Deformation of a magnetized quark star in Rastall gravity

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Abstract. We have investigated the influence of strong magnetic fields at the center of the quark stars which is an impact to shape of stars by using Einstein's theory of gravity, which uses the EoS CIDDM model. Our focus in this study is to compare the results of the deformation degree of quark stars from theory of gravity Einstein and Rastall, where on Rastall gravity introduced degree of freedom λ, where we use variations of λ = 1, 2, 3, 4 and 5, where Rastall gravity will reduced to Einstein theory of gravity at the value of $\lambda = 1$. We obtained for each EoS with different values of κ_1 indicate the degree of deformation tends toward prolate at the maximum mass of the same quark star for the both theories, the most significant difference on Rastall gravity is the maximum mass of quark stars increases with the high value of the λ we set.

1. Introduction

Until now, researchers have tried to develop the theory of modified gravity, like the Tensor-Scalar Theory gravity theory, f(R) gravity theory, Rastall Gravity and others (see reviews in Ref. [1-4]) to solve some problems of Einstein general relativity theory.

We chose the Rastall gravity because of its uniqueness in generalizing Einstein's theory of gravity. Peter Rastall argues there is a degree of freedom λ which makes $T^{\mu\nu}_{;\nu}$ not equal to zero in the equations of Einstein's gravitational field, with the aim of overcoming a problem such as if there is a new source around the relativistic object that makes $T^{\mu\nu}$ becomes impermanent (more fully discussed in Ref. [5]).

Rastall gravity was applied to the Neutron Stars by Oliveira on Ref. [5], where sets λ at intervals (1/3, 1/2), (1/2, 1) and (-1, 1/2). Until now, there are a small number of researchers who have use Rastall gravity on relativistic objects. In our research, we used the Rastall Gravity to explain the relation between λ and the magnetic field phenomenon in Quark Stars. We use the CIDDM model EoS Quark Stars which has been proposed by Qauli and Sulaksono [6]. For the effect of the magnetic field, we used the Hartle approaximation, which is expanded Multipole on the Schwarzschild metric [7], and this approach was used by Mallick and Schramm to explain the deformation of Neutron Star due to the presence of a high magnetic field at the center of the Neutron Star [8].

We carried out this study using Rastall's degrees of freedom λ = 1, 2, 3, 4 and 5, where $T^{\mu\nu} = 0$ at λ = 1 (reduced back to the Einstein Field equation). This study aims to compare each value of λ and see what value makes the Quark Stars more deformed.

2. Formalism

In this section we will briefly explain the formalism that we used in this research, we conducted starting from the Rastall gravity, adding magnetic field term to the Hartle approximation and EoS Quark Stars that we use. It should be noted that the formalism for the effect of the magnetic field and the EoS model that we use on this research is explained more detail in Ref. [6,8-10], so here we don't explain more detail on this paper.

We all know, for the static state of relativistic stars derived from Einstein's field equation and obtained TOV equation as follows

$$\frac{dP}{dr} = \frac{-G\varepsilon m}{r^2}\left[1+\frac{P}{\varepsilon}\right]\left[1+4\pi r^3\frac{P}{m}\right]\left[1-\frac{2Gm}{r^2}\right]^{-1}, \quad (1)$$

and

$$\frac{dm}{dr} = 4\pi r^2 \varepsilon. \quad (2)$$

Where equations (1) and (2) obtained for the set value $T^{\mu\nu}_{;\nu} = 0$. In Rastall gravity theory is assumed $T^{\mu\nu}_{;\nu} \neq 0$ as follows [3,4]

$$T^{\mu\nu}_{;\nu} = \frac{1-\lambda}{16\pi G} R^{;\mu}, \quad (3)$$

so the TOV equation can be rewritten as follows

$$\frac{d\not{P}}{dr} = \frac{-G\not{\varepsilon}\not{m}}{r^2}\left[1+\frac{\not{P}}{\not{\varepsilon}}\right]\left[1+4\pi r^3\frac{\not{P}}{\not{m}}\right]\left[1-\frac{2G\not{m}}{r^2}\right]^{-1}, \quad (4)$$

and

$$\frac{d\not{m}}{dr} = 4\pi r^2 \not{\varepsilon}, \quad (5)$$

where $\not{\varepsilon}$ and $\not{P}$ formulated as follows

$$\not{\varepsilon} = a_1\varepsilon + a_2 P, \quad (6)$$

$$\not{P} = a_2\varepsilon + a_3 P, \quad (7)$$

where

$$a_1 = \frac{3\lambda-1}{4\lambda-2}, a_2 = \frac{\lambda-1}{4\lambda-2}, a_3 = \frac{\lambda+1}{4\lambda-2}, \quad (8)$$

we set values of λ = 1,2,3,4 and 5.

