\rm Pl})^{1/3}$, c.f. eq. (4.186), dominates over the indirect dependence. As a consequence the bounds on the averaged squark mass decrease in this case for lower $M_{\rm GUT}$. This is displayed in figure 4.6 for the limits that come from the CP violation parameter ϵ_K. Interestingly, the weakest bounds are obtained for the standard values of the unification scale $M_{\rm GUT} \simeq 10^{16}$ GeV suggested by gauge coupling unification.

Let us finally turn to the limits that come from the yet unobserved branching ratio $\mathrm{BR}(\mu \to e\gamma)$. From eq. (4.191) and the leptonic analogue of (4.199) one obtains

$$\tilde{m}_{\tilde{l}} \gtrsim \frac{1}{\sqrt{1+g(t)\xi^2}} \left(\frac{|n|}{g_s}\right)^{1/4} \sqrt{\frac{|c_{y,F}|}{\eta}} \times 15.2 \text{ TeV} \tag{4.202}$$

If $M_{\rm GUT} \simeq 10^{16}$ GeV then one gets $\tilde{m}_{\tilde{l}} \gtrsim 10.6$ TeV, quite similar to the results obtained from non-constant closed string fluxes.



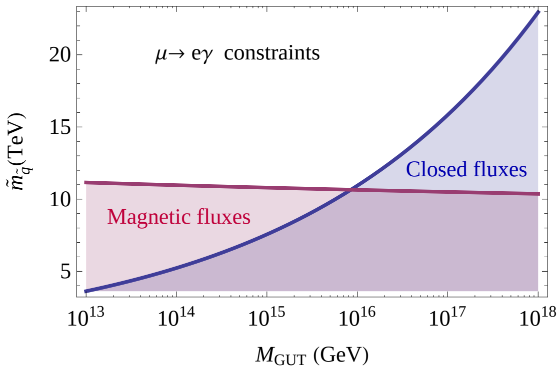


FIGURE 4.7: Bounds on the average slepton masses as a function open string flux n and unification scale $M_{\rm GUT}$. Contributions from non-constant open and closed string fluxes are shown.

	At $M_{\rm GUT}$	At TeV scale	Experimental constraint
$\left\lvert \mathrm{Re}\ \tilde{\delta}^{\rm d}_{12} \right\rvert$	$0.39\ a$ $0.37\ b$	$4.1 \times 10^{-2}\ a$ $3.9 \times 10^{-2}\ b$	$4.2 \times 10^{-2}\ \dfrac{\tilde{m}_{\tilde{q}}}{350\ \mathrm{GeV}}$
$\left\lvert \mathrm{Im}\ \tilde{\delta}^{\rm d}_{12} \right\rvert$	$0.39\ a$ $0.37\ b$	$4.1 \times 10^{-2}\ a$ $3.9 \times 10^{-2}\ b$	$1.8 \times 10^{-3}\ \dfrac{\tilde{m}_{\tilde{q}}}{350\ \mathrm{GeV}}$
$\left\lvert \tilde{\delta}^{\rm l}_{12} \right\rvert$	$0.39\ a$ $0.37\ b$	$1.9 \times 10^{-1}\ a$ $1.8 \times 10^{-1}\ b$	$4 \times 10^{-4}\ \left(\dfrac{\tilde{m}_{\tilde{l}}}{500\ \mathrm{GeV}}\right)^2$

Table 4.3: Prediction and experimental constraints for the mixing parameters δ_{12} coming from Δm_K, ϵ_K and BR$(\mu \to e\gamma)$ measurements/limits. The model-dependent parameters a and b are defined as $a = c_{y,G}/|n|^{1/2}$ and $b = c_{y,F}|n|^{1/2}/(g_s^{1/2}\eta)$ and are expected to be of order ~ 1.

In figure 4.7 we show a summary of the bounds on the selectron and smuon masses for different values of n and $M_{\rm GUT}$. Note that in settings like this, where slepton and squark masses unify at $M_{\rm GUT}$, having sleptons with masses of order ~ 10 TeV would imply much heavier squarks, with masses as large as ~ 25 TeV, quite above the bounds coming from the kaon system.

As a general summary of the numerical results obtained in this section for varying closed and open string fluxes, we present in table 4.3 the expected values of the real and imaginary parts of $\tilde{\delta}^{\rm d}_{12}$ and of $\tilde{\delta}^{\rm l}_{12}$, both at the $M_{\rm GUT}$ and the TeV scales, as well as the corresponding experimental limits.

4.2.3. Flavor non-universalities in F-theory matter curves

Open string fluxes are required in order to get chirality in the matter curves. In the previous sections we have denoted this magnetic flux as M for simplicity. However we have seen in 4.1.4 that in order to recover the MSSM spectrum a much richer strucure of magnetic fluxes than the one used so far is required. Here we provide for completeness the dictionary between the magnetic fluxes of the F-theory SU(5) local model of 4.1.4 and the general results for flavor violating soft terms obtained above. We also estimate the non-universalities coming from the trilinear terms.

Let us consider eq. (4.135) and discuss the case of sfermion masses. To simplify the discussion we set $q_s = 0$, since this is only required to be non-vanishing for having doublet-triplet splitting of the Higgss multiplet, but it plays no role in the sfermion sector. For the wavefunction of $\bar{\mathbf{5}}$ matter fields then we have $\lambda_y = 0$ and $\lambda_x = \lambda_+ \simeq -m^2 - \frac{1}{2}q_p^a$, and zero modes read

$$\Psi_{a_i^+} = \begin{pmatrix} -\frac{i\lambda_x}{m^2} \\ 0 \\ 1 \end{pmatrix} \chi^{\rm real}_{a_i^+} = \begin{pmatrix} i + \frac{iq_p^a}{2m^2} \\ 0 \\ 1 \end{pmatrix} \chi^{\rm real}_{a_i^+} \tag{4.203}$$

where $i = 1, 2, 3$ labels the three SM generations and

$$\chi^{\rm real}_{a_i^+} = \gamma^i_{a^+} \, m^{4-i} y^{3-i} \, e^{-m^2|x|^2} e^{-\frac{q_p}{2}|y|^2} \,, \tag{4.204}$$

The normalisation factors $\gamma^i_{a^+}$ are given by

$$||\gamma^i_{a^+}||^2 = \frac{1}{\pi^2(3-i)!} \frac{m^4}{m^4 + \lambda_+^2} \left(\frac{q_p}{m^2}\right)^{4-i} \,. \tag{4.205}$$

where we have extended the domain of integration to $\mathbb{C}^2$. This is indeed a good approximation in the limit on which the volume of the 4-cycle S is large. Inserting the local expansion (4.175) of the non-constant flux $G_{(0,3)}$ in eq. (4.135) and extending the domain of integration to $\mathbb{C}^2$, we get

$$m^2_{ij} = \frac{g_s \gamma_i \gamma_j}{4} \int_{\mathbb{C}^2} d^2x \, d^2y \, \Big[|\hat{G}_0|^2 \left(1 + G_y^* y + G_y \bar{y} + G_{y\bar{y}} |y|^2 + \ldots\right) \\ y^{3-i} \bar{y}^{3-j} \, e^{-2m^2|x|^2 - |q_p||y|^2} \Big] \tag{4.206}$$

where we have defined

$$|\hat{G}_0|^2 \equiv |G_0|^2 \left(1 - \left|\frac{q_p}{2m^2}\right|\right) \,. \tag{4.207}$$

Sizeable flavor non-diagonal transitions δ^{RR}_{ij} or δ^{LL}_{ij} that do not mix the left and right sectors generically arise from soft mass terms. In particular, the leading contributions to FCNC transitions come from the off-diagonal mass terms. For $\Delta F = 1$ soft masses we have

$$\begin{aligned} m^2_{ij} &= \frac{g_s \gamma_i \gamma_j}{4} \int_0^\infty 2\pi x \, dx \int_0^\infty 2\pi y \, dy \, |\hat{G}_0|^2 \left(G_y^* y + G_y \bar{y}\right) e^{-2m^2|x|^2 - |q_p||y|^2} y^{3-i} \bar{y}^{3-j} \\ &= \frac{g_s k}{4} \frac{|\hat{G}_0|^2 G_y}{\sqrt{|q_p|}} \,, \qquad \text{where} \qquad k \equiv \begin{cases} \sqrt{2} & \text{for} \quad i = 1, \; j = 2 \\ 1 & \text{for} \quad i = 2, \; j = 3 \end{cases} \end{aligned} \tag{4.208}$$

The off-diagonal $\Delta F = 2$ mass term m^2_{13} is proportional to higher derivatives of the 3-form flux and is therefore subleading with respect to m^2_{12} and m^2_{23}. The relevant quantity in the generation of FCNC effects in the Kaon system is

$$\delta^{\rm d}_{12} = \frac{m^2_{12}}{m^2_{\tilde{q}}} = \frac{\sqrt{2} G_y}{\sqrt{|q_p|}} = \frac{\sqrt{2} G_y}{\sqrt{|\tilde{M} - \frac{1}{3}\tilde{N}_Y|}} \tag{4.209}$$

whereas for the left-handed leptons we have

$$\delta_{12}^{\mathrm{L}} \;=\; \frac{m_{12}^2}{m_{\tilde{L}}^2} \;=\; \frac{\sqrt{2}G_y}{\sqrt{|q_p|}} = \frac{\sqrt{2}G_y}{\sqrt{|\tilde{M}+\frac{1}{2}\tilde{N}_Y|}}\,. \tag{4.210}$$

Hence, flavor violation induced by non-constant 3-form fluxes in this context is slightly larger for sleptons than for squarks. From now on the same estimations and constraints of section 4.2.2 apply here. Recall (4.186). The size of these flavor-violating terms depends on the variation of the closed string fluxes over S through $c_{y,G}$ and is inversely proportional to the square root of the open string flux, which is what determines the width of the wavefunctions. As expected, the more localised the wavefunction is, the smaller the amount of flavor violation. However, within the current approximation it is not possible to suppress the size of flavor-violating effects by making the open string flux q_p large, since the perturbative flux expansion that we are assuming in our computations would break down. Also in that limit the soft scalar masses in eq. (4.135) may become tachyonic. On the other hand, these flavor-violating effects may be suppressed if the closed string fluxes G vary slowly over S, namely if $c_{y,G}$ is small.

Finally flavor non-diagonal transitions δ_{ij}^{LR} mixing left and right also generically appear from soft trilinear scalar couplings with non-constant closed string fluxes. By reducing the DBI+CS action in the presence of closed string fluxes and backgrounds for Φ and F_2 we obtain

$$A_{ijk} \;=\; -3g_{\mathrm{YM}}\,g_s^{1/2}\int G^*\;\det(\vec{v}_\alpha,\vec{v}_\beta,\vec{v}_\gamma)\;f_{\alpha\beta\gamma}\;\chi^i_{a_p^+}\chi^j_{b_q^+}\chi^k_{c_r^+}\,\mathrm{dvol}_S \tag{4.211}$$

where $f_{\alpha\beta\gamma} = -i\mathrm{Tr}([E_\alpha,E_\beta],E_\gamma)$. When $G_{\bar{1}\bar{2}\bar{3}}$ varies over the 4-cycle S, flavor-dependent trilinear couplings appear. Once the Higgs boson takes a vev at the EW scale, these give rise to flavor-violating soft masses of the form δm^2_{LR}. Although they are suppressed by the Higgs vev, they still might be relevant since the experimental constraints for δm^2_{LR} are rather strong. Indeed, the relevant terms in the local expansion of the $G_{(0,3)}$ flux around the triple intersection point are in this case

$$G^* \;=\; G_0^*\sum_{n,m}G_{nm}\bar{x}^n\bar{y}^m \;+\;\ldots \tag{4.212}$$

When performing the integral (4.211) the rest of the terms in the expansion vanish. To leading order in the magnetic fluxes, the above expression becomes

$$A_{ij} \;\simeq\; \mathrm{const.}\,G_0^*\sum_{nm}G_{nm}\int\bar{x}^n\bar{y}^m\;\chi^i_{a_p^+}\chi^j_{b_q^+}\chi_{c_r^+}\,\mathrm{dvol}_S \tag{4.213}$$

where all the flavor independent factors have been absorbed in the constant in front of this expression. Note that we have set $k=1$ since the matter curve Σ_c only has one single generation corresponding to the Higgs, while the matter curves $\Sigma_{a,b}$ must accommodate three generations corresponding to the three chiral families of the SM $(i,j=1,2,3)$. The computation of this integral is cumbersome but we can easily estimate the order of magnitude of the flavor non-universalities that appear. Since G_{nm} scales as

$$G_{nm} \;\sim\; \frac{c_{nm}}{\mathrm{Vol}(B_3)^{\frac{n+m}{6}}} \tag{4.214}$$

with c_{nm} an adimensional parameter, making use of eqs. (3.33) and (3.34) we find for $c_{nm} \simeq 1$,

$$A_{ij} \;\simeq\; G_0^* \left(\frac{M_{\rm G}}{\alpha_{\rm G}\, M_{\rm Pl}\, n^{3/2}} \right)^{2-\frac{i+j}{3}} \tag{4.215}$$

The induced flavor-violating soft masses are given by

$$(\delta m^2_{LR})_{ij} \simeq \frac{A_{ij}\langle v\rangle}{m^2_{\rm soft}} \tag{4.216}$$

where $\langle v\rangle$ is the EW vacuum expectation value of the Higgs and $m^2_{\rm soft} \sim |G_0|^2$. Thus, from the above expressions we obtain that flavor-violating soft masses mixing the first two generations scale as

$$(\delta m^2_{LR})_{12} \;\sim\; \frac{\langle v\rangle}{\sqrt{m^2_{\rm soft}}} \frac{c_{12}}{n^{3/2}} \left(\frac{M_{\rm G}}{\alpha_{\rm G} M_{\rm Pl}} \right) \;\sim\; \frac{6}{\sqrt{m^2_{\rm soft}({\rm GeV})}} \tag{4.217}$$

where in the last equation we have used $M_{\rm G} \simeq 10^{16}$ GeV, $\alpha_{\rm G} = 1/24$, $M_{\rm Pl} \simeq 10^{19}$ GeV, $\langle v\rangle = 246$ GeV and $n, c \sim \mathcal{O}(1)$. If the SUSY breaking scale is of order $\sqrt{m^2_{\rm soft}} \sim 1$ TeV, then these flavor non-universalities are of order $10^{-2} - 10^{-3}$, whereas experimental bounds from $\mu \to e\gamma$ require $(\delta m^2_{LR})_{e\mu} < 10^{-5} - 10^{-6}$ for slepton masses of order 1 TeV, see [67–70, 85, 86]. This suggests again sefermion masses should be in the multi-TeV range.

4.2.4. Flavor violation, symmetries and the LHC reach

Given the stringent results obtained in 4.2.2 and 4.2.3, with squark and slepton masses above the ~ 10 TeV range, inaccessible to LHC, a natural question arises. Under what conditions these bounds may be released and allow for a SUSY spectrum within experimental reach at LHC?

The answer is obviously that those compactifications in which the fluxes vary very slowly on S will get relaxed bounds. Examples of such models are the toroidal Type IIB orientfolds (and orbifolds there-off) of e.g. [56, 88–90] in which indeed constant fluxes are used. On the other hand one may reasonably argue that such models are not completely realistic and in any event rather un-generic.

It could be argued though that we have to some extent assumed the most pessimistic scenario in which the variation of fluxes within the manifold S is linear in the local coordinates with coefficients $c_{y,G}$ and $c_{y,F}$ of order one. The bounds on squark masses are proportional to these coefficients so that if for some reason they are suppressed (say, $|c_{y,G}| \simeq |c_{y,F}| \simeq 1/4$) squarks could be accessible to LHC. One possibility is that some symmetry (e.g. $x, y \to -x, -y$) forbids the linear variation of the fluxes, see e.g. [91–99] for some recent papers on discrete flavour symmetries in string compactifications. In this case the first terms contributing to the flux expansion would be quadratic. We can repeat the analysis in this case to find, for non-constant closed string flux density a contribution to the mass difference (contributing through eq. (4.164) to $\tilde\delta^{\rm d}_{12}$)

$$\rho^{\rm d}_{12} \;=\; \frac{\delta m_2^2 - \delta m_1^2}{2\tilde m^2_{\tilde q}} \;=\; \frac{|G_{y\bar y}|}{2q|M_0|} \;=\; \frac{|c_{y\bar y,G}|}{\pi|n|}\left(\frac{5}{3}\right)^{1/2} \varrho^2 \;=\; 0.5\, \left|\frac{c_{y\bar y,G}}{n}\right| \left(\frac{M_{\rm GUT}}{M_{\rm Pl}\,\alpha_{\rm GUT}} \right)^{2/3} \tag{4.218}$$

that leads to the lower bound

$$\tilde{m}_{\tilde{q}} \gtrsim \frac{|c_{y\bar{y},G}|}{(1+g(t)\xi^2)|n|} \times \left(\frac{M_{\rm GUT}}{M_{\rm Pl}}\right)^{2/3} \times 35.9 \text{ TeV} . \tag{4.219}$$

For $M_{\rm GUT} \simeq 10^{16}$ GeV and parameters of order one, one gets a very weak bound with $\tilde{m}_{\tilde{q}} \gtrsim 34$ GeV. Note that to quadratic order there is no imaginary contribution to $\delta^{\rm d}_{12}$ so there is no contribution to CP violation from this correction. Similar results are obtained from the quadratic term coming from non-constant open string fluxes which yield

$$\rho^{\rm d}_{12} = \frac{\delta m_2^2 - \delta m_1^2}{2\tilde{m}^2_{\tilde{q}}} = \frac{M_{y\bar{y}}}{2q|m|} = \frac{4\pi|c_{y\bar{y},F}|}{\eta}\left(\frac{5}{3}\right)^{1/2} \frac{\alpha'}{\text{Vol}(S)^{1/2}} = 0.41\,\frac{|c_{y\bar{y},F}|}{\eta}\left(\frac{M_{\rm GUT}}{M_{\rm st}}\right)^2 \tag{4.220}$$

and gives rise to

$$\tilde{m}_{\tilde{q}} \gtrsim \frac{1}{1+g(t)\xi^2}\frac{1}{\sqrt{g_s}}\frac{|c_{y\bar{y},F}|}{\eta} \times 0.99 \text{ TeV} . \tag{4.221}$$

For $M_{\rm GUT} \simeq 10^{16}$ GeV and parameters of order one, one gets again a very weak bound with $\tilde{m}_{\tilde{q}} \gtrsim 105$ GeV, since there is no CP violating contribution to the kaon system in this case. In fact one can also check that similar limits to these may be obtained from the quadratic contribution to $\text{Im}\,\tilde{\delta}^{\rm d}_{13}$. In this case the quadratic contribution is complex and a contribution to CP violation in the B^0_d system exists. We skip this analysis here for simplicity.

Regarding the limits that come from the $\mu \to e\gamma$ rate, it is possible to check that one gets from the leptonic analogues of (4.218) and (4.220) the lower bounds

$$\begin{aligned} \tilde{m}_{\tilde{l}} &\gtrsim \sqrt{\frac{|c_{y\bar{y},G}|}{(1+g(t)\xi^2)|n|}} \times \left(\frac{M_{\rm GUT}}{M_{\rm Pl}}\right)^{1/3} \times 52 \text{ TeV} \qquad \text{(closed string flux)} \\ \tilde{m}_{\tilde{l}} &\gtrsim \frac{1}{\sqrt{1+g(t)\xi^2}}\frac{1}{g_s^{1/4}}\sqrt{\left|\frac{c_{y\bar{y},F}}{\eta}\right|} \times 8.6 \text{ TeV} \qquad \text{(open string flux)} . \end{aligned} \tag{4.222}$$

With $M_{\rm GUT} \simeq 10^{16}$ GeV and parameters of order one, the resulting limits on the averaged slepton mass are $\tilde{m}_{\tilde{l}} \gtrsim 3.4$ TeV from the first and $\tilde{m}_{\tilde{l}} \gtrsim 6$ TeV from the second. Altogether we see that the limits on squark masses are totally relaxed if linear terms are absent whereas those from lepton number violation are only somewhat released. This is mostly due to the fact that the RG dilution is larger for squarks than for leptons. Additional uncertainties in factors could allow for lighter slepton masses, within reach of LHC, but certainly the slepton limits are harder to relax.

In a different vein, it would be interesting to perform similar computations in other compactification schemes. For example, very similar results are expected in the case of type IIA models with intersecting D6-branes or their relatives, models based on M-theory compactifications on manifolds with G_2 holonomy. In particular, the three generations of SU(5) **5**-plets will arise at different intersections of a SU(5) stack with a U(1) stack. These intersections happen at different points in compact space and soft terms induced by (non-constant) closed string fluxes will be different for the different generations. Furthermore the intersection angles for the three generations (which are T-dual to the open string fluxes in IIB) will be generically different. This is the dual of having non-constant open string fluxes in the IIB side. All in all, the same structure that we find in a IIB/F-theory is expected in this other large class of compactifications. In the alternative models in which SM fields live on D3-branes located at singularities, the cause of non-universalities would not be the non-uniformity of fluxes but the generic non-isotropy of the compactifications.

Given the numerical uncertainties we cannot exclude squarks being discovered at the LHC. In particular we have shown how e.g. symmetries can substantially relax the obtained mass limits, although this relaxation seems more difficult in the case of the first two generations of sleptons. On the other hand a heavy SUSY spectrum seems to be preferred by the observed mass of the Higgs $m_H \simeq 126$ GeV. Our results seem to go in the same direction.

4.3. Intermediate SUSY breaking scale

String theory, by providing specific microscopic mechanisms of SUSY breaking, is able to constrain the huge parameter space a priori available in an effective approach of the MSSM. Up to here, we have studied the particular structure of soft terms and the presence of non-universalities arising from flux induced supersymmetry breaking. But it can also give us information about the SUSY breaking scale. We will see that closed string fluxes push this scale to a quite high value, around $10^{10} - 10^{13}$ GeV. Much effort in string phenomenology has been devoted to lower this SUSY scale to the TeV range for phenomenological reasons. However, the so far absence of SUSY at the TeV scale and the high mass of the Higgs boson are driving us to rethink about this choice. In this section we reanalyze the problem and discuss several phenomenological and theoretical hints pointing out indeed to an Intermediate scale of SUSY breaking. We also study the realisation of this scenario together with gauge coupling unification in Type IIB/F-theory compactifications. Furthermore, we compute the Higgs mass as a function of the SUSY breaking scale obtaining a very constrained result, $m_H = 126 \pm 3$ GeV for $M_{SS} \geq 10^{10}$ GeV, supporting the idea of SUSY being realised at a high scale.

4.3.1. Motivation

The only thing we know for sure about supersymmetry is that (if it exists) it has to be broken at some energy scale above M_{EW}. While there is no fundamental reason to pick out a specific scale (a priori any value from the current energy reached at the colliders to the Planck scale would be possible) the prefered and most popular scale is undoubtely the TeV-range. We call this scenario low energy or TeV-scale supersymmetry. The reason being is that low energy supersymmetry provides a solution to the EW hierarchy problem of the SM. Without supersymmetry or any other BSM extension, the Higgs mass receives quantum corrections depending quadratically in the cutoff of the theory. If this cutoff is taken to be the Planck scale, a huge fine-tuning of order 1 part in 10^{32} is required to keep the Higgs boson light. In the supersymmetric extension of the SM the quadratic divergences are cancelled by those coming from the corresponding superpartners. However this cancellation is succesful only if the superpartners are not very far from the EW scale. Otherwise, we have to appeal again to a fine-tuning on the mass parameters to keep the Higgs light. Therefore supersymmetry as a solution to the hierarchy problem must lay on the TeV range, implying that can be detected by the LHC. However this has not been the case so far. As of today, there is no sign of Supersymmetry (or any BSM physics) at the LHC. In fact, the LHC has pushed the mass of the squarks to the multi-TeV range, implying already a fine-tuning of one per mil in the Higgs mass. If this pressure persists, the proposal of low energy supersymmetry as a solution to the hierarchy problem will be strongly questionable and the original motivation for breaking supersymmetry at low energy will be lost. In that case, the two primary questions about SUSY have to be rethought: why we need SUSY even if it does not solve the hierarchy problem, and if so what is the scale of SUSY breaking chosen by nature. In what follows we attempt to answer these questions giving some phenomenological and theoretical motivations about an atractive alternative: supersymmetry broken at an Intermediate scale around $10^{10} - 10^{13}$ GeV.

4.3.1.1. Phenomenological motivations

Let us assume that there is no sign of TeV-scale supersymmetry at the next run of the LHC. From a phenomenological point of view, once the original motivation of the fine-tuning is lost, one could think of abandoning completely the idea of supersymmetry as an extension of the SM. So far all the predictions of the SM work extremely and surprisingly well so apparently there would be no reason to pursue with supersymmetry if it can not solve the hierarchy problem. For instance, one could think of recovering old non-susy GUT's. However these theories have several problems, like the loss of gauge coupling unification, too fast proton decay, or the absence of a candidate for dark matter.

In any event, one could bet on the SM and assume its validity up to the Planck scale, where an UV completion including quantum gravitational effects would then be required. However the recent discovery of the Higgs mass call into question this scenario. If one runs up via the renormalization group equations (RGE) the Higgs quartic coupling from the EW scale to higher energies, one can see how λ decreases with the energy due to the large top Yukawa coupling and in fact it seems to vanish at scales $10^{10} - 10^{13}$ GeV. A detailed study of the non-SUSY SM Higgs potential indicates that the theory becomes metastable before reaching the unification scale [100–107]. Although in principle there is no fundamental obstruction about living in a metastable vaccum as long as the lifetime of the vacuum is bigger than the age of the universe, this could be interpreted as a hint of the nature telling us that something is going on at that scale.

In addition, studies about the cosmological implications of the Higgs mass measurement show that a high scale of inflation might be incompatible with a metastable SM vacuum [108] (see [109–111] for recent work). Quantum fluctuations in the Higgs field during inflation might locally drive the Higgs vacuum expectation value to the unstable part of the potential. Therefore in the case that BICEP2 results [3] are confirmed and primordial gravitational waves are detected in forthcoming experiments, the resulting high scale of inflation would imply that new BSM physics is indeed required at an Intermediate scale to stabilize the SM vacuum.

Supersymmetry would be an excellent candidate for that new physics, since in the MSSM the quartic Higgs coupling is automatically positive definite and the stability of the SM would be ensured. This provides a new interesting motivation for supersymmetry in Particle Physics. If present at some intermediate scale, the role of supersymmetry would not be to stabilize the Higgs mass but the SM vacuum, which is not a less important task.

4.3.1.2. Theoretical motivations

As explained in the Introduction of the thesis, at some point one would like to have an ultraviolet completion of the SM, unifying both gravitational and gauge interactions. As of today the best candidate for that UV completion is String Theory, so a natural question is whether String Theory can tell us something about the existence and energy scale of SUSY.

Supersymmetry is one of the key ingredients of String Theory. It is absolutely necessary in the worldsheet in order to obtain fermions in the space-time. But SUSY in the worldsheet does not imply necessarily SUSY in the space-time, so to be fair, supersymmetry in the space-time is not strictly indispensable. However, it is highly recommended, since it protects the theory from undesired instabilities and allows to have a parametric controll over partially computable quantities that would not be possible otherwise. In fact, without supersymmetry we would not even have the necessary machinery and tools to do any reliable computation. This is the reason why all promising string theories have also supersymmetry in the space-time.

Nevertheless, supersymmetry can be broken in the compactification of the theory. In fact, String Theory does not force SUSY to be broken near the EW scale at all. It could also be consistently broken at some intermediate/high scale (below the compactification scale) so that the SUSY particles would be out of the LHC reach. Therefore, by ruling out low energy supersymmetry we are not ruling out any of the string theories.

One could then ask if there is a prefered value for the SUSY breaking scale within String Theory. The answer depends of course on the mechanism of SUSY breaking. We have argued in section 3.1 that a natural source of SUSY breaking in String Theory are the closed string fluxes. From the point of view of the effective supergravity action, these fluxes correspond to the auxiliary fields of closed string moduli. Supersymmetry is then broken in the (hidden) closed string sector and transmitted to the SM fields via gravity mediation. If this is the case, there is indeed a natural scale of SUSY breaking in the theory, as we proceed to explain in what follows.

Defining the scale of SUSY breaking as the size of the soft terms $M_{SS} \simeq m_{\tilde{q}}$, from the results of the previous section we have

$$M_{SS}^2 \simeq \frac{g_s}{4}|G_3|^2 \tag{4.223}$$

As was explained in 3.1 these fluxes are quantized and its size can be estimated by

$$\frac{1}{2\pi\alpha'}\int_{\gamma_j} G_3 = 2\pi f_j \to G_3 \simeq \frac{4\pi^2\alpha' f}{V_6^{1/2}} \tag{4.224}$$

where $\gamma_j \in B_3$ denotes the 3-cycle that supports the closed string fluxes and f the corresponding flux integer. This implies a natural scale of SUSY breaking given by

$$M_{SS} \simeq \frac{g_s^{1/2}}{\sqrt{2}} G_3 \;=\; \frac{f}{\pi}\frac{M_s^2}{g_s^{1/2}M_p} \tag{4.225}$$

where we have used eq.(3.33). Taking into account eq.(3.35) we get the SUSY breaking scale

$$M_{SS} \;\simeq\; \frac{f\, M_c^2}{2\pi\alpha_G^{1/2}M_p}\;. \tag{4.226}$$

in terms of the compactification/unification scale.

Notice that for $f \simeq 1$ and standard unifcation values $M_{GUT} \simeq 10^{16}$ GeV and $\alpha_{GUT}^{-1} \simeq 24$ we obtain $M_{SS} \simeq 6.5 \times 10^{12}$ GeV. Therefore the natural scale of SUSY breaking by closed string fluxes is actually quite high and lays within the range to provide a good solution to the stability problem of the SM vacuum. Remark though that this does not imply that low energy supersymmetry is not possible in this scenario. It should be noticed that the parameter f in general receives contributions from a large number of 3-cycles so that large cancellations can take place leading to $f \ll 1$ and lowering the scale of SUSY-breaking. This is similar to what occurs for the small superpotential parameter W_0 in KKLT vacua [8].[8] Notice though that in order to have a SUSY breaking scale around 1 TeV, we need a huge fine-tuning or suppression in the fluxes such that $f \sim 10^{-10}$. But if we wanted low energy SUSY indeed to solve the fine-tuning in the Higgs, it seems that we are just hidding this hierarchy problem into a fine-tuning on the fluxes in order to lower the scale of SUSY breaking. From this perspective, there are two special situations: low energy SUSY with a natural Higgs but with a fine-tuned scale of SUSY breaking, or a fine-tuned Higgs with a natural scale of SUSY breaking. Of course, it is not our call to choose what scenario is exhibited in nature, but of the experiments. If in the next run of the LHC there is still no sign of supersymmetry, the second posibility will score some points. And if supersymmetry is pushed to higher and higher energies, we should be open to think that maybe the EW scale is indeed fine-tuned and supersymmetry is present at a much higher scale. This fine-tuning of the EW scale could be understood in the context of the flux landscape of String Theory, perhaps selected on anthropic grounds. The problem would be very similar to that of the cosmological constant in flux compactifications [81–84].

So far, much work in String Phenomenology has gone in the direction of lowering the scale of SUSY breaking, resorting to huge cancellations between a large number of 3-cycles or warping factors to suppressed the effect of the fluxes over localised sectors (see [4] for details). Here we would like to follow the opposite direction. Let us consider that SUSY is broken indeed at the natural scale suggested by String Theory once we impose a standard unification scale, ie. $M_{SS} \simeq 10^{10} - 10^{13}$ GeV, and study what are the phenomenological implications of such a high scale.

4.3.2. Implications for the Higgs mass

Since the scalars are sensitive to the cutoff scale of the theory, the Higgs boson ocupies the first place in the study of the phenomenological implications of an Intermediate scale of SUSY breaking.

To start with, let us consider the general scenario in which M_{SS} is a free parameter, so the SM is extended to the MSSM above a certain scale M_{SS} not necessarily tied to the EW scale but possibly much higher. In this case, as we have already explained, a fine-tuning of the underlying theory is required in order to have a light Higgs. We can use this fine-tuning condition to fix the EW vacuum expectation value of the Higgs to its experimental result. Then the Higgs mass depends only on the quartic coupling, which is still a free parameter in the SM. However, since the SM will be extended to the MSSM at a

[8] In the large volume scenario of moduli stabilization [14–17], low energy supersymmetry is achieved without tuning W_0 but instead lowering the string scale. So if we want to keep a unification scale around 10^{16} GeV there is no other way than suppressing the fluxes or considering huge delicate cancellations such that $f \ll 1$.

higher scale, this quartic coupling will be given by the gauge couplings and the proportional composition of the MSSM Higgs doublets in the linear combination corresponding to the SM Higgs. Hence the Higgs mass is not a free parameter and can be computed once taking into account the running from M_{SS} to the EW scale.

In section 4.3.2.1 we start the discussion by being more precise about what we mean with the statement that the EW scale has to be fine-tuned. We will see how to quantify this fine-tuning in the Higgs mass matrix. In section 4.3.2.2 we describe the different steps required to compute the Higgs mass as a function of the SUSY-breaking scale M_{SS}, and show the results.

4.3.2.1. Fine-tuning of the Higgs

Let us consider a situation in which SUSY is broken at some high scale M_{SS} with $M_{EW} \ll M_{SS} \ll M_C$, where M_C is the unification/compactification scale. For previous work on a fine-tuned Higgs in a setting with broken SUSY at a high scale see e.g. [112–120, 124, 133, 138–140]. With generic SUSY breaking soft terms one is just left at low energies with the SM spectrum. In addition the scalar potential should be fine-tuned so that one Higgs doublet remains light and thus is able to trigger EW symmetry breaking. To see this let us recall what is the general form for the Higgs masses in the MSSM at the scale M_{SS},

$$\left(\begin{array}{cc} H_u \ , & H_d^* \end{array} \right) \left(\begin{array}{cc} m_{H_u}^2 & m_3^2 \\ m_3^2 & m_{H_d}^2 \end{array} \right) \left(\begin{array}{c} H_u^* \\ H_d \end{array} \right) . \tag{4.227}$$

where we will take m_3^2 real for simplicity. If all these mass terms were zero we would get two Higgs doublets in the massless spectrum. However this would require extra unnecessary fine-tuning. The *minimal Higgs fine-tuning* would only require a single Higgs doublet to remain at low-energies. This is achieved for a single fine-tuning

$$m_3^4 = m_{H_u}^2 m_{H_d}^2 \quad \text{at } M_{SS} \tag{4.228}$$

to get a zero eigenvalue in the above matrix. The massless eigenvector is then

$$H_{SM} \ = \ sin\beta \ H_u \ + \ cos\beta \ H_d^* \tag{4.229}$$

with

$$tan\beta \ = \ \frac{|m_{H_d}|}{|m_{H_u}|} \ . \tag{4.230}$$

One would say that no trace would be left from the underlying supersymmetry after breaking to the SM. However this is not the case [114]. Since dimension four operators are not affected (to leading order) by spontaneous SUSY breaking, the value $\lambda(M_{SS})$ of the Higgs self-coupling at the M_{SS} scale will be given in the MSSM by the (tree level) boundary condition

$$\lambda_{SUSY}(M_{SS}) \ = \ \frac{1}{4}(g_2^2 \ + \ g_1^2) \ cos^2 2\beta \tag{4.231}$$

which is inherited from the D-term scalar potential of the MSSM. Here $g_{1,2}$ are the EW gauge couplings and β is the mixing angle which defines the linear combination of the two $SU(2)$ doublets H_u, H_d of the MSSM which remains massless after SUSY breaking. Thanks to this boundary condition, for any given value of tanβ one can compute the Higgs mass as a function of the SUSY breaking scale M_{SS}.

Schematically, the idea is to run in energies the values of g_1, g_2 up to the given M_{SS} scale. For any value of tanβ one then computes $\lambda(M_{SS})$ from eq.(4.231). Starting with this value we then run down in energies and obtain the value for the Higgs mass from $m_H^2(Q) = 2v^2\lambda(Q)$. Threshold corrections at both the EW and SUSY scales have to be included. This type of computation for different values of tanβ was done e.g in ref. [107, 121–123]. We show results for a similar computation in fig.4.8 (grey bands) in section 4.3.2.2. The Higgs mass may have any value in a broad band below a maximum around 140 GeV. One may easily understand the general structure of these curves. The mass is higher for higher tanβ since the tree level contribution to the Higgs mass through eq.(4.231) is higher. On the other hand the Higgs mass slowly grows with larger M_{SS} as expected.

What we want to emphasize here is that the natural assumption of Higgs soft mass unification at the unification scale M_C, i.e.

$$m_{H_u}(M_C) \;=\; m_{H_d}(M_C) \tag{4.232}$$

leads to a much more restricted situation with trajectories converging to a very narrow strip in the $m_{Higgs} - M_{SS}$ plane rather than a wide band. Note that this equality is quite generic in most SUSY, unification or string models. In particular it appears in gravity mediation as well as in almost all SUSY breaking schemes, including those arising from compactified string theory, see e.g. [4].

One can then compute the value of tan$\beta(M_{SS})$ by running the ratio in (4.230) from the unification scale M_C down to the SUSY breaking scale M_{SS}. One computes the value of the Higgs self-coupling $\lambda(M_{SS})$ from eq.(4.231) and then runs down in energies to compute the Higgs mass for any given value of M_{SS}. In a general MSSM model we can compute this in terms of the underlying structure of soft terms at M_{SS}. In particular one expects generic SUSY-breaking soft terms of order M_{SS}. For definiteness we will assume here a universal structure of soft terms with the standard parameters m (3-d generation scalars masses), M (gaugino masses), A (3-d generation trilinear parameter) and μ (mu-term). As we will see in section 4.3.2.3, the results are very little dependent on this universality assumption which simplifies substantially the computations. This universality assumption is also consistent with the (weaker) assumption of Higgs mass unification, eq.(4.232).

Let us remark that in this approach the only relevant condition is $m_{H_u} = m_{H_d}$ at the unification scale M_C. There is no need for a *shift symmetry* which imposses $m_3^4 = m_{H_u}^2 m_{H_d}^2$ at the unification scale as in ref. [124], since then the fine-tuning would be destroyed by the running from M_C to M_{SS}. The idea is that enviromental selection should ensure that at the scale M_{SS} (not M_C) the fine-tuning condition $m_3^4 = m_{H_u}^2 m_{H_d}^2$ is impossed with high accuracy. Although for the time being these two conditions will be taken as assumptions, in section 4.3.4 we will see how indeed they can be naturally accomodated in Type IIB/F-theory models.

4.3.2.2. Higgs mass as a function of the SUSY breaking scale

The main question we want to address in this section is: what would be the mass of the Higgs boson if SUSY is broken at a high scale?

Although this is a quite generic question, the answer turns out to be much more constrained than expected. Under standard unification assumptions ($m_{Hu} = m_{Hd}$ at the unification scale M_c) it gives rise to a prediction of the Higgs mass as a function of the SUSY breaking scale, with a very little dependence on the soft SUSY breaking parameters.

We now turn to a description of the different steps required to compute the Higgs mass as a function of the SUSY-breaking scale M_{SS}.

Computing the couplings at M_{SS}. We start by computing the electroweak couplings at the M_{SS} scale. We take the central values for the masses (in GeV) and couplings at the weak scale

$$M_Z = 91.1876\ ,\ M_W = 80.385\ ,\ m_t = 173.1 \tag{4.233}$$

$$\sin^2\theta_W(M_Z) = 0.23126\ ,\ \alpha_{em}^{-1}(M_Z) = 127.937\ ,\ \alpha_3(M_Z) = 0.1184\ . \tag{4.234}$$

We will allow to vary the top mass with an error $m_t = 173.1 \pm 0.7$ GeV obtained from the average from Tevatron [125] and CMS and ATLAS results as in ref. [126]. We will neglect the error from α_3 which is much smaller than that from the top quark mass. To extract the value of the top Yukawa coupling $h_t(m_t)$ we take into account the relationship between the pole top-quark mass m_t and the corresponding Yukawa coupling in the $\overline{MS}$ scheme [127]

$$h_t(m_t) = \frac{m_t}{v}(1+\delta_t) \tag{4.235}$$

where the dominant one-loop QCD corrections may be estimated ([127], [107, 122])

$$\delta_t^{QCD}(m_t) = -\frac{4}{3\pi}\alpha_3(m_t) - 0.93\alpha_3^2(m_t) - 2.59\alpha_3^3(m_t) \approx -0.0605\ . \tag{4.236}$$

One then obtains $h_t(m_t) = 0.934$. We run now the couplings g_1,g_2 and h_t up to the given scale M_{SS}. We do this by solving the RGE at two loops for the SM couplings. Those equations are shown for completeness in appendix B.1.

Computing tanβ and $\lambda(M_{SS})$. With $g_{1,2}(M_{SS})$ at hand we want now to compute the value of $\lambda(M_{SS})$ from eq.(4.231). To do that we need to compute tan$\beta(M_{SS})$ from eq.(4.230), which in turn requires the computation of the running of the masses m_{H_u},m_{H_d} from the unification scale at which $m_{H_u} = m_{H_d}$ down to M_{SS}.

The value of the unification scale M_C is usually obtained from the unification of gauge coupling constants. In our case, with two regions respectively with the SM (below M_{SS}) and the MSSM (in between M_{SS} and M_C) the value of M_C is not uniquely determined. In fact it is well known that precise unification is only obtained for $M_{SS} \simeq 1$ TeV, as in standard MSSM phenomenology [128–130]. However, approximate unification around a scale $M_C \simeq 10^{14} - 10^{15}$ GeV is anyway obtained for much higher values of M_{SS}, even in the limiting case with $M_{SS} \simeq M_C$ in which case SUSY is broken at the unification scale, so a simple approach would be to take $M_C \simeq 10^{15}$ GeV to compute the runing of tanβ. We find more interesting instead to achieve consistent gauge coupling unification from appropriate threshold corrections. In particular, in a large class of string compactifications like F-theory $SU(5)$ GUT's there are small threshold corrections respecting the boundary condition at the GUT scale [53, 131–134]

$$\frac{1}{\alpha_1(M_C)} = \frac{1}{\alpha_2(M_C)} + \frac{2}{3\alpha_3(M_C)}\ . \tag{4.237}$$

This boundary condition is consistent (but more general) than the usual GUT boundary conditions $\alpha_3 = \alpha_2 = 5/3\alpha_1$. It arises for example from F-theory $SU(5)$ GUT's [25–29,135] once fluxes along the hypercharge direction are added to break the $SU(5)$ symmetry down to the SM [32, 33, 53] (see section 4.3.3.2). Using the RGE for gauge couplings in both SM and MSSM regions (at two loops for the gauge couplings and one loop for the top Yukawa) one finds that unification of couplings is neatly obtained at a scale M_C related with M_{SS} by the approximate relationship

$$\log M_C = -0.23 \log M_{SS} + 16.77 \ . \tag{4.238}$$

As one varies M_{SS} in the range 1 TeV-M_C one obtains $M_C \simeq 10^{16} - 10^{14}$ GeV. In section 4.3.3 we will delve into this issue of gauge coupling unification, discussing in more detail the threshold corrections that give rise to (4.237) and the computation of the gauge coupling unification condition (4.238) (black line in fig.4.13). In the matter at hand, to compute tan$\beta(M_{SS})$ we will use the unification scale M_C obtained from eq.(4.238). It is important to remark though that this has very little impact in the numerical results. There is no detailed dependence on the value of M_C as long as it remains in the expected $10^{14} - 10^{17}$ GeV region.

