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The interaction between dark energy and dark matter

Jian-Hua He, Bin Wang

Department of Physics, Fudan University, 200433 Shanghai, China

E-mail: wangb@fudan.edu.cn

Abstract. In this review we first present a general formalism to study the growth of dark matter perturbations in the presence of interactions between dark matter(DM) and dark energy(DE). We also study the signature of such interaction on the temperature anisotropies of the large scale cosmic microwave background (CMB). We find that the effect of such interaction has significant signature on both the growth of dark matter structure and the late Integrated Sachs Wolfe effect(ISW). We further discuss the potential possibility to detect the coupling by cross-correlating CMB maps with tracers of the large scale structure. We finally confront this interacting model with WMAP 5-year data as well as other data sets. We find that in the 1σ range, the constrained coupling between dark sectors can solve the coincidence problem.

1. Introduction

There has been a great deal of evidence at present indicating that our universe has entered an epoch of accelerated expansion[1, 2, 3]. Theoretical effects to interpret the phenomena in accordance with the observational data has to attribute the acceleration of the universe to a so-called dark energy(DE). The leading interpretation of such dark energy is a cosmological constant(CC), but such model suffers the cosmological constant problem and coincidence problem[4]. It is natural to consider the interaction between DE and DM and such consideration has been widely discussed[5] and it is expected that it can provide a mechanism to alleviate the coincidence problem [5, 6, 7, 8]. However, it was claimed that such interaction may cause serious instabilities of the perturbations among dark sectors[9]. Latter, it was clarified [10] that the instability depends on the choice of the forms of the coupling between dark sectors and there exists rooms to avoid such instability. Furthermore, such interacting models are shown viable in confront with observational data. [11, 12, 13, 14, 15].

Nevertheless, the DE is usually not supposed to clump in the formation of cosmic structures, and the most powerful way to unveil its nature is believed to investigate the expansion history of the universe. However, current observations of cosmic expansion cannot break the degeneracies among different approaches to explain the acceleration of the expansion. Considering the interaction between DE and DM, it was observed that due to the coupling, the DE involves in the growth of the cosmic structure [16, 17, 18].

On the other hand, the WMAP data showed the deficit of large scale power in the temperature map, in particular in the CMB quadrupole. The significant contribution to the fluctuations on these scales is the late time Integrated Sachs Wolfe (ISW) effect which is induced by the shift of CMB photons frequencies through the time evolving gravitational potential when the universe enters a rapid expansion phase once DE dominates. The late time ISW effect has the unique ability to probe the "size" of DE. Much effort has been put into determining the EoS and the speed of sound of DE [19, 20]. Moreover,as found in [11], the couplings also has significant influence on the ISW effect and so does the large scale power in the temperature map. In this short review article we present a short review on these results.

The review is organized as follows: In Section 2, we go over the formalism of the perturbation theory in the presence of interactions between dark sectors. In Section 3, we present the influence of interactions on the dark matter structure formation. In Section 4, we talk about the influence on ISW effect due to the interaction between dark sectors. In Section 5, we report the fitting results from WMAP 5-year and other observational data sets on the coupling. In the last Section, we conclude our main results.

2. Analytical formalism

In this section we review the basic formalism of the perturbation theory when dark sectors are in interactions. The detailed descriptions can be found in [10, 16]. The scalar perturbation of the metric at first order for spatially flat Friedmann-Robertson-Walker (FRW) background is of the form

$$ds^2 = a^2[-(1+2\psi)d\tau^2 + 2\partial_i B d\tau dx^i + (1+2\phi)\delta_{ij}dx^i dx^j + D_{ij}E dx^i dx^j], \tag{1}$$

where ψ, B, ϕ, E are scalar metric perturbations, a is the cosmic scale factor and the operator D_{ij} is defined as

$$D_{ij} = (\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2)$$

.

The energy-momentum tensor of a general perfect fluid can be represented as

$$T^{\mu\nu} = \rho U^\mu U^\nu + p(g^{\mu\nu} + U^\mu U^\nu), \tag{2}$$

For a two-component system which is composed of DE and DM, the conservation law is satisfied for the whole system, while for each component we have

$$\nabla_\mu T^{\mu\nu}_{(\lambda)} = Q^\nu_{(\lambda)} \tag{3}$$

where $Q^\nu_{(\lambda)}$ describes the interaction and λ denotes either DE or DM.