We applied the magnetic field effect to the Quark Stars by adding the $B^2/8\pi$ term to the energy-momentum tensor as explained in the Mallick and Schramm formalism [8] where we also had similar formalism with a slightly different equation in the Neutron Stars (for more the details explanation on Ref. [9,10]).

The EoS Quark Stars model we use is CIDDM model using SC (Scalar Coulomb) model with isospin independent parameter set κ_1 = (0.54, 0.45) and VC (Vector Coulomb) with κ_1 = (0.83, 0.63), for a more detailed explanation, refer to the Ref. [6].

3. Result and Discussion

In this section we will explain the results we get, that is the relation between mass and radius stars and the state of the star's profile at the maximum value.

Because there are multipole term due to added magnetic field effects on the Quark Stars and impact on the shape of the star, so the radius is categorized into two, namely Re (equatorial Radius) and Rp (Polar Radius). In Figure 1, we show the plot results for the Coulomb Scalar model with a value of κ_1 = 0.54, can be seen in Figure 1 (a), the Mass-Radius curve for the state without magnetic field for $\lambda = 1 - 5$. we can see the decrease in mass value and increase in radius value if the value of λ is raised. In Figure 1 (b) we take two data λ, namely 1 and 5, where we include the effect of changing the direction of the radius due to the presence of an anisotropic magnetic field in Quark Stars, where the black, red and blue lines respectively represent Radius, Equatorial and Polar Radius, can be seen in the movement for both values λ looks the same, which has the rule of $Re < R < Rp$, where stars are prolate due to the presence of a magnetic field. For the values of λ = 2,3, and 4, we do not include it because it has the same tendency as the two data, for more details can be seen in Table 1.

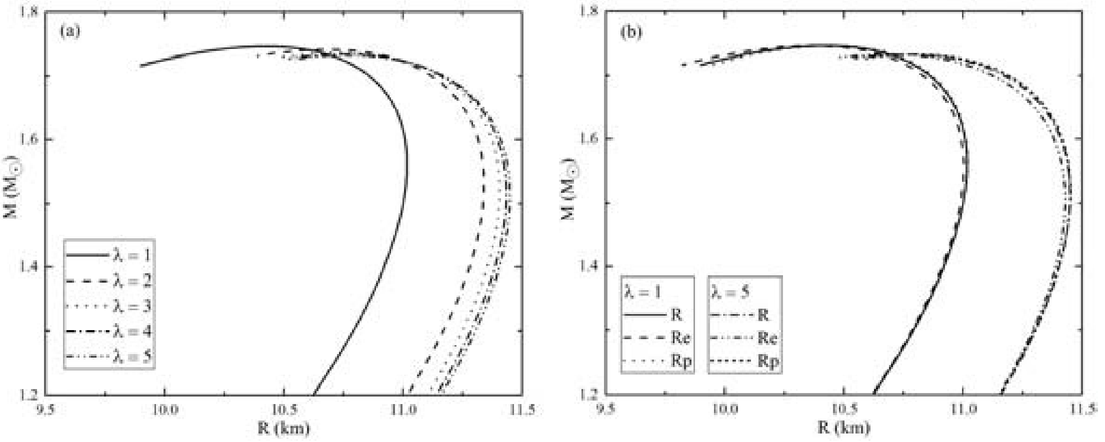

Figure 1. The mass-radius curve for EoS Scalar Coulomb model (κ_1=0.54), (a) without including magnetic field effect with set values of $\lambda = 1 - 5$, (b) including magnetic field effect for $\lambda = 1$ and 5.