To compute tanβ at M_{SS} one solves the RGE for the Higgs mass parameters m_{H_u}, m_{H_d}. At this point one needs to make some assumptions about the structure of the SUSY-breaking soft terms of the underlying MSSM theory. We will thus assume a standard universal SUSY breaking structure parametrized by universal scalar masses m, gaugino masses M and trilinear parameter A. The results are independent from the value of the B parameter which is fixed by the fine-tuning condition (4.228) at M_{SS}. Given these uncertainties it is enough to use the one-loop RGE for the soft parameters, which were analytically solved in ref. [77]. Thus one has tan$\beta(M_{SS}) = |m_{H_d}(M_{SS})|/|m_{H_u}(M_{SS})|$ with

$$\begin{aligned} m_{H_d}^2(t) &= m^2 + \mu^2 q^2(t) + M^2 g(t) \\ m_{H_u}^2(t) &= m^2 h(t) - k(t)A^2 + \mu^2 q^2(t) + M^2 e(t) + AMf(t) \end{aligned} \tag{4.239}$$

where m, M, A, μ are the standard universal CMSSM parameters at the unification scale M_C, $t = 2log(M_C/M_{SS})$ and q, g, h, k, e, f are known functions of the top Yukawa coupling h_t and the three SM gauge coupling constants. Except for regions with large tanβ, appearing only for low M_{SS}, one can safely neglect the bottom and tau Yukawa couplings, $h_b = h_\tau = 0$. For completeness these functions are provided in appendix B.2. The value taken for h_t to perform the running of soft terms is a bit subtle since at M_{SS} one has to match the h_t^{SM} value obtained from the SM running up to M_{SS} with the SUSY value h_t^{SUSY} which are related by

$$h_t^{SM} = h_t^{SUSY} sin\beta \ . \tag{4.240}$$

Since the value of h_t^{SUSY} depends on β through eq.(4.240), the computation of tanβ is done in a self-consistent way: a value is given to sin$\beta(M_{SS})$, h_t^{SUSY} is run up in energies and one has a tentative $h_t(M_C)$. One then runs m_{H_u}/m_{H_d} down in energies and computes tanβ at M_{SS}. When both values for β at M_{SS} agree the computation of tanβ is consistent.

Once computed the value of tanβ as described above, one then obtains the Higgs quartic coupling $\lambda(M_{SS})$ from eq.(4.231). In addition there are threshold corrections at M_{SS} induced by loop diagrams involving the SUSY particles. The leading one-loop correction is given by

$$\delta\lambda(M_{SS}) = \frac{1}{(4\pi)^2} 3h_t^4 \left(2X_t - \frac{X_t^2}{6}\right) \tag{4.241}$$

where h_t is the SUSY top Yukawa coupling at M_{SS} and the stop mixing parameter X_t is given by

$$X_t \;=\; \frac{(A_t - \mu \cot\beta)^2}{m_Q m_U} \;. \tag{4.242}$$

with $m_Q(m_U)$ the left(right)-handed stop mass. This term comes from finite corrections involving one-loop exchange of top squarks. There are further correction terms which are numerically negligible compared to this at least for not too low M_{SS}, in which case the SUSY spectrum becomes more spread and further threshold corrections become relevant, see e.g. [122]. We have computed the parameter X_t using the one loop RGE for the soft parameters that are provided in appendix B.2 and the value of tanβ obtained above.

Computing the Higgs mass Starting from $(\lambda + \delta\lambda)(M_{SS})$ one runs back the self-coupling down to the EW scale (using the SM RGE at two loops) and computes the Higgs mass at a scale Q (taken as $Q = m_t$) through

$$m_H^2 \;=\; 2v^2(\lambda(Q) + \delta^{EW}\lambda(Q)) \;, \tag{4.243}$$

where $v = 174.1$ GeV is the Higgs vev and $\delta^{EW}\lambda(Q)$ are additional EW scale threshold corrections. At one-loop these corrections are given by [136]

$$\delta^{EW}\lambda = -\frac{\lambda G_F M_Z^2}{8\pi^2\sqrt{2}}(\xi F_1 + F_0 + F_3/\xi) \approx 0.011\lambda \tag{4.244}$$

where $\xi = m_H^2/M_Z^2$ and the functions F_1, F_0 y F_3 depend only on EW parameters and are shown in appendix B.3 for completeness. This completes the computation procedure for the Higgs mass as a function of M_{SS}.

Figure 4.8 plots the value of m_H^2 as a function of M_{SS}. For definiteness we plot the results for universal soft terms with $M = \sqrt{2}m$, $A = -3/2M$. This choice of values is motivated by modulus dominance SUSY breaking in string scenarios, see e.g. [58], [4]. In particular, they arise from models in which the MSSM is localised at a system of intersecting 7-branes and closed string fluxes are the main source of SUSY breaking, as we discussed in section 4.1. However, as we will explain below, other different choices for soft parameters m, M, A lead to analogous results. The grey bands correspond to the computation of the mass for tanβ = 1, 2, 4, 50 and $X_t = 0$. The results are similar to those obtained in ref. [107, 121–123]. The other colored bands correspond to the Higgs mass values obtained under the assumption of Higgs parameter unification as in eq.(4.232). Results are displayed for a mu-term $\mu = -M/4, -M/2, -3/4M, -M$ with the value for X_t computed from the obtained running soft terms [9]. The width of the grey and colored bands corresponds to the error from the top quark mass. Finally the horizontal band corresponds to the average CMS and ATLAS results for the Higgs mass (we take $m_H = 125.5 \pm 0.54$, see [107]).

The figure shows that above a scale $\simeq 10^{10}$ GeV the value of the Higgs mass is contained in the range

$$m_H \;=\; 126 \;\pm\; 3 \; GeV \;. \tag{4.245}$$

This is remarkably close to the measured value at LHC and supports the idea that SUSY *and* unification underly the observed Higgs mass. This result is quite independent of the

[9]The results are very weakly dependent on the sign of μ through the X_t appearing in the threshold corrections.

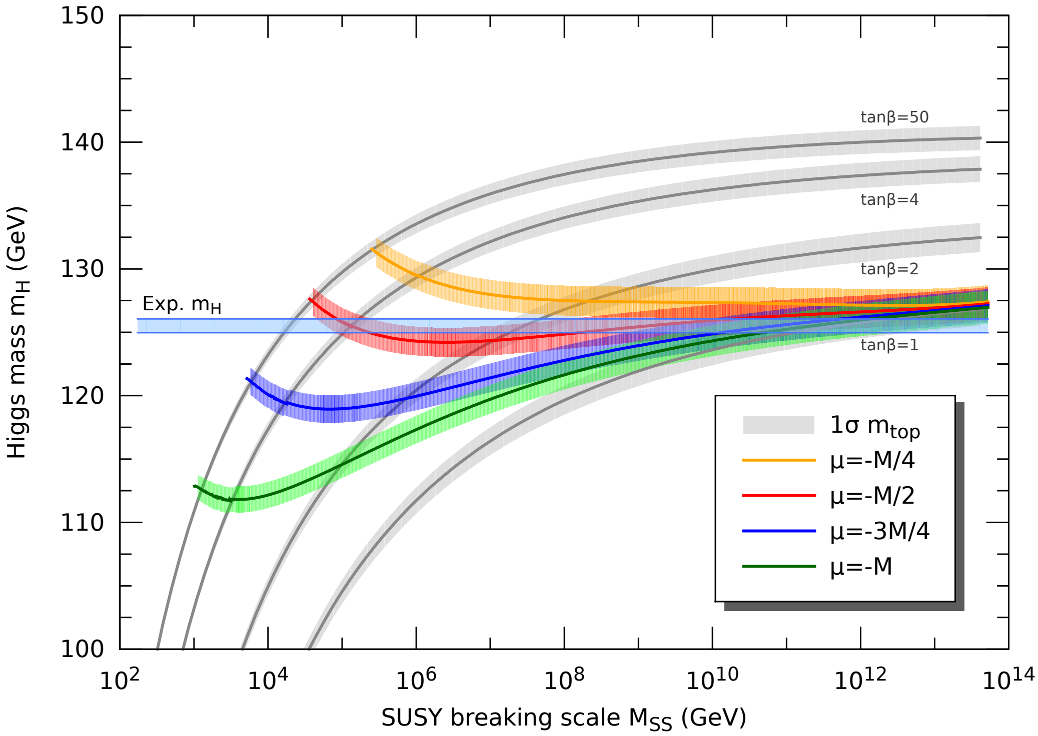


Figure 4.8: Higgs mass versus SUSY breaking scale M_{SS}. The grey bands correspond to the Higgs mass for different values of tanβ, for $X_t = 0$, without impossing unification of Higgs soft parameters. The other colored bands correspond to impossing tanβ values consistent with unification of soft terms, $m_{H_u} = m_{H_d}$. Results are shown for a choice of universal soft terms $M = \sqrt{2}m$, $A = -3/2M$ and four values for the μ-term. The stop mixing parameter X_t is computed from the given soft parameters. The width of the bands correspond to the error from the top quark mass which is taken to be $m_t = 173.1 \pm 0.7$. The horizontal band corresponds to the ATLAS and CMS average Higgs mass result.

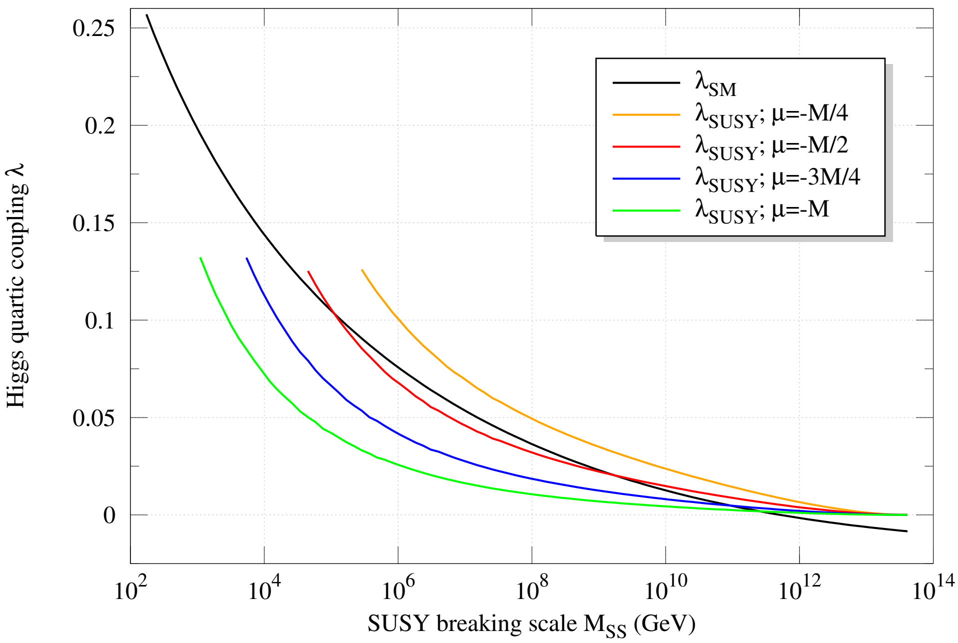


Figure 4.9: The black line shows the value of the SM self-coupling λ as a function of M_{SS}, using as input the LHC Higgs data. The remaining curves show values of λ_{SUSY} consistent with $m_{H_u}(M_C) = m_{H_d}(M_C)$ for different values of μ. When these λ_{SUSY} lines cross the λ curve the SUSY model is consistent with LHC Higgs data.

details of the soft terms. Below 10^9 GeV the Higgs mass becomes more model dependent. In particular the Higgs mass is reduced as $|\mu|$ increases. This is easy to understand from eq.(4.239) since for larger μ the ratio m_{H_u}/m_{H_d} approaches one, yielding tan$\beta \simeq 1$. One still gets a Higgs mass consistent with LHC results for not too large $|\mu|$. As one approaches $M_{SS} \simeq 10 - 100$ TeV one reaches the region of standard fine-tuned MSSM with a Higgs mass which may be as large as 130 GeV. As we approach that region our treatment becomes incomplete since some neglected SUSY threshold corrections beyond those in (4.244) become important, and the SUSY spectrum spreads out. However, that region corresponds to the well understood situation of the MSSM with a heavy SUSY spectrum with masses in the 10-100 TeV region.

Let us finally note that, within uncertainties, the figure also favours values for the SUSY breaking scale $M_{SS} \lesssim 10^{13}$ GeV.

One may also interpret graphically the above results in terms of the unification of the SM Higgs self-coupling λ_{SM} and the SUSY predicted self-coupling $\lambda_{SUSY} = (g_1^2 + g_2^2)cos^2 2\beta/4$. This is depicted in fig.4.9, in which we have not included the uncertainty from the m_t error to avoid clutter. Note that the dependence of λ_{SUSY} on M_{SS} is qualitatively similar to the running of λ_{SM}. This may be understood as follows. In the definition of λ_{SUSY}, $(g_1^2 + g_2^2)$ runs very little and remains practically constant. On the other hand one has $cos^2 2\beta = (m_{H_u}^2 - m_{H_d}^2)^2/(m_{H_u}^2 + m_{H_d}^2)^2$. The difference on the numerator goes like h_t^4, which is also the order of the leading correction to the λ_{SM} coupling.

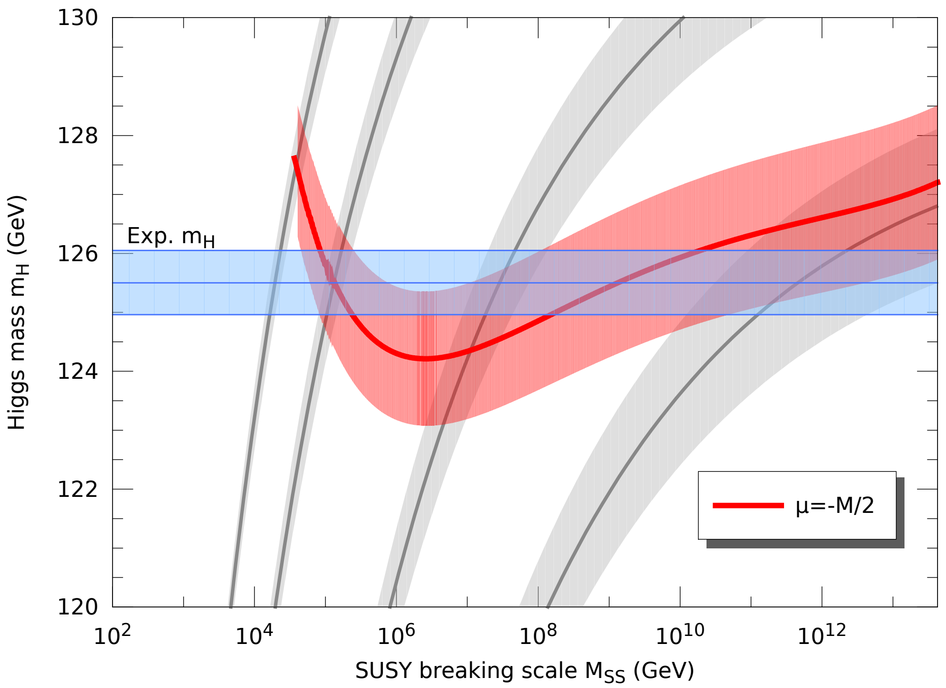


Figure 4.10: Higgs mass versus SUSY breaking scale M_{SS} for $\mu = -M/2$ (red band). Its width reflects the uncertainty on $m_t = 173.1 \pm 0.7$. The grey bands, as in fig.4.8 show the Higgs mass for several values of tan$\beta = 1, 2, 4, 50$ and are displayed to guide the eye.

4.3.2.3. Model dependence

In this section we discuss the dependence of our results on the specific structure of the underlying soft terms. With sufficiently precise information about the top quark and Higgs masses one might even obtain interesting constraints on the possible structure of the SUSY-breaking terms.

Let us concentrate first in the case with universal soft terms and $\mu = -M/2$ but still keeping the relationships $M = \sqrt{2}m$, $A = -3/2M$. As we said these values are interesting since, as discussed in ref. [58], they may be understood as arising from a Giudice-Masiero mechanism in a modulus dominance SUSY breaking scheme. In the the microscopic description of section 4.1 they correspond to the structure of soft terms induced by an isotropic configuration of fluxes (with $G_{\bar{1}\bar{2}\bar{3}} = S_{\bar{3}\bar{3}}/2$) on matter fields living at brane intersections. The dependence of the Higgs mass as a function of M_{SS} in this particular case is shown in fig.4.8 with the red band, a zoom is provided in fig.4.10. Given the uncertainties, in this particular case ($\mu = -M/2$) essentially any value for M_{SS} in the $10^4 - 10^{14}$ GeV region is consistent with the observed Higgs mass, although regions around $10^4 - 10^5$ and $10^8 - 10^{10}$ GeV are slightly favoured. This second possibility with $M_{SS} \simeq 10^{10}$ GeV will be explored in more detail in section 4.3.3.2 (see also [124, 137]) in which it will be argued that such intermediate SUSY breaking may be interesting for additional reasons coming from the Type IIB/F-theory compactifications.

It is interesting to explore how relaxing the above mentioned relationships $M =$

$\sqrt{2}m$, $A = -3/2M$ modify the results for the Higgs mass. In fig.4.11 we show how the prediction for the Higgs mass is changed as one varies the value of m away from $m = M/\sqrt{2}$. The figure remains qualitatively the same but one observes that as m/M increases the Higgs mass tends to be lighter. Above $M_{SS} \simeq 10^7$ GeV the Higgs mass remains in the region $m_H \simeq 126 \pm 3$ GeV. The effect of varying A away from $A = -3/2M$ is shown in fig.4.12. Although we have not included the error coming from the top quark mass to avoid clutter, one concludes that the overall structure remains the same and the Higgs mass stays around 126 ± 3 GeV for $M_{SS} \gtrsim 10^{10}$ GeV. However now values of M_{SS} in between 100 TeV and 10^{10} GeV are more easily consistent with the observed Higgs mass for particular choices of soft terms. Thus one can conclude that the specific structure of

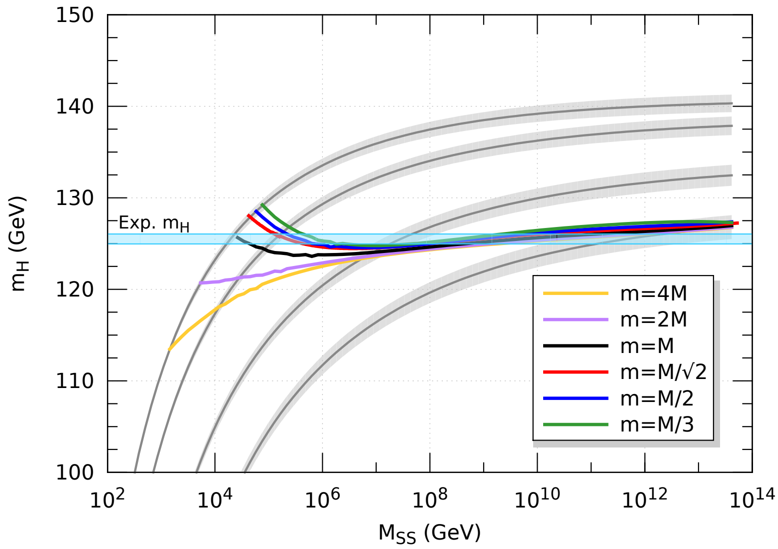


Figure 4.11: Higgs mass versus SUSY breaking scale M_{SS} for $\mu = -M/2$ and various values of the scalar mass parameter m in units of the gaugino mass M.

soft terms generally does not change qualitatively the main result of the previous section: If SUSY is broken at high energy and we assume unification on the Higgs soft masses, the SM Higgs mass is naturally centered around 126 GeV.

On the other hand, we could try to use the experimental value of the Higgs mass to constrain the possible structure of the soft terms. Unfortunately we do not have yet enough precision on the top quark and Higgs masses to really rule out any specific SUSY breaking mechanism. The way to proceed would be to compute $\lambda(M_{EW})$ from the experimental value of the Higgs mass and run up in energies to obtain $\lambda(M_{SS})$ and extract from it $tan\beta(M_{SS})$. Then, for a given value of M_{SS} one could put constraints on the values of the soft terms that would give rise to this $tan\beta(M_{SS})$ starting with $tan\beta(M_c) = 1$ at the unification scale. We leave this for future work when the experimental uncertainties on the top and Higgs masses could be notably improved.

Let us finally comment that our results do not directly apply to the case of Split SUSY [112,113,138–140] in which one has $M, \mu, \ll m$, since then the effect of light gauginos and Higgssinos should be included in the running below M_{SS}. In that case however it has been shown (see e.g. [121–123]) that split SUSY is only consistent with a 126 GeV Higgs

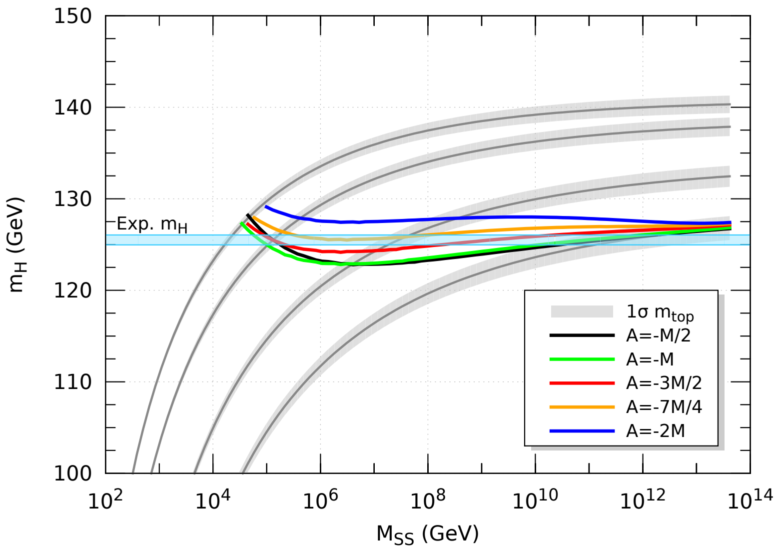


Figure 4.12: Higgs mass versus SUSY breaking scale M_{SS} for $\mu = -M/2$ and various values of the trilinear A parameter.

for $M_{SS} \lesssim 100$ TeV and no intermediate scale scenario is possible. Essentialy Split SUSY becomes a fine-tuned version of the standard MSSM. One relevant issue is also that in Split SUSY, due to the smallness of gaugino masses, in running down from the unification scale the scalar quarks of the third generation may easily become tachyonic, which restricts a lot the structure of the possible underlying SUSY breaking terms [138–140].

4.3.3. Gauge coupling unification

In the previous section we have computed the Higgs mass as a function of the SUSY breaking scale, being the latter a free parameter. The results are independent of the possible embedding in String Theory or in any ultraviolet completion of the MSSM. We have seen that an Intermediate SUSY breaking scale of $M_{SS} = 10^{10} - 10^{13}$ GeV leads to $m_H = 126 \pm 3$ GeV, indicating that the experimental value of the Higgs mass is indeed the most typical one if SUSY is broken at an Intermediate/High scale. The down side of this universality is that then the Higgs mass can not constrain much the SUSY scale. In order to be more specific, we need an extra input. From a phenomenological point of view, this extra input could come from cosmological data, since the scale at hand is far away from the energies attainable at the colliders. We will delve more in this point in chapter 5. However, one can also look for constraints from the UV theory to narrow the allowed range for M_{SS}. If closed string fluxes are the main source of SUSY breaking, they lead to a SUSY breaking scale between $M_{SS} = 10^{10} - 10^{13}$ GeV (assuming no fine-tuning or suppression at all on the fluxes). The exact value depends on the GUT scale (see (4.226)). Therefore we can try to be more explicit about M_{SS} by imposing correct gauge coupling unification and fixing M_C. We proceed to do that in what follows.

4.3.3.1. Hypercharge threshold corrections

One of the most appealing features of low energy SUSY is precisely the quite successful unification of the three guage couplings of the strong and electroweak interactions. However in our work we allow the scale of SUSY breaking M_{SS} to be a free parameter so a priori the unification gets worse as we take M_{SS} away from the TeV scale. For the extreme case in which $M_{SS} \approx M_c$ we have the SM below M_c and we know that coupling unification fails. On the basis of this one could conclude that gauge coupling unification forces M_{SS} to be close to the weak scale. Interestingly enough, F-theory GUT's described above can solve this problem because the breaking of the SU(5) symmetry via fluxes has a novel type of threshold corrections compared to the field theory case that have the form and size needed to reduce and even cancel the spliting of the gauge couplings.

To leading order the gauge kinetic function for the $SU(5)$ group within the 7-branes is given by the local Kähler modulus T whose real part is proportional to V_4, consistently with eq.(3.34). However in the presence of hypercharge fluxes f_Y the gauge kinetic functions get corrections [33, 53, 141]. These corrections can be obtained by dimensionally reducing the Dirac-Born-Infeld (DBI) + Chern-Simons (CS) action of the D7-branes in the presence of a background for the worldvolume field strength of the form [53]

$$F = \sum_{a=1}^{8} F^a_{SU(3)} \begin{pmatrix} \lambda_a/2 & 0 \\ 0 & 0 \end{pmatrix} + \sum_{i=1}^{3} F^i_{SU(2)} \begin{pmatrix} 0 & 0 \\ 0 & \sigma_i/2 \end{pmatrix} + \frac{1}{6} F_Y \begin{pmatrix} -2_{3\times 3} & 0 \\ 0 & 3_{2\times 2} \end{pmatrix} + \\ + (f_a + \frac{2}{5} f_Y) \begin{pmatrix} 1_{3\times 3} & 0 \\ 0 & 1_{2\times 2} \end{pmatrix} + \frac{1}{5} f_Y \begin{pmatrix} -2_{3\times 3} & 0 \\ 0 & 3_{2\times 2} \end{pmatrix} \tag{4.246}$$

where the capital letters F_G denote the four-dimensional gauge fields and the small letters f the internal background fluxes. f_a are fluxes along the U(1) contained in the U(5) gauge group of the 7-branes which are needed for technical reasons[10] but are not relevant in our discussion. The relevant terms of the DBI+CS action that will give rise to the Yang-Mills kinetic terms are

$$S \propto \mu_7 \int_{\Sigma_4 \times \mathbb{R}^{1,3}} \mathrm{STr} \left[\frac{1}{4} \left(1 + \frac{g_s^{-1}}{4} F_{ab} F_{ab} \right) F_{\mu\nu} F_{\mu\nu} \right] + \mu_7 \int_{\Sigma_4 \times \mathbb{R}^{1,3}} C_0 \wedge tr(F^4) \tag{4.247}$$

Inserting eq.(4.246) in eq.(4.247) and computing the traces for each gauge group of the SM, one can obtain that the tree level gauge kinetic functions are given by

$$\begin{aligned} 4\pi f_{SU(3)} &= T - \frac{1}{2}\tau \int_S f_a \wedge f_a \\ 4\pi f_{SU(2)} &= T - \frac{1}{2}\tau \int_S (f_a \wedge f_a + f_Y \wedge f_Y) \\ \frac{3}{5} 4\pi f_{U(1)} &= T - \frac{1}{2}\tau \int_S \left(f_a \wedge f_a + \frac{3}{5}(f_Y \wedge f_Y) \right) . \end{aligned} \tag{4.248}$$

where $\tau = \frac{1}{g_s} + iC_0$ is the complex dilaton. This implies that at the compactification scale one has the condition

$$\frac{1}{\alpha_1(M_c)} = \frac{1}{\alpha_2(M_c)} + \frac{2}{3\alpha_3(M_c)}. \tag{4.249}$$

[10] These fluxes are needed in order to satisfy the Freed-Witten anomaly condition since the D7-branes are wrapping a del-Pezzo surface which is non-Spin, see e.g. [142].

which is a generalization of the standard relationship $5/3\alpha_1 = \alpha_2 = \alpha_3$. In addition it turns out that in order to get rid of exotic matter massless fields beyond those of the MSSM the topological condition $\int f_Y \wedge f_Y = -2$ should be fulfilled [30–33], in which case one also obtains

$$\frac{3}{5}\frac{1}{g_s} \;=\; \frac{3}{5\alpha_1(M_c)} \;-\; \frac{1}{\alpha_3(M_c)} \;=\; \frac{3}{5}\left(\frac{1}{\alpha_2(M_c)} \;-\; \frac{1}{\alpha_3(M_c)}\right) \;. \tag{4.250}$$

Thus the size of the threshold corrections is determined by the inverse of the string coupling g_s. The corrections by themselves imply an ordering of the size of the fine structure constants at M_c given by

$$\frac{1}{\alpha_3(M_c)} \;<\frac{1}{\alpha_1(M_c)} \;<\frac{1}{\alpha_2(M_c)} \;. \tag{4.251}$$

We will neglect in what follows other possible sources of threshold corrections which are subleading in general. However, we will come back to this issue at the end of next section.

If one wants to keep $M_{SS} \approx 1TeV$ the corrections in eq.(4.248) may in fact spoil the standard joining of gauge coupling constants in the MSSM, which works quite well, unless they are suppressed by assuming $g_s \gg 1$. Furthermore the above ordering of couplings goes in the wrong direction if one wanted to use such corrections to further improve the agreement with experiment [53].

However as we have commented, in our setting with 1TeV $< M_{SS} < M_c$ the corrections have just the required form and size to get consistency with gauge coupling unification without the addition of any extra matter field beyond the MSSM (see also ref. [133]). The one-loop renormalization group equations lead to the standard formulae

$$\frac{1}{\alpha_i(M_c)} \;=\; \frac{1}{\alpha_i(M_{EW})} \;-\; \frac{b_i^{NS}}{2\pi}\, log\frac{M_{SS}}{M_{EW}} \;-\; \frac{b_i^{SS}}{2\pi}\, log\frac{M_c}{M_{SS}} \tag{4.252}$$

where b_i^{NS}, b_i^{SS} are the one-loop beta-function coefficients of the SM and the MSSM respectively. These are given by $(b_1, b_2, b_3)^{NS} = (41/6, -19/6, -7)$ and $(b_1, b_2, b_3)^{SS} = (11, 1, -3)$. Combining these equations and including the boundary condition (4.249) one obtains

$$\begin{aligned} 2\pi\,\left(\frac{1}{\alpha_1(M_{EW})} - \frac{1}{\alpha_2(M_{EW})} - \frac{2}{3\alpha_3(M_{EW})}\right) \quad &= \\ = \left(b_1^{NS} - b_2^{NS} - \frac{2}{3}b_3^{NS}\right) log\left(\frac{M_{SS}}{M_{EW}}\right) \quad &+ \quad \left(b_1^{SS} - b_2^{SS} - \frac{2}{3}b_3^{SS}\right) log\left(\frac{M_c}{M_{SS}}\right) \end{aligned} \tag{4.253}$$

In our case this yields

$$\frac{44}{3}\, log\frac{M_{SS}}{M_{EW}} \;+\; 12\; log\frac{M_c}{M_{SS}} \;=\; 2\pi\,\left(\frac{1}{\alpha_1(M_{EW})} - \frac{1}{\alpha_2(M_{EW})} - \frac{2}{3\alpha_3(M_{EW})}\right). \tag{4.254}$$

so we obtain a constraint between M_c and M_{SS} that has to be satisfied in order to have gauge coupling unification. For completeness we have improved the computation by using the 2-loop RGE for the gauge couplings in both SM and MSSM regions, obtaining the following approximate relation between M_c and M_{SS}

$$\log M_C = -0.23 \log M_{SS} + 16.77 \;. \tag{4.255}$$

This latter constraint is plotted in Figure 4.13 as a black line, although actually it changes very little compared to the one obtained just using the RGE at one loop (eq.(4.254)).

4.3.3.2. ISSB in F-theory GUT's

In what follows we combine the information collected about gauge coupling unification and flux-induced SUSY breaking to determine the prefered scale of SUSY breaking in flux F-theory compactifications.

Throughout this thesis we have seen how, in Type IIB/F-Theory compactifications, the different scales of the theory are not independent from each other. In the general setup where closed string fluxes are the main source of SUSY breaking one can find a relation between the unification scale M_c and the SUSY breaking scale M_{SS} given by eq.(4.226) and plotted in fig.4.13 as a red line. On the other hand gauge coupling unification is easily accomodated in F-theory. We have seen how the hypercharge flux that breaks the unification group to the SM induces some threshold corrections to the couplings that drives us to a corrected gauge unification condition given by eq.(4.254) and represented by the black line in fig.4.13. One can get consistent unification for values of M_{SS} up to slightly below 10^{14} GeV, which is required by the condition $M_{SS} < M_c$. The unification scale has also a lower bound at the same scale.

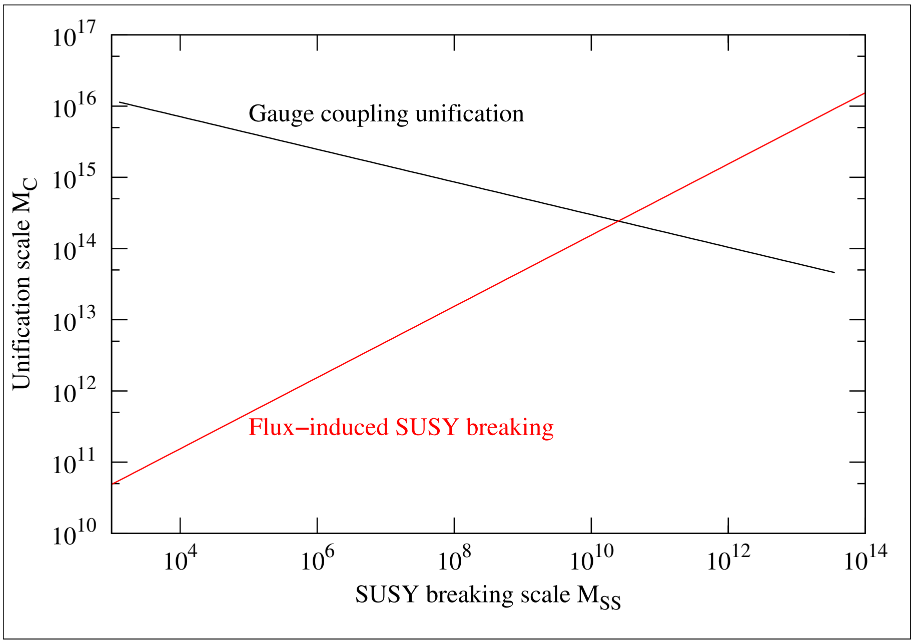


Figure 4.13: Constraints on M_{SS} and M_c from gauge coupling unification using the hypercharge threshold corrections (black line) and closed string flux induced SUSY breaking (red line).

Taking into account both conditions (gauge coupling unification and flux induced SUSY breaking) the scales of SUSY breaking and gauge unification are totally determined. As we can see in the fig.4.13 both lines intersect at

$$M_{SS} \simeq 2.5 \times 10^{10} \text{ GeV} \quad ; \quad M_c \simeq 2.4 \times 10^{14} \text{ GeV} . \tag{4.256}$$

Thus one gets correct coupling unification consistent with closed string flux SUSY breaking for SUSY broken at intermediate scale $\simeq 10^{10}$ GeV and a slightly low $SU(5)$ unification scale of order 10^{14} GeV. This immediately poses an apparent problem with proton decay that we will deal with in section 4.3.3.3.

It is also interesting to display the value of g_s as a function of M_{SS} from eq.(4.250). This is shown in fig. 4.14. For the values in eq.(4.256) one finds $g_s = 0.28$. This shows that the string coupling here is in a perturbative regime. On the other hand for values $M_{SS} \simeq 1$ TeV, corresponding to standard MSSM low-energy supersymmetry one needs $g_s \gg 1$. Note that in the context of F-theory the dilaton value g_s varies over different locations in extra dimensions and may be large or small, so both situations are possible in F-theory GUTs.

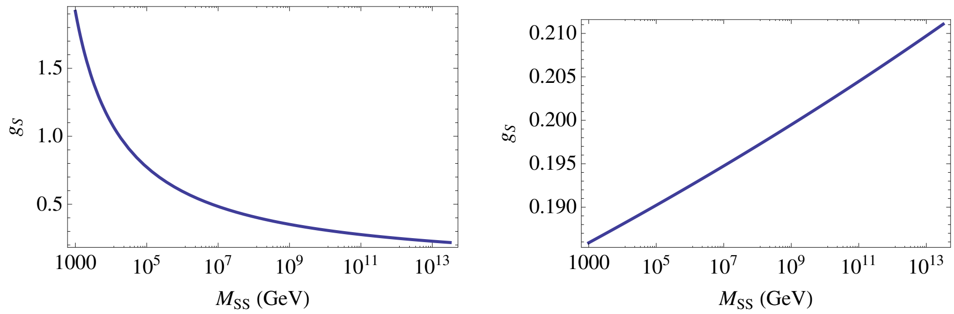


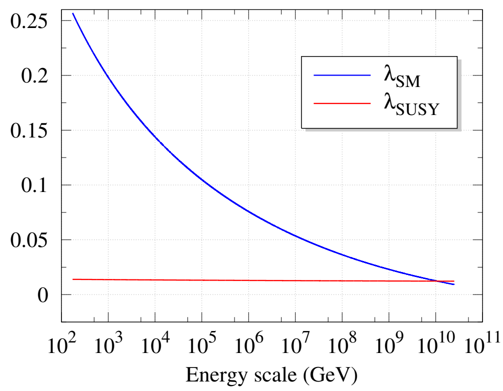
Figure 4.14: The string dilaton coupling constant versus M_{SS} for consistent gauge coupling unification. Left: With an MSSM content in the region $M_{SS} - M_c$; Right: With an additional vector-like triplet set $D + \overline{D}$ in that region.

Another interesting point is that the unification line (at one loop) in fig. 4.13 does not change if in the region $M_{SS} - M_c$ there are *incomplete* $SU(5)$ 5-plets, as equation (4.254) does not change. Thus for example the curve remains the same if the $SU(5)$ partner of the Higgs doublets, the triplets $D, \overline{D}$ transforming like $(3, 1, 1/3) + c.c.$, remain in the spectrum below M_c. These triplets are potentially dangerous since their exchange give rise to dimension 6 proton decay operators. The rate is above experimental limits unless $M_D \geq 10^{11}$ GeV [143], see section 4.3.3.3. That is why in GUTs one needs to perform some form of doublet-triplet splitting so that the Higgs fields remain light but the triplets are superheavy. In our case however these triplets will get a mass of order $M_{SS} \approx 10^{11}$ GeV anyhow so they may be tolerated below M_c and no doublet-triplet splitting is necessary. The presence of these triplets does however affect the size of the threshold corrections and g_s. In this case one gets typically smaller g_s which slowly grows as M_{SS} increases, see fig. 4.14. For $M_{SS} \simeq 10^{11}$ GeV one gets $g_s = 0.20$.

As discussed in section 4.3.2, an intermediate SUSY breaking scale with the only assumption of unification of the soft Higgs masses at M_c, automatically predicts a Higgs mass around 126 GeV. Following the procedure explained there, one can compute the Higgs mass for the scales given by eq.(4.256), obtaining

$$m_H \;=\; 126.1 \pm 1.2 \;\; GeV \qquad (4.257)$$

where the error includes only that coming from the top mass uncertainty. This is clearly consistent with the findings at ATLAS and CMS. In this scheme with an intermediate scale M_{SS} the Higgs self-coupling unifies with its SUSY extension as depicted in fig.4.15



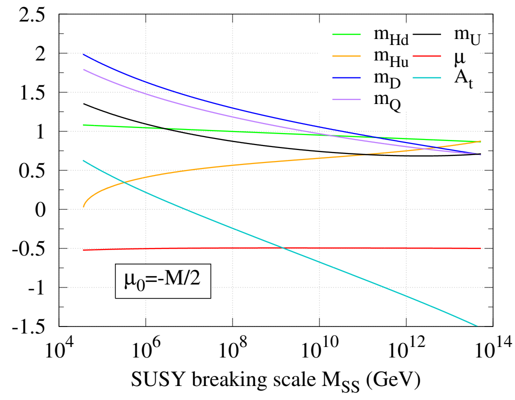


Figure 4.15: Left: Evolution of the SM Higgs selfcoupling $\lambda(t)$ and the combination $\lambda_{SUSY} = (g_1^2(t) + g_2^2(t))/4 \times cos^2(2\beta)(M_{SS})$ in the model with $\mu = -M/2$ and an intermediate scale $M_{SS} \approx 3 \cdot 10^{10}$GeV. They unify at M_{SS} where SUSY starts to hold. Right: Values of the 3-d generation squark soft masses $m_{Q,U,D}$ as well the Higgs mass parameters m_{H_u}, m_{H_d}, μ and trilinear A_t at the scale M_{SS} obtained from the running below the unification scale M_C.

(left). The soft masses evolve logarithmically from M_C down to M_{SS} as depicted in fig.4.15 (right). The value of tanβ increases as the value of $m^2_{H_u}$ decreases and $m^2_{H_d}$ remains almost constant, so that tanβ increases as M_{SS} decreases.

Let us remark again that this does not imply that in F-theory GUT's with closed string fluxes as the main source of SUSY breaking one is forced to break SUSY at 10^{11}GeV. If one wants low energy SUSY ($M_{SS} \sim 1TeV$) the usual procedure is to set $M_c \sim 10^{16}$GeV consistent with MSSM gauge coupling unification and consider that the effect of the fluxes is somehow suppressed, as we explained in section 4.3.1. Possible suppressions could come from fine-tuning in the flux or warping factors leading to a flux dilution. This is the implicit assumption in models with flux induced SUSY breaking, $M_s \simeq 10^{16}$ GeV and a standard SUSY solution to the hierarchy problem. However what we want to emphasize here is that avoiding any kind of fine-tuning or supression in the fluxes the most natural scale for SUSY breaking arising in this class of string compactifications corresponds to an intermediate SUSY breaking scale (ISSB)[11] instead of the usual low energy MSSM.

Before concluding, let us comment on other possible sources of threshold corrections to the gauge couplings beyond the classical contribution coming from the hypercharge fluxes considered here. In general, gauge couplings can also receive quantum corrections coming from KK massive modes running on internal loops, as emphasized in [144]. In [145] it was performed a detailed analysis considering both kind of corrections, but the results do not differ much from the ones obtained here and the conclusions are basically the same. On the other hand, the hypercharge gauge coupling might also receive corrections from mixing of the $U(1)_Y$ with hidden U(1)'s, as has been recently studied in [146]. Since this mixing is very model dependent, an alternative to fix M_c would be to focus only on α_2, α_3 and forgetting about α_1. We define then the unification scale as the one at which

[11]Note that the Intermediate Scale SUSY Breaking (ISSB) described above corresponds to a variant of the High Scale SUSY Breaking (HSSB) scheme of Hall and Nomura in ref. [114].