In the Fourier space the perturbed energy-momentum tensor reads [16]

$$\begin{aligned}
\delta'_\lambda + 3\mathcal{H}(\frac{\delta p_\lambda}{\delta\rho_\lambda} - w_\lambda)\delta_\lambda = -(1 &+ w_\lambda)kv_\lambda - 3(1+w_\lambda)\phi' + (2\psi - \delta_\lambda)\frac{a^2 Q^0_\lambda}{\rho_\lambda} + \frac{a^2\delta Q^0_\lambda}{\rho_\lambda}. \\
(v_\lambda + B)' + \mathcal{H}(1-3w_\lambda)(v_\lambda + B) &= \frac{k}{1+w_\lambda}\frac{\delta p_\lambda}{\delta\rho_\lambda}\delta_\lambda - \frac{w'_\lambda}{1+w_\lambda}(v_\lambda + B) + k\psi - \frac{a^2 Q^0_\lambda}{\rho_\lambda}v_\lambda \\
&- \frac{w_\lambda a^2 Q^0_\lambda}{(1+w_\lambda)\rho_\lambda}B + \frac{a^2\delta Q_{p\lambda}}{(1+w_\lambda)\rho_\lambda}.
\end{aligned} \tag{4}$$

v is the potential of three velocity and the prime denotes the derivative with respect to the conformal time τ. By constructing the gauge invariant quantities[21],

$$\begin{aligned}
\Psi &= \psi - \frac{1}{k}\mathcal{H}(B + \frac{E'}{2k}) - \frac{1}{k}(B' + \frac{E''}{2k}) \\
\Phi &= \phi + \frac{1}{6}E - \frac{1}{k}\mathcal{H}(B + \frac{E'}{2k})
\end{aligned}$$

$$\begin{aligned}
\delta\rho_\lambda^I &= \delta\rho_\lambda - \rho_\lambda' \frac{v_\lambda + B}{k} \\
\delta p_\lambda^I &= \delta p_\lambda - p_\lambda' \frac{v_\lambda + B}{k} \\
\Delta_\lambda &= \delta_\lambda - \frac{\rho_\lambda'}{\rho_\lambda} \frac{v_\lambda + B}{k} \\
V_\lambda &= v_\lambda - \frac{E'}{2k} \\
\delta Q_\lambda^{0I} &= \delta Q_\lambda^0 - \frac{Q_\lambda^{0'}}{\mathcal{H}}(\phi + \frac{E}{6}) + Q_\lambda^0 \left[\frac{1}{\mathcal{H}}(\phi + \frac{E}{6})\right]' \\
\delta Q_{p\lambda}^I &= \delta Q_{p\lambda} - Q_\lambda^0 \frac{E'}{2k}.
\end{aligned} \tag{5}$$

the perturbed Einstein equations can be written as:

$$\begin{aligned}
&\Phi = 4\pi G \frac{a^2}{k^2} \sum_\lambda \left(\Delta_\lambda + \frac{a^2 Q_\lambda^0}{\rho_\lambda} \frac{V_\lambda}{k} \right) \rho_\lambda \\
&k\left(\mathcal{H}\Psi - \Phi'\right) = 4\pi G a^2 \sum_\lambda \left(\rho_\lambda + p_\lambda\right) V_\lambda \\
&\Psi = -\Phi,
\end{aligned} \tag{6}$$

where we have neglected the anisotropic stress tensor $\Pi_j^i = 0$.

For the coupling, there is no fundamental theory that selects a specific form of the interaction between dark sectors. Here we introduce the phenomenological description of the interaction in the comoving frame[10, 16]

$$\begin{aligned}
Q_c^\nu &= \left[\frac{3\mathcal{H}}{a^2}(\xi_1\rho_c + \xi_2\rho_d), 0, 0, 0\right]^T \\
Q_d^\nu &= \left[-\frac{3\mathcal{H}}{a^2}(\xi_1\rho_c + \xi_2\rho_d), 0, 0, 0\right]^T,
\end{aligned} \tag{7}$$

where ξ_1, ξ_2 are small dimensionless constants and T is the transverse of the matrix. The positive sign of ξ_1, ξ_2 indicates that the energy transfers from DE to DM .