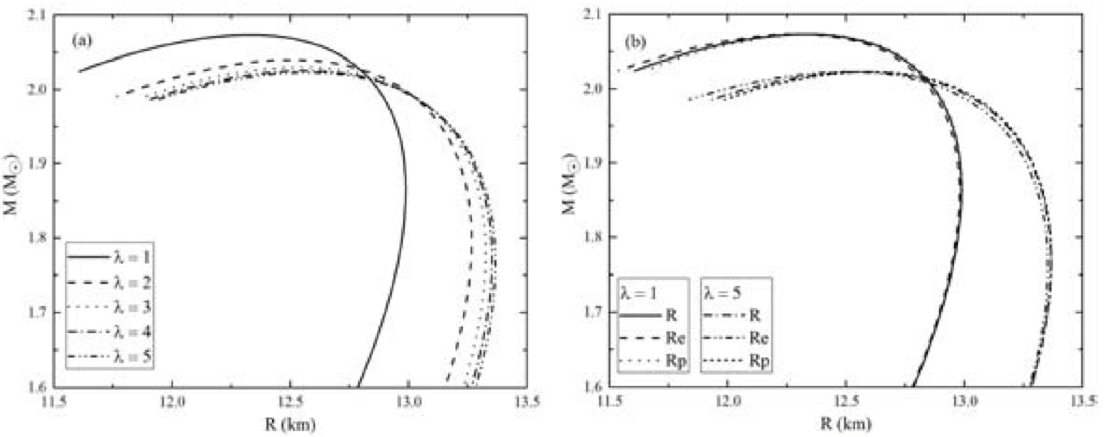

Figure 2. The mass-radius curve for EoS Scalar Coulomb model (κ_1=0.45), (a) without including magnetic field effect with set values $\lambda = 1 - 5$, (b) including magnetic field effect λ = 1 and 5.

In Figure 2, we plot the Massa-Radius curve for the Coulomb Scalar model with $\kappa_1 = 0.45$. Qauli and Sulaksono have reported that mass values will increase if the independent isospin parameters are reduced (for more details see Ref. [6]). However, if we compare it with Figure 1, the change in curve at the addition of the value λ is exactly same as Figure 2. It only differs in mass value. The same thing happens when added magnetic field effects.

Although the trends of the two graphs are the same, there is still a difference if we look at Table 1. where the value of ellipticity (e) indicates the degree of deformation of the stars geometry. There is an increase in the value of e along with an increase in the value of λ, this happens in both EoS Figure 1 and 2, but the value of e for Figure 1 is more significant than the figure 2, it means the small value of κ_1 have a significant deformation.

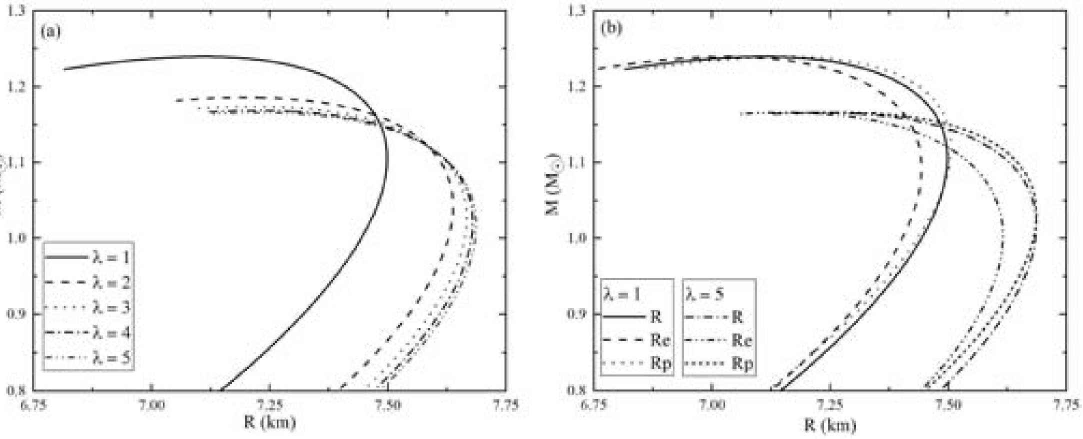

Figure 3. The mass-radius curve for EoS Vector Coulomb model (κ_1=0.83), (a) without including magnetic field effect with set values $\lambda = 1 - 5$, (b) including magnetic field effect λ = 1 and 5.