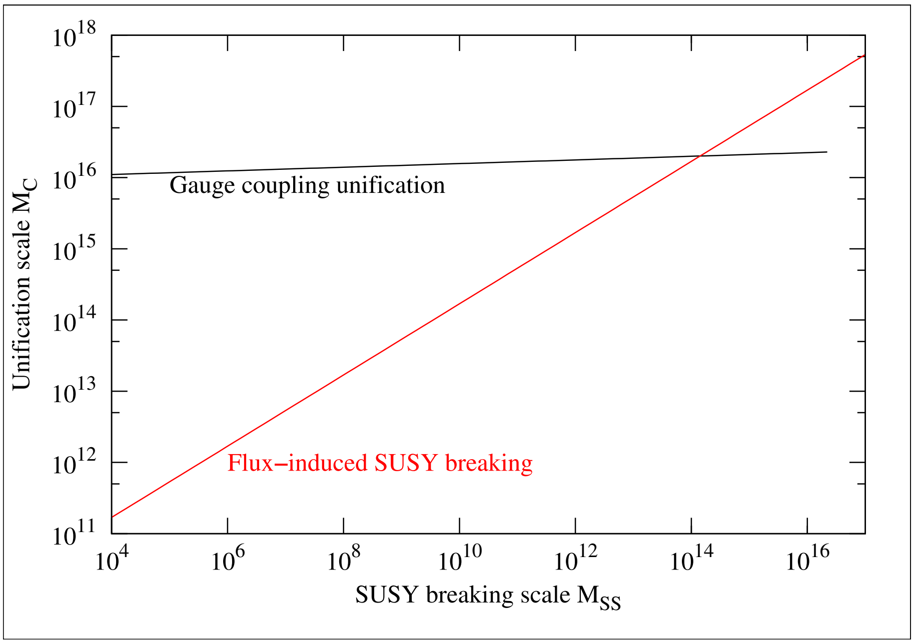


Figure 4.16: Constraints on M_{SS} and M_c from gauge coupling unification (black line) and closed string flux induced SUSY breaking (red line). M_c is defined as the value at which $\alpha_2 = \alpha_3$ due to possible unknown corrections to α_1 from U(1) mixing.

$\alpha_2 = \alpha_3$, considering that α_1 receives unknown threshold corrections from U(1) mixing so that coupling unification is satisified. The 2-loop unification condition then reads

$$log M_c = 0.025 log M_{SS} + 15.94 \qquad (4.258)$$

yielding $M_c \simeq 10^{16}$ GeV, roughly independent of M_{SS}. This scenario is plotted in fig.4.16. Once the condition for flux-induced SUSY breaking is taken into account, this yields

$$M_{SS} \;\simeq\; 1.5 \times 10^{14} \text{ GeV} \quad ; \quad M_c \;\simeq\; 2 \times 10^{16} \text{ GeV} \,. \qquad (4.259)$$

corresponding to a SUSY breaking scale in the upper limit to stabilize the SM vacuum.

4.3.3.3. Proton decay and axions

In this section we discuss some phenomenological issues arising in the Intermediate SUSY breaking scale scenario described above, regarding proton decay and possible candidates for dark matter.

Proton decay. As we already advanced with a unification scale as low as $M_c = 3 \times 10^{14}$ GeV there is a danger of dimension 6 operators giving rise to proton decay rates much faster than experiment. In standard field theory GUTs, the proton decay dim=6 operators obtained after integrating out the massive X, Y doublet of gauge bosons are [143]

$$O_1 = \frac{4\pi\alpha_G}{2M^2_{X,Y}} \overline{U^c_{aL}} \gamma^\mu Q_{aL} \overline{E^c_{bL}} \gamma_\mu Q_{bL} \tag{4.260}$$

$$O_2 = \frac{4\pi\alpha_G}{2M^2_{X,Y}} \overline{U^c_{aL}} \gamma^\mu Q_{aL} \overline{D^c_{bL}} \gamma_\mu L_{bL} \ . \tag{4.261}$$

The first operator arises from the exchange of the heavy gauge bosons with masses $M_{X,Y}$ between two 10-plets whereas the second from the exchange between a 10-plet and a 5-plet. Experimentally, the Super-Kamiokande limit on the chanel $p \to \pi^0 e^+$ gives an absolute lower limit $\tau_p > 5 \times 10^{33}$ years [147]. This corresponds to a bound on $M_{X,Y}$

$$M_{X,Y} \geq \sqrt{\frac{\alpha_G}{1/39}}\, 1.6 \times 10^{15}\ GeV \tag{4.262}$$

A value $M_{X,Y} = M_c = 3 \times 10^{14}$ GeV is 5 times smaller and that could pose a problem. In F-theory GUTs the same proton decay operators as above will appear, the difference now being that the symmetry is broken due to a hypercharge flux. Due to this fact the coefficients of the operators may change substantially, as we now discuss.

Indeed, considering proton decay in the context of F-theory SU(5) unification provides a new interesting mechanism to suppress proton decay. A microscopic computation of the above dimension 6 proton decay operator would involve first computing couplings of the form e.g. $\overline{U^c_{aL}} X_\mu Q_{aL}$ and then integrating out the massive doublet X, Y. The computation of such trilinear couplings is rather similar to the computation of Yukawa couplings, in the sense that it also involves a triple overlap of internal wavefunctions, namely

$$\Gamma^{ij}_1 = 2\, m_* \int_S (\Psi^i_{10})^\dagger \Psi^j_{10} \Phi_{X,Y} \qquad \Gamma^{ij}_2 = 2\, m_* \int_S (\Psi^i_{\bar{5}})^\dagger \Psi^j_{\bar{5}} \Phi_{X,Y} \tag{4.263}$$

where now $\Phi_{X,Y}$ are the internal wavefunctions of the broken SU(5) bosons X, Y. These form a doublet of massive gauge bosons with quantum numbers $(3, 2, 5/6) + c.c.$.

In standard 4d GUTs, the value of such couplings does not depend on the vev of the Higgs in the **24** of SU(5), and so it is exactly the same before and after SU(5) breaking (to leading order). Hence, one may extract the trilinear couplings like $\overline{U^c_{aL}} X_\mu Q_{aL}$ directly from the SU(5) Lagrangian as the strength by which SU(5) gauge bosons couple to chiral matter, namely $(4\pi\alpha_G)^{1/2}$.

Now, the key point for proton decay suppression in F-theory is the fact that the ingredient that triggers SU(5) breaking is not a vev for a scalar in the adjoint of SU(5), but the presence of the hypercharge flux F_Y along the GUT 4-cycle S. The mass of the X, Y gauge bosons is given by

$$M^2_{X,Y} = \frac{5\mu}{6\pi} \tag{4.264}$$

where $\mu = \sqrt{N_Y^2 + \tilde{N}_Y^2}$ measures the density of hypercharge flux, which we take constant for simplicity. The flux quantization condition implies that $5/3(F_Y/2\pi)$ is quantized in S (i.e., its integral over 2-cycles of S is an integer), so that $N_Y, \tilde{N}_Y \approx 6\pi/5\mathrm{Vol}_S^{-1/2}$ and indeed $M_{X,Y} \simeq M_c \simeq \mathrm{Vol}^{-1/4}$. Finding the wavefunctions in (4.263) involves solving a Dirac or Laplace equation for them, in which any flux threading S will enter. We then have that both the wavefunctions for chiral fields and massive gauge bosons X, Y depend on the internal fluxes on S, and in particular on the hypercharge flux F_Y. As a result, adding an hypercharge flux will necessarily change the value of the effective 4d couplings (4.263): while in the absence of F_Y such couplings must be $\propto \alpha_G^{1/2}$ in its presence they will have a new value.

To show that this new value will be suppressed with respect to $\alpha_G^{1/2}$ we need to compute explicetely the internal wavefunctions for the matter fields. The basics of the computation was explained in section 4.1.2 for a toy model with gauge group $U(3)$. Here we will use the more ellaborated wavefunctions obtained in the local SO(12) F-theory GUT model described in [42] to account for a realistic embedding of the SM. Such a model was also introduced in section 4.1.4 to compute the hypercharge dependence of the soft terms. Here we will try to be schematic, referring the reader to section 4.1.4 and to [42] (see also [34–41, 63] for more details on the subject). In F-theory SU(5) models there are basically two kinds of wavefunctions: the ones that are peaked at the matter curves of S, namely Ψ_{10}^i, $\Psi_{\bar{5}}^j$ and $\Phi_{H_{U,D}}$, and the ones that are spread all over the 4-cycle S, namely the SU(5) gauge bosons and in particular $\Phi_{X,Y}$. As they come from different sectors of the theory, these two kinds of wavefunctions feel the effect of the hypercharge flux in a different way.

Indeed, let us consider the wavefunctions involved in the coupling Γ_1 in (4.263). Solving for them in a local patch of S and assuming that the 4-cycle S is sufficiently large (see [42] for more details) we have that

$$\Psi_{10}^i = \begin{pmatrix} 0 \\ \vec{v} \end{pmatrix} \psi_{10}^i, \quad \psi_{10}^i = \gamma_{10}^i \, m_*^{4-i} \, x^{3-i} \, e^{-\frac{|M_x + q_Y \tilde{N}_Y|}{2}|x|^2} e^{-m^2|y|^2 - q_S \mathrm{Re}(x\bar{y})} \tag{4.265}$$

$$\Phi_{X,Y} = \gamma_{X,Y} \, m_* \, e^{-\frac{5}{12}\mu(|x|^2+|y|^2)} \tag{4.266}$$

where (x, y) stand for local complex coordinates of the 4-cycle S, and we have assumed that matter curve supporting the chiral fields **10** is given by $\Sigma_{10} = \{y = 0\}$. The hypercharge dependence of the wavefunction Ψ_{10} is encoded in the hypercharge value q_Y and in $q_S = N_F + q_Y N_Y$, so that for a non-vanishing F_Y particles with different hypercharge have different wavefunctions. Here M_x, $\tilde{N}_Y$, N_Y and N_F stand for densities of fluxes threading the 4-cycle S, and in particular M_x is the density of the flux necessary to have three families of **10**'s along Σ_{10}. The parameter m^2 stands for the slope of the intersection between the SU(5) 4-cycle S and the U(1) 7-brane intersecting S in Σ_{10}. Such intersection scale is typically of the order of the fundamental scale of F-theory m_* ($\simeq M_s$ in a perturbative IIB orientifold), which implies that Ψ_{10}^i are highly peaked along the matter curve $\Sigma_{10} = \{y = 0\}$. Finally, $\vec{v}$ is a three-dimensional vector that depends on m^2 and the flux densities, and the γ's are normalization factors that insure that such fields are canonically normalized.

Both $\vec{v}$ and the quantities that appear in the exponential factor of ψ_{10}^i are family independent: the only dependence of the family index i corresponding to the power of x (the matter curve Σ_{10} coordinate) that appears in the wavefunction. It has been found [34–41] that with this prescription (that assigns the power x^2 to the first family, etc.) one can reproduce the mass hierarchy between families observed in nature.

Notice that the fact that M_x, $\tilde{N}_Y$ and m^2 are non-zero gives a gaussian profile to these wavefuctions, and this allows to carry the integral for Γ_1 by replacing S with $\mathbb{R}^4$. This is important since otherwise we would need geometrical information about the full manifold B_3, which is in general not available. Notice also that the wavefunction for the boson X, Y is only affected by the hypercharge flux density μ, and that in the limit

$\mu \to 0$ we recover a constant wavefunction. This is to be expected, since at this limit the SU(5) symmetry is restored and X, Y become massless gauge bosons, which always have a constant profile.

Given these facts we are now ready to compute the coupling Γ_1 above. First notice that in the limit $\mu \to 0$ the integral is trivial in the sense that $\Phi_{X,Y} = \gamma_{X,Y} m_*$ is constant, since

$$2m_* \int_S (\Psi_{10}^i)^\dagger \Psi_{10}^j \Phi_{X,Y} = 2\gamma_{X,Y} m_*^2 \int_S (\Psi_{10}^i)^\dagger \Psi_{10}^j \approx \alpha_G^{1/2} \delta^{ij} \tag{4.267}$$

where used that for $\mu = 0$, the normalization factor is simply $\gamma_{X,Y} = \mathrm{Vol}_S^{-1/2} m_*^{-2} \approx \alpha_G^{1/2}$. Hence in this limit we recover the result expected from SU(5) gauge invariance.

This result is no longer true when $\mu \neq 0$ and so the wavefunction $\Phi_{X,Y}$ has a non-trivial profile. Then one finds that there is a suppression in the above coupling which is family dependent, and bigger for lower families. Indeed, to get an estimate of this coupling it is useful to take the approximation $m^2 \sim m_*^2 \gg M_x, \mu$ and treat the Gaussian profile $\exp(-m^2|y|^2)$ as a δ-function in the coordinate y, which is nothing but asking that the matter wavefunctions Ψ_{10}^i are fully localized in Σ_{10}. That is, we take the limit $m^2 \to \infty$ in which

$$(\psi_{10}^i)^* \psi_{10}^j \;\to\; \gamma_{10}^i \gamma_{10}^j m_*^{8-i-j} \bar{x}^{3-i} x^{3-j}\, e^{-|M_x + \bar{q}_Y \tilde{N}_Y||x|^2} \frac{\pi}{m^2} \delta(y) \tag{4.268}$$

and so the integral must be basically taken over Σ_{10}. Here $\bar{q}_Y = (q_{Y_p} + q_{Y_q})/2$ is the mean value of hypercharge for the two particles of the 10-plot participating in the amplitude. Taking into account that in this limit the normalization factors are [42]

$$\gamma_{10}^i \;=\; \frac{1}{\sqrt{2(3-i)\pi}} \left(\frac{|M_x + q_Y \tilde{N}_Y|}{m_*^2} \right)^{\frac{4-i}{2}} \qquad \gamma_{X,Y} \;=\; \frac{1}{\sqrt{2\pi}} \frac{5\mu}{6m_*^2} \tag{4.269}$$

we obtain that

$$\begin{aligned} 2m_* \int_S (\Psi_{10}^i)^\dagger \Psi_{10}^j \Phi_{X,Y} \;&=\; \delta^{ij} \frac{5\mu}{6\sqrt{2}\pi m_*^2} \left(\frac{|M_x + q_{Y_p} \tilde{N}_Y|^{1/2} |M_x + q_{Y_q} \tilde{N}_Y|^{1/2}}{|M_x + \bar{q}_Y \tilde{N}_Y| + \frac{5}{12}\mu} \right)^{4-i} \\ &\approx\; \delta^{ij} \alpha_G^{1/2} \left(\frac{|\sigma^2 + (\frac{5}{12}\tilde{N}_Y)^2|^{1/2}}{\sigma + \frac{5}{12}\mu} \right)^{4-i} \end{aligned} \tag{4.270}$$

where we have defined $\sigma = |M_x + \bar{q}_Y \tilde{N}_Y|$ and used $|q_{Y_p} - q_{Y_q}| = 5/6$ and $\mu \approx 6\pi/5\mathrm{Vol}_S^{-1/2}$. Nevertheless, this result can be reproduced with a bit more of effort without taking the δ-function approximation.

Since $\mu > \tilde{N}_Y$, the coupling (4.270) is indeed suppressed with respect to the 4d GUT result $\alpha_G^{1/2}$, and the suppression is bigger the lighter the family. Since we are interested in proton decay operators one could in principle focus on the first family $i = 1$, in which by assuming $M_x \approx N_Y 5/12 \approx \tilde{N}_Y 5/12$ we already obtain a suppression factor of around 1/5, and much bigger if $N_Y > \tilde{N}_Y$. In fact, being more rigorous, we would really need to take into account the fact that the actual physical first generation wave functions will be proportional to a linear combination of the $x^2, x, 1$ monomials. Even if this extra terms are present, one expects the first generation to be dominated by the x^2 monomial with a small contamination (related to mixing angles) from the other two. In any event, the presence of a suppression will be generic.

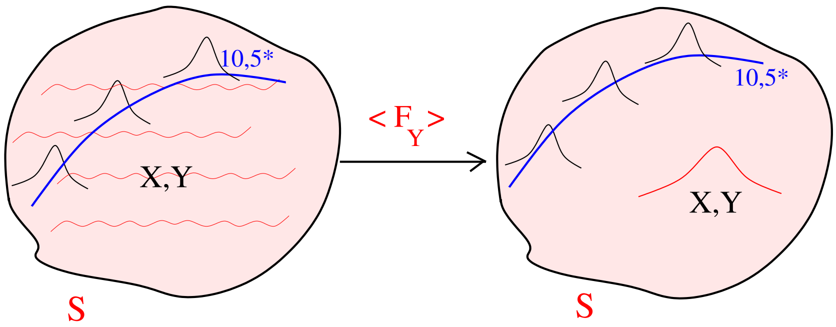


Figure 4.17: Coupling of $SU(5)$ off-diagonal gauge bosons X, Y. Before symmetry breaking by hypercharge fluxes the wave function of X, Y is extended over the whole 4-cycle S. After the hypercharge flux F_Y in introduced their wavefunction is localized and their coupling to $10, \bar{5}$ fields is supressed.

The fact that the suppression factor is bigger for each family can be given an intuitive understanding, since in F-theory families with smaller Yukawa couplings are those that have a higher polynomial degree x^n in their wavefunction (see eq.(4.265)). Such higher power gives a compensating effect to the localization that arises from the family independent exponential factor $\exp(-a|x|^2)$, that tends to localize the triple overlap around $x = 0$. The lighter the family the bigger the compensating effect, thus the smaller the coupling.

This understanding of the coupling strength in terms of exponential factors gives yet another mechanism for suppressing the dimension six proton decay operators. Indeed, notice that in (4.266) we have described the wavefunction for the massive X, Y bosons in terms of a Gaussian function on S peaked at $x = y = 0$. However, that the wavefunction $\Phi_{X,Y}$ peaks there is in fact a choice that we have made biased by the local patch description of our F-theory model setup. Unlike for the wavefunctions Ψ^j_{10}, whose equations of motion force them to be localized at the matter curve $\Sigma_{10} = \{y = 0\}$, there is nothing special about $y = 0$ for the wavefunctions of the gauge bosons X, Y which only depends on the hypercharge flux F_Y and on the geometry of the 4-cycle S. Only these two factors will determine where the peak of the wavefunction $\Phi_{X,Y}$ is, so there is a priory no reason to think that it will be peaked at any matter curve. Now, if the wavefunction $\Phi_{X,Y}$ is not peaked at $y = 0$ but somewhere else the δ-function in (4.268) will yield an extra suppression upon integration on the complex coordinate y, as the wavefunction density for $\Phi_{X,Y}$ will be exponentially suppressed away from its peak.

To summarize, F-theory SU(5) models have naturally suppressed dimension 6 proton decay operators, because the mechanism that breaks the SU(5) symmetry - the hypercharge flux F_Y - also affects the couplings where these operators come from. Indeed, the presence of the hypercharge flux deforms the wavefunction profile for the fields $\mathbf{10}$, $\bar{\mathbf{5}}$ and X, Y, as illustrated in figure 4.17. In particular it affects the X, Y bosons, which instead of being massless gauge bosons extended evenly over the whole 4-cycle S, are due to F_Y massive modes peaked at some point of it. Such localization effect indeed changes the value of the couplings (4.263) as we have shown in the computation above. Moreover, for the sake of simplicity we assumed above that the peak of the X, Y wavefunction lied on top of the matter curve Σ_{10} where the 10-plet resides. There is no reason for this assumption to hold in a global description of our setup, so the X, Y wavefunction will in general be suppressed in the region of Σ_{10} and there will be a further suppression to the coupling of X, Y to quarks-leptons. It is easy to see that any of these suppression mechanisms allow to have a rate for proton decay consistent with experimental limits. Note however that the precise value of the coefficient of the operators depends on the details (i.e. local fluxes) of the model. Still these results allow for the possible detection of proton decay through e.g. the channel $p \to \pi^0 e^+$, typical of non-SUSY unification, in future proton decay experiments.

If Higgs triplets $D, \overline{D}$ with a mass $M_D \simeq M_{SS} \simeq 10^{11}$ GeV are present in the spectrum, there will appear additional contributions to proton decay close to the present experimental limits [143]. They would come from the exchange of the scalar fields $\tilde{D}, \tilde{\overline{D}}$ among quarks and leptons of the first and second generations from Yukawa couplings, with $p \to \mu^+ K^0$ the dominant channel. In field theory GUTs these Yukawa couplings are directly related to the Yukawa couplings of the Higgs doublets due to the $SU(5)$ symmetry. In our case however the relevant D-field Yukawas are different to those of the Higgs, again due to the presence of the hypercharge flux [42]. One still expects those Yukawas to be of the same order of magnitude, i.e. of order 10^{-5} for the first generation. The combination of a massive D-field with the smallness of Yukawa couplings make these extra dimension 6 contributions compatible with experimental bounds, given the uncertainties.

Note in closing that dimension 5 proton decay operators are very much suppressed in the present framework due to the large mass of the SUSY partners. Additional sources of proton decay could appear if the underlying MSSM contains dimension 4 R-parity violating couplings. These could give rise to new dimension 6 operators by the exchange of sfermions but the rate will be again suppressed by the large mass of the SUSY partners combined with the expected smallness of the R-parity violating couplings involving the first generations.

Axions. The strong CP problem is a naturality problem with no obvious anthropic solution. In this sense it is quite satisfactory that string theory has natural candidates for the axion solution of the strong CP problem. As shown in eq.(4.248) the imaginary part of the local Kähler modulus Im T has axionic couplings to the QCD gauge bosons, and hence is in principle an axion candidate which could solve the strong CP problem [12]. In the Type IIB/F-theory scheme under discussion it is an important point the decoupling of the local GUT physics sitting on the local S 4-cycle from the global physics of the full six extra dimensions. A good model for this structure is considering the CY manifold $\mathbf{P}^4_{[1,1,1,6,9]}$ in ref. [14, 15] with one *small* Kahler modulus T and one *big* Kahler modulus T_b with Kahler potential

$$K \;=\; -2log(t_b^{3/2} \;-\; t^{3/2})\;. \tag{4.271}$$

with $t = 2ReT$ and $t_b = 2ReT_b$. Here one takes $t_b \gg t$ and take both large so that the supergravity approximation is still valid. In the F-theory context the analogue of these moduli t, t_b would correspond to the size of the 4-fold S and the 6-fold B_3 respectively.

One can compute the associated axion scale F_a from the kinetic term of the modulus T (see e.g. [4])

$$F_a^2 \;=\; \frac{M_p^2}{4\pi(8\pi^2)^2}\frac{\partial K(T,T^*)}{\partial T\partial T^*} \;=\; \frac{M_p^2}{4\pi(8\pi^2)^2}\frac{3t^{-1/2}}{8t_b^{3/2}} \tag{4.272}$$

[12] The τ complex dilaton scalar has also axionic couplings but Imτ gets generically massive in the presence of closed string fluxes.

where in the last equality we have used eq.(4.271), which correctly features the decoupling of the local $SU(5)$ physics from the global properties of the compact manifold. For the local modulus one has $t = 1/\alpha_G$ and

$$t_b = \frac{V_6^{2/3}}{g_s \alpha'^2 (2\pi)^4} = \left(\frac{\alpha'^{1/2} g_s^{1/4}}{\sqrt{8}} M_p \right)^{4/3} \tag{4.273}$$

where in the last equality we have used eq.(3.33). Using eq.(3.35) one finally obtains

$$F_a = \left(\frac{18}{\pi^2} \right)^{1/4} \frac{M_c}{16\pi^2} . \tag{4.274}$$

Note that the axion decay constant is directly related to the compactification scale (or the string scale via eq.(3.35)) and hence may be naturally low. This is to be contrasted to the heterotic model-independent axion ImS whose axionic coupling is directly tied to the Planck scale through $F_a^{het} = \alpha_G M_p/(8\pi^{3/2}) \simeq 10^{16}$ GeV (see e.g. [4] and references therein). In our case, for the preferred value $M_c = 2.4 \times 10^{14}$ GeV one obtains

$$F_a \simeq 1.8 \times 10^{12} \text{ GeV} . \tag{4.275}$$

This is an interesting value since F_a it is in the allowed QCD invisible axion range. It is at the upper limit of the allowed window, which is in fact required for the axion to be a viable dark matter candidate. This is also fortunate because in this scheme there are no light neutralinos as in the MSSM or split SUSY which could play the role of dark matter.

The mass of the axion is given through standard formulae by (see. e.g. [148])

$$m_a = \frac{z^{1/2}}{(1+z)} \frac{f_\pi m_\pi}{F_a} = \frac{0.6 \times 10^3 \ \mu\text{eV}}{F_a/(10^{10}\text{GeV})} \tag{4.276}$$

where we have taken $z = m_u/m_d = 0.56$. For the F_a value in (4.275) one gets an axion mass

$$m_a \simeq 3.3 \ \mu\text{eV} . \tag{4.277}$$

Due to the underlying $SU(5)$ symmetry the coupling of the axion to photons is directly related to F_a by a factor $sin^2\theta_W = 3/8$ (this is analogous to the DFSZ axion case [149,150]). In particular, defining the (normalized) axion-photon coupling as

$$\frac{G_{a\gamma\gamma}}{4} \ a \ F^\gamma \wedge F^\gamma \tag{4.278}$$

one obtains

$$G_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi F_a} \left(\frac{8}{3} - \frac{2}{3} \frac{(4+z)}{(1+z)} \right) \simeq 0.42 \times 10^{-15} (\text{GeV})^{-1} . \tag{4.279}$$

These values are not far from the limits obtained from searches with the microwave cavity experiment ADMX for cosmic axion dark matter [151]. They obtain

$$\frac{|G_{a\gamma\gamma}|}{m_a/\mu\text{eV}} < 5.7 \times 10^{-16} (\text{GeV})^{-1} \sqrt{\frac{0.45 \text{ GeV}/cm^3}{\rho_{DM}}} \tag{4.280}$$

for m_a in a range $m_a = 1.9 - 3.55$ μeV. Here ρ_{DM} is the local dark matter density. In our case we have $|G_{a\gamma\gamma}|/(m_a/\mu\text{eV}) \simeq 1.3 \times 10^{-16}$ (GeV^{-1}). The upgrading of ADMX should be able to test the axion parameters of the present scheme [13]. This would be an important test of these ideas.

Let us finally comment that a possible problem for the axion in the local modulus T to become a QCD axion is moduli fixing. Indeed one may wonder whether the dynamics fixing the moduli could also give a large mass to Im T. However this is not necessarily the case see e.g. [153–155].

4.3.4. Higgs finetuning in Type IIB/F-theory GUT's

As we have seen, an Intermediate SUSY breaking scale is consistent both with gauge coupling unification from appropiate threshold corrections and flux-induced SUSY breaking by 3-form fluxes. Besides, it gives rise to a Higgs mass of approximately $\simeq$ 126GeV, consistent with LHC results. In the computation of the Higgs mass (described in detail in section 4.3.2) we made two assumptions that are worthy of further study: universality on the soft Higgs masses

$$m_{H_u}^2 \;=\; m_{H_d}^2 \qquad \text{at } M_c \tag{4.281}$$

and the fine-tuning condition

$$m_3^4 \;=\; m_{H_u}^2 m_{H_d}^2 \quad \text{at } M_{SS} \tag{4.282}$$

required to keep light a Higgs doublet to be identified with the SM Higgs field. The question is whether there is any SUSY/string based scheme in which these two conditions are satisfied naturally. The first condition points at an underlying symmetry under the exchange of H_u and H_d. The second condition does not necessarily imply any underlying symmetry, but rather a fine-tuning constraint which has to be satisfied if we want to have a light Higgs. The latter could just have an anthropic explanation in a string landscape of possibilities.

The aim of this section is to discuss how these two conditions can be accomodated in string Type IIB/F-theory compactifications (see [124, 137] for similar work).

4.3.4.1. Universal Higgs soft masses

The universality condition (4.281) is generic and can naturally appear in any type II configuration in which the D-brane system contains an $N = 2$ subsector with the HIggs fields living in an $N = 2$ hypermultiplet. In F-theory GUT's the matter fields are localized in the extra dimensions at the so called matter curves, which in the perturbative picture of Type IIB correspond to intersections of D7-branes. In order to get chirality one turns on a magnetic flux in the worldvolume of the branes such that the matter curve is charged under it and just one of the two matter fields survives, ending up with a chiral spectrum. If we want the two Higgs doublets to come from the same matter curve Σ_H we need that the integral of the gauge field flux over the curve Σ_H vanishes. In that case we get a non-chiral $N = 2$ subsector of the theory that can be identified with the Higgs sector. In the typical MSSM constructions within F-theory this usually happens the other way around

[13] See ref. [152] and references therein for recent ideas of about axions in the context of fine-tuning.

(see [24–26, 28, 29] for reviews). In order to solve the doublet-triplet splitting problem people turn on a net hypercharge flux on the curve such that just one of the two doublets survives while the triplets remain uncharged an then typically adquire a large mass of the order of the string scale. Thus these constructions require the existence of two different matter curves 5_H and $\bar{5}_H$, one for each Higgs supersymmetric doublet. However, in our scheme SUSY will be broken at high energy so the triplets will already take a large mass and we do not need to worry about them. Therefore we can consider $\int_{\Sigma_H} F_Y = 0$ and get H_u and H_d living in the same matter curve. In that case any source of SUSY breaking will induce generically a mass matrix of the form (4.227) symmetric under the exchange of the two Higgses, satisfying the condition (4.281).

Even if both Higgses live in different matter curves, they can still have approximately the same mass. Let us consider the expressions for the soft masses derived in (4.137). Throughout the previous sections we have discussed two possible sources of non-universalities: hypercharge dependence and flavor mixing from non-constant fluxes. Matter curves feeling different amount of hypercharge flux will lead to matter fields with different soft masses, as we discussed in section 4.1.4. However, since both up and down Higgses have the same (absolute value of) hypercharge, the resulting soft masses are also the same. Therefore, the only way to get an splitting between m_{H_u} and m_{H_d} is by allowing for non-constant densities of the closed and/or open string fluxes. Since both Higgses live in different curves slightly localised at different loci in the internal dimensions, they will feel a different flux density and therefore get a slightly different soft mass. However, this splitting can not be very pronounced because both Higgs curves are expected to be very close to each other in order to recover the experimental values for the entries of the CKM matrix. Therefore the condition $m_{H_u} \simeq m_{H_d}$ is expected to be approximately satisfied in any event.

4.3.4.2. Tuning a light Higgs

In order to have a light Higgs scalar h_{SM} at scales $M_{\mathrm{EW}} \ll M_s$ we need to fine-tune $m_3^4 = m_{H_u}^2 m_{H_d}^2$ at the scale M_{SS} at which SUSY is broken, with soft terms of order M_{SS}. If we had an exact symmetry implying (4.282) at the unification scale M_c (like an exact shift symmetry, see [124, 137]) the running from M_c to M_{SS} would spoil that condition. Therefore the idea is to start at the string scale with $m_{H_u}^2 m_{H_d}^2 > m_3^4$, such that MSSM loop corrections lead eventally to $m_3^4 = m_{H_u}^2 m_{H_d}^2$ at the SUSY-breaking scale M_{SS}, so that a massless Higgs doublet survives. In the flux compactifications of Type IIB and F-theory studied in this thesis these soft terms are related to the flux background, so we can translate this condition into a fine-tuning condition in the flux background of the vacuum. This vacuum could then be anthropically selected within a string landscape of possibilities in order to satisfy eq.(4.282).

Another type of fine-tuning, based on anthropic arguments, was previously put forward by Weinberg as a potential explanation of the smallness of the cosmological constant (c.c.). In that case the existence of a huge landscape of string theory vacua, parametrized by a large number of discrete choices for fluxes in type IIB string theory, makes plausible the existence of vacua with small (and slightly positive) c.c. [81]. In the simple KKLT setting [8] such fine-tuning is possible because of two ingredients: i) there is a large number of 3-form flux choices, making possible to fine-tune a constant superpotential in the effective action and ii) there is an uplift mechanism provided by anti-D3 branes trapped on flux throats with tunable wrapping factor. The latter might be replaced by D7-branes with self-dual magnetic fluxes, such that they carry an effective anti-D3-brane charge. This scheme has been generalised in different directions, as in the LARGE volume scenario [14–17]. It is also fair to say that the introduction of anti-D3 branes to uplift the vacuum energy to a sufficiently long metastable deSitter vacuum is under debate nowdays. Although at the moment there is not a complete example fulfilling all the phenomenological requirements, it is reasonable to think that type IIB string theory vacua with fluxes and D-brane sources is sufficiently rich to allow for a landscape solution to the c.c. problem.

It is then natural to ask whether type IIB string theory also allows for a simultaneous fine-tuning of the Higgs mass. If so, has anything to do with the c.c. fine-tuning in e.g. the KKLT scheme? What would be in this case the microscopic description of the tuning? There is of course an obvious difference with the fine-tuning of the c.c., namely the smaller amount of tuning that is required. Indeed, the fine-tuning of the electroweak scale is much less severe, with $(M_{\rm EW}/M_X)^2$ of order $(10^{-2})^2 - (10^{-14})^2$, where M_X is either the string scale M_s or the SUSY-breaking scale M_{SS}. This is to be compared to the $(10^{-30})^4$ tuning required for the (almost) cancellation of the c.c.

It is very difficult at present to give a complete answer to the above questions. More modestly, in this subsection we would like to display the different leading microscopic contributions to the Higgs mass matrix that we can envisage in the bottom-up context of the present thesis. The question that we would like to address is whether flux-induced soft masses allow for this structure.

For definiteness we consider the case in which the SM fermions and scalars are localised on matter curves (or intersecting 7-branes) on a 4-fold S wrapped by a stack of 7-branes, as in local SU(5) F-theory models. For simplicity we take the case of a non-chiral Higgs matter curve in which a μ-term is generated by an ISD 3-form flux $S_{(2,1)}$ and the dominant source of SUSY-breaking also comes from ISD fluxes. According to our results in previous sections, at the string scale the Higgss mass matrix (4.227) has a qualitative structure of the form

$$m^2_{\rm Higgs} \;=\; \frac{g_s}{8}\begin{pmatrix} 2|G_{(0,3)}|^2+\frac{1}{4}|S_{(2,1)}|^2 & -G_{(0,3)}S_{(2,1)} \\ -G^*_{(0,3)}S^*_{(2,1)} & 2|G_{(0,3)}|^2+\frac{1}{4}|S_{(2,1)}|^2 \end{pmatrix} + \mathcal{O}(\langle F_2\rangle^2) \\ +\; \mathcal{O}(S_{(1,2)},G_{(3,0)}) \;+\; \ldots \quad (4.283)$$

The first term shows the structure of soft terms that we found in section 4.1.2 for the Higgs field on a matter curve with ISD fluxes. The reader can check that this matrix is positive definite and hence has only positive eigenvalues. Thus, at the unification scale there is no zero eigenvalue, and thus no light Higgs, for arbitrarily large 3-form fluxes. On the other hand, as we have already argued, the RG running from M_s down to the SUSY-breaking scale M_{SS} should lead to $\det(M_{\rm Higgs})^2 = 0$ so that a light Higgs scalar becomes possible. In fact it can be shown that this choice of soft terms when applied to the MSSM leads to a massless Higgs upon running down to an intermediate SUSY breaking scale $M_{SS} \simeq 10^{10} - 10^{12}$ GeV, for certain ranges of the μ-parameter. This is consistent with the assumption of closed string fluxes as the main source of SUSY breaking.

The simultaneous presence of $G_{(0,3)}$ fluxes breaking SUSY and $S_{(2,1)}$ fluxes inducing a supersymmetric mass can also be understood from the $N = 1$ 4d supergravity action, given by a Kahler potential invariant under shifts of the SM Higgs field, plus an additional supersymmetric μ-term in the superpotential. Even if the Kahler potential is shift invariant, the μ-term breaks the shift symmetry in the scalar potential, leading to $\det(M_{\text{Higgs}})^2 > 0$ at M_c. The zero eigenvalue arises dynamically only after running the soft masses down, at a scale roughly given by the relation between the μ-term and the Giudice-Masiero contribution (or microscopically, between both kind of fluxes). Both parameters must be tuned to yield a zero eigenvalue exactly at M_{SS}. We will further discuss about the suprgravity description in section 5.2.

It is important to remark that the condition (4.282) has to be satisfied with a precision of 16 orders of magnitude at M_{SS}. Therefore all subleading corrections (and not only the leading contribution coming from the 3-form fluxes) become important. It is here where the delicate fine-tuning required between all the contributions is manifest.

There are various types of corrections in eq. (4.283). The factor $\mathcal{O}(\langle F_2\rangle^2)$ denotes corrections quadratic in the magnetic fluxes that appear in non-chiral Higgs matter curves, such as those computed in section 4.1.1.2. Those corrections may have different origins, as we have already discussed. For instance, they may encode contributions induced by distant anti-branes, computed in section 4.1.3. We can illustrate those by summing over the contributions of n distant stacks of N_i anti-D3-branes located at distances r_{0i} from the SM 7-branes,

$$\delta m^2_{\text{Higgs}} = \frac{2\sigma^4}{\pi}\sum_i^n \frac{N_i}{Z_{0i}}\frac{F_-^2}{r_{0i}^6}\begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix} \tag{4.284}$$

with $Z_{0i} = 1 - g_s N\sigma^2\pi^{-1}r_{0i}^{-4}$. These corrections are higher order in the magnetic flux since, as we have already mentioned, only in the presence of magnetic flux F_- in the worldvolume of the 7-branes the backreaction of anti-D3-branes is felt by 7-branes. Analogous contributions could be induced by distant 7-branes with self-dual magnetic fluxes F_+ in their worldvolume.

There may be also contributions from IASD closed string 3-form fluxes, denoted by $\mathcal{O}(S_{(1,2)}, G_{(3,0)})$ in eq. (4.283). In fact, specific scenarios of moduli fixing include additional 7-branes with gaugino condensation or instanton effects that generate superpotentials which are crucial in fixing the Kähler moduli of the compactification. It was shown in [62] that such non-perturbative effects generate both ISD and IASD 3-form fluxes as part of their backreaction.

The size of the various contributions to eq. (4.283) is very model-dependent. For instance, in certain class of LARGE volume compactifications the main source of SUSY-breaking is modulus domination [14–17], being locally given by the contribution of ISD 3-form fluxes above. In others, including the original KKLT scenario, the contribution of distant anti-D3-branes and IASD fluxes turns out to be non-negligible. We can make a naive estimate of the relative size of ISD 3-form flux contribution with respect to that of distant anti-D3-branes. Considering uniform fluxes $G_{(0,3)} \simeq \alpha'/R^3$, we expect flux-induced soft terms of order

$$m^2_{\text{Higgs}} \simeq \frac{g_s\alpha'^2}{2R^6} \simeq \frac{M_s^4}{g_s M_{\text{Pl}}^2} \tag{4.285}$$

where we have used the type IIB equation

$$M^2_{\text{Pl}} = \frac{8\text{Vol}(B_3)}{(2\pi)^6 g_s^2\alpha'^4} \tag{4.286}$$

with $\text{Vol}(B_3) \approx (2\pi R)^6$ the volume of the compact space. On the other hand, assuming that the distance between the branes r_{0i} is of the order of the size of the CY, we can replace $r_{0i} \sim R$ and from eq. (4.284) we obtain that the contribution of anti-D3-branes to the Higgs mass matrix scales as

$$\delta m^2_{\rm Higgs} \ \sim \ \frac{M_s^4}{g_s M_p^2} \ \times \left(nN\sigma^2 F_-^2 \right) \ . \tag{4.287}$$

The contribution of distant anti-D3-branes to soft masses is thus comparable to that of 3-form fluxes, except for the fact that the first are suppressed by the magnetic flux factor $\sigma^2 F_-^2$. The latter is assumed to be small if the open string fluxes are diluted, so that the 3-form flux contribution is expected to dominate in many situations. Nevertheless 3-form fluxes might be diluted at the position of the SM 7-branes or alternatively the local 3-form flux could be fine-tuned.

The above discussion shows the abundance of possible contributions to the fine-tuning of the Higgs mass. Even in cases where ISD 3-form fluxes dominate SUSY-breaking, the contributions from open string magnetic fluxes, distant anti-branes or IASD fluxes can probably not be neglected in what concerns the Higgss fine-tuning. All of them are important, along with loop corrections, as long as the SUSY breaking scale is much above $1 - 10$ TeV. For instance, if $M_{SS} \simeq 10^{11}$ GeV, a fine-tuning of 16 orders of magnitude is required in which all these effects can potentially become important. In particular, the same anti-D3-branes which play a role in (almost) cancelling the c.c. in KKLT and related scenarios, generically influence the fine-tuning of the Higgss mass. In this regard, one important point to remark is that the Higgss mass is really directly sensitive to the *local* values of closed and open string flux densities, rather than to the integrated fluxes. Of course, in a putative compactification with all moduli fixed, the full geometry (including also the local values of fluxes near the SM branes) depend on the global features of the compactification such as the integer flux quanta, and therefore the Higgs mass, like the c.c., will eventually depend on the flux integers.

5

From String Theory to Cosmology

In this chapter we turn to describe the construction of inflationary models within String Theory. We start section 5.1 with a review of the basic concepts of inflation, followed by a detailed discussion about the extreme sensitivity of inflation to UV physics and the problems arising in the effective field theory approach. We finish the section by discussing the realization of large field inflation in string theory and the different proposals to get a transplanckian field range for the inflaton. In section 5.2 we propose and study in detail a new inflationary model called Higgs-otic inflation, in which the MSSM Higgs sector leads to a 2-field inflationary model interpolating between chaotic and linear inflation. We embed the model in Type IIB orientifold compactifications in which the inflaton corresponds to the position modulus of a D7-brane. We also discuss the effects of higher order corrections and compute the relevant cosmological parameters.

5.1. Inflation in String Theory

Our current understanding of the universe is encoded in the so called standard model of cosmology (ΛCDM), which provides a succesful explanation of the observation of the cosmic microwave background (CMB), the distribution of large scale structure, the abundance of light atoms and the accelerating expansion of the universe. The CMB is the most precise picture that we have of our universe, when it was only 300000 years old. At first glance, it reflects that our universe is in overall homogeneous and isotropic, the two basic principles which lay behind the modern cosmology. But more interestingly, the temperature anisotropies of the CMB are the product of tiny density fluctuations of the primordial universe which are indeed the origin of all structure of the universe. The understanding of the evolution from these primordial density fluctuations to the formation of the large structure we observe today (billion of years later) is a success of the modern cosmology. However the origin of these primordial fluctuations is a mistery for conventional cosmology. Besides, the experimental measurements combined with the ΛCDM model show that our universe can only arise from some very extremely special and fine-tuned initial conditions in order to reproduce the overall homogeneity, isotropy and flatness we see today. The theory of inflation solves dynamically this puzzle of initial conditions at the same time that it provides an understanding of the physical origin of all structure of our universe. Inflation refers to an early period of accelerating expansion of the universe (only 10^{-34} seconds after Big Bang) which modifies the causal structure of spacetime. This would allow our universe to arise from generic initial conditions, solving the puzzle of the initial conditions. In addition, microscopic quantum fluctuations during the inflationary period would be the seed of the cosmological formation structure today. Nevertheless, in our eagerness to know, a remaining question stays: what is the microphysical origin of inflation? And here is where String Theory, as a consistent UV quantum completion of gravity, comes into play.

5.1.1. Inflation basics

Inflation can be defined as a period at which the comoving Hubble radius decreases, or equivalently a period of accelerating expansion,

$$\frac{d}{dt}\left(\frac{1}{aH}\right) < 0 \rightarrow \ddot{a} > 0 \tag{5.1}$$

where we have used the definition of the Hubble parameter, $H = \frac{\dot{a}}{a}$ (see [156–159] for reviews of inflation in the context of string theory).