3. Structure formation

The growth of structure happens in small scale. In the subhorizon approximation, from the perturbed Einstein equations eq(6) we can get the "Poission equation"

$$-\frac{k^2}{a^2}\Psi = \frac{3}{2}H^2\left\{\Omega_m\Delta_m + (1-\Omega_m)\Delta_d\right\} \tag{8}$$

and from the equations of matter perturbations eq(4), we obtain the density evolution equations for DM and DE respectively

$$\begin{aligned}
\frac{d^2 ln\Delta_m}{dlna^2} &= -\left(\frac{dln\Delta_m}{dlna}\right)^2 - \left[\frac{1}{2} - \frac{3}{2}w(1-\Omega_m)\right]\frac{dln\Delta_m}{dlna} - (3\xi_1 + 6\frac{\xi_2}{r})\frac{dln\Delta_m}{dlna} \\
&+ \frac{3[exp(ln\frac{\Delta_d}{\Delta_m}) - 1]}{r}\left\{\xi_2 + 3\xi_1\xi_2 + 3\xi_2^2/r + \xi_2(\frac{dlnH}{dlna} + 1) - \xi_2\frac{dlnr}{dlna}\right\} \\
&+ 3\frac{\xi_2}{r}\frac{dln\Delta_d}{dlna}exp(ln\frac{\Delta_d}{\Delta_m}) + \frac{3}{2}\left[\Omega_m + (1-\Omega_m)exp(ln\frac{\Delta_d}{\Delta_m})\right] \quad,
\end{aligned} \tag{9}$$

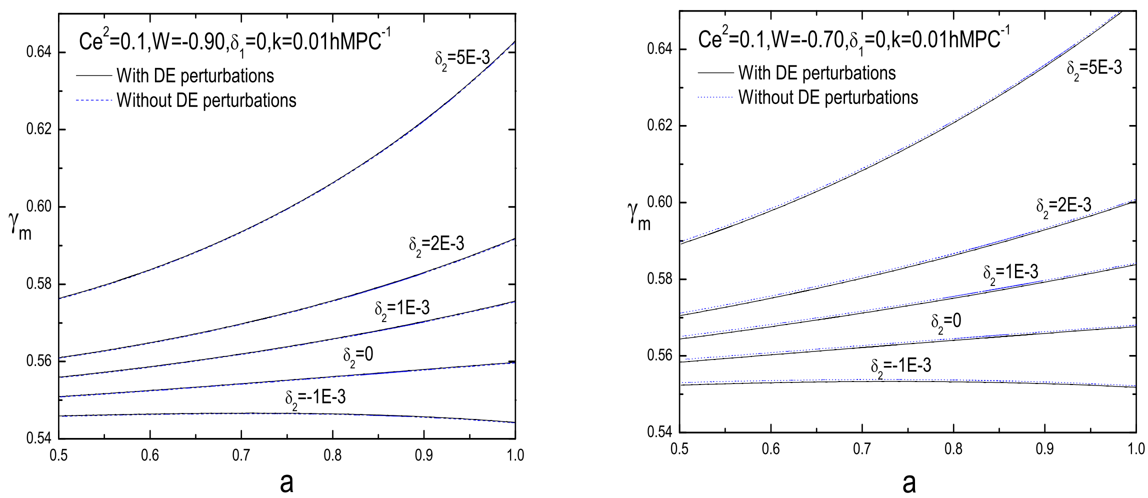


Figure 1. The growth index behavior when the interaction between DE and DM presents. Solid lines are for the result with DE perturbation, while dotted lines are for the result without DE perturbation.

$$\begin{aligned}
\frac{d^2ln\Delta_d}{dlna^2} &= -\left(\frac{dln\Delta_d}{dlna}\right)^2 - \left[\frac{1}{2} - \frac{3}{2}w(1-\Omega_m)\right]\frac{dln\Delta_d}{dlna} - \frac{k^2C_e^2}{a^2H^2} \\
&+ (1+w)\frac{3}{2}\left[\Omega_m exp(ln\frac{\Delta_m}{\Delta_d}) + (1-\Omega_m)\right] - 3\xi_1\left[(\frac{dlnH}{dlna}+1)r + \frac{dr}{dlna}\right]exp(ln\frac{\Delta_m}{\Delta_d}) \\
&+ \left[3\xi_2 + 6\xi_1 r + 6w - 3C_a^2 + 3(C_e^2 - C_a^2)\frac{\xi_1 r + \xi_2}{1+w} + \frac{C_e^2}{1+w}\frac{dln\rho_d}{dlna}\right]\frac{dln\Delta_d}{dlna} \\
&- 3r\xi_1 exp(ln\frac{\Delta_m}{\Delta_d})\frac{dln\Delta_m}{dlna} + 3(\frac{dlnH}{dlna}+1)(w - C_e^2) + 3\xi_1(\frac{dlnH}{dlna}r + r + \frac{dr}{dlna}) \\
&+ 3\left[w - C_e^2 + \xi_1 r(1 - exp(ln\frac{\Delta_m}{\Delta_d}))\right]\left[(1-3w) - 3\frac{C_e^2 - C_a^2}{1+w}(1 + w + \xi_1 r + \xi_2)\right] \\
&+ 3\left[w - C_e^2 + \xi_1 r(1 - exp(ln\frac{\Delta_m}{\Delta_d}))\right]\left[-3(\xi_1 r + \xi_2) - \frac{C_e^2}{1+w}\frac{dln\rho_d}{dlna}\right] \quad . \qquad (10)
\end{aligned}$$