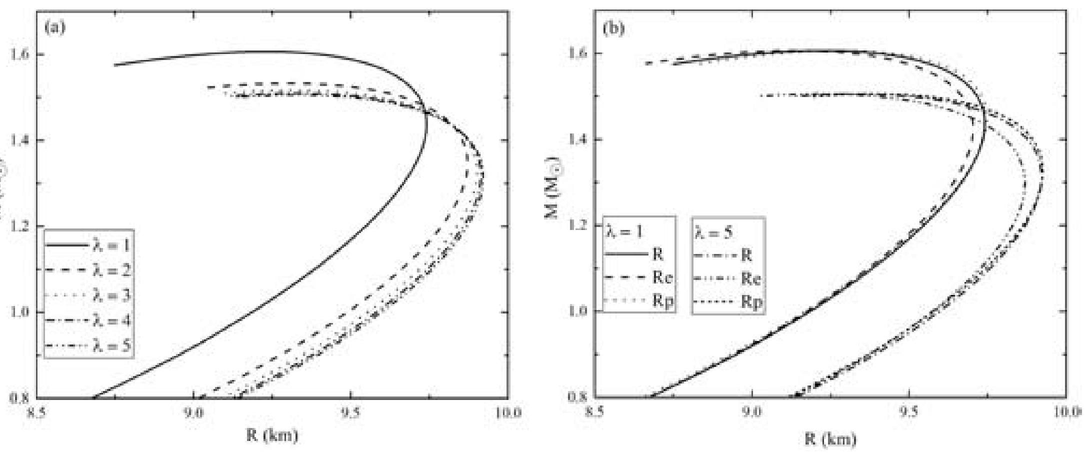

Figure 4. The mass-radius curve for EoS Vector Coulomb model (κ_1=0.63), (a) without including magnetic field effect with set values $\lambda = 1 - 5$, (b)) including magnetic field effect λ = 1 dan 5.

For Figure 3 and 4 represent the EoS Vector Coulomb model with κ_1 values respectively 0.83 and 0.63. If we compare between SC and VC, there is a difference in pressure at the center of the star, wherein for VC has a very high pressure to gain maximum mass compared to the SC as we can see in Table 1.

In Figure 3. there is a difference in the value of e, which is compared with the other three models, the degree of deformation for VC (κ_1 = 0.83) decreases with increasing value of λ (can be seen in Table 1), which may be due to very high pressure in the center of the star.

Table 1. The maximum value of EoS Quark Stars profile.

EoS Model	λ	Pc (MeVfm^{-3})	M (M_{sun})	R (km)	Re (km)	Rp (km)	e
SC (κ_1=0.54)	1	273	1.7462	10.4272	10.3749	10.4552	0.1246
	2	336	1.7414	10.6855	10.6169	10.7203	0.1399
	3	351	1.7365	10.7337	10.6614	10.7698	0.1430
	4	357	1.7339	10.7560	10.6821	10.7926	0.1442
	5	374	1.7323	10.7321	10.6552	10.7706	0.1476
SC (κ_1=0.45)	1	194	2.0733	12.3299	12.2928	12.3497	0.0959
	2	247	2.0402	12.5033	12.4497	12.5303	0.1133
	3	258	2.0305	12.5440	12.4867	12.5723	0.1165
	4	263	2.0261	12.5610	12.5021	12.5899	0.1179
	5	266	2.0235	12.5697	12.5090	12.5989	0.1187
VC (κ_1=0.83)	1	660	1.2396	7.1066	7.0314	7.1522	0.1830
	2	809	1.1861	7.1988	7.1230	7.2396	0.1787
	3	858	1.1740	7.2050	7.1302	7.2446	0.1769
	4	862	1.1687	7.2213	7.1461	7.2602	0.1766
	5	867	1.1657	7.2282	7.1529	7.2668	0.1763
VC (κ_1=0.63)	1	388	1.6069	9.2264	9.1351	9.2821	0.1772
	2	483	1.5331	9.3022	9.1944	9.3608	0.1877
	3	500	1.5169	9.3257	9.2150	9.3841	0.1889
	4	514	1.5099	9.3265	9.2144	9.3851	0.1899
	5	519	1.5060	9.3342	9.2213	9.3926	0.1900

4. Conclusion

In conclusion, the impact of including the Rastall λ degree of freedom in the Quark Stars significantly influences the star's profile, where the addition of the value λ increases the pressure in the center of the star and results in the maximum mass reduction of the star and raises the star radius. If the magnetic field factor is included, it can make the degree of star deformation higher where the star goes into a prolate shape, where the magnetic field exits from the direction of the z-axis Quark Stars. From table 1 it can be ascertained that significant deformations are found in VC κ_1=0.63 in λ = 5, where the higher value of e is 0.19.

An important note from the results of our research exists an oddity in the EoS VC model (κ_1 = 0.83), wherein the model if a given value of λ makes the pressure increase drastically to get the maximum mass value, impact of a decreases the degree of deformation only for that model. Most likely due to very high pressure at the center of Quark Stars (P_c = 867 MeV/fm^3) impact in stars on the magnetic field that we apply to that model will be difficult to deform. For better ensure, we need to investigate the Quark Stars Profile at κ_1 values around 0.83.

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