The background $\dot{H}$ required for inflation can be parametrized in terms of a scalar degree of freedom, which corresponds to the Goldstone boson associated with the spontaneous breaking of time translational invariance. That is the reason being inflation is usually described by a scalar field ϕ called the inflaton rolling down a potential (see fig.5.1). The dynamics of the scalar field yields an approximate de Sitter background with a "clock", ie. ϕ is an order parameter (or clock) which parametrizes the time-evolution of the inflationary energy density. The action for the inflaton can be written as

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right) \tag{5.2}$$

Acceleration of the universe occurs when the potential energy $V(\phi)$ dominates over the

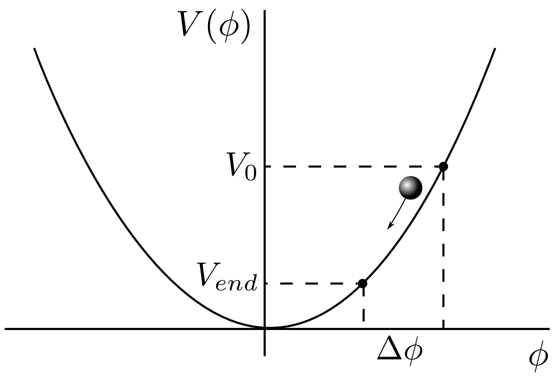


Figure 5.1: Scalar particle rolling down a potential and yielding inflation.

kinetic energy $\frac{1}{2}\dot{\phi}^2$. This can only be achieved for a sufficiently long period of time if in addition the second time derivative of the field $\ddot{\phi}$ is small enough. These two conditions can be expressed as constraints on the shape of the inflationary potential,

$$\epsilon = \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2 < 1 \tag{5.3}$$

$$\eta = M_p^2\frac{V''}{V} < 1 \tag{5.4}$$

where V' and V'' stand for first and second derivatives of the potential with respect to ϕ. The smallness of the above parameters is known as the slow-roll conditions for inflation. Inflation ends when these conditions are violated, ie. $\epsilon(\phi_{end}) \sim 1$, implying that the kinetic energy starts dominating over the potential energy. Then hot Big Bang occurs: the inflaton starts oscillating around the minimum, converting all the inflationary energy via decays in SM degrees of freedom. This process is known as reheating, a rich and complicated subject by itself. The duration of the inflationary period is parametrized in terms of the number of efolds, defined as

$$N = \int_t^{t_{end}} H dt = \int_{\phi_{end}}^{\phi} \frac{d\phi}{\sqrt{2\epsilon}} \tag{5.5}$$

The fluctuations observed in the CMB must be created 50-60 efolds before the end of inflation, in order to solve the horizon problem.

So far we have only reviewed the classical dynamics of a scalar field rolling down a potential, leading to a background evolution $\bar{\phi}(t)$ responsible for the accelerating expansion of the universe. However the origin of the cosmological structure formation is indeed in the quantum fluctuations around this background. We refer to [158] for a pedagogical review on how these quantum fluctuations during inflation are converted into fluctuations of the primordial power spectrum. Intuitively, these fluctuations give rise to a local delay in the time at which inflation ends, ie. different parts of the universe will end inflation at slightly different times inducing relative density fluctuations. Since we can have fluctuations both in the scalar field $\phi(t)$ and the metric $g_{\mu\nu}(t)$ they will be translated into density fluctuations as well as gravitational waves. In order to compare the theoretical predictions with the cosmological observations it is useful to compute the power spectrum of both scalar and tensor fluctuations, which is often approximated by the power law

$$\Delta_s^2(k) = A_s(k_*) \left(\frac{k}{k_*}\right)^{n_s(k_*)-1} \quad , \quad \Delta_t^2(k) = A_t(k_*) \left(\frac{k}{k_*}\right)^{n_t(k_*)} \tag{5.6}$$

where k_* is the wave length of a pivot scale. Deviations from this power law leads to non-gaussianities which are predicted to be small in single slow-roll inflations and are also highly constrained by the cosmological observations. The primordial scalar and tensor tilt, n_s and n_t respectively, measure the scale dependence of the power spectrum. Any deviation from perfect scale invariance ($n_s = 1$ and $n_t = 0$) is an indirect proof of inflation and can be related to the slow-roll parameters

$$n_s = 1 + 2\eta - 6\epsilon \quad , \quad n_t = -2\epsilon \tag{5.7}$$

evaluated 50 or 60 efolds before inflation ends. The amplitudes of the cosmological perturbations created by inflation are

$$A_s(k) = \frac{1}{8\pi^2}\frac{H^2}{M_p^2}\frac{1}{\epsilon} \ , \quad A_t(k) = \frac{2}{\pi^2}\frac{H^2}{M_p^2} \tag{5.8}$$

which have to be evaluated at the moment at which the corresponding fluctuation exits the horizon (ie. $k = aH$), corresponding to 40-60 efolds before inflation ends (the exact value depends on the inflationary model and the physics of reheating). An useful quantity is the tensor-to-scalar ratio,

$$r = \frac{A_t}{A_s} = 16\epsilon \tag{5.9}$$

which can be related to the energy scale of inflation

$$V_0^{1/4} \sim \left(\frac{r}{0.01}\right)^{1/4} 10^{16} GeV \tag{5.10}$$

According to experimental Planck results [160] of the power spectrum of CMB temperature fluctuations, the best fit value for the scalar amplitude is

$$A_s = (2.196 + 0.051 - 0.060) \times 10^{-9} \tag{5.11}$$

while the spectral index has been measured with a high precision obtaining

$$n_s = 0.9603 \pm 0.0073 \tag{5.12}$$

consistent with slow-roll inflation. The recent combined results of Planck/BICEP2 [161] yield un upper limit for the tensor to scalar rario, $r < 0.12$ at 95% level of confidence.

5.1.2. Lyth bound and UV sensitivity

Inflationary physics is extremely sensitive to UV physics, specially in models which involve transplanckian excursions of the inflaton. To that end, let us first comment on the Lyth bound [162], which relates tensor modes with field displacements of ϕ,

$$\frac{\Delta\phi}{M_p} = \mathcal{O}(1) \left(\frac{r}{0.01}\right)^{1/2} \tag{5.13}$$

It is remarkable that observable tensor modes $r > 0.01$ require transplanckian field excursions of the inflaton. This allows to distinguish between two classes of inflationary models, small field inflation ($\Delta\phi < M_p$) and large field inflation ($\Delta\phi > M_p$), with non-detectable or detectable tensor modes respectively.

The notion of effective field theories is the core on which we have built the current understanding and formulation of the physical theories describing the behaviour of nature. The idea is to integrate out all massive modes above a certain energy scale Λ to get an effective theory which is valid up to the cutoff Λ. In this way we can work in condensed matter physics without considering the internal degrees of freedom of the atoms, or in nuclear physics without considering quark physics. UV physics has two effects over the effective theory in the IR: quantum corrections that renormalize the couplings in the effective theory and new higher order dimensional non-renormalizable operators suppresed by inverse powers of the cutoff. In general, the first effect can give rise sometimes to naturalness or hierarchy problems while the second one becomes important only when we approach the cutoff. However, the delicate flatness required for inflation is extremely sensitive to any UV correction, so the naturalness problem is generically present in any effective inflationary model. In addition, for displacements of $\Delta\phi \sim \Lambda$ the infinite tower of higher dimensional operators becomes important and can not be neglected, which makes the effective theory intractable. These two effects give rise to the weak and strong eta-problem respectively, as we proceed to explain in the following (for a recent review see e.g. [158] and references therein).

- Weak eta-problem: renormalizable operators

The typical imprint of UV physics in the effective theory is the renormalization of the IR couplings of the light states of the effective theory coming from the heavy fields running in internal loops. In general scalar masses receive quantum corrections which drive the masses to the cutoff scale,

$$\Delta m^2 \sim \Lambda^2 \tag{5.14}$$

unless they are protected by some symmetry. These corrections imply in turn corrections on the slow-roll eta-parameter of order

$$\Delta\eta \sim \frac{\Delta m^2}{V} \sim \frac{\Lambda^2}{3H^2} \tag{5.15}$$

where we have used $H^2 \approx \frac{1}{3}V(\phi)$ with H being the Hubble scale. Since the cutoff of the theory is always above the Hubble scale $\Lambda \geq H$, the mass term is radiatively unstable leading to a violation $\Delta\eta \geq \mathcal{O}(1)$ of the slow-roll condition and preventing succesful inflation. This problem is present in both small and large field inflationary models. In the absence of symmetries, a severe fine-tuning between quantum corrections and the bare value of the mass term is required to keep $\eta < 1$. This naturalness problem is similar to the hierarchy problem of the EW Higgs mass. However, here the presence of supersymmetry is not enough to solve it. During inflation supersymmetry is spontaneously broken by the positive vacuum energy, so the corrections to the inflaton mass will be at least of order of the Hubble scale, implying still $\Delta\eta \sim \mathcal{O}(1)$. Frecuencies above the Hubble scale will not be sensible to the expanding background and the corresponding loop corrections cancel each other, but the same does not apply to frequencies below the Hubble scale. Therefore supersymmetry diminishes the eta-problem but can not solve it completely. The way out is to impose some additional global symmetry preserved by the effective lagrangian and only slightly broken to generate the potential. In other words, in order to forbid the presence of all perturbative renormalizable couplings, the inflaton should be a pseudo-goldstone boson with an approximate shift symmetry of the form $\phi \to \phi + a$. All perturbative quantum corrections will be then suppresed by the parameter responsible for the weak breaking of the shift symmetry giving rise to small corrections to the eta-parameter.

- Strong eta-problem: non-renormalizable operators

In addition to the renormalizable part of the lagrangian, any effective theory contains higher dimensional operators

$$\mathcal{L}_{eff}(\phi) = \mathcal{L}_R(\phi) + \sum_i c_i \frac{\mathcal{O}_i(\phi)}{\Lambda^{\delta_i - 4}} \tag{5.16}$$

suppressed by inverse powers of the cutoff scale. If they disappear when the cutoff is send to infinity $\Lambda \to \infty$ then the theory can be succesfully decoupled from UV physics. In general they are negligible unless we approach energies close to the cutoff scale. In other contexts these operators can be used to estimate the scale of new physics or to put constraints on the possible UV embedding. In inflation, they easily can spoil the delicate flatness required of the potential. For instance let us consider the dimension six operator

$$\mathcal{O}_6 = c_6 V_R(\phi)\frac{\phi^2}{\Lambda^2} \tag{5.17}$$

where V_R contains the renormalizable terms of the potential. Even if the renormalizable potential preserves an approximate shift symmetry this does not have to be the case of the higher dimensional operators (the UV theory might not preserve the symmetry). Therefore the mass term receives corrections of order the Hubble scale since $V_R \sim H^2$, implying again $\Delta\eta \sim 1$.

Not all symmetries of the IR theory can be realized in a consistent UV theory. Whether a desired symmetry in the IR remains in the UV completion is a question that can not be addressed in the effective theory. Here lies the importance of having an UV completion of inflation in a consistent quantum theory of gravity.

For small field inflation, $\Delta\phi < M_p$, operators of dimension $\delta > 6$ can be neglected (assuming the conservative cutoff $\Lambda = M_p$). Therefore to address the eta-problem it is enough to consider operators with $\delta \leq 6$. The exact threshold at which operators of higher dimensions become important depends on the model and in particular on the ratios Λ/M_p and ϕ/M_p. For large field inflation the problem is much more dramatic. For $\Delta\phi > M_p$ all higher dimensional operators become important even far below the cutoff, so in order to avoid the eta-problem one would need to fine-tune the full infinite tower of non-renormalizable operators. This makes the effective theory untractable.

For both small and large field inflation, assumptions in the UV theory are required to guarantee the presence of a symmetry protecting the potential. However for large field inflationary models the problem is more pressing because it can not be solved by simply fine-tuning a finite number of parameters. It is required a careful analysis of the symmetries preserved by the UV completion to determine the precise form and value of the survival non-renormalizable operators. We need then a theory valid at scales of order M_p which accounts simultaneously for quantum effects and gravity. The best candidate for such a theory is String Theory.

Kaloper-Sorbo lagrangian. Before turning to the discussion of inflationary models within String Theory, let us comment on a proposal at the level of the effective theory to protect the potential from non-desired UV corrections [163–165] (see also [166–170]). We have seen that a shift symmetry on the inflaton can help to avoid the presence of UV corrections which spoil inflation. However, from the bottom-up approach, to impose an approximate global shift symmetry in the IR is not enough to guarantee the absence of higher order dimensional operators. It is known that global symmetries will probably be broken by gravity in the UV completion, unless they are promoted to gauge symmetries. The natural following step is then trying to gauge the global symmetry. The standard way to gauge a shift symmetry is by introducing a vector gauge field A_μ. Then the shift symmetry is promoted to a local gauge symmetry and the axion becomes the Stuckelberg field for A_μ. This induces a gauge invariant mass for the gauge field. However, what we want for inflation is rather the opposite, to give a mass for the axion in a shift invariant way. There is an alternative way to gauge the global symmetry and give mass to the axion without breaking explicitly the shift symmetry and without adding more degrees of freedom. The way is to introduce a coupling between the axion and a gauge three-form $C_{\mu\nu\rho}$ (see [168]). A three form in four dimensions has no propagating degrees of freedom. Nevertheless, it can give rise still to an electric field in the vacuum F_4 which in the absence of sources can take any constant value. In the presence of 2-branes (domain walls) the value of F_4 changes by an integer quantity at the location of the membrane. The action is given by

$$S = \int d^4x\sqrt{-g}\left(\frac{M_p^2}{2}R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{48}F_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma} - \frac{\mu}{24}\phi\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma}\right) \tag{5.18}$$

Upon integrating out the four-form field via its equation of motion we get the following potential for the scalar,

$$V = \frac{1}{2}(q+\mu\phi)^2 \tag{5.19}$$

where q is an integration constant related to the charge of the membranes under the three-form field. The variation on ϕ is cancelled by a shift on q. Therefore, although the axion gets a mass the shift symmetry remains unbroken in the lagrangian. This can also be understood from the dual picture, in which a three-form gets a mass after eating a two-form field. The lagrangian in this dual picture is still gauge invariant, implying that the action (5.18) remains shift symmetric. Only when we select a specific vacuum (specific value of q) the shift symmetry is spontaneously broken. The presence of this underlying shift symmetry protects the potential from non-desired corrections. For instance, it implies that the couplings of the axions with other fields can only appear via derivative couplings, so they will not induce radiative corrections to the mass. Higher dimensional operators are also under controll. They must respect the gauge symmetry of the three-form field (and consequently the shift symmetry of ϕ) so they can only come as powers of the 4-form field strength over the cutoff, $\frac{F^{n+2}}{M_p^{2n}}$. It can be checked that they will give rise, upon integrating out the 4-form, to corrections going as powers of the potential itself, $\frac{V^n}{M_p^{4n}}$ [163–165], instead of the naive expansion in terms of the field ϕ performed in (5.16). As long as the potential remains subplanckian $V < M_p^4$, these corrections will be subleading and will not spoil inflation. We will show in 5.2.3.2 that our inflationary model can be reduced to an effective Kaloper-Sorbo lagrangian, leading to a consistent string embedding of this scenario.

5.1.3. Large field inflation in String Theory

From now on we will focus on large field inflationary models. They are interesting by themselves due to their extremely sensitivity to UV physics, which can be seen as an opportunity to test the physics at closely Planck scales via cosmological observations. We will also see that the embeding of large field inflation in String Theory is not trivial and forces us to push the theory to the boundary of our knowledge. In addition, recent BICEP2 observations [3] point to a large tensor to scalar ratio favoring large field inflation, although the results are under debate and need to be confirmed.

The prototypical example of large field inflation is chaotic inflation [171], in which the inflationary potential is quadratic on the field,

$$V(\phi) = \frac{1}{2}m_I^2\phi^2 \tag{5.20}$$

and the cosmological observables read

$$r = \frac{8}{N}\ , \quad n_s - 1 = -\frac{2}{N}\ , \quad N \simeq \frac{1}{4}\left(\frac{\phi_0}{M_p}\right)^2 \tag{5.21}$$

Transplanckian field excursions of the inflaton about $\Delta\phi \sim 10 - 15M_p$ are required in order to get 50-60 efolds. Besides, it implies using (5.10) a quite high scale of inflation,

$$V_0^{1/4} \simeq 10^{16} GeV \tag{5.22}$$

and a Hubble scale of order $H \simeq 10^{14}$ GeV. These energy scales can also be used to estimate the inflaton mass, obtaining $m_I = 10^{12} - 10^{13}$ GeV, which is also consistent with the cosmological bounds on the amplitude of the density scalar perturbations.

If the BICEP2 results are confirmed, it would the first experimental hint about a scale of new physics beyond the Standard Model. We want to remark that a high scale of inflation could also be an indication of a high scale of supersymmetry breaking. Assuming a similar height of the potential for inflation and SUSY breaking $V_{\text{infl}} \sim V_{SS}$, we get

$$V_{SS} \simeq (m_{3/2} M_p)^2 \Rightarrow M_{SS} \simeq \frac{V_0^{1/2}}{M_p} \simeq 10^{13} GeV \tag{5.23}$$

where we have used that the gravitino mass $m_{3/2}$ gives also the typical size of SUSY breaking soft terms. This estimation is reinforced if we consider that the inflationary potential is indeed generated by the same source responsible for supersymmetry breaking. In chaotic inflation the inflaton mass would then correspond to the scale of the SUSY breaking soft terms $m_I \sim M_{SS} \sim 10^{12} - 10^{13}$ GeV. This is the idea behind our proposal in section 5.2, where closed string fluxes (responsible for supersymmetry breaking) also induce the inflationary potential.

The scheme of chaotic inflation is simple and attractive, but requires an implementation in which trans-Planckian inflation excursions make sense, which in turn requires a consistent theory of quantum gravity. Our most firm candidate for such a theory is string theory, and indeed string models with large field inflation have been constructed in the last decade, see [158, 159] for reviews. There are mainly two research branches, depending if one wants to preserve a discrete shift symmetry in the background solution (natural inflation for one or multiple fields) or fully spontaneously break it upon choosing a vacuum (monodromy inflation). We briefly comment on both possibilities and extend the discussion for the latter, since it will be the one used in section 5.2.

- Natural inflation and multiple axion models

  Let us consider an axion ϕ whose continous shift symmetry is broken to a discrete periodicity $\phi \to \phi + 2\pi$ by non-perturbative effects. They induce a periodic scalar potential given by
  $$\mathcal{L} = f^2(\partial_\mu\phi)^2 - \Lambda^4 cos(\phi) \tag{5.24}$$
  where f is the so called decay constant. For small perturbations it is reduced to chaotic inflation with a quadratic potential. The maximum physical displacement in field space is given by $2\pi f$, so a transplanckian decay constant $f > M_p$ is required to get transplanckian field excursions of the inflaton. This leads to a problem when one tries to embed the model in String Theory. In [172] it was shown for several examples that in order to have a transplanckian decay constant for a single axion in String Theory, one is always forced to go beyond the controlled perturbative regime of the effective theory. New contributions, usually in the form of higher harmonics to the potential, become then relevant at scales $\mathcal{O}(M_p)$, reducing the effective field range to a subplanckian value. People have tried to evade these difficulties by

considering models of multiple axions in which one might hope to engineer a direction in the moduli space with an enhanced effective field range. However there is not a completely succesful embedding of any of these models yet in string theory (see [173, 174] for the original proposals of N-flation and lattice alignment). The relation between the Weak Gravity Conjecture [175] and the difficulties found to get transplanckian field ranges in String Theory has ben discussed recently in [176–183]. Concretely, [178] study the effect of gravitational instantons over the different proposals of large field inflation with multiple axions. The results show that parametrically large transplanckian decay constants can not be realised for any direction in the moduli space in any quantum theory of gravity, and belong instead to the swampland of string theory. However, there is not a priori fundamental obstruction for slightly transplanckian decay constants in specific cases, and the stringy realisation of these models is still under debate.

- Axion monodromy inflation

  Let us consider a scalar with a periodic moduli space, typically an axion. Even if all directions in the moduli space are subplanckian one can get transplanckian field excursions by travelling several times in the compact dimension. The idea is to add an extra contribution to the vacuum energy which increases each time the inflaton completes a period. In other words, one can unfold the periodic moduli space of the axion due to extra ingredients like space-time filling branes, allowing for the required large field excursions. This class of models go under the name of *axion monodromy inflation* because the corresponding potential grows as the axionic inflaton completes a cycle [184, 185]. An intuitive picture of these models is given

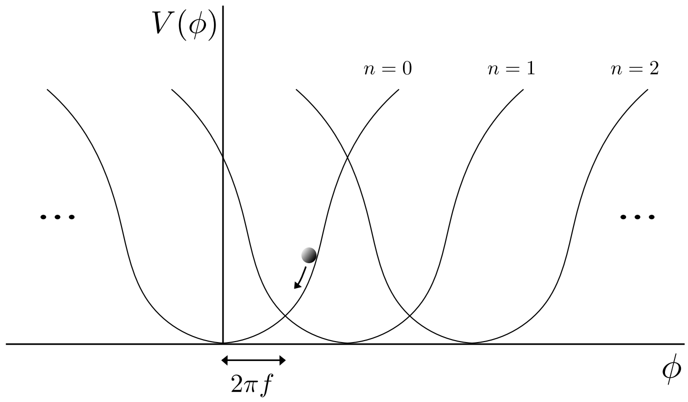


Figure 5.2: System of multibranches of an axion monodromy potential.

by the system of branches of fig.5.2. Each branch corresponds to a specific solution of the theory. Even if the inflaton is defined modulo $2\pi f$, once a specific vacuum is selected one can continue going up in the corresponding branch increasing the energy and reaching large (even transplanckian) field values. The shift symmetry is spontaneously broken by the vacuum, although it is preserved by the full theory. This underlying shift symmetry is enough to keep under control the appearance of Planck suppressed terms in the potential of the form $V_p \simeq M_p^{(4-p)}\phi^p$ with $p > 4$. They will be at least suppresed by the parameter controlling the small breaking of the shift symmetry. If one continous increasing the energy indefinitely there are two effects that must be taken into account: the modification of the structure of branches at large energies due to the backreaction with gravity, and tunneling effects to branches of smaller vacuum energy. However, it has been argued that these effects are negligible for the scales and field ranges required for inflation, so a slight enhancement of the field range to transplanckian values sounds reasonable.

Some of the first such axion models [186, 187] made use of non-SUSY configurations of NS-brane-antibrane pairs in type IIB theory (see [188] for a related F-theory construction). This structure was required in order to cancel unwanted D3 tadpoles and makes the stability of these models difficult to handle. More recently it has been realised that the same idea can be implemented in SUSY configurations if the monodromy is induced by an F-term potential for the axion [189]. Typical examples of this framework, dubbed *F-term axion monodromy inflation*, involve closed string axions whose potential is created by the presence of closed string background fluxes, see [189–194] for concrete realisations. A further novelty of this framework is that one can also implement the monodromy idea in models identifying the inflaton with either continuous Wilson lines or their T-dual, D-brane position moduli, see [189, 195–197]. In the latter case large inflaton excursions correspond to a D-brane position going around some cycle in the internal compact space.

Another advantage of F-term axion monodromy is that it allows to connect with the 4d axion monodromy framework developed in [163–165]. Indeed, it was found in [189] that upon dimensional reduction one obtains an effective Kaloper-Sorbo Lagrangian describing the coupling of an axion with a non-dynamical four-form. As we explained in section 5.1.2 the presence of this four-form creates a quadratic potential for the inflaton which is protected against dangerous corrections to the slow-roll potential that arise upon UV completion of the theory. Up to now, the Lagrangian (5.18) has been obtained from F-term axion monodromy constructions involving either closed string axions or open string axions arising from massive Wilson lines [189] (see also [170]). As part of our analysis we will see that (5.18) can also be reproduced from models where the inflaton is a D-brane position modulus, which is one specific realisation of our scenario in 5.2.

We have seen that natural candidates for large field inflatons are axion-like fields, which are indeed abundant in string compactifications. Typical examples of such axions are the scalars coming from dimensionally reducing the NSNS and RR fields. In Type IIB possible axionic candidates are those coming from B_2, C_2 and C_4 (recall eqs.(3.5)-(3.6)) and wilson lines of the open string sector. Another possible candidate from the closed string sector is the universal axion C_0. All these fields present a continous shift symmetry (broken by non-perturbative effects or the addition of fluxes) coming from the gauge invariance in higher dimensions. However, one can also consider geometric fields which present an approximate shift symmetry in some corners of the moduli space. This shift symmetry is inherited from the properties of the compactification. In particular, it is the remnant of the monodromy discrete transformations (which leave invariant the Kahler potential) around special points of the moduli space. This allows us to extend the list of candidates also to the complex structure U^a moduli and the D7-brane position moduli. In [198] it was performed a systematic study of the special points in the complex structure moduli space of CY manifolds at which axions (potential candidates for inflation) may arise. Here we will focus on the case of D7-brane position moduli.

Before concluding this section it is important to remark the challenges arising in string inflation. As commented, scalar fields (known as moduli) are abundant in string compactifications. For simplicity, it is usually assumed that all remaining scalars (except for the inflaton) are heavier than the Hubble scale and then can be integrated out during inflation. However, this is easier said than done. And even if we manage to keep the inflaton light over all the other moduli of the compactification, we still have to keep controll over the contributions coming from integrating out these fields, in order to avoid the weak and strong eta-problem. Therefore, strictly speaking the moduli stabilization problem can not be decoupled from inflationary dynamics. Needless to say that if there is some scalar with mass $m^2 < H^2$ it has to be included in the dynamics of inflation. For a few fields this is doable but for many fields it becomes technically unfeasible. Unlike the SM of particle physics, inflation is intimately related to gravity and all the moduli of the compactification. This difficulties the construction of succesfull inflationary models in string theory. But it is also a motivation to do it, because it forces us to develop a better understanding of global aspects of the compactification and go beyond our confortable perturbative decoupled models. Moreover, the study of inflation in String Theory leads us to think about fundamental and interesting questions regarding the set of vacua yielding the landscape of String Theory. Thus these difficulties can be seen as an opportunity to study the theory at the frontiers of knowledge, and perhaps even being able to test some predictions via cosmological observations.

5.2. Higgs-otic inflation

Higgs-otic inflation refers to theories in which the inflaton is a complex scalar giving rise to gauge symmetry breaking, while attaining large field inflation. The most obvious and natural candidate for that is the SM Higgs field itself. Nevertheless the same idea may be applied to other BSM fields introduced for other purposes, as we briefly discuss below. The essential ingredient is the identification of a complex inflaton with the position moduli of some Dp-brane system in string compactifications. The motion of the brane corresponds to the gauge symmetry breaking through a scalar vev and the scalar potential is induced upon switching on closed string fluxes. They lead to a quadratic (chaotic) potential which is flattened for large values of the field. In this section we propose and discuss the main phenomenological features of Higgs-otic inflation as well as study in detail a specific realization in Type IIB orientifold compactifications.

5.2.1. Setting the idea

As of today, the only scalar detected experimentally is the Higgs boson. Its discovery at LHC completes the minimum set of particles required for a consistent understanding of the properties of the SM of Particle Physics. In a different direction, evidence is mounting in favour of the existence of a second fundamental scalar in the theory, the inflaton. Given these two inputs, an obvious question has been around for some time: Can the Higgs boson be identified with the inflaton?. Before we knew the value of the Higgs boson mass this possibility looked unlikely, since the Higgs potential is quartic with no obvious region which could lead to slow roll inflation (see e.g. [199] for a review and references therein). However, as we said, for a Higgs mass value around 126 GeV the Higgs self coupling λ evolves down to zero at a scale $10^{11} - 10^{13}$ GeV. In fact, if one takes a 2σ uncertainty for the measured value of the top-quark mass and α_{strong}, it could still be possible that we have $\lambda \simeq 0$ close the the Planck scale M_p. It has been proposed that this could be the signal of some new conformally invariant physics [200], [201–204]. In this case inflation could also appear with the inflaton identified with the SM Higgs if non-minimal gravitational couplings of type $\int \alpha |h|^2 R$ are assumed. While it has been debated whether this scheme has problems with unitarity (see e.g. [205] and references therein), for appropriate values of the parameters one may still obtain a Starobinsky-like inflation with negligible tensor perturbations. See also [206] for a SUSY Higgs inflation with small field leading also to small tensor perturbations.

Here we propose a new scenario, dubbed Higgs-otic inflation, in which the inflaton can be identified with the Higgs boson, with minimal couplings to gravity and giving rise to large field inflation. The key point that makes possibe these features is the assumption of an Intermediate/High SUSY breaking scale which is identified with the scale of the inflaton mass.

Let us quickly review the results obtained in section 4.3 to clarify notation and set the basis of the forthcoming identification of an MSSM Higgs boson with the inflaton. As we discussed in section 4.3.2, admitting the possible presence of Higgs mass fine-tuning, one can consider leaving the scale of soft masses M_{SS} as a free parameter and ask for consistency with the measured Higgs mass. We found that that if the MSSM SUSY-breaking scale is $M_{SS} \simeq 10^9 - 10^{13}$ GeV, and a fine-tuned SM Higgs survives below that scale, then one necessarily gets $m_h = 126 \pm 3$ GeV, in agreement with LHC data. Imposing gauge coupling unification and flux-induced isotropic SUSY breaking further points to a Higgs with a mass around 126 GeV. This is true if one assumes the unification boundary condition for the two MSSM doublets $m_{H_u} = m_{H_d}$, but no other further input. One could then interpret the observed Higgs mass as indirect evidence for large scale SUSY breaking in a unification scheme. The fine-tuned light SM Higgs is obtained from the general MSSM Higgs mass matrix

$$\left(\begin{array}{cc} H_u \ , & H_d^* \end{array} \right) \left(\begin{array}{cc} m_{H_u}^2 & m_3 \\ m_3^* & m_{H_d}^2 \end{array} \right) \left(\begin{array}{c} H_u^* \\ H_d \end{array} \right) . \tag{5.25}$$

If one fine-tunes $|m_3|^2 = m_{H_u}^2 m_{H_d}^2$, there are massless ($H_L$) and massive ($H_M$) eigenstates

$$H_L \ = \ \sin\beta \, e^{i\gamma/2} H_u - \cos\beta \, e^{-i\gamma/2} H_d^* \quad , \quad H_M \ = \ \cos\beta \, e^{i\gamma/2} H_u + \sin\beta \, e^{-i\gamma/2} H_d^* \ , \tag{5.26}$$

with

$$\tan\beta \ = \ \frac{|m_{H_d}|}{|m_{H_u}|} \tag{5.27}$$

and $\gamma = \mathrm{Arg}\, m_3$. All these quantities must be evaluated at the soft mass scale $M_{SS} \simeq 10^{10} - 10^{13}$, below which all the SUSY spectrum decouples and just the SM survives. Note in particular that at some unification scale $M_c > M_{SS}$ one might expect $m_{H_u}(M_c) = m_{H_d}(M_c)$ (i.e. $\tan\beta = 1$), and that then the running from M_c down to M_{SS} will make $|\tan\beta(M_{SS})|$ slightly larger than one. Moreover at such scale M_c both scalars H_L, H_M will be massive, although one will still have $m_{H_M} \gg m_{H_L}$ due to the short running in between M_c and M_{SS}.

The fact that large quadratic terms appear for the Higgs fields above M_{SS} suggests to study whether such fields can indeed lead to some form of chaotic inflation. If that were the case, the inflaton would have a large mass of order $M_{SS} \simeq 10^{10} - 10^{13}$ GeV. This question is interesting by itself, but would become particularly relevant if the indications of BICEP2 of large tensor perturbations [3] were confirmed. In section 5.1 we proposed to identify the large SUSY breaking scale suggested by the measured Higgs mass with the inflaton mass suggested by the BICEP2 data. This indeed would be very attractive and economical, connecting two apparently totally independent physical phenomena, the Higgs mass with possible cosmological tensor perturbations.

In fig.5.3 we plot the running of the Higgs mass parameters from M_c down to M_{SS}. In the left plot we see the running of $|m_3|$ and $m_{H_u}m_{H_d}$. When both curves intersect the fine-tuning condition is satisfied and we have a massless eigenvalue at the SUSY breaking scale M_{SS}. This is also depicted in the right plot, in which although both mass eigenstates are massive at M_c, one of them (H_L) becomes massless after the running from M_c down to M_{SS}. To correctly interpret these figures recall that the running stops at the point M_{SS} in which all SUSY-particles become massive and one is left just with the SM at energies below that given value of M_{SS}.

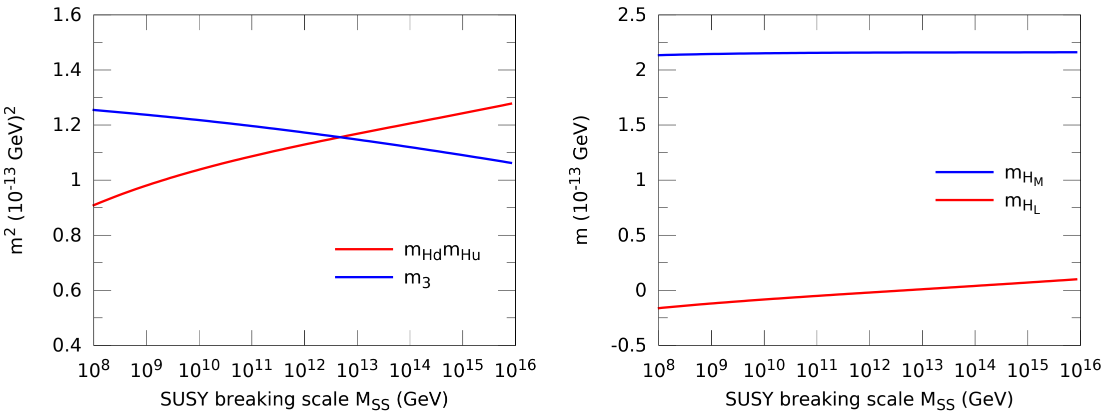


Figure 5.3: Running from M_c down to M_{SS} of the parameters of the Higgs mass matrix (left) and of the mass eigenvalues m_{H_M} and m_{H_L} (right).

In addition to the mass terms there is the $SU(2) \times U(1)$ D-term contribution to the scalar potential given by

$$V_{SU(2)} = \frac{g_2^2}{8}\left(|H_u|^4 + |H_d|^4 + 2|H_u|^2|H_d|^2 - 4|H_uH_d|^2\right) \tag{5.28}$$

$$V_{U(1)} = \frac{g_1^2}{8}\left(|H_u|^4 + |H_d|^4 - 2|H_u|^2|H_d|^2\right) \tag{5.29}$$

where we have

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} , \qquad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} \tag{5.30}$$

all four fields being complex. Note that here $H_uH_d = (H_u^+H_d^- - H_u^0H_d^0)$, so in general $|H_uH_d|^2 \neq |H_u|^2|H_d|^2$. The $SU(2)$ piece of the potential is however minimised if the charged fields have no vev, in which case $|H_uH_d|^2 = |H_u|^2|H_d|^2$, so that the complete potential is then given by (with now only neutral components included)

$$\begin{aligned} V &= m_{H_M}^2|H_M|^2 \;+\; m_{H_L}^2|H_L|^2 \;+\; \frac{g_1^2+g_2^2}{8}\left(|H_u|^2-|H_d|^2\right)^2 \\ &= m_{H_M}^2|H_M|^2 \;+\; m_{H_L}^2|H_L|^2 \\ &+ \frac{g_1^2+g_2^2}{8}\left(\cos 2\beta(|H_M|^2-|H_L|^2)+2\sin 2\beta\,\mathrm{Re}(H_L H_M^*)\right)^2 \end{aligned} \tag{5.31}$$

with $m_{H_L}(M_{SS}) \simeq 0$. At this level the H_L eigenvalue is (approximately) massless and H_M decouples below M_{SS}, leading to the following SM quartic potential at M_{SS}

$$V \;=\; \frac{g_1^2+g_2^2}{8}\cos^2 2\beta |H_L|^4 \; . \tag{5.32}$$

For $\tan\beta(M_{SS}) \simeq 1$, as implied by the $m_{H_u}(M_c) = m_{H_d}(M_c)$ boundary condition, one has $\cos 2\beta \simeq 0$, explaining why the SM Higgs self-coupling seems to vanish at the M_{SS} scale. This in turn explains, after running the Higgs self coupling down to the EW scale, why $m_{H_L} \simeq 126$ GeV.

Note that the D-term potential has a general neutral flat direction given by

$$\sigma = |H_u| = |H_d| \qquad\qquad H_u = e^{i\theta}H_d^* \tag{5.33}$$

with $\sigma \in \mathbf{R}^+$ and θ the relative phase of H_u and H_d^*. Denoting $H_u = |H_u|e^{i\theta_u}$ and $H_d = |H_d|e^{i\theta_d}$ then $\theta = \theta_u + \theta_d$. Since at M_{SS} one has $\tan\beta \simeq 1$, it is useful to define the doublet linear combinations

$$h \;=\; \frac{e^{i\gamma/2}H_u - e^{-i\gamma/2}H_d^*}{\sqrt{2}} \quad , \quad H \;=\; \frac{e^{i\gamma/2}H_u + e^{-i\gamma/2}H_d^*}{\sqrt{2}} \; . \tag{5.34}$$

Then at M_{SS} the SM doublet is approximately given by $h \simeq H_L$ whereas $H \simeq H_M$ is massive. Note that for the neutral components of h and H one has

$$H \;=\; \sqrt{2}\sigma\cos\left(\frac{\theta+\gamma}{2}\right)e^{i(\theta_u-\theta_d)/2} \quad , \quad h \;=\; i\sqrt{2}\sigma\sin\left(\frac{\theta+\gamma}{2}\right)e^{i(\theta_u-\theta_d)/2} \; , \tag{5.35}$$

where $\theta = \theta_u + \theta_d$, and the universal phase on both fields may be rotated away through a hypercharge rotation. Then

$$|H| \;+\; i|h| \;=\; \sqrt{2}\sigma e^{i\frac{\theta+\gamma}{2}} \; . \tag{5.36}$$

Along the above mentioned flat direction the potential is reduced to quadratic terms. This suggests to consider these neutral Higgs fields $|h|$, $|H|$ (or σ, θ) as candidates to give rise to inflation in the manner prescribed by chaotic inflation, as we will describe below.

Before considering specific embeddings of our scheme let us briefly discuss the scale structure of a large field inflation string model, see figure 5.4. The fundamental scale is the string scale which is in the region $M_s \simeq 10^{16} - 10^{18}$ GeV. The (reduced) Planck scale is $M_p \simeq 10^{18}$ GeV and the inflaton initial value Φ_* is typically of order 10-15 M_p to obtain the appropriate number of e-folds. Using field theory and a scalar potential makes sense only at energies below the compactification/unification scale M_c, which should be sufficiently below M_s so that the 10d action we start with makes sense. The Hubble scale at inflation is $H_I \simeq 10^{14}$ GeV and the inflaton mass is $m_I \simeq 10^{13}$ GeV. In the Higgs-otic scenario the latter is also of the order of the SUSY breaking scale M_{SS}.

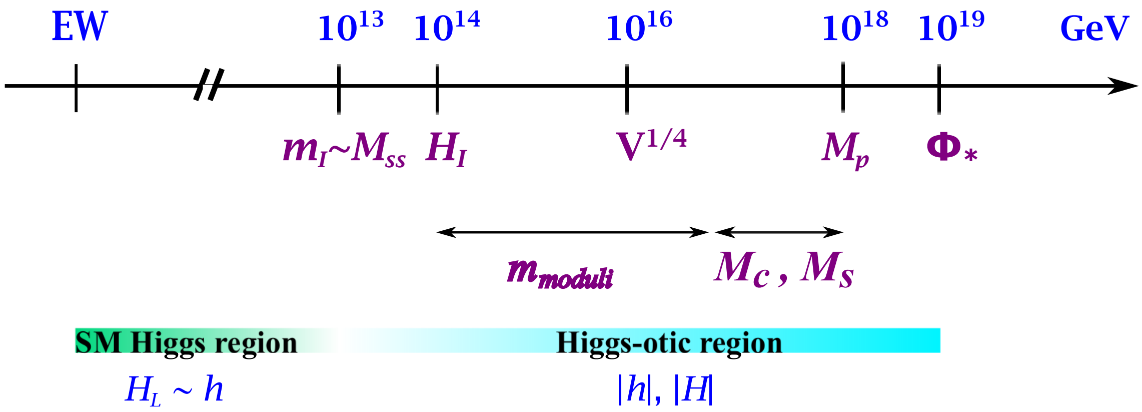


Figure 5.4: Energy scales in the Higgs-otic Inflation scenario. Below 10^{13} GeV the light degrees of freedom in the Higgs sector are given by the $SU(2)$ doublet H_L. Above this scale $SU(2)$ is broken and they lie within the neutral components of h and H.

5.2.2. String theory embeddings

In order to allow for consistent large field inflaton/Higgs, we will search for string constructions in which a MSSM Higgs sector of doublets H_u, H_d appear. We want the neutral components of these doublets to be associated with either continuous Wilson lines or position D-brane moduli. In this chapter we will provide examples of both possibilities. The first example is a compact $\mathbf{Z}_4$ toroidal heterotic orbifold in which Higgs fields are identified with certain scalars in the untwisted sector. In the second example we will identify the Higgs scalars with the position moduli of a $D7$-brane in a IIB orientifold with $\mathbf{Z}_4$ singularities. The subsequent analysis will focus on this second possibility since the addition of ingredients that give rise to monodromy is better understood.

5.2.2.1. The MSSM Higgs system in heterotic orbifolds

As a first example we will consider a Heterotic compactification in which a MSSM-like Higgs sector appears. We start with the $Spin(32)$ heterotic string compactified on a $\mathbf{T}^2 \times \mathbf{T}^2 \times \mathbf{T}^2$ torus, with each 2-torus defined in terms of an $SO(4)$ lattice. The model is subject to a twist in the compact dimensions defined by a $\mathbf{Z}_4$ shift $v = 1/4(1,1,-2)$ acting on the lattices as $\pi/2$ rotations in the first two tori and a reflection $z_3 \to -z_3$ in the third torus. The embedding of this twist in the $Spin(32)$ weight lattice is given by the 16-dimensional shift (see e.g. [207] for notation and examples)

$$V \;=\; \frac{1}{4}(1,1,1,2,2,0\;;\;1,1,3,0,0,0,0,0,0,0)\;, \tag{5.37}$$

where the SM group $SU(3) \times SU(2)$ lives in the first five entries. In addition we add discrete order-4 Wilson lines a_1 and a_2 around the first and second torus respectively, with

$$a_1 \;=\; \frac{1}{4}(1,1,1,1,1,-1\;;\;0,0,-1,1,0,0,0,0,0,0) \tag{5.38}$$

$$a_2 \;=\; \frac{1}{4}(-1,-1,-1,-1,-1,1\;;\;0,0,-1,1,2,0,0,0,0,0)\;. \tag{5.39}$$

As required both $4V$ and $4a_1$,$4a_2$ belong to the $Spin(32)$ weight lattice. The shift and Wilson lines verify the modular invariance constraints (see e.g. [207])

$$4 \times \left((V \ \pm a_1 \ \pm a_2)^2 \ - \ v^2\right) \ = \ 2s, \quad s \in \mathbf{Z} \ , \tag{5.40}$$

which automatically guarantee anomaly cancellation. The projections $P.V = n$, $P.a_1 = m$, $P.a_2 = q$, with $P_I \in \Lambda_{Spin(32)}$ and $n, m, q \in \mathbf{Z}$, give us the invariant gauge group which is

$$SU(3) \times SU(2) \times U(1) \times (SO(10) \times SU(2)' \times U(1)^6) \ . \tag{5.41}$$

The chiral matter fields in the untwisted sector are obtained from P_I verifying $P.V = -1/4$ (mod integer) but $P.a_i \in \mathbf{Z}$ for the first two complex planes and $P.V = 1/2$ mod integer for the third. One gets

$$2(3, 2) \ + \ 2(\bar{3}, 1) \ + \ (1, 2) \ + \ (1, \bar{2}) \ + \ hidden \tag{5.42}$$

under the SM gauge group $SU(3) \times SU(2)$. By hidden we denote matter fields not transforming with respect to this SM group. Note there is a minimal set of Higgs fields, which is vector like, and can be identified with the H_u, H_d scalars of the MSSM. They are associated to the third complex plane. In addition the untwisted sector contains two generations of left- and right-handed quarks, associated to the first two complex planes. In addition to the above matter fields, there will be additional ones from the θ, θ^2 and θ^3 twisted sectors. They will provide for the rest of the two MSSM generations plus additional stuff, cancelling all anomalies. We will not display those since they are not relevant for our purposes.

As discussed in refs. [208–212] the vevs of untwisted fields in an orbifold along D-flat directions correspond to switching on continuous Wilson lines in the underlying torus, in this case along the third torus. So this is an example of a consistent global string construction in which MSSM-like Higgs vevs are parametrised by continuous Wilson lines.

The inflation potential is however flat so far. In order to obtain a potential (and hence a mass) for the Higgs/inflaton system we would need some source of monodromy. A natural source could be the presence of some sort of fluxes, like those geometric fluxes present in the definition of massive Wilson lines given in [189]. However our understanding of fluxes in heterotic compactifications is still quite incomplete compared to that in type IIB compactifications. This is why in the next section we turn to the description of the Higgs/inflaton system in type IIB orientifolds.

Before turning to the IIB case let us recall what is the structure of the Kähler potential involving untwisted matter and moduli fields in $\mathbf{Z}_{2N}$ orbifolds in which one complex plane (i.e., the third) suffers only a twist of order 2. In this case the untwisted matter fields associated to the third complex plane are vector like, i.e., chiral matter multiplets A, B with opposite gauge quantum numbers, like is the case for H_u, H_d in the MSSM. This is what happens in the $\mathbf{Z}_4, \mathbf{Z}_6', \mathbf{Z}_8'$ and $\mathbf{Z}_{12}'$ heterotic orbifolds, (see e.g. [207]). Then the Kähler potential has a contribution of the form

$$K \ = \ -\log\left[(T_3 + T_3^*)(U_3 + U_3^*) \ - \ \frac{\alpha'}{2} \ (A + B^*)(A^* + B)\right] , \tag{5.43}$$

where T_3 and U_3 are the Kähler and complex structure modulus of the $\mathbf{T}^2$ in the third complex direction. In the above $\mathbf{Z}_4$ example we will have that $A + B^* = H_u + H_d^*$. The consequences of this structure, which is also present in the type IIB orientifold model of next subsection, will be discussed in sections 5.2.3.5 and 5.2.5.1.

5.2.2.2. The MSSM Higgs system in type IIB orientifolds

In this second example we will concentrate on type IIB compactifications with $O3/O7$ orientifold planes, in which the addition of RR and NS 3-form fluxes is at present best understood. The addition of these fluxes will give rise to the desired monodromy for the inflaton/Higgs. This is so for the position moduli of D7-branes which are directly sensitive to the presence of ISD closed string 3-fluxes.[1] In what follows we will thus concentrate on the case in which one identifies the Higgs/inflaton field with a D7 position modulus in a IIB orientifold

In particular, we will consider a type IIB O3/O7 orientifold with a stack of D7-branes sitting on a $\mathbf{Z}_4$ singularity, with a local geometry of the form $(X \times \mathbf{T}^2)/\mathbf{Z}_4$, with X some complex two-fold. The D7-branes are transverse to the $\mathbf{T}^2$ and are initially located at its origin, on top of the singularity. The D7-branes wrap the compact 4-cycle X which may be taken to be $\mathbf{T}^4$ for simplicity, but whose structure will not be crucial for the relevant Higgs sector. We will consider this setting as a local model and do not care much about global RR tadpoles.

Examples of D-brane models in the case where $X = \mathbf{T}_4$ have been given in [45,213]. Such orbifold has a geometric action of the form

$$\theta: \ (z_1, z_2, z_3) \ \mapsto \ (e^{-2\pi i/4} z_1, e^{-2\pi i/4} z_2, e^{2\pi i/2} z_3) \ = \ (-iz_1, -iz_2, -z_3) \tag{5.44}$$

encoded in the shift vector $v = \frac{1}{4}(1, 1, -2)$, as in the previous heterotic example. We then consider a stack of N D7-branes extended over the first two complex coordinates, and such that the action of the orbifold generator θ on the Chan-Paton degrees of freedom is

$$\gamma_{\theta,7} = \text{diag} \ \ (\mathbf{1}_{n_0}, i\mathbf{1}_{n_1}, -\mathbf{1}_{n_2}, -i\mathbf{1}_{n_3}) \tag{5.45}$$

with $\sum_{i=1}^{4} n_i = N$. Implementing the standard procedure (see e.g. [207]) one obtains the following spectrum for open strings in the 77 sector:

$$\begin{array}{ll} \textbf{Vector Multiplets} & \prod_{i=1}^{4} U(n_i) \\ \textbf{Chiral Multiplets} & \sum_{r=1}^{3} \sum_{i=1}^{4} (n_i, \bar{n}_{i+4v_r}) \end{array} \tag{5.46}$$

where the index i is to be understood mod 4.

Let us now consider the case where $n_0 = 1$, $n_1 = 3$, $n_2 = 2$, $n_3 = 0$. The spectrum in the 77 sector is then given by a gauge group $U(3) \times U(2) \times U(1)$ and matter spectrum

$$2 \times (\bar{3}, 1)_{+1} \ + \ 2 \times (3, \bar{2})_0 \ + \ (1, \bar{2})_{+1} \ + \ (1, 2)_{-1} \tag{5.47}$$

where the subscript stands for the charge under the U(1) of the 0^{th} node.

What is more relevant for us is how these representations arise in terms of the original stack of D7-branes and its fields, which correspond to three adjoints $(A_{\bar{1}}, A_{\bar{2}}, \Phi)$ of U(6). After performing the orbifold projection we obtain that these matrices get projected down to off-diagonal entries that contain the above matter fields. More precisely

$$A_{\bar{i}} \ = \left(\begin{array}{ccc} \mathbf{0}_3 & Q_L^i & \\ & \mathbf{0}_2 & \\ U_R^i & & 0 \end{array} \right) \qquad \Phi \ = \left(\begin{array}{ccc} \mathbf{0}_3 & & \\ & \mathbf{0}_2 & H_u \\ & H_d & 0 \end{array} \right) \tag{5.48}$$

[1] The case of D3-branes (or rather anti-D3-branes) would be more subtle since they may feel the presence of ISD fluxes only through the back-reaction of the geometry, see [52].

where we used standard notation to label the matter fields.[2] In particular the hypercharge generator is given by the non-anomalous U(1) combination

$$Q_Y \;=\; -\frac{Q_3}{3} - \frac{Q_2}{2} - Q_1 \tag{5.49}$$

where Q_n is the generator for $U(1) \subset U(n)$. This justifies the following notation for the Higgs sector

$$H_u \;=\; (1,2)_{-1} \qquad H_d \;=\; (1,\bar{2})_1 \tag{5.50}$$

The other two $U(1)$'s within the local model are anomalous and become massive through the GS mechanism. From (5.48) one can compute the Yukawa couplings of this system by using the D7-brane superpotential formula

$$W \;=\; \mathrm{tr}\left([A_{\bar{1}}, A_{\bar{2}}]\Phi\right) \;\rightarrow\; Q_L^2 H_u U_R^1 - Q_L^1 H_u U_R^2 \tag{5.51}$$

or simply orbifold CFT techniques. Here superindices denote generations. Notice that the representation H_d does not enter in the superpotential, which is to be expected because the representation D_R will only appear when we include fractional D3-branes that cancel the twisted tadpoles of the model. One can also compute the D-term potential of this model from $V_D \sim \mathrm{tr}\ DD^\dagger$ with $D = [A_{\bar{1}}, A_1]+[A_{\bar{2}}, A_2]+[\Phi, \bar{\Phi}]$. From here one obtains the D-term quartic potential described in section 5.2.1.

The twisted tadpole cancellation conditions allow for sets of D7-branes with traceless contribution to quit the singularity and to travel to the bulk. In particular if one of the two $U(2)$ branes combines with the $U(1)$ brane, they do not give net contribution to the tadpole and can travel through the bulk, in particular they can travel over through $\mathbf{T}^2$ in the z_3 direction. They should do that in a way consistent with the $\mathbf{Z}_4$ symmetry, which acts on z_3 through a the reflection $z_3 \rightarrow -z_3$, and so the two wandering D7-branes should travel at mirror locations z_3 and $-z_3$ respectively. When that happens, the 4 D7-branes remaining on the singularity have gauge group $U(3)\times U(1)$ whereas the wandering couple carries a single U(1). Taking into account that the GS mechanism gave masses to two U(1)'s, a single $U(1)_{em}$ remains unbroken, corresponding to electromagnetism. All in all there is a symmetry breaking process

$$U(3)\times U(2)\times U(1) \rightarrow SU(3)\times SU(2)\times U(1)_Y \rightarrow SU(3)\times U(1)_{em}\ , \tag{5.52}$$

whereas the first symmetry breaking is due to the GS mechanism, and the last one is due to the Higgs mechanism induced by the wandering pair of branes.

The fact that the wandering D7's can travel freely through T^2 corresponds to the existence of a flat direction $|\langle(1,\bar{2})\rangle| = |\langle(\bar{1},2)\rangle|$, i.e., $|H_u| = |H_d^*|$. The position of the D7-brane as it moves in the third $\mathbf{T}^2$ is parametrised by the vevs (σ,θ). In particular one has for this coordinate[3]

$$z_3^2 \;=\; (2\pi\alpha')^2\sigma^2 e^{i\theta} \;=\; (2\pi\alpha')^2 H_u H_d \;=\; (2\pi\alpha')^2\frac{(|H|+i|h|)^2}{2}e^{-i\gamma} \tag{5.53}$$

Thus $2\pi\alpha'\sigma$ corresponds to the distance of the wandering D7-branes to the branes remaining at the $\mathbf{Z_4}$ singularity. This separation corresponds to spontaneous gauge symmetry

[2] In (5.48) we have made a change of basis so that (5.45) reads $\gamma_{\theta,7} = \mathrm{diag}\ (i\mathbf{1}_3, -\mathbf{1}_2, 1)$.

[3] Note that it is z_3^2 , which is invariant under the $\mathbf{Z}_2$ reflection, which is well defined in the orbifold quotient space, rather than z_3 itself.

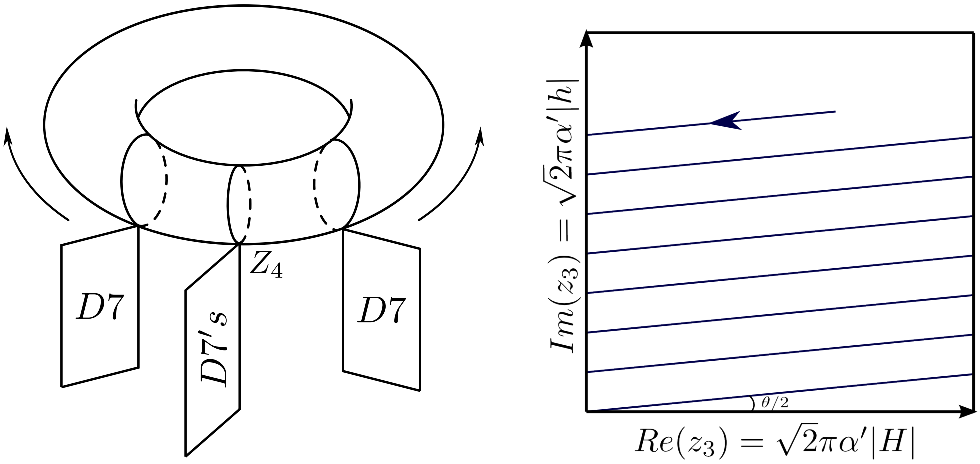


Figure 5.5: Left:A possible trajectory of the inflaton/Higgs D7-brane cycling around the $\mathbf{T}^2$ before fluxes are turned on. Right: A pictorial sketch of the system of D7-branes.

breaking. A possible trajectory of the wandering-D7/Higgs/inflaton branes over $\mathbf{T}^2$ is illustrated in figure 5.5, where we assume $\gamma = 0$. The open strings going from the $D7$ to the singularity will give rise to massive $W^{\pm}, Z^0$ gauge bosons and their SUSY partners. In particular, consider a D7-brane at the point $z_3 = x + iU_3 y$, where iU_3 is the complex structure of the third $\mathbf{T}^2$. Then the mass formula for the open string states between the singularity and the D7-brane is given by

$$M^2 = \frac{1}{(2\pi\alpha')^2} \left| z_3 - (w_1 + iU_3 w_2) 2\pi R \right|^2 , \tag{5.54}$$

where $w_{1,2}$ are the winding numbers around the two cycles of the transverse $\mathbf{T}^2$, whose radius along x is given by R. We thus obtain $M^2 = \sigma^2$ for $w = 0$ and small x, so that the mass is controlled by $\langle\sigma\rangle$.[4]

The massive states include not only $W^{\pm}, Z^0$, but also three massive scalars $H^{\pm}, h^0$, which are the scalars included in the $N = 1$ SUSY massive vector multiplets. The counting of degrees of freedoms is as follows: We start with 8 real scalars from H_u, H_d. Three of them become goldstone bosons, whereas other three ($H^{\pm}, h^0$), complete a massive vector multiplet. The two remaining scalars are massless at this level, and correspond to the two neutral scalars from σ, θ, which parametrise the position of the D7 wandering branes through the third $\mathbf{T}^2$. In the model the 2 families of quarks become also massive due to the Yukawa couplings in eq.(5.51).

Note that the Higgs vev σ may be arbitrarily large, even larger than the Planck scale. This however *does not lead to new states with masses larger than* M_p. In particular this applies to the massive $W^{\pm}, Z^0$ boson and their partners, which can never get masses larger than the KK scale of the $\mathbf{T}^2$. Indeed, as shown in eq.(5.54), for $|z_3| > \pi R$, the lightest states to be identified with these bosons correspond to winding numbers $w_{1,2} \neq 0$, and no longer to the initial states with $w_{1,2} = 0$. In this sense the *effect of the inflaton/Higgs vev in this string context is very mild*, not deforming the structure of the KK/string spectra in a substantial manner. This is to be contrasted to a purely 4d field theory model of the

[4]The familiar factor proportional to the square of the gauge coupling appears upon normalising the fields canonically.

MSSM in which the gauge boson masses *are* proportional to the vev of the scalar and hence would produce masses larger than M_p, with physics difficult to control, if at all.

Note that an interesting property of the wandering D7-branes is that, as the position varies and the inflaton vev decreases, the masses of $W^{\pm}$, Z^0 etc. decrease in an oscillating manner, since the distance of the brane to the singularity also oscillates. In some particular limits in which the brane travel along one of the axis or the diagonal, these fields become periodically massless as the vev of the inflaton decreases. This is however not generically the case, and it will not be the case in the relevant Higgs-otic model.

5.2.3. Effective inflationary potential

In the previous section we have discussed how a vev within the MSSM Higgs sector may be understood in terms of the motion of a D7-brane on a $\mathbf{T}^2$. However, up to now the full scalar potential is flat along such D-term flat direction. We will now induce mass terms for the inflaton/Higgs as required in oder to obtain a chaotic-like potential. To do that we will consider the case in which there are imaginary self-dual (ISD) 3-form fluxes G_3 acting as a background. As is well known, such classes of ISD fluxes are solutions of the type IIB 10d equations of motion in warped Calabi-Yau backgrounds [7]. In such type IIB compactifications there are two types of ISD fluxes, with tensor structure $G_{(0,3)}$ and $G_{(2,1)}$ respectively. The first class breaks SUSY and induces SUSY-breaking soft-terms: scalar and gaugino masses. The second class preserves SUSY and may induce supersymmetric F-term masses to the chiral multiplets. These flux-induced terms were analysed in section 4.1 and in refs. [45, 46, 52, 58, 214]. In our discussion below we will consider the generic case in which both classes of fluxes are turned on simultaneously. More precisely, we will consider the following closed string background

$$
\begin{aligned}
ds^2 &= Z(x^m)^{-1/2}\eta_{\mu\nu}d\hat{x}^\mu d\hat{x}^\nu + Z(x^m)^{1/2}ds^2_{\text{CY}} \\
\tau &= \tau(x^m) \\
G_3 &= \frac{1}{3!}G_{lmn}dx^l \wedge dx^m \wedge dx^n \\
\chi_4 &= \chi(x^m)d\hat{x}^0 \wedge d\hat{x}^1 \wedge d\hat{x}^2 \wedge d\hat{x}^3 \\
F_5 &= d\chi_4 + *_{10}d\chi_4
\end{aligned}
\tag{5.55}
$$

with $\tau = C_0 + ie^{-\phi}$ the 10d axio-dilaton, Z a warp factor that depends on the internal coordinates x^m, and ds^2_{CY} the Ricci-flat metric of the internal covering space, namely $\mathbf{T}^4 \times \mathbf{T}^2$. Finally, $G_3 = F_3 - \tau H_3$ is the complexified three-form flux, with F_3 and H_3 the RR and NSNS fluxes respectively. As mentioned before we take this flux to be of the form $G_3 = G_{(0,3)} + G_{(2,1)}$, and in particular we choose $G_{(0,3)} = G_{\bar{1}\bar{2}\bar{3}}\, d\bar{z}_1 \wedge d\bar{z}_2 \wedge d\bar{z}_3$ and $G_{(2,1)} = G_{\bar{1}\bar{2}3}\, dz_1 \wedge dz_2 \wedge d\bar{z}_3$, as these are the two fluxes that are invariant under the $\mathbf{Z}_4$ action (5.44). Since we are considering only ISD 3-form fluxes, the background dilaton τ must be holomorphic in order to satisfy the IIB supergravity equations of motion. For simplicity we will consider τ to be constant, although our results can easily be generalised for a non-constant profile.

The potential for the fields living in the D7-brane worldvolume can be obtained by evaluating the D7-brane DBI+CS action in the above background, as we now describe.

5.2.3.1. Flux induced scalar potential from DBI+CS

We will consider again the toroidal setting and compute the effect of the G_3 fluxes on the $U(6)$ adjoint complex scalar existing in the model in the previous section before orbifolding. This adjoint contains off-diagonal components containing the $H_{u,d}$ fields of interest, which we will display at the end.

The effective action for the microscopic fields of a system of D7-branes in the 10d Einstein frame is given by the Dirac-Born-Infeld (DBI) + Chern-Simons (CS) actions

$$S_{DBI} = -\mu_7 g_s^{-1}\mathrm{STr}\left(\int d^8\xi\sqrt{-\det(P[E_{MN}]+\sigma F_{MN})\det(Q_{mn})}\right) \tag{5.56}$$

$$S_{CS} = \mu_7 g_s \mathrm{STr}\left(\int d^8\xi P\left[-C_6\wedge B_2 + C_8\right]\right) \tag{5.57}$$

where

$$E_{MN} = g_s^{1/2}G_{MN} - B_{MN} \qquad Q^m_n = \delta^m_n + i\sigma[\phi^m,\phi^\rho]E_{\rho n} \qquad \mu_7 = (2\pi)^{-3}\sigma^{-4}g_s^{-1} \tag{5.58}$$

and $\sigma = 2\pi\alpha'$. Here M, N are D7-brane worldvolume indices and $P[\cdot]$ denotes the pullback of the 10d background onto such worldvolume, while m, n are indices transverse to the D7-brane. Finally, 'STr' stands for the symmetrised trace over gauge indices.[5]

The D7 world volume spectrum compactified to 4d contains before orbifolding two adjoints $A_{1,2}$ which come from 8d vectors and an adjoint Φ which parametrises the D7-position and that will be the subject of our interest. The determinant in the DBI action can be factorised between Minkowski and the internal space (labelled by μ,ν and a, b indices respectively) and after some calculations we obtain

$$\det(P\left[E_{MN}\right]+\sigma F_{MN}) = -g_s^4 f(B)^2\left[1+2Z\sigma^2 D_\mu\Phi D^\mu\bar\Phi + \frac{1}{2g_s}\sigma^2 Z F_{\mu\nu}F^{\mu\nu}\right] \tag{5.59}$$

and

$$\det(Q_{mn}) = 1 - \frac{Zg_s\sigma^2}{2}[\Phi_m,\Phi_n]^2 \tag{5.60}$$

where

$$f(B)^2 = 1 + \frac{1}{2}Z^{-1}g_s^{-1}B_{ab}B^{ab} - \frac{g_s^{-2}}{4}Z^{-2}B_{ab}B^{bc}B_{cd}B^{da} + \frac{g_s^{-2}}{8}Z^{-2}\left[B_{ab}B^{ab}\right]^2 \tag{5.61}$$

The details of the computation can be found in Appendix A. Recall that Z is a possible warp factor which we will often set to unity when doing explicit computations. Nevertheless, a non-constant warp factor might have interesting phenomenological consequences, as we will briefly discuss later on. The contribution coming from (5.60) will give rise to the usual D-term potential. Since this term does not change formally when including the α' corrections, we will skip it in the computation below and restore it only at the end of the section, to avoid clutter.

For simplicity we are not considering neither Wilson lines nor magnetic fluxes on the branes worldvolume, that is, we are setting $\langle A_a\rangle = 0$. In our configuration only the

[5] The parameter σ in here should not be confused with the inflaton field σ defined in eq.(5.33).

adjoint field Φ will take a non-zero vacuum expectation value, which will parametrise the position of the D7-branes in their transverse space z_3 via the equation

$$\det\left(\langle\Phi\rangle - \sigma^{-1} z_3 I\right) = 0\ . \tag{5.62}$$

For this reason, in (5.59) we have already neglected all the terms that are not relevant for the scalar potential (like BF, FF and $[A,\Phi]$ couplings), since they vanish for $\langle F\rangle = \langle A\rangle = 0$. Notice however that we have kept all those depending only on B to all orders. The reason is that, in the presence of a background three-form flux H_3, changing the vev of Φ induces a B-field on the D7-brane worldvolume. Hence, since our model of inflation the vev $\langle\Phi\rangle$ is going to take large values, we cannot neglect the dependence on B to any order in the DBI expansion.

Let us for now ignore the contribution coming from $\det(Q_{mn})$, which gives the D-term scalar potential. Then, plugging (5.59) into the DBI action (5.56) we obtain

$$S_{DBI} = -\mu_7 g_s \mathrm{STr}\int d^8\xi \sqrt{f(B)^2\left[1 + 2Z\sigma^2 D_\mu\Phi D^\mu\bar\Phi + \frac{1}{2} Z g_s^{-1}\sigma^2 F_{\mu\nu}F^{\mu\nu}\right]} \tag{5.63}$$

with $f(B)$ the same as in (5.61). One can check that whenever the B-field is a $(2,0)+(0,2)$-form on the D7-brane internal worldvolume $f(B)$ can be written as

$$f(B)\ =\ 1 + \frac{1}{2} Z^{-1} g_s^{-1} B^2 \tag{5.64}$$

where we have denoted $B^2 \equiv B_{ab}B^{ab}/2$ and used that $4B_{ab}B^{bc}B_{cd}B^{da} = \left[B_{ab}B^{ab}\right]^2$. This implies that all corrections in α', which appear as powers of the B-field in $f(B)^2$, can be completed into a perfect square. The reason is the underlying supersymmetry of the system, which imposes that for a worldvolume flux $\mathcal{F}$ which is a self-dual two-form on the D7-brane internal dimensions the D7-brane gauge kinetic function must be holomorphic on the axio-dilaton τ, while for an anti-self-dual two-form it must be anti-holomorphic. In both cases (ours being the second) no square roots should appear multiplying $F_{\mu\nu}F^{\mu\nu}$, because there are none multiplying $F_{\mu\nu}\tilde F^{\mu\nu}$. We refer to Appendix A for further details.

Even if Φ is supposed to take large vacuum expectation values their derivatives must remain small, since we are interested in slow-roll dynamics. We can then expand the square root neglecting higher orders in $\partial_\mu\Phi$, obtaining

$$S_{DBI} = -\mu_7 g_s \mathrm{STr}\int d^8\xi f(B)\left[1 + Z\sigma^2 D_\mu\Phi D^\mu\bar\Phi + \frac{1}{4} Z g_s^{-1}\sigma^2 F_{\mu\nu}F^{\mu\nu} + \mathcal{O}(\partial^4)\right] \tag{5.65}$$

where we have taken the same approximation for A_μ and its derivatives.

In order to proceed further we have to express the B-field in terms of the fluctuations of the 8d field Φ. Recalling that $G_3 = F_3 - \tau H_3$ (with $F_3(H_3)$ being the RR(NSNS) 3-form flux), we can integrate

$$dB_2 = \frac{\mathrm{Im}G_3}{\mathrm{Im}\tau} \tag{5.66}$$

to obtain the B-field induced on the brane due to the presence of a constant G_3 background flux. The result for the B-field components is given by

$$B_{12} = \frac{g_s\sigma}{2i}\left(G^*_{(0,3)}\Phi - G_{(2,1)}\bar\Phi\right) \quad ; \quad B_{\bar1\bar2} = -\frac{g_s\sigma}{2i}\left(G_{(0,3)}\bar\Phi - G^*_{(2,1)}\Phi\right) \tag{5.67}$$

where recall that, in tensorial notation the (0,3)-form flux corresponds to components $G_{\bar{1}\bar{2}\bar{3}}$ while the (2,1)-form flux to $G_{\bar{1}2\bar{3}}$. From now on we will denote the fluxes as $G \equiv G_{\bar{1}\bar{2}\bar{3}}$ and $S \equiv \epsilon_{3jk} G_{3\bar{j}\bar{k}}$ for simplicity in the notation. Plugging this in (5.64) we get that $f(B)$ becomes

$$f(\Phi) = 1 + \frac{Z^{-1} g_s \sigma^2}{4} |G^* \Phi - S\bar{\Phi}|^2 \ , \tag{5.68}$$

Let us now consider the Chern-Simons piece. From the equations of motion of type IIB supergravity one can derive the following relations between the RR fields and the 3-form fluxes

$$dC_6 = H_3 \wedge (C_4 + \frac{1}{2} B_2 \wedge C_2) - *\mathrm{Re}\, G_3 \tag{5.69}$$

$$dC_8 = H_3 \wedge C_6 - *\mathrm{Re}\, d\tau \tag{5.70}$$

Integrating these equations and using that the background for the dilaton is constant, we obtain the following RR 6-form and 8-form potentials

$$(C_6)_{12} = -\frac{Z^{-1}\sigma}{2i} (G^*\Phi - S\bar{\Phi}) \tag{5.71}$$

$$(C_8)_{1\bar{1}2\bar{2}} = \frac{Z^{-1} g_s \sigma^2}{4} \left((|G|^2 + |S|^2)|\Phi|^2 - 4G^* S^* \Phi^2 + \text{c.c.} \right) \tag{5.72}$$

Plugging these expressions in the Chern-Simons action of the D7-branes we get

$$S_{CS} = \mu_7 g_s \mathrm{STr} \int d^8\xi \left(-\frac{Z^{-1} g_s \sigma^2}{4} |G^*\Phi - S\bar{\Phi}|^2 \right) \tag{5.73}$$

which combined with the DBI part results in the following 8d action

$$S_{8d} = -\mu_7 g_s \mathrm{STr} \int d^8\xi \left(f(\Phi) \left(Z\sigma^2 D_\mu \Phi D^\mu \bar{\Phi} + \frac{1}{4} Z g_s^{-1} \sigma^2 F_{\mu\nu} F^{\mu\nu} \right) - \tilde{V}(\Phi) \right) \tag{5.74}$$

where the scalar potential is given by

$$\tilde{V}(\Phi) \ = \ 2(f(\Phi) - 1) \ = \ \frac{Z^{-1} g_s \sigma^2}{2} |G^*\Phi - S\bar{\Phi}|^2 \tag{5.75}$$

In this last step we have also subtracted the D7-brane tension (which is cancelled by the contribution of the orientifold planes). Notice that once done so the DBI and the CS parts of the action contribute the same amount to the scalar potential, so we cannot neglect the contribution from the CS action, as is oftentimes done in the literature.

Finally, integrating over the internal $\mathbf{T}^4$ wrapped by the D7-branes (using that the internal profile of the wavefunctions for Φ is constant, see [54,215]) and rescaling the fields such that

$$\Phi \to \Phi (V_4 \mu_7 g_s Z \sigma^2)^{-1/2} \quad ; \quad A_\mu \to A_\mu (V_4 \mu_7 Z^{-1} \sigma^2)^{-1/2} \tag{5.76}$$

we obtain the following 4d effective Lagrangian

$$\mathcal{L}_{4d} = \mathrm{STr} \left(f(\Phi) D_\mu \Phi D^\mu \bar{\Phi} + \frac{1}{4 g_{YM}^2} F_{\mu\nu} F^{\mu\nu} - V(\Phi) - \frac{1}{2} g_{YM}^2 [\Phi, \bar{\Phi}]^2 \right) \tag{5.77}$$

where we have restored the D-term. Notice that all the dependence of the D-term on the higher order corrections is absorbed in $g_{YM}^{-2} = V_4\mu_7 Z^{-1}\sigma^2 f(\Phi)$, with V_4 being the volume of the internal $\mathbf{T}^4$. The rescaled scalar (F-term) potential and $f(\Phi)$ become

$$V(\Phi) = \frac{Z^{-2}g_s}{2}|G^*\Phi - S\bar{\Phi}|^2 \ , \tag{5.78}$$

$$f(\Phi) = 1 + \frac{Z^{-2}(V_4\mu_7)^{-1}}{4}|G^*\Phi - S\bar{\Phi}|^2 \ . \tag{5.79}$$

As expected, this potential looks like a quadratic potential for the adjoint scalars. However, one has to take into account the field redefinition required to have canonical kinetic terms in eq.(5.77), which becomes important for large values of $\langle\Phi\rangle$. As we will describe in section 5.2.4, this redefinition modifies the large Φ behaviour of the system, which turns close to a linear potential. Note that this *flattening* effect is similar to that obtained in previous examples of monodromy inflation models [185,186,216]. It is however important to realise that in the present case the flattening effect is purely due to the field redefinition, and not to the square root of the DBI action. In fact notice that the CS piece suffers the same flattening effect with no square root involved whatsoever.

5.2.3.2. Kaloper-Sorbo Lagrangian

While it may not be obvious from the above discussion, the system of D7-branes described above is an example of F-term axion-monodromy inflation model [189], in the sense that for small values of $\langle\Phi\rangle$ the scalar potential can be understood as a standard F-term potential. This has already been shown for the case of D7-branes in smooth Calabi-Yau geometries, see for instance [45,217,218]. For the orbifold model of interest to this paper the connection with $N = 1$ supergravity turns out to be more involved, but as we will show in section 5.2.3.5 a similar result applies. Hence, we can also consider this model as an example of F-term monodromy inflation.

Now, as pointed out in [189], in general models based on F-term axion monodromy have a direct connection with the 4d effective framework developed in [163–165], which features a Lagrangian of the form (5.18). Following [189], it is for instance straightforward to obtain the Kaloper-Sorbo Lagrangian from a heterotic or type I model where the inflaton is a massive Wilson line in a twisted torus, this being the most direct way to give a mass to the Higgs system of the model of section 5.2.2.1. Nevertheless, a similar derivation for F-term monodromy models where the inflaton is a D-brane position has so far not been worked out.

In order to see how to derive the 4d Lagrangian (5.18) from a model of wandering D7-branes, let us consider a single D7-brane transverse to z_3 and in the presence of the ISD three-form fluxes $G \sim G_{\bar{1}\bar{2}\bar{3}}$ and $S \sim G_{12\bar{3}}$. Now, looking at the DBI action in the Yang-Mills approximation we have that

$$\mu_7 \int \frac{1}{2}(\sigma F_2 + B_2)\wedge *_8(\sigma F_2 + B_2) \ = \ \mu_7 \int \frac{1}{2}\sigma^2 F_6 \wedge *_8 F_6 + \sigma B_2 \wedge F_6 + \ldots \tag{5.80}$$

where we have only kept terms that depend on $F_6 = dA_5$, the magnetic dual of $F = dA$. If we assume that the D7-brane has a position modulus ϕ, then it means that the four-cycle S_4 wrapped by the D7-brane contains a (2,0)-form ω_2 [219], in which we can expand the magnetic potential A_5 as

$$A_5 \ = \ iC_3 \wedge \bar{\omega}_2 - i\bar{C}_3 \wedge \omega_2 \tag{5.81}$$

where C_3 is a complex three-form in 4d. For instance, if $S_4 = \mathbf{T}^4$ such (2,0)-form will be given by $\omega = dz_1 \wedge dz_2$. Plugging this decomposition into the kinetic term for A_5 in (5.80) and performing dimensional reduction we obtain

$$\mu_7 \sigma^2 \frac{1}{2} \int_{\mathbb{R}^{1,3} \times S_4} F_6 \wedge *_8 F_6 \quad \to \quad \rho \int_{\mathbb{R}^{1,3}} d^4x \, |dC_3|^2 \,, \qquad \rho \, = \, \mu_7 \sigma^2 \int_{S_2} \omega_2 \wedge *_4 \bar{\omega}_2 \tag{5.82}$$

which is nothing but the complex generalisation of the term $\int |F_4|^2$ in (5.18), in the sense that $F_4 = dC_3$ is now a complex four-form in 4d.

Let us now dimensionally reduce the second term in the rhs of (5.80). By taking into account that

$$B_2 \, = \, \frac{g_s \sigma}{2i} (G^* \phi - S \bar{\phi}) \, \omega_2 + \text{c.c.} \tag{5.83}$$

as derived in the previous section we obtain

$$\mu_7 \sigma \int_{\mathbb{R}^{1,3} \times S_4} B_2 \wedge F_6 \quad \to \quad -g_s \rho \int_{\mathbb{R}^{1,3}} \phi (G^* dC_3 - S^* d\bar{C}_3) + \text{c.c.} \tag{5.84}$$

where we have used that $*_4 \omega_2 = -\omega_2$. Again, we obtain a generalisation of the axion-four-form term $\int \phi F_4$ in (5.18), where a complex scalar ϕ couples to the four-form $F_4 = dC_3$ and its complex conjugate via the presence of fluxes. Notice that a similar expression was found in [170] for the coupling of a complex scalar to a complex four-form. In our case we find a more general expression, in the sense that ϕ can couple to both $\bar{F}_4$ and F_4 due to the respective presence of supersymmetric (S) and non-supersymmetric (G) background fluxes respectively.

From this Lagrangian and following the general philosophy of [163–165] one finds that after integrating out F_4 the potential generated for the scalar field ϕ is given by

$$V(\phi) \, = \, \frac{g_s}{2} |G^* \phi - S \phi^*|^2 \tag{5.85}$$

just as found in the previous section when setting $Z = 1$, as we have done here. Of course this will only be the potential in the small field regime, receiving corrections for large values of $\langle \phi \rangle$. Nevertheless, due to the symmetry properties of the Kaloper-Sorbo Lagrangian such corrections can only arise in powers of the initial scalar potential $V(\phi)$ and not of the field ϕ itself, see [163–165] and also [167–169]. In our analysis of the previous section we have seen that this is the case, occurring in the form of a redefinition for the kinetic term of ϕ, and giving rise to flattening effect for the potential. In section 5.2.5.1 we will discuss from an independent, string theoretical viewpoint why the Planck suppressed corrections to the inflaton potential should be of this form.

Finally, in this section we have only discussed the appearance of the Kaloper-Sorbo Lagrangian for the case of a single D7-brane with an Abelian gauge group. This is indeed the case of interest in our Higgs-otic D7-brane model, since away from the orbifold singularity we have a single wandering D7-brane. We nevertheless expect that a similar result applies to the non-Abelian case, given that the results of the previous section involving the large field corrections, flattening etc. are valid for any U(N) gauge group or even orbifolds thereof. Such non-Abelian analysis is however beyond the scope of this paper and we hope to return to this problem in the near future.

5.2.3.3. Estimation of the scales of the model

The coefficient of the quadratic term in the inflation potential, and hence the inflaton mass, is determined by the size of the fluxes. We can try to estimate the size of the fluxes in terms of the energy scales in the theory, assuming an approximate isotropic compactification.

Since the 3-form fluxes have to be quantised over the internal 3-cycles γ_j that they wrap, they are expected to scale as

$$\frac{1}{2\pi\alpha'}\int_{\gamma_j} G_3 = 2\pi n_j \rightarrow G_3 \simeq \frac{4\pi^2\alpha' n}{V_6^{1/2}} \tag{5.86}$$

where V_6 is the volume of the internal dimensions and n are integer quanta. Using the following identities from type IIB compactifications for the Planck mass and the compactification/unification scale [207]

$$m_p^2 = (8\pi)M_p^2 = \frac{8M_s^8 V_6}{(2\pi)^6 g_s} \quad , \quad M_c = M_s\left(\frac{2\alpha_G}{g_s}\right)^{1/4} \quad , \tag{5.87}$$

where we have defined the compactification/unification scale as $M_c = 1/R_c$ with $V_4 = (2\pi R_c)^4$, we find

$$G_3 = \frac{n}{\pi}\frac{M_c^2}{\alpha_G^{1/2} m_p} \ . \tag{5.88}$$

One can then estimate the scale of SUSY breaking which is given by

$$M_{SS} = \frac{Z^{-1}g_s^{1/2}}{\sqrt{2}}G_3 = \frac{Z^{-1}n}{\pi}\frac{M_s^2}{g_s^{1/2}m_p} \tag{5.89}$$

For $n \sim O(1)$ one gets $M_{SS} \sim 10^{12} - 10^{13}$ GeV if $M_s \simeq 10^{16}$ GeV. Thus the above simple dimensional argument implies a SUSY breaking scale of the required order so that the SM Higgs potential is saved from its instability.

We have seen that the effect of considering higher order corrections on Φ is the presence of a function $f(\Phi)$ multiplying the kinetic terms given by

$$f(\Phi) = 1 + \frac{Z^{-2}(V_4\mu_7)^{-1}}{4}|G^*\Phi - S\bar{\Phi}|^2 \ . \tag{5.90}$$

For small field this function is approximately 1 and we recover canonically normalised kinetic terms. To estimate how important is the effect for large field we define the parameter $\hat{G} \equiv Z^{-1}V_4^{-1/2}\mu_7^{-1/2}G_3$ and using (5.88) we get

$$[\hat{G}] = [Z^{-1}V_4^{-1/2}\mu_7^{-1/2}G_3] \simeq 0.3Z^{-1}g_s^{-1/2}n\frac{1}{M_p} \tag{5.91}$$

For $n \sim O(1)$ one obtains $\hat{G} \sim 0.3\frac{1}{M_p}$, so this effect becomes appreciable approximately for $\langle\Phi\rangle > 7M_p$. We can also write the SUSY breaking scale in terms of $\hat{G}$ such that

$$M_{SS}^2 = V_4\mu_7 g_s|\hat{G}|^2 \sim 0.05M_s^4|\hat{G}|^2 \tag{5.92}$$

so $\hat{G}$ gives us the relation between the SUSY breaking scale and the string scale. This relation will be useful later on when checking that the potential energy never becomes larger than the string scale.

5.2.3.4. The Higgs/inflaton scalar potential

Even if the analysis in section 5.2.3.1 is done for an adjoint field of a U(N) gauge theory, it also applies after we have made an orbifold projection that converts the adjoint into a set of bifundamental fields charged under the orbifolded gauge group. In particular, we may consider the $\mathbf{Z}_4$ orbifold projection of section 5.2.2.2 and hence take Φ to be the 6×6 matrix containing the Higgs system of the model

$$\Phi = \begin{pmatrix} \mathbf{0}_3 & & \\ & \mathbf{0}_2 & H_u \\ & H_d & 0 \end{pmatrix} \tag{5.93}$$

as in (5.48). Then applying the results from section 5.2.3.1 we obtain the standard D-term contribution to the scalar potential and the F-term contribution which is given by

$$V(\Phi) = \mathrm{STr}\left(\frac{Z^{-2}g_s}{2}|G^*\Phi - S\bar{\Phi}|^2\right) \ , \tag{5.94}$$

which in terms of the bifundamental fields H_u, H_d gives rise to

$$V = \frac{Z^{-2}g_s}{2}\left[(|G|^2+|S|^2)(|H_u|^2+|H_d|^2) - 4\mathrm{Re}(G^*S^*H_uH_d)\right] \tag{5.95}$$

once we trace over the gauge indices. This potential can be rewritten in terms of the combinations

$$h \ = \ \frac{e^{i\gamma/2}H_u - e^{-i\gamma/2}H_d^*}{\sqrt{2}} \quad \text{and} \quad H \ = \ \frac{e^{i\gamma/2}H_u + e^{-i\gamma/2}H_d^*}{\sqrt{2}} \tag{5.96}$$

where $\gamma = \pi - \mathrm{Arg}(GS)$ as

$$V = \frac{Z^{-2}g_s}{2}\left[(|G|-|S|)^2|h|^2 + (|G|+|S|)^2|H|^2\right] \tag{5.97}$$

Note that, at this level, before field rescaling to canonical kinetic terms, the potential has the structure of double chaotic inflation. Note also that for $|S| = |G|$, h becomes massless. Thus, if eventually we want to fine-tune a massless SM Higgs, we would need to be close to a situation where $|S| = |G|$. The subsequent running from M_c down to the scale M_{SS} of soft parameters will give rise to a massless SM Higgs.

We may now write this potential in terms of the real scalars (σ, θ) which we defined in eq.(5.33). They parametrise the neutral Higgs along the D-flat direction. One finds

$$V(\sigma,\theta) = Z^{-2}g_s(|G|^2+|S|^2)\left(1 - A\cos\tilde{\theta}\right)\sigma^2 \tag{5.98}$$

where we have defined

$$A = \frac{2|SG|}{|G|^2+|S|^2} \qquad \text{and} \qquad \tilde{\theta} \ = \theta - \mathrm{Arg}(GS) \ . \tag{5.99}$$

Note that $0 \leq A \leq 1$ and one also has

$$A \ = \ \frac{m_H^2 - m_h^2}{m_H^2 + m_h^2} \ = \ |cos2\beta| \ ; \ \frac{m_H}{m_h} \ = \ \sqrt{\frac{1+A}{1-A}} \ , \tag{5.100}$$

with $tan\beta = m_H/m_h$. The potential in eq.(5.98) will be our inflation potential. It is essentially a quadratic potential in σ modulated by the dependence on $\tilde{\theta}$. Note however that we still have to include the effect that the kinetic terms are non-canonical and field dependent, as we will discuss later. However, the qualitative structure of the scalar potential can already be discussed at this point.

Roughly speaking, the shape of the potential depends on the value of the parameter A which parametrises the relative size of both types of ISD fluxes. In figure 5.6 we show the structure of the scalar potential for three characteristic values $A = 0.1, 0.5, 0.95$. For

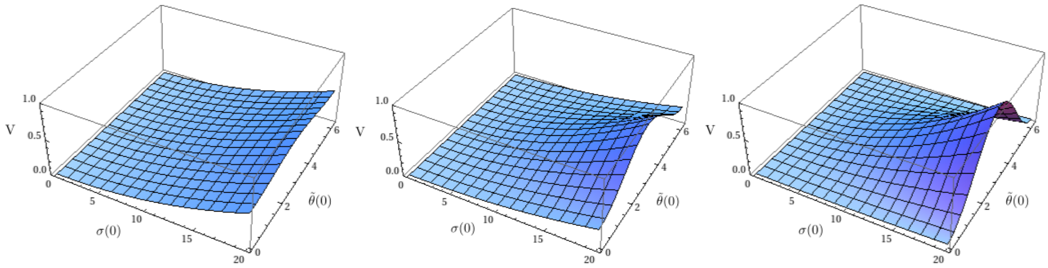


Figure 5.6: Scalar potential for three different values of A, $A = 0.1$ (left), $A = 0.5$ (centre) and $A = 0.9$ (right).

$A \simeq 0$, which can happen if either G or S vanish, the potential is simply given by

$$V = \frac{Z^{-2}g_s}{2}|G|^2(|H|^2 + |h|^2) = Z^{-2}g_s|G|^2\sigma^2 \tag{5.101}$$

which is $\tilde{\theta}$-independent. This case will be essentially identical to a single inflaton case with a chaotic, quadratic potential for σ (before flattening). This case with $A \simeq 0$ is depicted in the left plot in figure 5.6. Getting the same result with either $G = 0$ or $S = 0$ is expected by symmetry arguments, since a D7-brane which is point-like in the third complex plane cannot locally distinguish between the real and imaginary parts of z_3, and both choices of fluxes are related by interchanging z_3 by $\bar{z}_3$.

For the case $A = 1$ one has the fluxes related as $|G| = |S|$, and h is massless. The potential is then given by

$$V \;=\; 4Z^{-2}g_s|G|^2\cos^2(\tilde{\theta}/2)\sigma^2 \;=\; 2Z^{-2}g_s|G|^2|H|^2 \tag{5.102}$$

This corresponds to the right plot in figure 5.6. This choice of fluxes corresponds to a non-supersymmetric situation in which the NSNS 3-form flux H_3 only has a leg in one of the real directions of the transverse space, so the other direction is a flat direction for the D7-branes. In the 4d effective theory this is reflected by the presence of a massless real scalar which is given by $|h|$.

These two cases $A = 0, 1$ are limiting cases in which the potential reduces to a single field inflation model. For a generic choice of fluxes, one expects a situation in between, with both scalars playing an important role in inflation. In section 5.2.4 we will compute the slow-roll parameters first for the cases $A = 0, 1$ and then for the general 2-field inflation case.

Notice however that if we want to have a massless eigenstate at the SUSY breaking scale (in order to get a light SM Higgs), A is not a free parameter anymore. In terms of the mass parameters of the Higgs mass matrix in (5.25), A parametrises the ratio between the off-diagonal entries $|m_3|$ and the diagonal ones $m^2_{H_u} = m^2_{H_d}$ at M_c. Thus a massless eigenstate implies $|m_3|^2 = m^2_{H_u} m^2_{H_d}$ which corresponds indeed to $A = 1$ as we already commented. However, as we discussed in section 5.2.1, we need the eigenstate to become massless at $M_{SS} \sim 10^{12} - 10^{13}$ Gev and not at the inflation scale $\sim 10^{16}$ GeV, so A needs to be slightly lower than 1. We have computed the running between both scales and obtained that the optimal value to have a zero eigenvalue at M_{SS} is $A \simeq 0.83$, corresponding to $m_H/m_h = 3.28$. We take here the unification scale M_c as the scale at which $\alpha_2 = \alpha_3$. Of course this result depends on the exact value of M_{SS} which is in turn parametrised by the global factor in the potential, whose size was estimated in section 5.2.3.3 obtaining $M_{SS} \sim 10^{12} - 10^{13}$ Gev. In figure 5.7 we plot the value of A that we need to start with in order to have a light SM Higgs boson, as a function of the SUSY breaking scale. We have also imposed to get the experimental value of the top and Higgs mass at the EW scale. We can see that for $M_{SS} \sim 10^{12} - 10^{13}$, we have $0.8 < A < 0.85$, so in any case, we will be in a situation quite close to the single field case $A = 1$, in which the heavy Higgs H is the scalar which plays the role of the inflaton.

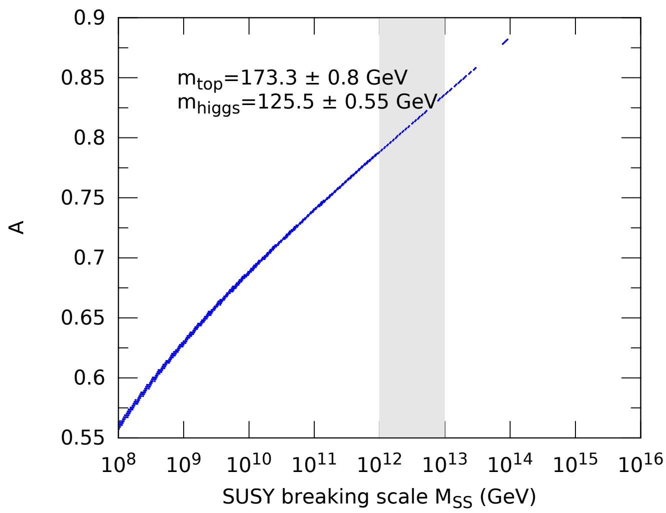


Figure 5.7: The required value of A in order to have a massless eigenstate at M_{SS} as a function of the SUSY breaking scale.

5.2.3.5. $N = 1$ supergravity description

Before turning to the computation of the slow roll parameters, let us compare the scalar potential of the previous section with the one that we would have obtained from a $N = 1$ supergravity computation. As we will see, upon introducing the appropriate Kähler potential and superpotential one recovers an F-term scalar potential with the same structure as the one found microscopically via the D7-brane action. The exact matching does however only occur for small values of the inflaton vev. For large field values there will be α' corrections that the supergravity approach fails to capture, and can only be seen by means of our previous DBI+CS analysis.

In eq.(5.43) we showed the Kähler potential for the Higgs fields in a $\mathbf{Z_4}$ heterotic orbifold. It is easy to convince oneself (e.g. by application of S-duality and T-duality along the third complex plane) that the corresponding Kähler potential for the type IIB model with a stack of D7's is given by

$$K_H = -\log[(S+S^*)(U_3+U_3^*) - \frac{\alpha'}{2}|H_u+H_d^*|^2] - 3\log(T+T^*) \tag{5.103}$$

where S is the complex type IIB dilaton. We have also added the well known Kähler moduli dependent piece in terms of a diagonal Kähler moduli field T (i.e. we are taking $(T_i+T_i^*)=(T+T^*)$, $\forall i$). We have also set the other matter fields $A_{1,2}=0$ since they do not play any role in the discussion and also the complex structure moduli to $U_1=U_2=1$. These simplifications are not important and the general case can be easily included in the discussion. The important point is that this dependence of the Kähler potential on T yields a no-scale structure for the F-term scalar potential, typical of type IIB compactifications with ISD fluxes [7].

In fact, it is well known that the effect of ISD fluxes on D7-brane fields can be understood macroscopically in terms of an $N=1$ supergravity description in which the SUSY-breaking effects are induced by the auxiliary fields of the Kähler moduli,see [45, 46, 52, 58, 214]. In our case the relevant superpotential in this effective description includes a constant term W_0 and a μ-term

$$W = W_0 + \mu H_u H_d . \tag{5.104}$$

Due to the no-scale structure of the Kähler potential, the scalar potential is simply given by

$$V = e^K (K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W}) \tag{5.105}$$

where the indices run over the dilaton and complex structure moduli. Let us assume that the above potential is minimised when

$$D_S W = 0 \quad ; \quad D_U W = 0 \tag{5.106}$$

which implies $V_0=0$. Moreover, as mentioned before, we assume that supersymmetry breaking comes from the Kähler moduli sector, namely

$$F^t = e^{K/2} K^{\bar{T}T} D_T W = -\frac{W_0}{\sqrt{st}} \neq 0 , \tag{5.107}$$

where $s=(S+S^*), t=(T+T^*)$. This is nothing but the assumption of modulus dominance SUSY breaking in type IIB which was studied in detail in [45, 46, 52, 58, 214]. Plugging all these data in standard $N=1$ sugra formulae [57] leads to a bilinear scalar potential of the form

$$V = (m_{H_u}^2 + \hat{\mu}^2)|H_u|^2 + (m_{H_d}^2 + \hat{\mu}^2)|H_d|^2 + B\hat{\mu} H_u H_d + \text{h.c.} \tag{5.108}$$

where $\hat{\mu}$ is the Higgsino mass with fields canonically normalised, and

$$m_{H_u}^2 = m_{H_d}^2 = |M|^2 , \; \hat{\mu} = \frac{W_0 + \mu s}{t^{3/2}\sqrt{s}} , \; B = -2M , \tag{5.109}$$

where

$$M = -\frac{W_0^*}{t^{3/2}\sqrt{s}} , \tag{5.110}$$

is a universal gaugino mass. Note that the physical μ-term $\hat{\mu}$ has two contributions, one coming from the original μ-term of the superpotential, and the other arising after SUSY breaking from the Kähler potential via a Giudice-Masiero mechanism, which is implicit in the form of the Kähler potential. All in all the scalar potential is given by

$$V = (|M|^2 + |\hat{\mu}|^2)(|H_u|^2 + |H_d|^2) - 2M\hat{\mu}H_uH_d + \text{h.c.} \tag{5.111}$$

This scalar potential is identical to the one we derived from explicit fluxes eq.(5.95) upon the identifications

$$G^* = \left(\frac{g_s}{2}\right)^{-1/2} \frac{W_0^*}{\sqrt{s}t^{3/2}} \quad , \quad S^* = -\left(\frac{g_s}{2}\right)^{-1/2} \frac{W_0 + \mu s}{\sqrt{s}t^{3/2}} \tag{5.112}$$

which implies $M = -\frac{g_s}{2}G^*$ and $\hat{\mu} = -\frac{g_s}{2}S^*$, in agreement with the results of [45].

Finally we can write the scalar potential in terms of the fields H, h obtaining[6]

$$V = \left[(|\hat{\mu}| + |M|)^2|H|^2 + (|\hat{\mu}| - |M|)^2|h|^2\right] \tag{5.113}$$

Note that in the absence of an explicit μ-term one has $\hat{\mu} = -M^*$ so that the h doublet is massless. So from the $N = 1$ sugra point of view, the desired situation with $m_h^2 \ll m_H^2$ would correspond to a suppressed explicit μ-term in the superpotential. This limit with a massless h field corresponds in terms of fluxes to a situation with $G = -S^*$. It is interesting to have this $N = 1$ sugra description for this equality which could be unmotivated from a microscopic point of view. Note finally that we will also have a similar situation whenever $|W_0| = |W_0 + \mu s|$.

Since the $N = 1$ sugra formalism is quite familiar one may be tempted to discuss inflation only in terms of the above formulae (see e.g. [220] for a recent two-field analysis in no-scale supergravity). The structure would be just the one of double chaotic inflation. However this $N = 1$ sugra formulation misses important α' stringy corrections. On the other hand, the DBI+CS D7-brane action on which we have based our analysis contains corrections to all orders in α', and so include all higher order terms in the expansion on the Higgs field vevs. These higher order terms are missed by the sugra formulation. In particular, the flattening of the inflation potential due to the kinetic field redefinitions is such an α' correction, and the sugra scalar potential would only capture the first term in the α' expansion.

5.2.4. Computing slow roll parameters for large inflaton

In this section we compute the slow-roll dynamics of our inflation model and the resulting cosmological observables. We first review the generalisation of the slow roll parameters to multiple field inflationary models in which the kinetic terms are not canonically normalised. Then we will solve the slow roll equations of motion and show the results for different values of A, distinguishing between the single field and two-field cases.

[6] In terms of H and h (5.103) reads $-\log[(S + S^*)(U_3 + U_3^*) - \alpha'(\cos^2(\gamma/2)|H|^2 + \sin^2(\gamma/2)|h|^2)]$. It is then quite remarkable that the scalar potential is independent of which combination of H and h appears in the Kähler potential.

5.2.4.1. Slow roll equations of motion

In the previous section we derived the effective action for the Higgs/inflaton sector obtaining for a general choice of fluxes a two-field inflation model. The 4d effective Lagrangian in terms of the neutral Higgs scalars H_u, H_d is given by

$$\mathcal{L}_{4d} = f(H_u, H_d)(|D_\mu H_u|^2 + |D_\mu H_d|^2) - V_F(H_u, H_d) - V_D(H_u, H_d) \tag{5.114}$$

where we have explicitly separated the F-term (5.95) and D-term (5.32) contribution of the potential. The function multiplying the kinetic terms is given also in terms of the F-term potential such that

$$f = 1 + \frac{(V_4 \mu_7 g_s)^{-1}}{2} V_F \ . \tag{5.115}$$

We saw that the D-term potential is minimised for

$$H_u = H_d^* e^{i\theta} \ , \ |H_u| = |H_d| = \sigma \tag{5.116}$$

with $\theta = \theta_u + \theta_d$. Thus in terms of the remaining scalar degrees of freedom σ, θ the potential becomes

$$V_F = Z^{-2} g_s (|G|^2 + |S|^2)(1 - A\cos\tilde{\theta})\sigma^2 \tag{5.117}$$

as we derived in (5.102). Recall that $\tilde{\theta} = \theta - \text{Arg}(GS)$ and A gives the relative size of the moduli of the fluxes (see (5.99)). The kinetic terms read

$$|D_\mu H_u|^2 + |D_\mu H_d|^2 \to 2(D_\mu \sigma)^2 + \frac{\sigma^2}{2}(D_\mu \theta)^2 \tag{5.118}$$

implying the following 4d effective Lagrangian for the fields σ, θ,

$$\mathcal{L}_{4d} = f(\sigma, \theta)\left(2|D_\mu \sigma|^2 + \frac{\sigma^2}{2}(D_\mu \theta)^2\right) - Z^{-2} g_s (|G|^2 + |S|^2)(1 - A\cos\tilde{\theta})\sigma^2 \tag{5.119}$$

One could think that the first step is to absorb the prefactor $f(\sigma, \theta)$ in a redefinition of the fields in order to have canonically normalised kinetic terms. Comparing with the general form of a Lagrangian of multiple fields

$$\mathcal{L}_{4d} = \frac{1}{2} G_{ab}(\phi) D_\mu \phi^a D^\mu \phi^b - V(\phi) \tag{5.120}$$

this is equivalent to ask if there exists an appropriate field redefinition such that $G_{ab} = \delta_{ab}$, where in our case the metric is given by

$$G_{ab} = \begin{pmatrix} 4f(\sigma, \theta) & 0 \\ 0 & \sigma^2 f(\sigma, \theta) \end{pmatrix} \tag{5.121}$$

This is always possible for a single field, making a field redefinition of the form

$$\phi' = \int d\phi f^{1/2}(\phi) \tag{5.122}$$

where we have assumed $G_{\phi\phi} = f(\phi)$. However, in general this can not be done globally (i.e. for all values of ϕ) for two or more fields simultaneously. Notice that G_{ab} transforms as a rank two tensor under field redefinitions of the form $\phi \to f(\phi)$ and is positive definite, so it can be interpreted as a metric on the moduli space parametrised by the fields. Therefore a change of variables which brings the metric to the flat metric $G_{ab} = \delta_{ab}$ can only be done globally if the curvature scalar vanishes everywhere. In fact, the metric (5.121) is conformal to the flat metric, so the Ricci scalar of curvature will be proportional to the Hessian of the function f. It can be checked that this scalar vanishes

$$R \propto \frac{1}{f}\Delta(\mathrm{Ln}\, f) = 0 \tag{5.123}$$

if the function f can be written as $f = |h(z_3)|^2$ where $h(z_3)$ is a holomorphic function on z_3. By absorbing all the global factors in the potential into a single overall parameter given by

$$|\hat{G}|^2 = Z^{-2}(V_4\mu_7)^{-1}(|G|^2 + |S|^2) \tag{5.124}$$

as in section 5.2.3.3 we obtain the function

$$f = 1 + \frac{|\hat{G}|^2}{2}(1 - A\cos\tilde{\theta})\sigma^2 \tag{5.125}$$

Then, recalling that $z_3 = (2\pi\alpha')\sigma e^{i\theta/2}$, we see that f is not a holomorphic function in general so it does not exist any field redefinition that canonically normalises simultaneously both fields σ and θ. Therefore for the general 2-field case we will have to keep track of the non-flat metric all over the computation of the slow roll parameters.

The scalar equations of motion for several inflaton fields are given by

$$\ddot{\phi}^a + \Gamma^a_{bc}(\phi)\dot{\phi}^b\dot{\phi}^c + 3H\dot{\phi}^a = -G^{ab}\frac{\partial V(\phi)}{\partial\phi^b}\ . \tag{5.126}$$

with H being the Hubble constant. The slow roll condition for inflation implies that the potential energy has to be dominant with respect to the kinetic energy over the whole inflationary trajectory, so we can drop the first two terms in (5.126) leading to the well known slow roll equations of motion

$$3H\dot{\phi}^a = -G^{ab}\frac{\partial V(\phi)}{\partial\phi^b}\ . \tag{5.127}$$

This is a good approximation whenever the slow roll parameters ϵ, η remain smaller than one. The generalisation of the ϵ parameter for multiple field inflation is given by (see e.g. [156])

$$\epsilon = \frac{M_p^2}{2}G^{ab}\frac{V'_aV'_b}{V^2} \tag{5.128}$$

where the primes denotes derivatives with respect to the fields $V'_a = \frac{\partial V}{\partial\phi^a}$. The η parameter would correspond though to the smallest eigenvalue of the matrix of second derivatives of the potential given by

$$N^a_b = M_p^2\frac{G^{ac}V''_{cb}}{V} \tag{5.129}$$

where $V''_{cb} = \frac{\partial V'_a}{\partial\phi^b} - \Gamma^a_{bc}V'_a$ is the covariant derivative.

The ϵ-parameter can also be defined in the multi-field case in terms of the number of efolds as

$$\epsilon = \frac{1}{2}G_{ab}\frac{d\phi^a}{dN_{\text{efolds}}}\frac{d\phi^b}{dN_{\text{efolds}}}\ . \tag{5.130}$$

This implies the following formula that we will use to compute N_{efolds} in terms of ϵ,

$$N_* = \int_{\phi_0^2}^{\phi_{end}^2} \frac{1}{\sqrt{2\epsilon}} \sqrt{G_{11}\left(\frac{d\phi^1}{d\phi^2}\right)^2 + G_{22}}\, d\phi^2 \tag{5.131}$$

in the two field case.[7] Finally, the scalar spectral index and the tensor to scalar ratio are defined as in section 5.1.1 (for single field) but using the multi-field generalisation of ϵ and η explained here.

Below we show the results first for the single field limit cases ($A = 0$ and $A = 1$) and then for a general two field case with arbitrary A, but with special focus on the case of special interest $A \simeq 0.83$.

5.2.4.2. Single field limit cases

We showed in section 5.2.3.4 that for specific choices of fluxes the potential reduces to a single field inflationary potential where the inflaton has a clear geometric interpretation. In particular, we get the potential

$$V = Z^{-2} g_s (|G|^2 + |S|^2)\phi^2 \tag{5.132}$$

with $\phi \equiv \sigma$ for $A = 0$ ($G = 0$ or $S = 0$) or $\phi \equiv |H|$ for $A = 1$ ($|G| = |S|$). Recall that the position of the D7-branes in the transverse torus is parametrised by

$$z_3 = 2\pi\alpha'\ \sigma e^{i\theta/2} = 2\pi\alpha' \frac{1}{\sqrt{2}}(|H| + i|h|)e^{-i\gamma/2} \tag{5.133}$$

If $A = 0$ the inflaton σ parametrises the distance of the travelling D7-branes to the singularity $\mathbf{Z}_4$, while if $A = 1$ the inflaton corresponds to the distance along one of the 1-cycles of the torus, the orthogonal 1-cycle being a flat direction.

Before taking into account the field redefinition the potential is quadratic on the fields, corresponding to a soft mass induced by breaking SUSY with the closed string fluxes. However, since we are interested in large field values, higher order corrections to the potential become important and can not be neglected. These corrections were computed from the DBI+CS action of the D7-brane and their effect is to induce non-canonical kinetic terms, with a prefactor

$$f = 1 + \frac{(V_4\mu_7 g_s)^{-1}}{2} V = 1 + \frac{|\hat{G}|^2}{2}\phi^2 \tag{5.134}$$

where we have again defined $|\hat{G}|$ by (5.124). In the single field case, the kinetic term can always be canonically normalised by an appropriate redefinition of the field. Therefore the effect of the higher order corrections can be encoded on a field redefinition given by

$$\varphi = \int d\phi f^{1/2}(\phi) \tag{5.135}$$

which becomes important for large field. Inserting (5.134) in (5.135) we get

$$\varphi = \frac{1}{2\sqrt{2}}|\phi|\sqrt{2 + |\hat{G}|^2|\phi|^2} + \frac{1}{\sqrt{2}}|\hat{G}|^{-1}\sinh^{-1}[|\hat{G}||\phi|/\sqrt{2}] \tag{5.136}$$

[7] Note that here ϕ^2 stands for ϕ^b with b=2, so it is not an exponent but an index.

In fig.5.8 we plot the new normalised field φ in terms of the old one ϕ. Notice that for large field this yields

$$\varphi \simeq \frac{1}{2\sqrt{2}}|\hat{G}|\phi^2 \tag{5.137}$$

and the potential becomes linear in the new normalised field φ. Hence the effect of the higher order corrections is indeed a flattening of the potential. In fig.5.9 we plot the scalar potential in terms of the new canonically normalised field, for different values of $\hat{G}$. The bigger $\hat{G}$ is, the sooner the flattening effect takes place. To work this plot out we have used the fact that the overall factor in the potential (which parametrises the SUSY breaking scale) is related to $|\hat{G}|$ by

$$M_{SS}^2 = Z^{-2} g_s(|G|^2 + |S|^2) = V_4 \mu_7 g_s |\hat{G}|^2 \simeq 0.05 g_s M_s^4 |\hat{G}|^2 \tag{5.138}$$

where M_s is the string scale. Hence the scalar potential interpolates between quadratic and linear depending on the SUSY breaking scale (through $\hat{G}$). For $|\hat{G}| > 1/M_p$ the potential becomes bigger than the string scale during inflation (i.e. $V^{1/4} > M_s^4$) and the computation is inconsistent, since new KK and string modes should be taken into account.

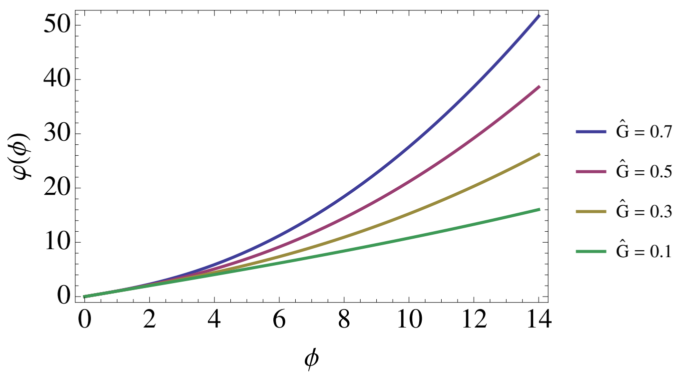


Figure 5.8: Field redefinition (new field φ vs old field ϕ) for different values of $\hat{G}$.

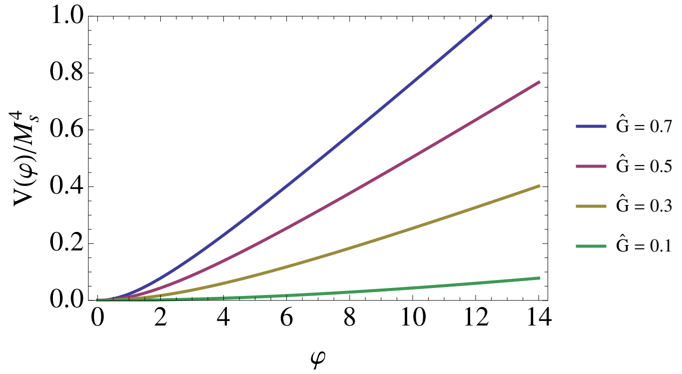


Figure 5.9: Scalar potential in terms of the canonically normalised field φ for different values of $\hat{G}$.

Let us compute now the tensor-to-scalar ratio r and the scalar spectral index n_s. We compute the field value ϕ_0 at which inflation starts by imposing to get between 50 and 60 efolds before inflation ends. Notice that inflation ends when $\epsilon(\phi_{end}) = 1$. Once we

N_{efolds}	φ_{end}	φ_0	r	n_s
60	1.38	13.38	0.080	0.972
50	1.38	12.33	0.098	0.966

Table 5.1: Results for $\hat{G} = 0.3/M_p$ in isotropic compactifications.

know the initial value ϕ_0, we can compute r and n_s by evaluating the slow roll parameters at $\phi = \phi_0$. We plot the result in fig.5.10. The result for Higgs-otic inflation (red band) has been superimposed over the figure with the Planck experimental exclusion limits and some inflationary models in the literature. Remark those corresponding to quadratic and linear potentials, given respectively by black and yellow points. Our model interpolates precisely between both of them, recovering a quadratic potential in the small $\hat{G}$ limit, and a linear potential in the large $\hat{G}$ limit. There is a special value for $\hat{G}$ (corresponding to the blue line inside the red band) given by considering generic fluxes in an isotropic compactification, as estimated in section 5.2.3.3. It corresponds to $\hat{G} \simeq 0.3/M_p$, implying a SUSY breaking scale around $10^{12} - 10^{13}$ GeV (depending on the exact value of the string scale). The numerical results for $\hat{G} \simeq 0.3/M_p$ are shown in table 5.1. Notice that the field range is given in units of the reduced Planck mass M_p. We can see that the prediction for the tensor to scalar ratio is around $r \simeq 0.09$.

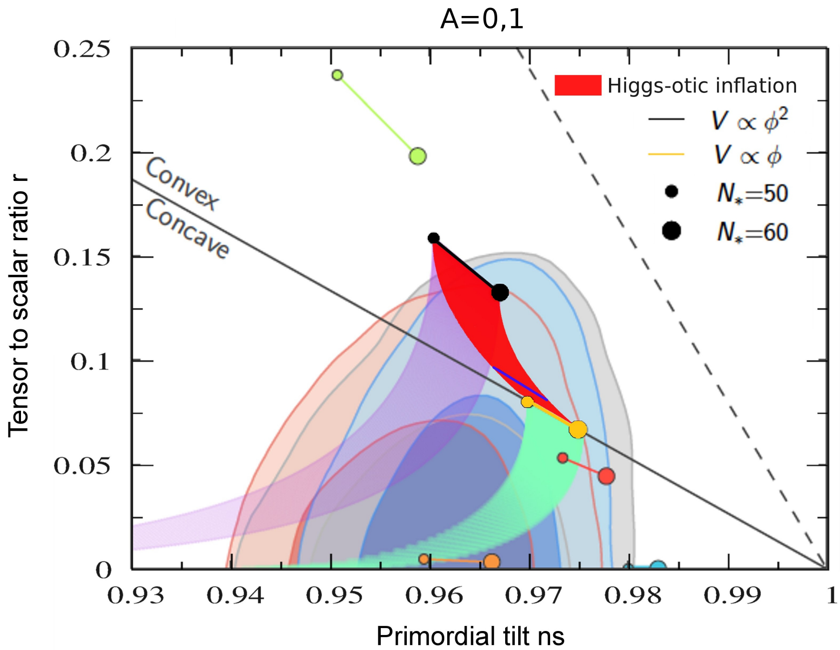


Figure 5.10: Tensor to scalar ratio vs scalar spectral index for $A = 0, 1$ in Higgs-otic inflation (red band).

Finally one could also wonder about the density of scalar perturbations. These have been measured experimentally by Planck obtaining an order of magnitude of

$$P_s = \frac{V}{24\pi^2 M_p^4 \epsilon} \sim \left(\frac{\delta\rho}{\rho}\right)^2 \sim (10^{-5})^2 \tag{5.139}$$

Using that $V = M_{SS}\phi(\varphi)^2$ where M_{SS} is the SUSY breaking scale and taking into account the field redefinition $\phi(\varphi)$, we can use the experimental result for the density scalar perturbations to estimate the SUSY breaking scale. The result is $M_{SS} \simeq 10^{12} - 10^{13}$ GeV depending on the exact value of the string scale, in agreement with the assumption of closed string fluxes as the main source of SUSY breaking. More precisely, for $\hat{G} \simeq 0.3/M_p$ fixed, we obtain $M_{SS} \simeq 3 \cdot 10^{12}$ GeV.

We can also estimate the number of times that the inflaton has to travel along the torus. For simplicity let us assume that the overall internal space is a direct product of the internal 4d space wrapped by the D7-branes and the transverse torus such that

$$\mathrm{Vol}(B_3) = \mathrm{Vol}(X_4)\mathrm{Vol}(\mathbf{T}^2) \tag{5.140}$$

where also $X_4 = \mathbf{T}^4$. Then $\mathrm{Vol}(X_4) = (2\pi R_c)^4$ and $\mathrm{Vol}(\mathbf{T}^2) = (2\pi r)^2$. The position of the branes is parametrised by

$$z_3 = 2\pi\alpha'\langle\varphi\rangle \tag{5.141}$$

and the inflaton completes a period when $\langle\varphi\rangle_0 = \frac{r}{\alpha'}$. Using eq.(5.140) and the identities (5.87) one period along the transverse torus is given by

$$\langle\varphi\rangle_0 = \frac{1}{2\pi\alpha'}\left(\frac{\mathrm{Vol}(B_3)}{\mathrm{Vol}(X_4)}\right)^{1/2} = \frac{g_s^{1/2} m_p}{2\alpha_G^{-1/2}} \sim 0.5 g_s^{1/2} M_p \tag{5.142}$$

Hence if we need $\Delta\varphi \simeq 10 M_p$, we will need about 20 periods. Of course this is the worst case in which we are assuming the same radius for both cycles of the torus and that the inflaton is circling only around one of them. In general

$$\Delta\varphi = \frac{R}{\alpha'}|m + iU_3 n| \tag{5.143}$$

with m, n the number of periods along both 1-cycles, so the effective number of periods can be considerably smaller (although always bigger than 1).

Note that all these $A = 0, 1$ results are independent on whether the inflatons have the quantum numbers of the MSSM Higgs bosons. If they were describing any other scalar field, but still corresponding to the position of a D7-brane in such closed string flux background, then their potential would be described by the analysis of section 5.2.3.1 or and orbifold thereof and the same results would apply. However, the case in which the inflaton is a Higgs field is further constrained by known Higgs physics. In particular, for Higgs-otic inflation we are interested in obtaining a massless eigenstate at the SUSY breaking scale that could play the role of SM Higgs boson, so we need a specific choice of fluxes satisfying $A \simeq 0.83$. This leads us to the two-field inflation case. However, if we start with initial conditions such that $< H > \gg < h >$ (implying $H_u = H_d^*$) the inflaton is mostly H and the analysis here described is a good approximation. For generic initial conditions, however, both fields are relevant for inflation and a more general analysis is needed. We turn now to describe the more general case of two fields.

5.2.4.3. The general 2-field Higgs/inflaton case

In this section we deal with the more general and interesting case of the two field inflationary potential.

Results for small field. As a first approximation we assume that the fields take only small values such that the function f is approximately $f(\sigma,\theta) \approx 1 + \ldots$ and we do not have to worry about the field redefinition. Notice that this is not consistent for our inflationary model in which the fields necessarily have to take large trans-planckian values in order to obtain of the order of 60 efolds during inflation. But this simplification allows us to solve analytically the equations of motion making easier the presentation of the new features that arise in a 2-field inflationary model with respect to the previous single field case. It is also a good approximation for very small values of $\hat{G}$. In the next subsection we will deal with the more general case including the field redefinition and obtaining a flattening of the potential. This will imply a reduction in the tensor to scalar ratio obtained in this subsection.

Neglecting the field redefinition coming from higher order corrections on α' in the DBI+CS action, the metric is simply given by $G_{ab} = \mathrm{diag}(4, \sigma^2)$. This leads the following slow roll equations of motion

$$\frac{d\sigma(t)}{dt} = -c\ \sigma(t)(1 - A\cos\tilde{\theta}(t)) \tag{5.144}$$

$$\frac{d\tilde{\theta}(t)}{dt} = -c\ 2A\sin\tilde{\theta}(t)\ . \tag{5.145}$$

where $c = Z^{-2} g_s(|G|^2 + |S|^2)/6H$. These equations can be solved analytically, obtaining

$$\sigma(t) = \sigma(0)e^{-c(1+A)t}\left(\frac{1 + e^{4\mathrm{Act}}\cot\left(\frac{\tilde{\theta}(0)}{2}\right)^2}{1 + \cot\left(\frac{\tilde{\theta}(0)}{2}\right)^2}\right)^{1/2} \tag{5.146}$$

$$\tan\left(\frac{\tilde{\theta}(t)}{2}\right) = e^{-2\mathrm{Act}}\tan\left(\frac{\tilde{\theta}(0)}{2}\right) \tag{5.147}$$

which can be combined to obtain the slow roll trajectory $\sigma(\tilde{\theta})$. This trajectory will be independent of the parameter c, recovering the well known result that the observables $r, n_s, N_{\mathrm{efolds}}$ are independent of the global factor of the potential in chaotic-like inflation models. Instead, these observables will depend only on the relative size of the fluxes parametrised by A.

By looking at the above equations, we can see that the phase remains unchanged $\tilde{\theta}(t) = \tilde{\theta}(0)$ for the case $A = 0$, while $\sigma(t) = \sigma(0)e^{-ct}$. This is the typical exponentially decreasing behaviour of single field inflation and we recover the results described in the previous section. The case $A = 1$ is a bit special since the minimum of the potential is at $\tilde{\theta} = 0$ for any value of σ, including $\sigma \neq 0$, which implies that at the end of inflation the gauge group $SU(2) \times U(1)$ remains broken. This is an unwanted situation, since we want to maintain the SM gauge symmetry unbroken after inflation. So this particular limit would not be viable generically. This case can also be reduced to single field inflation as we explained in the previous section. Here we are going to focus on an intermediate situation in which A takes a value in between 0 and 1, so both fields may be important for inflation.

There is a novel feature of the 2-field case comparing with single field inflation: the dependence of the results on the initial conditions $\sigma(0)$ and $\theta(0)$. Depending on which initial point on the field space inflation starts, the slow roll trajectory will be different giving rise to different values of the cosmological observables. Although one of the initial conditions can be fixed by imposing a specific number of efolds (as in single field inflation) the other one remains as a free parameter. This extends the range of possibilities but in principle also makes the model less predictive.

As we argued above, for the SM Higgs to be fine-tuned and (approximately) corresponding to the h linear combination we need to have $m_h^2 \ll m_H^2$ at the string scale. This corresponds to a value of $A \simeq 1$. In fact in section 5.2.3.4 we estimated the required value of A in order to have a vanishing SM Higgs eigenvalue at a scale $\simeq 10^{13}$ GeV, obtaining a value around $A = 0.83$. For this case of interest ($A = 0.83$) we have plotted the trajectory followed in the $(\sigma, \tilde{\theta})$-plane in fig.5.11, for different initial values $\tilde{\theta}(0)$. We see that for an initial value at the top of the hill ($\tilde{\theta}(0) \simeq \pi$, $\sigma \simeq 7$) the inflaton goes downhill in the σ direction keeping $\tilde{\theta}$ almost constant. Eventually the opposite happens and the phase goes fast to zero. For initial values at large $\sigma(0)$ but smaller $\tilde{\theta}(0)$ both σ and $\tilde{\theta}$ decrease simultaneously. For small values of $\tilde{\theta}(0)$ the inflaton goes fast to $\tilde{\theta} = 0$ and then goes downhill in σ.

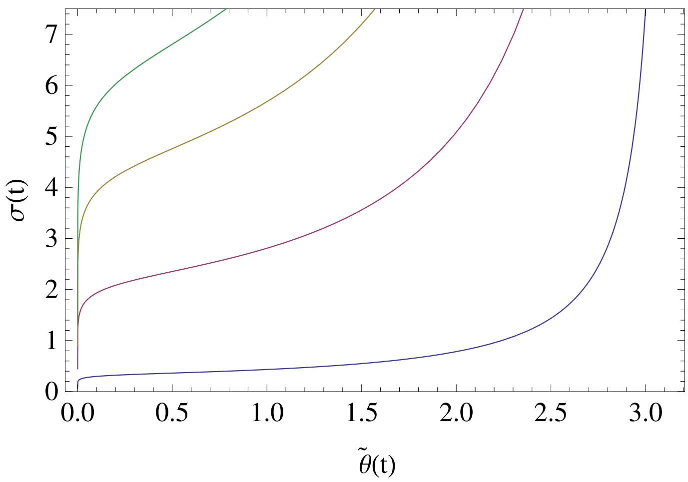


Figure 5.11: Trajectory $\sigma(\tilde{\theta}(t))$ described by the slow roll eqs. of motion for $A = 0.83$ and the different initial values $\tilde{\theta}(0) = 3, 3\pi/4, \pi/2, \pi/4$.

By using (5.128) we get the following formula for the slow roll ϵ parameter,

$$\epsilon = \frac{M_p^2}{2\sigma^2}\left(1 + A^2 \frac{\sin^2\tilde{\theta}}{(1 - A\cos\tilde{\theta})^2}\right) \tag{5.148}$$

Given a value for A and for the initial conditions $\sigma(0), \tilde{\theta}(0)$, we can compute the ϵ-parameter along the inflationary trajectory $\sigma(\tilde{\theta})$. The result is shown in fig.5.12 for the same choices of trajectories depicted in fig.5.11, and this time also for different values of A. Inflation ends when this parameter becomes order 1, or alternatively when both fields reach their minima.

Replacing the metric in (5.131) we get the following formula for the number of efolds,

$$N_{\text{efolds}} = \int_{\tilde{\theta}(0)}^{\tilde{\theta}_{end}} \frac{1}{\sqrt{2\epsilon(\tilde{\theta}, A, \sigma(0), \tilde{\theta}(0))}} \sqrt{4\left(\frac{d\sigma(\tilde{\theta})}{d\tilde{\theta}}\right)^2 + \sigma(\tilde{\theta})^2}\, d\tilde{\theta} \tag{5.149}$$

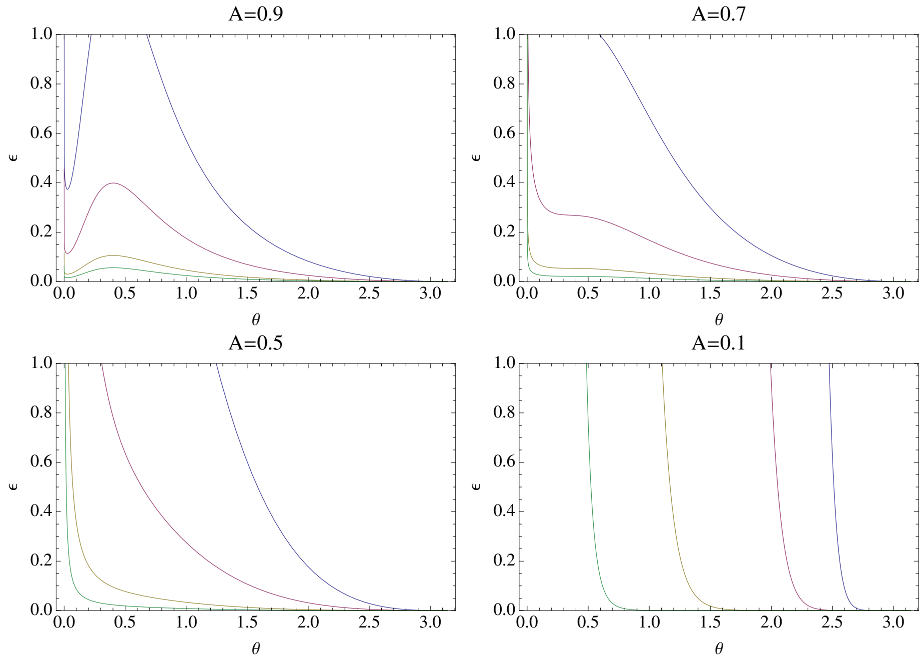


Figure 5.12: The slow roll parameter ϵ as a function of $\tilde{\theta}$ for different values of A and different possible trajectories.

The value $\tilde{\theta}_{end}$ is the one at which $\epsilon = 1$ and inflation ends. For some choices of initial conditions, we can see that ϵ remains $\epsilon < 1$ until the fields almost reach the minimum of the potential, so $\tilde{\theta}_{end} \simeq 0$. Finally the tensor-to-scalar ratio is proportional to the ϵ-parameter evaluated at the beginning of inflation,

$$r = 16\epsilon|_{\tilde{\theta}(0),\sigma(0)} \tag{5.150}$$

and the same for the primordial tilt,

$$n_s = 1 + 2\eta|_{\tilde{\theta}(0),\sigma(0)} - 6\epsilon|_{\tilde{\theta}(0),\sigma(0)} \tag{5.151}$$

We have studied the possible trajectories in our parameter space that give rise to $N_{\text{efolds}} = 50 - 60$ before inflation ends. This constraint implies a curve in the parameter space of initial conditions $(\tilde{\theta}(0), \sigma(0))$ for each value of A (fig.5.13). Note that the number of efolds (for $A < 1$) is almost independent of $\tilde{\theta}(0)$. All the dependence comes from the fact that ϵ, η do depend on $\tilde{\theta}(0)$, and thus $\tilde{\theta}_{end}$ may be different for different initial values $\tilde{\theta}(0)$. The dependence of N_{efolds} on A also comes from the slight dependence of $\tilde{\theta}_{end}$ on A. Therefore, the behaviour for $A < 1$ is quite similar to that of $A = 0$, in which σ is the only inflaton. For $A = 1$ the situation changes drastically and N_{efolds} only depends on $H(0)$, being this field the inflaton. Notice that in this case 60 efolds are obtained if $H(0) = 11M_p$. Taking into account the definition of canonically normalised fields (5.120) for which the physical field would actually be $\sqrt{2}H$, this implies a physical field range of $15.5M_p$, as usual in chaotic inflation. Therefore we recover the results of chaotic inflation in the cases $A = 0, 1$.

Although the behaviour of N_{efolds} does not differ much from the single field cases, the results for r and n_s do. Let us explain the reason. We can use the constraint of getting 50-60 efolds to fix one of the initial conditions $(\sigma(0))$, as we can see in fig.5.14

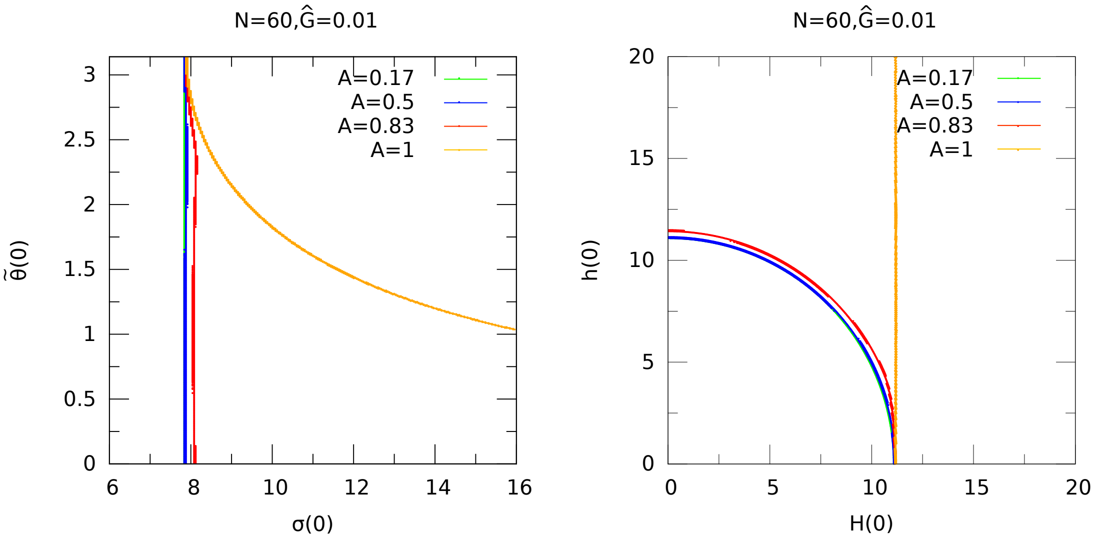


Figure 5.13: Possible initial values that will give rise to $N_{\text{efolds}} = 60$. Each curve corresponds to a different value for $A = 1, 0.83, 0.5, 0.17$. For simplicity in the right plot we have assumed $\text{Arg}(GS) = 0$.

(left). In the single field cases this determines completely r and n_s, but here we have another free parameter, the other initial condition $\tilde{\theta}(0)$. We can then plot r and n_s in terms of $\tilde{\theta}(0)$ obtaining the functions depicted in fig.5.14 (right) for the case $A = 0.83$. It is clear that these observables do depend on $\tilde{\theta}(0)$. The minimum value for r that we can get corresponds to the result of chaotic inflation ($r \simeq 0.13$), while the freedom of choosing $\tilde{\theta}(0)$ allows us to get bigger values for the tensor to scalar ratio. However if we impose the experimental constraint for the primordial tilt n_s only the region $\tilde{\theta}(0) > 1.7$ survives, implying $0.13 < r < 0.15$ again.

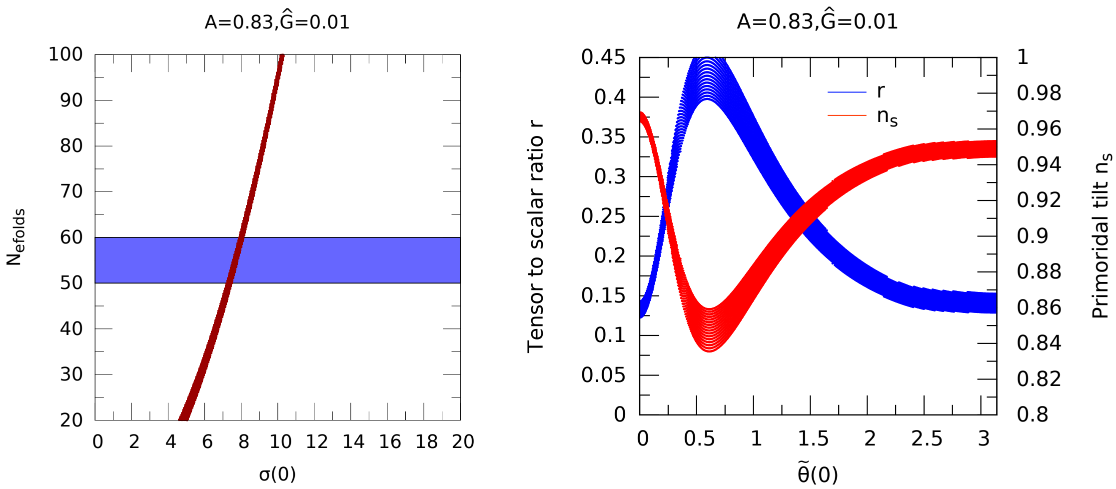


Figure 5.14: Left: Number of efolds vs the initial point $\sigma(0)$ for $A = 0.83$ before flattening. Right: Tensor-to-scalar ratio (blue curve) and scalar spectral index (red curve) as functions of the initial point $\tilde{\theta}(0)$ for $A = 0.83$.

In fig.5.15 we plot the value of the tensor-to-scalar ratio (left) and the scalar spectral index (right) without imposing a specific N_{efolds} in the parameter space of initial conditions for the relevant case $A = 0.83$. It has been imposed that the potential energy remains lower that the string scale ($V^{1/4} < M_s$). This implies a lower bound in r and an upper bound in n_s. Therefore although we allowed for more than 60 efolds, we could not get parametrically smaller values for r. It has also been superimposed a black band corresponding to the set of initial points which gives rise to $50 < N_{\text{efolds}} < 60$, to guide the eye. Notice however that the region from the black band to the right part of the plot is also allowed (whenever the potential remains subplanckian) corresponding to $N_{\text{efolds}} > 60$ and a smaller r. These values for r will decrease in the next section when including the flattening of the potential.

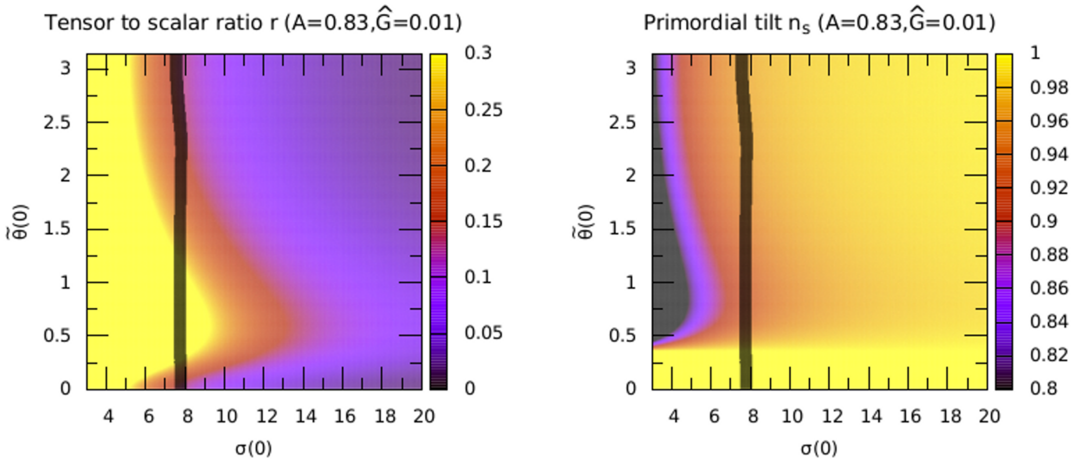


Figure 5.15: Left: Tensor-to-scalar ratio (contour plot) in the parameter space of initial conditions for $A = 0.83$ before flattening. Only plotted those points which imply a potential $V^{1/4} < M_s$. The black band corresponds to those points which lead to 50-60 efolds. Right: The same for the primordial tilt.

Results for large field. Once we take into account higher order corrections in he DBI+CS action, the kinetic terms turn out to be non-canonically normalised. The metric in the field space is given by (5.121)

$$G_{ab} = \begin{pmatrix} 4f(\sigma, \tilde{\theta}) & 0 \\ 0 & \sigma^2 f(\sigma, \tilde{\theta}) \end{pmatrix} \tag{5.152}$$

with

$$f = 1 + \frac{|\hat{G}|^2}{2}(1 - A\cos\tilde{\theta})\sigma^2 \tag{5.153}$$

In the previous section we neglected this effect assuming $\hat{G}$ small. There the results did not depend on $\hat{G}$ because this parameter entered only as a global factor in the scalar potential. In this section we consider the most general case in which both $\hat{G}$ and A can take arbitrary values. Hence, we have to deal with a two field inflationary model in which the kinetic terms are not canonically normalised, so we will use the generalisation for the slow roll parameters derived in 5.2.4.1. Now the results will also depend on $\hat{G}$ (and so in the SUSY breaking scale) as it enters in the field redefinition above. As we explained in the single field cases, the effect of the field redefinition will be a flattening of the potential giving rise to a decrease in the tensor to scalar ratio (more important as $\hat{G}$ increases). The structure will no longer be that of double chaotic inflation.

In the following we show the results for $\hat{G} = 1/M_p$, corresponding to the biggest value for $\hat{G}$ that still implies a potential energy lower than the string scale. For this value, in the single field cases the potential was almost linear, so here we expect to recover the results of linear inflation for $A = 0, 1$. We show the same plots than in the previous section but now for $\hat{G} = 1/M_p$, to highlight the flattening effect. Notice in fig.5.16 that 60 efolds are achieved now when $H(0) \simeq 4.7M_p$ for $A = 1$. This field is not canonically normalised, so in order to compare with the physical field we have to compute the field redefinition, possible in this single field case. In fact, the result is $H'(0) \simeq 11M_p$, recovering the result for linear inflation.

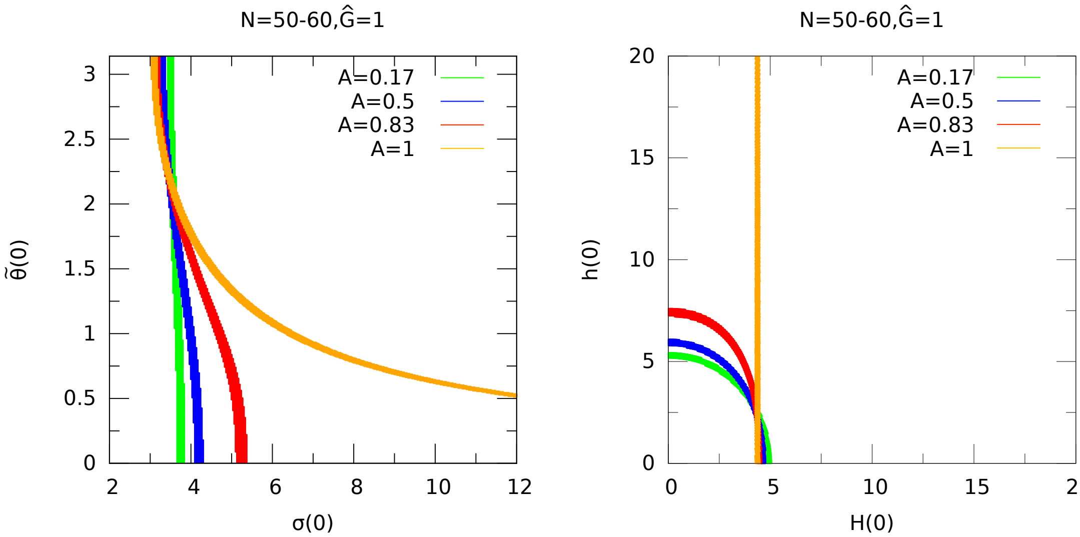


Figure 5.16: Possible initial values that will give rise to $N_{\text{efolds}} = 50 - 60$. Each curve corresponds to a different value for $A = 1, 0.83, 0.5, 0.17$.

In fig.5.17 we plot N_{efolds}, r and n_s for $A = 0.83$. The tensor to scalar ratio is smaller than in the previous section for a bigger range of $\tilde{\theta}(0)$. In fact, after imposing the experimental bound on n_s, the value for r is constrained to the range 0.07 - 0.1, corresponding again to the result of a single field with a linear potential. Therefore, after imposing the experimental constraints, the results look quite similar to the single field case, as in the previous section.

In fig.5.18 we illustrate the decrease on the tensor to scalar ratio due to the flattening of the potential. Notice also that the bound of getting $V^{1/4} < M_s$ is stronger, and the value $\hat{G} = 1/M_p$ corresponds to the limit case in which this bound is still satisfied.

All these figures show the results for $\hat{G} = 1/M_p$. The figures of the previous section can be recovered in this general analysis by fixing $\hat{G}$ small, around $\hat{G} \simeq 0.01/M_p$. For intermediate values of $\hat{G}$ we would have an intermediate situation between both sections. Recall that we have four free parameters in the model, two of them giving the absolute and

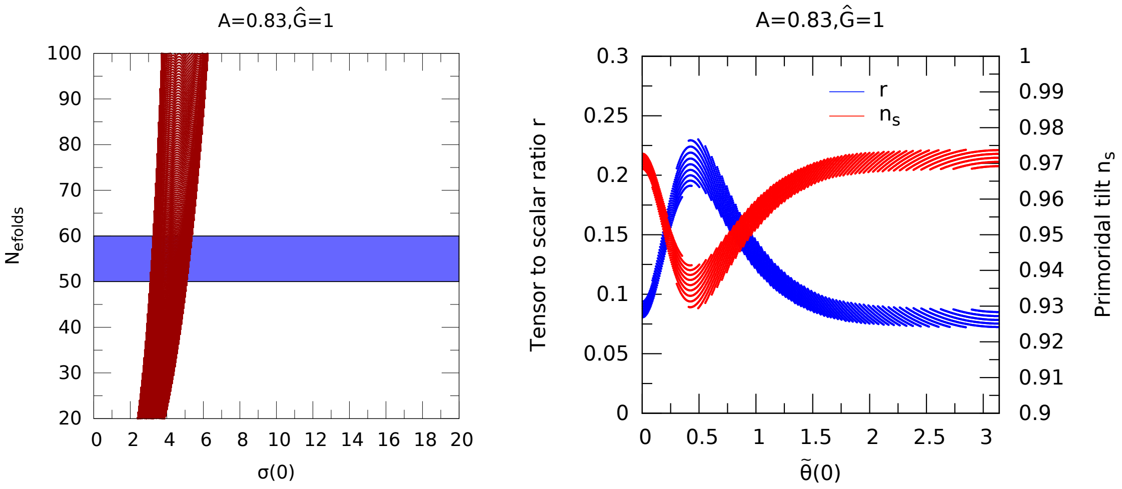


Figure 5.17: Left: Number of efolds vs the initial point $\sigma(0)$ for $A = 0.83$. Right: Tensor-to-scalar ratio and scalar spectral index as functions of the initial point $\tilde{\theta}(0)$ for $A = 0.83$.

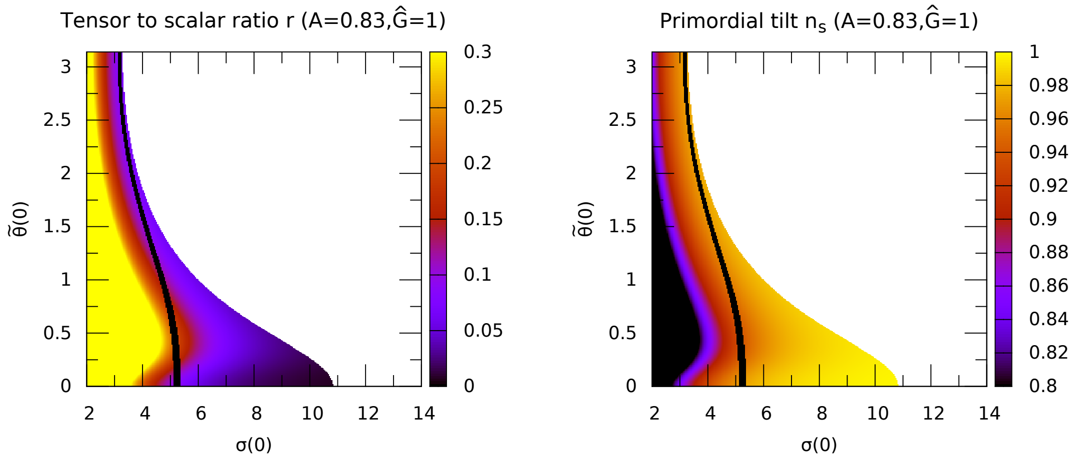


Figure 5.18: Left: Tensor-to-scalar ratio (contour plot) in the parameter space of initial conditions for $A = 0.83$. Blank regions in the plots correspond to those points where $V^{1/4} > M_s$. The black band corresponds to those points which lead to 50-60 efolds. Right: The same for the primordial tilt.

relative size of the fluxes ($\hat{G}$ and A), and the other two parametrising the initial conditions of the two fields. We have seen that the initial conditions can be highly constrained by imposing a specific number of efolds and the experimental bound on the primordial tilt n_s. The relative size of the fluxes (given by A) is fixed by imposing the condition of getting a massless eigenstate at M_{SS} which could play the role of the SM Higgs boson. Hence, only $\hat{G}$ remains as a free parameter. Although we are assuming an intermediate scale of SUSY breaking around $M_{SS} = 10^{11} - 10^{13}$ GeV (consistent with the density scalar perturbations), this flexibility still has a big impact in the results of the cosmological observables.

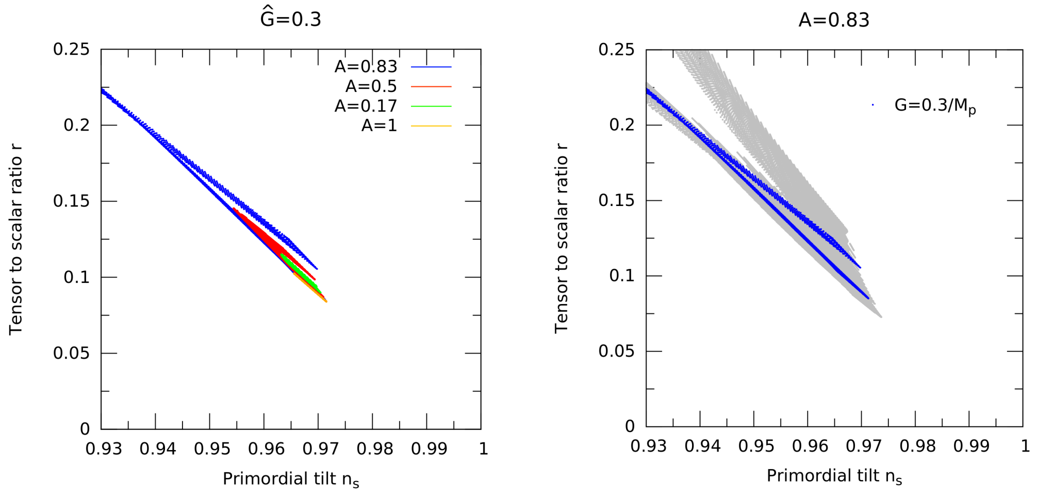


Figure 5.19: Allowed region of the parameter space (r vs n_s) that gives rise to 50-60 efolds. Left: For different values of $A = 1, 0.83, 0.5, 0.17$ and $\hat{G} = 0.3/M_p$. Right: For $A = 0.83$ and any $\hat{G}$ (grey points). The blue points corresponds to $\hat{G} = 0.3/M_p$.

In fig.5.19 (right) we plot all the values for r and n_s that we can get for any possible value of $\hat{G}$. We only require to get between 50 and 60 efolds during inflation, and a light SM Higgs (so $A = 0.83$). The minimum value for the tensor to scalar ratio that we can get is that one of linear inflation, around $r \simeq 0.07$. We have marked in blue those points that corresponds to an isotropic compactification with generic fluxes, ie. $\hat{G} \simeq 0.3/M_p$.

For completeness, we also show the results for r and n_s for different values of A and $\hat{G} = 0.3/M_p$ (fig.5.19 (left)). Although for arbitrary values of A the inflaton could not be identified with a Higgs boson, the results still apply for a generic D7-brane position modulus playing the role of the inflaton in such a closed string background.

In figure 5.20 we plot our final results in the $r - n_s$ plane superimposed over the experimental Planck exclusion limits. The data correspond to $A = 0.83$ and arbitrary $\hat{G}$. The color pattern from red to blue refers to the density of points, being the red regions the most populated. This could have been anticipated from figure 5.17, where most of the initial conditions gave rise to values of r and n_s closer to the single field prediction. The spreading of the results to smaller values of the spectral index is due to the freedom on the choice of the initial conditions, but the blue region corresponds to very fine-tuned values of θ_0 and the majority of the points is localised at $n_s \simeq 0.96 - 0.97$ and $r \simeq 0.07 - 0.12$. It is remarkable that the most populated regions are indeed those in better agreement with experiments.

5.2.5. Inflaton potential corrections, backreaction and moduli fixing

We will consider here in turn several properties of our inflaton system concerning corrections and the possible back-reaction of the closed string sector on the potential. We first discuss the Planck suppressed corrections to the inflaton potential, which are under control and fully given by the DBI+CS action. We then study the possible induction of D3-brane RR-tadpoles for non-vanishing values of the Higgs/inflaton. We show how there is a delicate cancellation coming from the closed string 10d action which sets to zero such tadpoles. Finally, we briefly discuss the issue of the moduli fixing potential and how one

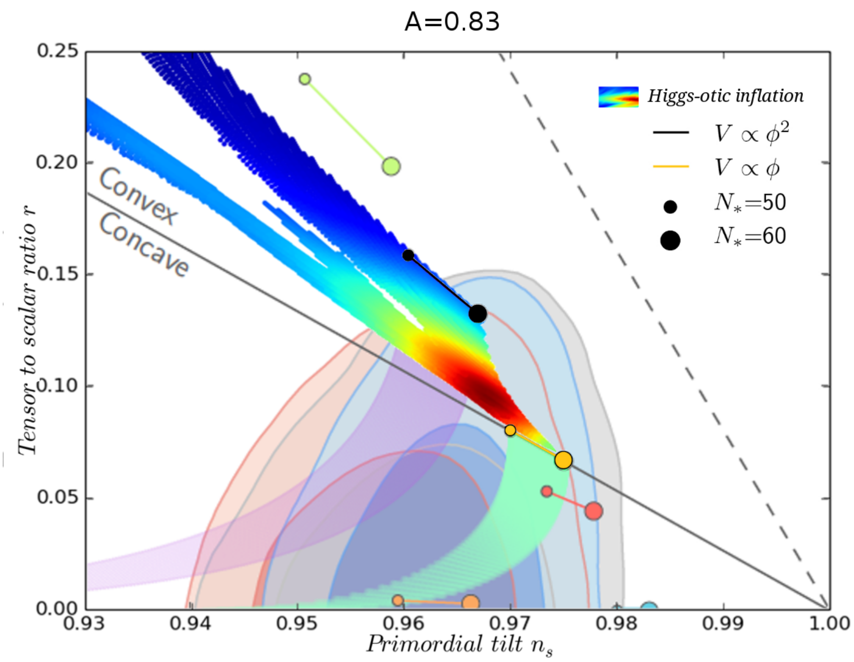


Figure 5.20: Tensor to scalar ratio and spectral index for Higgs-otic inflation with $A = 0.83$ and arbitrary $\hat{G}$. The data is superimposed over the recent Planck exclusion limits. The color pattern (from red to blue) corresponds to (higher or lower) density of initial condition points.

could hope to separate their dynamics from that of the inflaton sector.

5.2.5.1. Planck mass suppressed corrections

Higher dimensional Planck-supressed operators, i.e. terms of the form (ϕ^{4+2n}/M_p^{2n}) correcting the inflaton potential are a potential danger for the slow-roll conditions. The simplest such corrections to a an inflation potential V_0 are possible terms of the form

$$V_n \simeq V_0 \times \left(\frac{\Phi^2}{M_p^2}\right)^n \tag{5.154}$$

with $n > 0$. Such terms can give large contributions to the slow-roll parameters driving $\epsilon, \eta \simeq 1$ for transplanckian excursions of the inflaton. To avoid the presence of such terms it is customary to assume the existence of a shift symmetry under which $\phi \to \phi + c$ and the Kähler potential remains invariant.

The presence of such a symmetry helps also in trying to solve a second related problem, the η problem in $N = 1$ supergravity. The idea is that the pre-factor e^K appearing in the supergravity potential will tend to give a large (of order H) mass to the inflaton, once one expands K to leading order in the inflaton field. In the case of chaotic inflation this problem is not severe because m_I needs to be only one order of magnitude smaller than H, which can easily be done by a modest fine-tuning. If the inflaton does not appear explicitly in the Kähler potential, as happens in the presence of a shift symmetry, such mass term for the inflaton does not appear to leading order.

The effective action of string axions are known to possess shift symmetries which could protect the inflation potentials against these effects. In fact such shift symmetries are typically part of larger non-compact groups leaving invariant the $N = 1$ supergravity effective action. These large continuous groups are broken by instanton effects down to discrete (infinite) groups which are 4d duality groups in general. These shift symmetries are particularly welcome in models with large inflaton excursions, in which one expects that the above Planck-supressed corrections could be very important.

In the case we are considering here, the $N = 1$ supergravity η problem is mixed up with the fine-tuning required to have a massless SM doublet left below the SUSY-breaking scale M_{SS}. There is a fine-tuning in the flux parameters such that both a SM doublet survives and the inflaton mass is slightly below H. Both fine-tunings cannot be disentangled.

Concerning the first problem, in the case we studied above in which the inflaton is a D7-brane modulus corrections to the inflation potential of the type (5.154) do indeed appear. The important point however is that those corrections are computable to all orders in inverse Planck masses and are under control. Indeed, in our case the inflaton/Higgs fields are open string fluctuations and their action, including their interaction with closed string moduli are given to all orders in α' by the DBI+CS action.[8] For illustrative purposes let us look at the DBI+CS action for the $U(N)$ adjoint that we discussed in section 5.2.3.1. There we see that the full effect of those corrections is just a field redefinition. In particular one gets a structure of the form

$$\mathcal{L}_{4D} \;=\; \mathrm{STr}\left(\left[1+\frac{\kappa}{2}V_0(\Phi)\right]\; D_\mu\Phi D^\mu\bar{\Phi} \;-\; V_0(\Phi)\; + ...\right) . \tag{5.155}$$

with $\kappa = (V_4\mu_7 g_s)^{-1}$ a constant. After a field redefinition one sees that corrections will always appear in powers of the initial fiducial potential V_0. Thus indeed large corrections to the potential of the form in (5.154) do appear but in the D-brane case here considered these corrections are under control and lead to a flattening of the potential, i.e., the potential becomes of linear type rather than quadratic, leading to a new potential consistent with slow-roll.

It is interesting to consider the $N = 1$ sugra avatar of this property. We have seen how the Kähler potential involving the Higgs/inflaton fields is

$$K_H = \;-\log[(S+S^*)(U_3+U_3^*) \;-\; \frac{\alpha'}{2}\;|H_u+H_d^*|^2] \;-\; 3\log(T+T^*)\;. \tag{5.156}$$

As shown in refs. [221–224] for the S-dual heterotic case, the Lagrangian described by this Kähler potential is invariant under a $SL(2,\mathbf{Z})_{U_3}$ geometric symmetry associated to reparametrisation of the corresponding $\mathbf{T}^2$. In particular it is easy to check that under

[8] Corrections in α' to the non-Abelian DBI action which describes our MSSM system are to date not fully understood. However, notice that for large values of the inflaton, the inflationary system is described by a single D7-brane plus orbifold images. Thus, all α' corrections relevant to inflation should be captured by those of the Abelian DBI action.

the continuous transformations

$$U_3 \longrightarrow \frac{aU_3 - ib}{icU_3 + d} \tag{5.157}$$

$$S \longrightarrow S - \frac{ic}{2}\frac{H_u H_d}{icU_3 + d} \tag{5.158}$$

$$H_u \longrightarrow \frac{H_u}{icU_3 + d} \tag{5.159}$$

$$H_d \longrightarrow \frac{H_d}{icU_3 + d}\ ,\ a, b, c, d \in \mathbf{R} \tag{5.160}$$

with $ac - bd = 1$, the Kähler potential transforms like

$$K_H \longrightarrow K_H + \log|icU_3 + d|^2\ . \tag{5.161}$$

The latter is a Kähler transformation, so the Lagrangian will be invariant under it even if the Kähler potential is not. One can also easily check that the addition of a μ-term does not spoil this symmetry. In fact the low-energy effective action is invariant under these continuous symmetries, while the discrete subgroup with $a, b, c, d \in \mathbf{Z}$ is preserved to all orders in perturbation theory or sigma model expansions and it is only broken spontaneously once the moduli are fixed. Note that these transformations act both on the moduli and the Higgs/inflaton fields so that e.g. the particular dependence on the combination $H_u + H_d^*$ is dictated by the symmetries. In particular the linear combinations of Higgs fields transform as

$$H_u \pm H_d^* \longrightarrow \frac{d(H_u \pm H_d^*) + ic(U_3 H_d^* \mp U_3^* H_u)}{|icU_3 + d|^2}\ . \tag{5.162}$$

In the case with $a = d = 0$, $b = 1, c = -1$, one has $U_3 \to 1/U_3$ and

$$e^{i\gamma/2}H_u \pm e^{-i\gamma/2}H_d^* \longrightarrow -i\left(\frac{e^{-i\gamma/2}H_d^*}{U_3^*} \mp \frac{e^{i\gamma/2}H_u}{U_3}\right)\ . \tag{5.163}$$

For a square torus $U_3 = 1$ and the above transformation just exchanges the fields h and H. This is somehow expected because in this case the transformation $U_3 \to 1/U_3$ corresponds to the exchange of the two cycles of the torus. The transformation also implies a shift of the complex dilaton $S \to S - \frac{1}{2}H_u H_d$, as expected from the fact that $H_u H_d$ parametrises the wandering D7-brane position. Finally, an analogous symmetry $SL(2, \mathbf{Z})_S$ acting on the complex dilaton S exists. The transformations look the same as the ones above replacing $U_3 \leftrightarrow S$. However this S-duality symmetry is in general broken by quantum corrections.

A direct consequence of the modular symmetries is that, since the Kähler potential is *not* invariant and only the Lagrangian and the potential are, one expects corrections of higher order in α' to appear as powers of the potential itself, that is

$$\delta V_H \simeq \sum_{n>1} \frac{(V_0)^n}{(M_p)^{4(n-1)}} \tag{5.164}$$

Indeed this is consistent with the higher order corrections in α' given by the DBI+CS action that we studied, and with the fact that such action is related to a Kaloper-Sorbo 4d effective Lagrangian. As stressed, in our case these Planck suppressed corrections are known and give rise to the flattening of the inflaton potential at large field.

Note that a corollary of this discussion is that using the (two-derivative) $N = 1$ sugra standard formalism does not capture these corrections leading to the flattening of the scalar potential for large fields. This is particularly the case for any model in which the inflaton is an open string mode.

Let us finally comment that above the inflaton mass, where SUSY couplings are recovered, the loop corrections to the potential are only expected to lead to logarithmic corrections with small coefficients, and should not modify in any important way the shape of the potential. Among these loop corrections one expects the presence of minute modulations for the overall linear potential at large field. They would arise from the fact that, as we mentioned at the end of section (5.2.2.2), as the D7-brane position varies over the torus, the masses of the massive fields $W^{\pm}, Z^0, H^{\pm}$ etc. oscillate. This oscillation should induce in turn one-loop minute field dependent oscillations on the inflaton mass parameters.

5.2.5.2. Backreaction and induced RR-tadpoles

In general one expects that wandering D7-branes may lead to some level of backreaction in the surrounding geometry. This is a well known fact present in all perturbative IIB orientifolds with D7-branes. However our setting is initially supersymmetric, with SUSY broken spontaneously, and in this sense is more stable that settings in which there are both branes and antibranes and SUSY is broken at the string scale. In any event, taking into account this back reaction would require to go to a F-theory setting. We will have nothing to add concerning this issue other than pointing out that it would be interesting the embedding of this type of models into an F-theory background. For previous proposals of large field inflation models within F-theory see [188, 195–197, 225].

Other than that, the presence of a non-vanishing vev for the inflaton may have also an impact on the surrounding geometry. In particular as we have seen in section 5.2.3.1 the inflaton vev induces a background for the B-field in the D-brane worldvolume, which in turn leads to induced lower dimensional D-brane RR charges. This fact has already appeared in previous monodromy inflation models, leading to the introduction of brane-antibrane pairs to cancel the tadpoles. We show here that this is not the case in our setting, and therefore there is no need to introduce anti-branes.

The different sources of D3-brane tadpole in a type IIB compactification with O3/O7-planes are captured by the 5-form Bianchi identity as follows

$$d\tilde{F}_5 \;=\; F_3 \wedge H_3 + \sum_i \delta_6(p^i_{D3}) + \sum_j \delta_4(\pi^j_{D5}) \wedge B + \sum_k \delta_2(S^k_{D7}) \wedge \frac{1}{2}B^2 + \ldots \tag{5.165}$$

ignoring factors of α', etc. Here p^i_{D3} runs over all points where D3-branes are located, π^j_{D5} over the 2-cycles where D5-branes are located, and S^k_{D7} over the divisors wrapped by the D7-branes. The ∂_{2n}'s are $2n$-form bump functions localised on their respective worldvolumes.[9] Finally, the dots represent similar delta function contributions of opposite sign that come from the O3 and O7-planes. Typically, it is this negative contribution that allows for the integral of the r.h.s of (5.165) over the compact manifold X_6 to vanish, in

[9]In general the D7 and D5-branes will be magnetised by an open string worldvolume flux F, so one should replace $B \to \mathcal{F} = B + F$ everywhere in (5.165). For the sake of simplicity we will stick to the above notation, the generalisation being straightforward.

agreement with the fact that $\tilde{F}_5$ should be globally well-defined. If this integral over X_6 does not vanish we say that we have a D3-brane tadpole.

The problem arises when we have a non-trivial H_3 in our compactification, because then the contribution from D5-branes and D7-branes, which depends on the pull-back of the B-field in their worldvolume, is position-dependent. Hence it is not clear if, given that we can solve the tadpole condition in one particular point in open string moduli space $\{p^i_{D3}, \pi^j_{D5}, S^k_{D7}\}$, we can solve it for a different point $\{p^{i\prime}_{D3}, \pi^{j\prime}_{D5}, S^{k\prime}_{D7}\}$. In other words, when we move a D7-brane from S_{D7} to S'_{D7} the pull-back of B^2 on its worldvolume changes, and so does its induced D3-brane charge. It then seems that, during inflation, we will generate a D3-brane tadpole as soon as we move the D7-brane from its initial location.

In the following we will show that this is not the case. Basically, when we move D5 and/or D7-branes their induced D3-brane charge changes, but the contribution to the D3-brane tadpole coming from $F_3 \wedge H_3$ changes by a similar amount. Both effects cancel each other upon integration over X_6, and so $\tilde{F}_5$ is always well-defined and there is no tadpole. We will show this first for the case where we only have D5-branes in our model (which is unrealistic in SUSY compactifications) and then for the more interesting case of models with D7-branes.

Magnetised D5-branes. Let us consider the case where in our compactification there are only space-time filling D3-branes, D5-branes and fluxes (F_3, H_3). Take a D5-brane in a 2-cycle π_2 and move it to a new location π_2' within the same homology class. The difference in the contribution to the D3-brane tadpole can be measured by the integral

$$\int_{X_6} \partial_4(\pi_2') \wedge B - \int_{X_6} \partial_4(\pi_2) \wedge B \;=\; \int_{\pi_2'} B - \int_{\pi_2} B \;=\; \int_{\Sigma_3} H_3 \tag{5.166}$$

where Σ_3 is a 3-chain such that $\pi\Sigma_3 = \pi_2' - \pi_2$. So in general we see that the contribution to the D3-brane tadpole changes when we move one or several D5-branes.[10]

We should however take into account that, in the presence of D5-branes, F_3 is not a harmonic form, which is the case when we only have D3-branes. On the contrary, it satisfies the equation

$$dF_3 \;=\; \sum_j \delta_4(\pi^j_{D5}) \tag{5.167}$$

which we assume corresponds to a globally well-defined but non-closed F_3. As a result, when we move the D5-brane from π_2 to π_2' the field strength F_3 will change because (5.167) changes. Let us represent by F_3 the background flux with the D5 located at π_2, and by F_3' the flux with the D5 located at π_2' and $\Delta F_3 = F_3' - F_3$. Then it is easy to see that

$$d\Delta F_3 \;=\; \delta_4(\pi_2') - \delta_4(\pi_2) \tag{5.168}$$

Moreover notice that, even if non-closed, F_3 and F_3' are quantised 3-forms on X_6. Hence so is ΔF_3, and this fact together with (5.168) can be used to show that [146]

$$\int_{X_6} \Delta F_3 \wedge \omega_3 \;=\; - \int_{\Sigma_3} \omega_3 \tag{5.169}$$

[10] Together with this D5 we should move its orientifold image on $\Omega\mathcal{R}(\pi_2)$. Taking this into account will not change much the discussion, so we will ignore the effect of orientifold images in the following.

for any closed 3-form ω_3, and where again $\pi\Sigma_3 = \pi_2' - \pi_2$ is a 3-chain describing the deformation of the D5-brane location.

We can now use (5.169) to prove that the D3-brane tadpole induced by the background fluxes changes. Indeed, assuming that there are no NS5-branes in our compactification H_3 is a harmonic form and we can apply (5.169). Hence

$$\int_{X_6} F_3' \wedge H_3 - \int_{X_6} F_3 \wedge H_3 \;=\; \int_{X_6} \Delta F_3 \wedge H_3 \;=\; -\int_{\Sigma_3} H_3 \tag{5.170}$$

This is precisely the opposite as the previous change (5.166), so tadpoles still cancel when we change the D5-brane position.

Magnetised D7-branes. Let us now consider the case where we have D3-branes and D7-branes, as in the inflationary model of section 5.2.2.2, and that we move one of the latter as $S_4 \to S_4'$. The change in D3-brane tadpole is given by

$$\frac{1}{2}\left[\int_{X_6} \partial_2(S_4') \wedge B^2 - \int_{X_6} \partial_2(S_4) \wedge B^2\right] \;=\; \frac{1}{2}\left[\int_{S_4'} B^2 - \int_{S_4} B^2\right] \;=\; \int_{\Sigma_5} H_3 \wedge B \tag{5.171}$$

with Σ_5 a 5-chain with $\pi\Sigma_5 = S_4' - S_4$ and describing the above deformation.

Because the D7-branes are magnetised by the B-field they carry a D5-brane charge, and so again F_3 is not a closed 3-form. Instead it must satisfy the equation

$$dF_3 \;=\; \sum_k \delta_2(S^k_{D7}) \wedge B \;=\; dF_1 \wedge B \tag{5.172}$$

where we have used that

$$dF_1 \;=\; \sum_k \delta_2(S^k_{D7}) \tag{5.173}$$

So when we move a D7-brane as $S_4 \to S_4'$, the RR fluxes (F_1, F_3) change to (F_1', F_3') and we can define $(\Delta F_1, \Delta F_3)$ as their difference. In particular we have that

$$d\Delta F_3 \;=\; \partial_2(S_4') \wedge B - \partial_2(S_4) \wedge B \;=\; d\Delta F_1 \wedge B \tag{5.174}$$

Now it is ΔF_1 the flux that is quantised, and applying the reasoning of [146] we get

$$\int_{X_6} F_1 \wedge \omega_5 \;=\; -\int_{\Sigma_5} \omega_5 \tag{5.175}$$

for any closed 5-form ω_5 and Σ_5 defined as above. In particular we can take $\omega_5 = B \wedge H_3$. Putting all these things together we arrive at the following variation for the background flux D3-brane charge

$$\int_{X_6} F_3' \wedge H_3 - \int_{X_6} F_3 \wedge H_3 \;=\; \int_{X_6} \Delta F_3 \wedge H_3 \;=\; \int_{X_6} \Delta F_1 \wedge B \wedge H_3 \;=\; -\int_{\Sigma_5} B \wedge H_3 \tag{5.176}$$

which again cancels the variation (5.171) and guarantees D3-brane tadpole cancellation.

5.2.5.3. Decoupling of moduli fixing from inflation sector

The DBI+CS derived inflaton scalar potential that we used assumes implicitly that all the other moduli of the theory, in particular the complex dilaton S and Kähler (T^i) and complex structure (U^a) moduli are fixed at a scale well above the inflation scale. That is, we are assuming a full scalar potential of the form

$$V(\sigma, \theta; S, T^i, U^a) \;=\; V_{\text{inflation}}(\sigma, \theta; S, T^i, U^a) \;+\; V_{\text{moduli}}(S, T^i, U^a) \tag{5.177}$$

In particular we are assuming that the potential barriers fixing S, T^i, U^a are such that the inflaton scalar potential does not modify in a substantial manner the moduli dynamics. This may proof hard for an inflaton scale $\simeq 10^{16}$ GeV as suggested by BICEP2, since that would require the compactification M_c and string scale M_s not much below the reduced Planck scale $M_p \simeq 10^{18}$ GeV. This is a general problem for all string inflation models with large field inflation, see [193, 194, 197].

Here we would only like to add that the string models with the inflaton identified with open string moduli may be more flexible than closed string axion models in this regard. Indeed, the inflaton dynamics is localised in a D-brane sector of the theory rather than in the bulk. Then, as shown in eq.(5.91), the local G_3 flux felt by the D7's (fixing the inflaton mass) may be suppressed compared to the flux felt by the moduli in the bulk by a warp factor $Z^{-1/2}$. In this way the barriers of the potential V_{moduli} could be substantially higher than those in $V_{\text{inflation}}$. This would help in understanding the decoupling of the moduli fixing dynamics from the inflaton dynamics in a natural way.

5.2.6. Some further cosmological issues

Our study of the cosmological perturbations induced in the Higgs-otic scenario has been incomplete in several respects. In particular, while single inflaton models predict a Gaussian and adiabatic spectrum, it is well known that multi-inflaton models may in general give rise to non-Gaussianities as well as isocurvature (entropy) perturbations [226, 227], and such effects may significantly alter the observational signatures of a given model. The Higgs inflaton potential here studied has two fields involved in inflation, σ and θ, so that in principle one can think that non-Gaussianities and/or isocurvature perturbations could arise. Concerning non-Gaussianities, one does not expect any effect in our scheme since it is known that 2-field models yield non-linear parameters f_{NL} proportional to the slow roll parameters ϵ, η, see e.g. [229, 230]. On the other hand, a full analysis of the isocurvature perturbations have been carried out in [231] while this thesis was about to be finished. As expected, adiabatic and isocurvature perturbations form a coupled system and there is super-horizon evolution of the curvature perturbations. This leads in general to a relative increase of adiabatic perturbations and consequently to a reduction of the tensor to scalar ratio r compared to the computation here. The range of variation of n_s gets smaller and is centered around the region alowed by Planck data with a tensor to scalar ratio in a range $r = 0.08 - 0.12$. Moreover, the isocurvature component is always very suppressed at the end of inflation, consistent with upper Planck bounds. Therefore the predictivity of the model is increased compared to the adiabatic approximation, more in line with the results of the joint Planck/BICEP analysis [161].

Another interesting issue is that of reheating, which is expected to be quite efficient in this Higgs-otic scenario. At the end of inflation the universe is extremely cold and a reheating process occurs in which the inflaton oscillates around its minimum. The inflaton transfers all its energy through its decay into relativistic particles. The inflaton must couple to the SM particles which will end up in thermal equilibrium and give rise to the big-bang initial conditions. A generic problem in string cosmologies in which the inflaton is identified with a closed string mode (like e.g. an axion) is that the inflaton reheats predominantly into hidden sector fields or moduli rather than into SM fields. In our case, obviously, the inflaton is a Higgs field which will decay predominantly into top quarks and gauge bosons and this problem is automatically avoided. The decay rate will typically be of order

$$\Gamma_H \ \simeq \ \frac{h^2 m_I}{8\pi} \ , \tag{5.178}$$

with h the top Yukawa coupling or a gauge coupling and $m_I \simeq M_{SS} \simeq 10^{13}$ GeV is the inflaton mass, which is of the order of the SUSY breaking scale M_{SS}. Perturbative reheating ends when the expansion rate of the universe given by the Hubble constant $H = \sqrt{8\pi\rho/3M_p^2}$ is of order of the total inflaton decay rate. The SM interactions are strong enough so that thermal equilibrium is reached with a reheating temperature (see e.g. [232–234])

$$T_R \ \simeq \ 0.2\sqrt{\Gamma_H M_p} \ \simeq \ 10^{13} \ GeV \ , \tag{5.179}$$

where we have set $h \simeq 1/2, m_I \simeq 10^{13}$ GeV. This is high enough so that leptogenesis may take place in the usual way at an intermediate scale.

6 Conclusions

In this thesis we have studied different aspects of Particle Physics and Cosmology within the framework of Type IIB/F-theory compactifications in String Theory. As a key ingredient, we can remark the 3-form closed string fluxes, which are the main source of supersymmetry breaking as well as the origin of the inflationary potential in our model. These fluxes are generically present in Type IIB compactifications and are essential to stabilize the dilaton and the complex structure moduli. Its presence can also induce soft SUSY breaking terms via gravity mediation in the D-brane open string sector, where the MSSM fields are supposed to live.

In section 4.1 we have studied the pattern of flux-induced soft SUSY breaking terms over fields living in a system of D7-branes. We have followed a bottom-up approach expanding the DBI+CS effective action of the D7-brane in the presence of local densities for the closed string background, which parametrize our ignorance of the full compact space. For the case of bulk worldvolume fields (position moduli and wilson lines of the branes) we have performed a careful analysis considering the most general closed string background including simultaneously both ISD and IASD 3-form fluxes and also magnetic worldvolume fluxes on the branes. The computation is then generalized for the most phenomenologically interesting case of chiral bifundamental fields living at the intersections of 7-branes. To compute these soft terms, we combine for first time information about the closed string background and the local wavefunctions of the matter fields, the latter previously used in the computation of the Yukawa couplings. The results for ISD fluxes are consistent with those obtained from the 4d $N = 1$ supergravity effective action in the case of modulus dominated SUSY breaking. However we provide a microscopic interpretation of the supergravity variables in terms of the closed and open string fluxes, which allows us to go beyond the simpler situations. We also apply the results to the local F-theory SU(5) GUT model and show that in general the soft terms will be hypercharge dependent due to the presence of the hypercharge flux necessary to break the GUT group to the SM.

Our microscopic computation shows that the soft terms come from the overlap integral of the internal wavefunctions in the presence of a closed string background. Since different families live at slightly different regions in the internal space, non-constant flux densities induce flavor non-universalities on the soft terms, as we have studied in section 4.2. These flavor non-universalities are strongly constrained from FCNC transitions and CP violation at low energy, requiring sfermion masses in the multi-TeV region for natural background parameters.

In section 4.3 we start studying the typical size of the soft terms induced by closed string fluxes. For natural values of the background this scale turns out to be around $M_{SS} \simeq 10^{10} - 10^{13}$ GeV, depending on the exact value of the compactification/GUT scale. Imposing gauge coupling unification using some threshold hyperharge dependent corrections coming from F-theory implies $M_{SS} \simeq 10^{10}$ GeV and $M_{GUT} \simeq 10^{14}$ GeV. The prize to pay in this scenario is the fine-tuning of the Higgs mass. Such a high scale of supersymmetry can not solve the EW hierarchy problem. However it is not so different from the fine-tuning which is usually implicitely assumed in the closed string fluxes to lower the SUSY breaking scale at least seven orders of magnitude from its natural value to the EW scale. Therefore, in this thesis we take the less conventional choice in which the EW scale is fine-tuned but the SUSY breaking scale stays at its most natural value in Type IIB, ie. $M_{SS} \simeq 10^{10} - 10^{13}$ GeV. This proposal is further motivated by the absence of any sign of supersymmetry at the LHC so far. If the situation persists after the next run of the LHC, the alternative of low energy supersymmetry as a solution to the hierarchy problem will be strongly questionable. The fine-tuning of the Higgs mass might have then an anthropic explanation in a flux landscape of possibilities within string theory. Interestingly, the role of supersymmetry at such a high scale would not be to stabilize the Higgs mass but the SM vacuum, which otherwise becomes metaestable (or even inestable due to the fluctuations during inflation) before reaching the Planck scale.

Following with this approach, we have computed the Higgs mass as a function of the SUSY breaking scale, considering the latter as a free parameter from M_{EW} to M_{GUT}. Under the standard unification assumption of universal soft Higgs masses, the result turns out to be highly constrained. For $M_{SS} \simeq 10^{10} - 10^{13}$ we get that the Higgs mass is centered around $m_H \simeq 126 \pm 3$ GeV, consistent with LHC results. Below that scale the mass depends more on the details of the SUSY breaking mass parameters and the Higgs mass tends to the value of a standard fine-tuned MSSM scenario with $m_H \lesssim 130$ GeV. The predictivity for high scales of SUSY is remarkable, since the SM by itself would have allowed for any value from 100 GeV to 1 TeV approximately. This supports the idea that the experimental value of the Higgs mass might be an indirect evidence of an underlying supersymmetry (and some sort of unification) at a large scale.

Obviously, this scale of SUSY breaking is very far from the reach of any particle collider we can imagine to build. However, this does not imply that supersymmetry has no detectable implications for physics. For instance it could have implications for inflation, by being the source of the inflationary potential. In chapter 5 we work under the assumption that this Intermediate SUSY breaking scale indeed corresponds to the inflationary scale suggested by BICEP2 results. The fact that at this scale we have already the MSSM Higgs sector with soft masses of order 10^{13} GeV naturally suggests the identification of the inflaton with an MSSM Higgs boson, giving rise to a sort of chaotic inflation with a quadratic potential. We have developed this proposal in section 5.2 under the name of Higgs-otic inflation, which refers to models in which a scalar particle is responsible for both gauge symmetry breaking and inflation. Models with detectable gravitational waves and a high scale of inflation are known as large field inflation because a transplanckian field range for the inflaton is indeed required. This makes the models extremely sensitive to UV physics and a consistent embeding in a quantum theory of gravity like String Theory becomes essential. We have embedded Higgs-otic inflation in String Theory, identifying the inflaton with a D7-brane position modulus parametrizing the position of the brane in a transverse torus. We show specific examples and study in detail a IIB orientifold in a toroidal compactification with D7-branes at singularities. The inflationary potential is induced from 3-form fluxes and can be computed from the DBI+CS action to all orders in α', while the transplanckian field range is achieved in a similar way to monodromy inflation. The result is a 2-field inflationary quadratic potential with non-canonical kinetic terms. We have shown that the non-canonical kinetic terms come from summing over all α' corrections, which in the effective theory correspond to the naively dangerous Planck-suppressed operators. Interestingly, the effect of all these corrections is a flattening of the potential after field redefinition to get canonical kinetic terms. This behaviour can be traced back to the duality simmetries associated to reparametrizations of the torus, protecting the potential over transplanckian distances. From the point of view of the effective theory, our model can also be reduced to a Kaloper-Sorbo-like lagrangian which exhibits the protectection from UV corrections. Finally we have computed the cosmological parameters including the spectral index and the tensor-to-scalar ratio obtaining $r = 0.08 - 0.12$, consistent with combined PLANCK/BICEP results.

More clear now, the Higgs boson has been the bridge which has allowed us to jump from Particle Physics to Cosmology, always in the search of the specific vacuum of String Theory hosting our universe.

7 Conclusiones

En esta tesis hemos estudiado diferentes aspectos de Física de Partículas y Cosmología derivados de compactificaciones de Type IIB/F-theory en Teoría de Cuerdas. Como hilo conductor podemos destacar los flujos de cuerda cerrada, siendo la fuente principal tanto de ruptura de SUSY como del potencial inflacionario en nuestro modelo. Estos flujos se encuentran genéricamente en compactificaciones de Type IIB, siendo esenciales para la estabilización del dilatón y los moduli de estructura compleja. Su presencia también puede inducir términos de ruptura de SUSY (mediada por gravedad) en sectores de cuerdas abiertas de D-branas, donde se encuentran los campos del MSSM.

En la sección 4.1 hemos estudiado la estructura de los términos de ruptura de SUSY (*soft terms*) inducidos por los flujos sobre campos viviendo en un sistema de D7-branas. Hemos seguido la *bottom-up approach* expandiendo la acción efectiva DBI+CS de la D7-brana en presencia de densidades locales del fondo de cuerda cerrada, el cual parametriza nuestra ignorancia sobre el espacio compacto al completo. Para campos viviendo en las dimensiones expandidas por las branas (posiciones y wilson lines) hemos realizado un estudio detallado considerando la presencia simultánea de flujos de 3-forma ISD y IASD junto con flujos magnéticos de cuerda abierta en las branas. Después hemos generalizado el cálculo a campos chirales bifundamentales viviendo en las intersecciones de 7-branas, de mayor interés fenomenológico. Para calcular los *soft terms* hemos combinado por primera vez información sobre el fondo de cuerda cerrada y las funciones de onda locales de los campos de materia, estas últimas usadas en el cálculo de los acoplos de Yukawa. Los resultados para flujos ISD son consistentes con los obtenios a partir de la acción efectiva de supergravedad N=1 en 4d en el caso de ruptura de SUSY dominada por un campo Kahler. Sin embargo, nuestra formulación proporciona la interpretación microscópica de las variables de supergravedad en términos de los flujos de cuerda cerrada y abierta, lo que permite ir más allá de los casos más sencillos. También hemos aplicado los resultados al modelo local SU(5) GUT de F-theory y mostrado cómo en general los soft terms van a depender de la hipercarga del campo en cuestión debido a la presencia del flujo de hipercarga necesario para romper el grupo GUT al grupo gauge del SM.

Nuestro derivación microscópica de los soft terms muestra que estos vienen de integrales de solapamiento de las funciones de onda interna en presencia del fondo de cuerda cerrada. Como las diferentes familias de partículas se encuentran localizadas en regiones ligeramente distintas del espacio interno, densidades de flujo no constantes en el espacio dan lugar a no universalidades en los soft terms, como hemos estudiado en la sección 4.2. Estas violaciones de universalidad están fuertemente restringidas debido a las cotas experimentales en transiciones FCNC y violación de CP a baja energía, lo que implica que para valores naturales de los flujos las masas de los sfermiones deben ser por lo menos del orden de varios TeV.

En la sección 4.3 empezamos estudiando el tamaño típico de los soft terms inducidos por flujos de cuerda cerrada. Para valores naturales de los parámetros la escala de los soft terms resulta estar en torno a $M_{SS} \simeq 10^{10} - 10^{13}$ GeV, dependiendo del valor exacto de la escala de compactificación/unificación. Si además imponemos unificación de los acoplos gauge usando las correcciones de umbral dependientes de la hipercarga que vienen de F-theory, obtenemos $M_{SS} \simeq 10^{10}$ GeV y $M_{GUT} \simeq 10^{14}$ GeV. El precio a pagar en este escenario es el fine-tuning de la masa del bosón de Higgs. Una escala tan alta de supersimetría no puede resolver el problema de las jerarquías. Sin embargo, este fine-tuning no es tan diferente del que normalmente se asume de manera implícita en los flujos de cuerda cerrada para bajar la escala de ruptura de SUSY al menos siete órdenes de magnitud desde su valor natural a la escala EW. Por tanto, en esta tesis hemos tomado una alternativa poco convencional en la cual la escala EW está fine-tuned pero la escala de ruptura de SUSY se encuentra a su valor natural en Type IIB, es decir, $M_{SS} \simeq 10^{10} - 10^{13}$ GeV. Esta propuesta se ve a su vez motivada por la ausencia de cualquier signo de supersimetría en el LHC por ahora. Si la situación continúa así tras el próximo conjunto de medidas a mayor energía en el LHC, habrá que cuestionarse seriamente la alternativa de supersimetría a baja energía como solución al problema de las jerarquías. En tal caso, el fine-tuning del Higgs podría tener una explicación antrópica dentro del conjunto de posibles soluciones (*flux landscape*) de Teoría de Cuerdas. Es interesante que el papel de supersimetría a esas altas energías ya no sería estabilizar la masa del Higgs sino estabilizar el vacío del SM, el cual si no pasa a ser metaestable (o incluso inestable debido a las fluctuaciones durante inflación) antes de alcanzar la escala de Planck.

Siguiendo con esta idea, hemos calculado la masa del Higgs en función de la escala de ruptura de SUSY, considerando esta última un parámetro libre que varía desde M_{EW} hasta M_{GUT}. Bajo la hipótesis usual de tener masas soft universales para el Higgs en M_{GUT}, el resultado pasa a estar muy restringido. Para $M_{SS} \simeq 10^{10} - 10^{13}$ obtenemos que la masa del Higgs tiene que estar en torno a $m_H \simeq 126 \pm 3$ GeV, consistente con los resultados del LHC. Por debajo de esta escala en cambio el resultado depende en mayor medida de los detalles de los parámetros de ruptura de SUSY y recuperamos el estándar MSSM fine-tuned con $m_H \lesssim 130$ GeV. La predictividad que se obtiene con SUSY a alta energía es remarcable, en especial si se tiene en cuenta que el SM por sí mismo habría permitido cualquier valor entre 100 GeV y 1 TeV aproximadamente. Esto apoya la idea de que el valor de la masa del Higgs medida experimentalmente puede ser una evidencia indirecta de la presencia de supersimetría (y unificación) a alta energía.

Claramente, esta escala de SUSY está demasiado lejos del alcance de cualquier acelerador de partículas que nos podamos imaginar construir. Sin embargo, esto no implica que supersimetría no tenga implicaciones detectables para la física. Podría tener por ejemplo implicaciones en inflación, si la ruptura de SUSY es la fuente del potencial inflacionario. En el capítulo 5 trabajamos bajo la hipótesis de que esta escala intermedia de ruptura de SUSY de hecho corresponde a la escala de inflación sugerida por los resultados de BICEP2. El hecho de que a esta escala tenemos el sector de Higgs del MSSM con masas soft del orden de 10^{13} GeV sugiere de manera natural identificar el inflaton con un bosón de Higgs del MSSM. Esta idea es propuesta y posteriormente desarrollada en la sección 5.2 bajo el nombre de *Higgs-otic inflation*, refiriéndonos a modelos en los que un mismo escalar es responsable a la vez de la ruptura de simetría de un grupo gauge y de inflación.

Modelos que predicen ondas gravitacionales detectables y escalas altas de inflación se conocen como *large field inflation*, pues requieren valores transplanckianos para el inflaton. Esto provoca que los modelos sean extremedamente sensibles a la física en el UV y que un embedding adecuado en una teoría consistente de gravedad cuántica como Teoría de Cuerdas se vuelva imprescindible. En esta tesis hemos embebido Higgs-otic inflation en Teoría de Cuerdas, identificando el inflatón con un campo parametrizando la posición de una D7-brana en un toro transverso. Mostramos ejemplos concretos estudiando en detalle una compactificación toroidal con orientifolds en Type IIB y con D7-branas en singularidades. El potencial inflacionario se genera debido a la presencia de flujos de 3-forma de cuerda cerrada y puede calcularse a partir de la acción de DBI+CS a todos órdenes en α'; mientras que distancias transplanckianas se consiguen de manera similar a *monodromy inflation*. El resultado es un potencial cuadrático de dos campos con términos cinéticos no canónicos. Hemos visto que los términos cinéticos no canónicos son producto de sumar sobre todas las correcciones en α', las cuales dan lugar a operadores de dimensión mayor a priori peligrosos en la teoría efectiva. Es interesante que el efecto de todas estas correcciones resulta ser aplanar el potencial una vez que redefinimos los campos para tener términos cinéticos canónicos. Este comportamiento es el remanente de las simetrías de dualidad asociadas a reparametrizaciones del toro, protegiendo el potencial en distancias transplanckianas. Desde el punto de vista de la teoría efectiva, nuestro modelo se puede reducir a un lagrangiano tipo Kaloper-Sorbo poniendo de manifiesto de nuevo la protección sobre correcciones del UV. Finalmente hemos calculado los parámetros cosmológicos incluyendo el índice espectral y la relación entre fluctuaciones tensoriales y escalares, obteniendo $r = 0{,}08 - 0{,}12$, consistente con los resultados combinados de PLANCK/BICEP.

Más evidente ahora, el bosón de Higgs ha sido el puente que nos ha permitido unir Física de Partículas y Cosmología, siempre en la búsqueda de esa solución específica de Teoría de Cuerdas que corresponde a nuestro universo.

A
The DBI+CS computation

The effective action for the microscopic fields of a system of D7-branes in the 10d Einstein frame is given by the Dirac-Born-Infeld (DBI) + Chern-Simons (CS) actions

$$S = -\mu_7 g_s^{-1}\mathrm{STr}\int d^8\xi\,\sqrt{-\det\left(P[E_{MN}+E_{Mi}(Q^{-1}-\delta)^{ij}E_{jN}]+2\pi\alpha' F_{MN}\right)det(Q^i_j)}$$
$$+\,\mu_7\mathrm{STr}\int P[C_8 + C_6\wedge\mathcal{F}_2+\frac{1}{2}C_4\wedge\mathcal{F}_2\wedge\mathcal{F}_2] \quad \text{(A.1)}$$

where

$$E_{MN} = g_s^{1/2}G_{MN} - B_{MN} \quad ; \quad Q^i_j = \delta^i_j + i2\pi\alpha'[\Phi^i,\Phi^k]E_{kj} \quad \text{(A.2)}$$
$$\sigma = 2\pi\alpha' \quad ; \quad \mu_7 = (2\pi)^{-3}\sigma^{-4}g_s^{-1} \quad ; \quad \mathcal{F}_2 = 2\pi\alpha' F_2 - B_2 \quad \text{(A.3)}$$

$P[\cdot]$ denotes the pullback of the 10d background onto the D7-brane worldvolume and 'STr' is the symmetrised trace over gauge indices. The indices M, N denote the directions extended by the D7-brane while i, j denote the transverse directions.

Neglecting derivative couplings[1], the determinant in the DBI action can be factorised between Minkowski and the internal space as follows

$$\det(P[E_{MN}]+\sigma F_{MN}) = g_s^4\det\left(\eta_{\mu\nu}+2Z\sigma^2\partial_\mu\Phi\partial_\nu\bar{\Phi}+Z^{1/2}g_s^{-1/2}\sigma F_{\mu\nu}\right)$$
$$\cdot\det\left(g_{ab}+Z^{-1/2}g_s^{-1/2}\sigma F_{ab} - Z^{-1/2}g_s^{-1/2}B_{ab} - \sigma^2([A_a,\Phi][A_b,\bar{\Phi}]+[A_a,\bar{\Phi}][A_b,\Phi])\right) \quad \text{(A.4)}$$

where μ, ν label the 4d non-compact directions and a, b the internal D7-brane dimensions. We have neglected the cross terms mixing internal and transverse coordinates in the pullback of the DBI action because they are only relevant if some of the 4-cycles are non-trivially fibered in the normal direction, which we do not consider here for simplicity.

[1]This is justified as long as the the fields have constant profiles in the internal 4-cycle, as in toroidal compactifications. Notice though that even if this is not the case, these contributions in general will induce a mixing only between massive modes, so it is still a good approximation if we are interested in the lightest modes.

Then, using the matrix identity

$$\begin{aligned}\det(1+\varepsilon M) &= 1+\varepsilon\,\mathrm{tr}\, M-\varepsilon^2\left[\frac{1}{2}\mathrm{tr}\, M^2-\frac{1}{2}(\mathrm{tr}\, M)^2\right] \\ &+ \varepsilon^3\left[\frac{1}{3}\mathrm{tr}\, M^3-\frac{1}{2}(\mathrm{tr}\, M)(\mathrm{tr}\, M^2)+\frac{1}{6}(\mathrm{tr}\, M)^3\right] \\ &- \varepsilon^4\left[\frac{1}{4}\mathrm{tr}\, M^4-\frac{1}{8}(\mathrm{tr}\, M^2)^2-\frac{1}{3}(\mathrm{tr}\, M)(\mathrm{tr}\, M^3)\right. \\ &+ \left.\frac{1}{4}(\mathrm{tr}\, M)^2(\mathrm{tr}\, M^2)+\frac{1}{24}(\mathrm{tr}\, M)^4\right]+\mathcal{O}(\varepsilon^5)\end{aligned} \tag{A.5}$$

we obtain on the one hand that

$$-\det\left(\eta_{\mu\nu}+2Z\sigma^2\partial_\mu\Phi\partial_\nu\bar{\Phi}+Z^{1/2}g_s^{-1/2}\sigma F_{\mu\nu}\right)=1+2Z\sigma^2\left(\partial_\mu\Phi\partial^\mu\bar{\Phi}-\frac{g_s^{-1}}{4}F_{\mu\nu}F^{\mu\nu}\right) \tag{A.6}$$

where we have neglected terms with more than two derivatives in Minkowski[2]. This is consistent with the slow-roll condition that will be imposed on the system in section 5.2. On the other hand we have that

$$\det\left(g_{ab}+Z^{-1/2}g_s^{-1/2}\mathcal{F}_{ab}\right) = \det(g_{ab})\,f(\mathcal{F})^2 \tag{A.7}$$

where $\mathcal{F}_{ab}=\sigma F_{ab}-B_{ab}$ and

$$f(\mathcal{F})^2=1+\frac{1}{2}Z^{-1}g_s^{-1}\mathcal{F}_{ab}\mathcal{F}^{ab}-\frac{g_s^{-2}}{4}Z^{-2}\mathcal{F}_{ab}\mathcal{F}^{bc}\mathcal{F}_{cd}\mathcal{F}^{da}+\frac{g_s^{-2}}{8}Z^{-2}\left[\mathcal{F}_{ab}\mathcal{F}^{ab}\right]^2 \tag{A.8}$$

Notice that for simplicity in the l.h.s. of (A.7) we have not included couplings of the form $[A,\Phi]$ which will not be relevant for the F-term contribution of the scalar potential. These terms, together with the quartic term coming from

$$det(Q_{ij})=1+g_s\sigma^1 Z([\Phi,\bar{\Phi}])^2 \tag{A.9}$$

will give rise to the D-term potential, as discussed in the main text. It is important to remark that, unlike in (A.6), when deriving (A.7) we have not made any approximation. Indeed by taking

$$M = g^{-1}\mathcal{F} \qquad \text{and} \qquad \varepsilon = (g_s Z)^{-1/2} \tag{A.10}$$

and using the fact that M is a 4×4 matrix it is easy to see that the expansion of eq.(A.5) ends at order ε^4. Finally, using that

$$\mathrm{tr}\ g^{-1}\mathcal{F} = \mathrm{tr}\ \mathcal{F}^t g^{-1\,t} = -\,\mathrm{tr}\ g^{-1}\mathcal{F} \tag{A.11}$$

so that $\mathrm{tr}\ M=\mathrm{tr}\ M^3 = 0$, we are led to the above result. Up to here, the computation applies both to section 4.1 and 5.2, but differ from now on. In section 4.1 we proceed by Taylor expanding the square root of the DBI action and keeping only the terms which in the presence of a closed/open string background lead to renormalizable couplings in the 4d effective theory. The computation can be found in the main text. However, this local

[2]Notice that in the computation of the soft SUSY breaking terms in section 4.1 this is not an approximation but the right result, because we are interested in the soft lagrangian which is given only by renormalizable couplings.

expansion is not valid for 5.2, in which we are interested in the vacuum energy density during inflation, when the scalar Φ takes large field values. In order to be consistent with large field inflation, we must not expand in powers of Φ but keep all the terms in the position modulus. In what follows we show that in the presence of only self or anti-self dual fluxes, the computation simplifies enormously obtaining a nice analytic result without any further approximation.

The discussion of 5.2 and eqs.(5.59) and (5.61) in the main text follow by simply replacing here $\mathcal{F} \to -B$. In fact, for a 4×4 matrix M satisfying that tr $M =$ tr $M^3 = 0$, we also have the identity

$$\det(1+\varepsilon M) \;=\; 1-\varepsilon^2\frac{1}{2}\mathrm{tr}\;M^2+\varepsilon^4\det M \tag{A.12}$$

which is easy to prove by looking at the characteristic polynomial of M. This allows us to write

$$\det(1+\varepsilon M) \;=\; 1+\varepsilon^2\mathcal{F}^2+\varepsilon^4\frac{1}{4}\left(\mathcal{F}\wedge\mathcal{F}\right)^2 \tag{A.13}$$

where the square of a p-form ω is defined as $\omega\cdot\omega$ with

$$\omega_p\cdot\chi_p \;=\; \frac{1}{p!}\omega_{a_1\dots a_p}\chi^{a_1\dots a_p} \tag{A.14}$$

Now, whenever $\mathcal{F}$ is a self or antiselfdual two form

$$\mathcal{F} \;=\; \pm *_4\mathcal{F} \tag{A.15}$$

we will have that

$$(\mathcal{F}\wedge\mathcal{F})^2 \;=\; (\mathcal{F}\wedge *_4\mathcal{F})^2 \;=\; \left(\mathcal{F}^2 d\mathrm{vol}_{S_4}\right)^2 \;=\; (\mathcal{F}^2)^4 \tag{A.16}$$

and so

$$\det(1+\varepsilon M) \;=\; \left(1+\frac{1}{2}\varepsilon^2\mathcal{F}^2\right)^2 \tag{A.17}$$

obtaining a perfect square. This will be the case for our wandering D7-brane system, since there $\mathcal{F}=-B$ will be a $(2,0)+(0,2)$ form due to (5.67).[3]

Putting everything together we find that the relevant part of the DBI action is given by

$$S_{DBI} = -\mu_7 g_s \mathrm{STr}\int d^8\xi\sqrt{\det(g_{ab})f(\mathcal{F})^2\left(1+2Z\sigma^2 D_\mu\Phi D_\mu\bar{\Phi}+\frac{1}{2}Zg_s^{-1}\sigma^2F_{\mu\nu}F^{\mu\nu}\right)} \tag{A.18}$$

Expanding this expression to second order in 4d derivatives and setting $\mathcal{F}=-B$ we obtain

$$S_{DBI} = -\mu_7 g_s \mathrm{STr}\int d^8\xi\sqrt{\det g}f(B)\left[1+Z\sigma^2D_\mu\Phi D^\mu\bar{\Phi}+\frac{1}{4}Zg_s^{-1}\sigma^2F_{\mu\nu}F^{\mu\nu})\right] \tag{A.19}$$

[3]To connect with the derivation of the perfect square in eq.(5.64) notice that in our case we have the identity

$$\det M \;=\; -\frac{1}{4}\mathrm{tr}\;M^4+\frac{1}{8}(\mathrm{tr}\;M^2)^2$$

and that $\mathcal{F}$ (anti)selfdual translates into $4\,\mathrm{tr}\;M^4=(\mathrm{tr}\;M^2)^2$ so that finally $16\det M=(\mathrm{tr}\;M^2)^2=4B^2$.

which is the expression used in the main text (c.f.(5.65)) where for simplicity $\sqrt{\det g} = 1$ has been taken. Notice that the same function which leads to the scalar potential, also multiplies the kinetic term. This function is the result of summing over all the terms in an expansion of the position modulus (or equivalently, all the higher order corrections in α'). This relation between the potential and the kinetic term is characteristic of the DBI action.

B

Renormalization group equations and threshold corrections

B.1. RGE for the gauge couplings

Here we first present the renormalization group equations at two loops for the SM couplings (the three gauge couplings, the top Yukawa and the Higgs quartic coupling).

$$\frac{dg_1}{dt} = \frac{1}{(4\pi)^2}\frac{41}{6}g_1^3 + \frac{g_1^3}{(4\pi)^4}\left(\frac{199}{18}g_1^2 + \frac{27}{6}g_2^2 + \frac{44}{3}g_3^2 - \frac{17}{6}h_t^2\right) \tag{B.1}$$

$$\frac{dg_2}{dt} = -\frac{1}{(4\pi)^2}\frac{19}{6}g_2^3 + \frac{g_2^3}{(4\pi)^4}\left(\frac{9}{6}g_1^2 + \frac{35}{6}g_2^2 + 12g_3^2 - \frac{3}{2}h_t^2\right) \tag{B.2}$$

$$\frac{dg_3}{dt} = -\frac{1}{(4\pi)^2}7g_3^3 + \frac{g_3^3}{(4\pi)^4}\left(\frac{11}{6}g_1^2 + \frac{9}{2}g_2^2 - 26g_3^2 - 2h_t^2\right) \tag{B.3}$$

$$\begin{aligned}\frac{dh_t}{dt} = {} & \frac{1}{(4\pi)^2}h_t\left(\frac{9h_t^2}{2} - \frac{17g_1^2}{12} - \frac{9g_2^2}{4} - 8g_3^2\right) + \frac{1}{(4\pi)^4}h_t\left(-12h_t^4 + \frac{6\lambda^2}{4} - \right.\\ & -\frac{12}{2}\lambda h_t^2 + \frac{131}{16}g_1^2h_t^2 + \frac{225}{16}g_2^2h_t^2 + 36g_3^2h_t^2 + \frac{1187}{216}g_1^4 - \frac{23g_2^4}{4} - \\ & \left. -108g_3^4 - \frac{3}{4}g_1^2g_2^2 + 9g_2^2g_3^2 + \frac{19}{9}g_3^2g_1^2\right)\end{aligned} \tag{B.4}$$

$$\begin{aligned}\frac{d\lambda}{dt} = {} & \frac{1}{(4\pi)^2}\left(12\lambda h_t^2 - 9\lambda\left(\frac{g_1^2}{3} + g_2^2\right) - 43h_t^4 + \frac{3}{4}g_1^4 + \frac{3}{2}g_2^2g_1^2 + \frac{9}{4}g_2^4 + 12\lambda^2\right) + \\ & + \frac{1}{(4\pi)^4}2\left(-\frac{312}{8}\lambda^3 + \frac{36}{4}\lambda^2\left(g_1^2 + 3g_2^2\right) - \frac{1}{2}\lambda\left(-\frac{629}{24}g_1^4 - \frac{39}{4}g_1^2g_2^2 + \right.\right.\\ & \left. + \frac{73g_2^4}{8}\right) + \frac{305g_2^6}{16} - \frac{289}{48}g_1^2g_2^4 - \frac{559}{48}g_1^4g_2^2 - \frac{379}{48}g_1^6 - 32g_3^2h_t^4 - \\ & -\frac{8}{3}g_1^2h_t^4 - \frac{9}{4}g_2^4h_t^2 + \frac{1}{2}\lambda h_t^2\left(\frac{85}{6}g_1^2 + \frac{45g_2^2}{2} + 80g_3^2\right) + \\ & \left. + g_1^2h_t^2\left(-\frac{19}{4}g_1^2 + \frac{21g_2^2}{2}\right) - \frac{144}{4}\lambda^2h_t^2 - \frac{3}{2}\lambda h_t^4 + 30h_t^6\right)\end{aligned} \tag{B.5}$$

And finally the RGE (at 2 loops for gauge couplings, leading order in h_t) for the SUSY case:

$$\begin{aligned}
\frac{dg_1}{dt} &= \frac{11g_1^3}{(4\pi)^2} + \frac{g_1^3}{(4\pi)^4}\left(\frac{199}{9}g_1^2 + 9g_2^2 + \frac{88}{3}g_3^2 - \frac{26}{3}h_t^2\right) && \text{(B.6)}\\
\frac{dg_2}{dt} &= \frac{g_2^3}{(4\pi)^2} + \frac{g_2^3}{(4\pi)^4}\left(3g_1^2 + 25g_2^2 + 24g_3^2 - 6h_t^2\right) && \text{(B.7)}\\
\frac{dg_3}{dt} &= -\frac{3g_3^3}{(4\pi)^2} + \frac{g_3^3}{(4\pi)^4}\left(\frac{11}{3}g_1^2 + 9g_2^2 + 14g_3^2 - 4h_t^2\right) && \text{(B.8)}\\
\frac{dh_t}{dt} &= \frac{h_t}{(4\pi)^2}\left(6h_t^2 - \frac{13g_1^2}{9} - 3g_2^2 - \frac{16g_3^2}{3}\right) && \text{(B.9)}
\end{aligned}$$

B.2. RGE solutions for the soft terms

In the following we display all the functions that appear in the solution of the RGE for the Higgs mass parameters m_{Hu} and m_{Hd} (see ref. [77]).

First we define the functions

$$E(t) \;=\; (1+\beta_3 t)^{16/(3b_3)}(1+\beta_2 t)^{3/(b_2)}(1+\beta_1 t)^{13/(9b_1)} \quad , \quad F(t) = \int_0^t E(t')dt' \qquad \text{(B.10)}$$

with $\beta_i = \alpha_i(0)b_i/(4\pi)$ and $t = 2\log(M_c/M_{SS})$. The beta-functions coefficients for the SUSY case are $(b_1, b_2, b_3) = (11, 1, -3)$ and we define $\alpha_0 = \alpha(0) = \alpha_i(0) = g_i^2(0)/(4\pi^2)$ for $i = 2, 3$, $\alpha_1(0) = (3/5)\alpha(0) = g_1^2(0)/(4\pi^2)$ where α_0 is the unified coupling at M_c. In our case the couplings do not strictly unify, only up to 5% corrections. In the numerical computations we take the average value of the three couplings at M_c, which is enough for our purposes.

We then define the functions in eqs.(4.239)

$$\begin{aligned}
q(t)^2 &= \frac{1}{(1+6Y_0F(t))^{1/2}}(1+\beta_2 t)^{3/b_2}(1+\beta_1 t)^{1/b_1} \;;\; h(t) = \frac{1}{2}(3/D(t)-1)\\
k(t) &= \frac{3Y_0F(t)}{D(t)^2} \;;\; f(t) \;=\; -\frac{6Y_0H_3(t)}{D(t)^2} \;;\; D(t) \;=\; (1+6Y_0F(t)) \qquad \text{(B.11)}\\
e(t) &= \frac{3}{2}\left(\frac{(G_1(t)+Y_0G_2(t))}{D(t)} \;+\; \frac{(H_2(t)+6Y_0H_4(t))^2}{3D(t)^2} \;+\; H_8\right)
\end{aligned}$$

where $Y_0 = Y_t(0)$ and $Y_t = h_t^2/(4\pi)^2$. The functions $g, H_2, H_3, H_4, G_1, G_2$ and H_8 are independent of the top Yukawa coupling, only depend on the gauge coupling constants and are given by

$$
\begin{aligned}
g(t) &= \frac{3}{2}\frac{\alpha_2(0)}{4\pi}f_2(t) \;+\; \frac{1}{2}\frac{\alpha_1(0)}{4\pi}f_1(t) \\
H_2(t) &= \frac{\alpha_0}{4\pi}\left(\frac{16}{3}h_3(t) \;+\; 3h_2(t) \;+\; \frac{13}{15}h_1(t)\right) \\
H_3(t) &= tE(t) \;-\; F(t) \\
H_4(t) &= F(t)H_2(t) \;-\; H_3(t) \\
H_5(t) &= \frac{\alpha_0}{4\pi}\left(-\frac{16}{3}f_3(t) \;+\; 6f_2(t) \;-\; \frac{22}{15}f_1(t)\right) \\
H_6(t) &= \int_0^t H_2(t')^2E(t')dt' \\
H_8(t) &= \frac{\alpha_0}{4\pi}\left(-\frac{8}{3}f_3(t) \;+\; f_2(t) \;-\; \frac{1}{3}f_1(t)\right)
\end{aligned}
$$

$$
\begin{aligned}
G_1(t) &= F_2(t) \;-\; \frac{1}{3}H_2(t)^2 \\
G_2(t) &= 6F_3(t) \;-\; F_4(t) \;-\; 4H_2(t)H_4(t) \;+\; 2F(t)H_2(t)^2 \;-\; 2H_6(t) \\
F_2(t) &= \frac{\alpha_0}{4\pi}\left(\frac{8}{3}f_3(t) \;+\; \frac{8}{15}f_1(t)\right) \\
F_3(t) &= F(t)F_2(t) \;-\; \int_0^t E(t')F_2(t')dt' \\
F_4(t) &= \int_0^t E(t')H_5(t')dt'
\end{aligned}
\tag{B.12}
$$

where $f_i(t)$ and $h_i(t)$ are defined by

$$
f_i(t) \;=\; \frac{1}{\beta_i}(1 \;-\; \frac{1}{(1+\beta_i t)^2}) \;;\; h_i(t) \;=\; \frac{t}{(1+\beta_i t)} \;. \tag{B.13}
$$

The low energy of the top mass may be obtained from the solutions of the one-loop renormalization group equations, devided into two pieces, SUSY and non-SUSY, i.e. (here $Y_t = h_t^2/(16\pi^2)$)

$$
Y_t(m_t) \;=\; sin^2\beta Y_t(M_{SS})\frac{E'(t_{EW})}{(1+(9/2)sin^2\beta Y_t(M_{SS})F'(t_{EW}))} \tag{B.14}
$$

where

$$
Y_t(M_{SS}) \;=\; Y_t(M_c)\frac{E(t_{SS})}{(1+6Y_t(M_c)F(t_{SS}))} \tag{B.15}
$$

The functions E, F are as defined above, with $t_{SS} = 2log(M_c/M_{SS})$ and $t_{EW} = 2log(M_{SS}/M_{EW})$, while the functions E', F' are analogous to E, F but replacing the b_i and anomalous dimensions by the non-SUSY ones, i.e.

$$
E'(t) \;=\; (1+\beta_3' t)^{8/(b_3^{NS})}(1+\beta_2' t)^{9/(4b_2^{NS})}(1+\beta_1' t)^{17/(12b_1^{NS})} \;\;,\;\; F'(t) = \int_0^t E'(t')dt' \tag{B.16}
$$

with $\beta_i' \;=\; \alpha_i(M_{SS})b_i^{NS}/(4\pi)$, $b_i^{NS} \;=\; (41/6, -19/6, -7)$ and $t \;=\; t_{EW}$. For the anomalous dimensions we have made the change in the definition of $E(t)$ $(13/9, 3, 16/3)$

$\rightarrow (17/12, 9/4, 8)$. And we take the value of $h_t(m_t)$ computed in eq.(4.235) taking into account the threshold corrections at electroweak scale. For this particular computation we take actually as electroweak scale the top mass, so $t_{EW} = 2log(M_{SS}/m_t)$.

Finally, in order to compute the value of the stop mixing parameter X_t we need the following equations for the running of the soft parameters:

$$\begin{aligned}
A_t(t) &= \frac{A}{D(t)} + M(H_2(t) - \frac{6Y_0 H_3(t)}{D(t)}) \\
\mu(t) &= \mu_0 q(t) \\
m_4^2(t) &= M^2(-3\frac{\alpha_2(0)}{4\pi} f_2(t) + \frac{\alpha_1(0)}{4\pi} f_1(t)) \\
m_5^2(t) &= -\frac{1}{3} m^2 + M^2(-\frac{8}{3}\frac{\alpha_3(0)}{4\pi} f_3(t) + \frac{\alpha_2(0)}{4\pi} f_2(t) - \frac{5}{9}\frac{\alpha_1(0)}{4\pi} f_1(t)) \\
m_D^2(t) &= m^2 + 2M^2(\frac{4}{2}\frac{\alpha_3(0)}{4\pi} f_3(t) + \frac{3}{4}\frac{\alpha_2(0)}{4\pi} f_2(t) + \frac{1}{36}\frac{\alpha_1(0)}{4\pi} f_1(t)) \\
m_U^2(t) &= \frac{2}{3} m_{Hu}^2(t) - \frac{2}{3}\mu^2(t) - m_5^2(t) \\
m_Q^2(t) &= \frac{1}{2} m_D^2(t) - \frac{1}{2} m_4^2(t) + \frac{1}{2} m_U^2(t) \qquad \text{(B.17)}
\end{aligned}$$

B.3. Threshold corrections at the EW scale

The functions appearing in the computation of the threshold corrections to the Higgs self-coupling at the weak scale are given by [136]:

$$F_1 = 12\log\left[\frac{Q}{m_h}\right] + \frac{3\log[\xi]}{2} - \frac{1}{2} Z\left[\frac{1}{\xi}\right] - Z\left[\frac{c_W^2}{\xi}\right] - \log\left[c_W^2\right] + \frac{9}{2}\left(\frac{25}{9} - \frac{\pi}{\sqrt{3}}\right) \qquad \text{(B.18)}$$

$$\begin{aligned}
F_0 = & -12\log\left[\frac{Q}{M_Z}\right]\left(1 + 2c_W^2 - \frac{2m_t^2}{M_Z^2}\right) + \frac{3c_W^2 \xi \log\left[\frac{\xi}{c_W^2}\right]}{\xi - c_W^2} + 2Z\left[\frac{1}{\xi}\right] + 4c_W^2 Z\left[\frac{c_W^2}{\xi}\right] + \\
& + \frac{3c_W^2 \log\left[c_W^2\right]}{s_W^2} + 12 c_W^2 \log\left[c_W^2\right] - \frac{15}{2}\left(1 + 2c_W^2\right) - \\
& - \frac{3m_t^2\left(2Z\left[\frac{m_t^2}{M_Z^2 \xi}\right] - 5 + 4\log\left[\frac{m_t^2}{M_Z^2}\right]\right)}{M_Z^2} \qquad \text{(B.19)}
\end{aligned}$$

$$\begin{aligned}
F_3 = & \; 12\log\left[\frac{Q}{M_Z}\right]\left(1 + 2c_W^4 - \frac{4m_t^4}{M_Z^4}\right) - 6Z\left[\frac{1}{\xi}\right] - 12 c_W^4 Z\left[\frac{c_W^2}{\xi}\right] - 12 c_W^4 \log\left[c_W^2\right] + \\
& + 8\left(1 + 2c_W^4\right) + \frac{24 m_t^4}{M_Z^4}\left(Z\left[\frac{m_t^2}{M_Z^2 \xi}\right] - 2 + \log\left[\frac{m_t^2}{M_Z^2}\right]\right) \qquad \text{(B.20)}
\end{aligned}$$

where $\xi = m_h^2/M_Z^2$, $c_W = \cos\theta_W$, $s_W = \sin\theta_W$ and

$$Z(z) = \begin{cases} 2\zeta \arctan\left[\frac{1}{\zeta}\right] & for \quad z > 1/4 \\ \zeta \log[\frac{1+\zeta}{1-\zeta}] & for \quad z < 1/4 \end{cases} \qquad \text{(B.21)}$$

where $\zeta = \sqrt{Abs[1-4z]}$. In the computation we have taken the central experimental values for M_Z, m_t and s_W given by eqs.(4.233,4.234) and the tree level value for the Higgs mass, i.e. $m_h^2 = 2\lambda v^2$ with $v = 174.1$.

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