We introduce the growth index γ with the definition [22]

$$\gamma_m = (\ln\Omega_m)^{-1}\ln\left(\frac{a}{\Delta_m}\frac{d\Delta_m}{da}\right). \qquad (11)$$

The growth index is generically not constant which was first emphasized and investigated in terms of cosmological parameters in [23]. The growth index has been argued as a useful way in principle to distinguish the modified gravity models from DE models [24, 25]. In Fig 1, we present our numerical results when we incorporate the interaction between DE and DM. The solid lines are for the results with DE perturbation, while the dotted lines are for the results without DE perturbation. It is clearly shown that the growth index got more influenced from the interaction between dark sectors than the DE perturbation. Although the enhancement of the growth index due to the interaction is clearly shown, the available accuracy from the observations such as DUNE[26] etc. is calculated for the ΛCDM model and may not be true for our interacting cosmology, since the current values of Ω_{c0} may not be typically equal in the two cases as argued in [17]. However, our result is interesting, as it opens the possibility that the future measurement of the growth factor may be helpful to reveal the presence of the interaction between DE and DM.

4. ISW effect

The ISW effect can be classified into early and late time effects. The early ISW effect takes place from the time following recombination to the time when radiation is no longer dynamically significant, which gives clues about what has happened in the universe from the radiation domination to matter domination. The late time ISW effect arises when DE becomes dynamically important and has the unique ability to probe the "size" of DE. When DE becomes non-negligible, the gravitational potential decays. When a photon passes through a decaying potential well, it will have a net gain in energy and thus leads to the late time ISW effect. The late time ISW effect is a significant contribution to the large scale power in the temperature map of CMB.

However, such late time ISW effect can not be measured directly from observations. The most efficient way is to investigate the galaxies-ISW correlation. The angular correlation function for galaxies-ISW and galaxies-galaxies correlation are represented as

$$C_l^{gg} = 4\pi \int \frac{dk}{k} \mathcal{P}_\chi(k) I_l^{g*}(k) I_l^g(k) \tag{12}$$

$$C_l^{gI} = 4\pi \int \frac{dk}{k} \mathcal{P}_\chi(k) I_l^{g*}(k) \Delta_l^{ISW}(k), \tag{13}$$

where

$$\Delta_l^{ISW} = \int_{\tau_i}^{\tau_0} d\tau j_l(k[\tau_0 - \tau]) e^{\kappa(\tau_0)-\kappa(\tau)} [\Psi' - \Phi'], \tag{14}$$

is the contribution due to the change of the gravitational potential when photons passing through the universe on their way to earth, namely the ISW effect. The integrand $I_l^g(k)$ for galaxy densities reads,

$$I_l^g(k) = \int dz b_g(z) \Pi(z) (D_{gc} + D_{gb}) j_l[k\chi(z)]. \tag{15}$$

Here $b_g(z)$ is the galaxy bias, $\Pi(z)$is the redshift distribution and $\chi(z)$ is the conformal distance, or equivalently the look-back time from the observer at redshift $z = 0$,

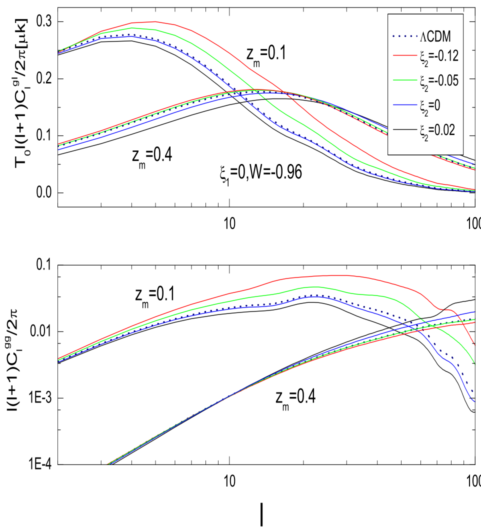
$$\chi(z) = \int_0^z \frac{dz'}{H(z)} = \int_{\tau_i(z)}^{\tau_0} d\tau. \tag{16}$$

We assume $b(z) \sim 1$ for simplicity and adopt the redshift distribution of the form[27],

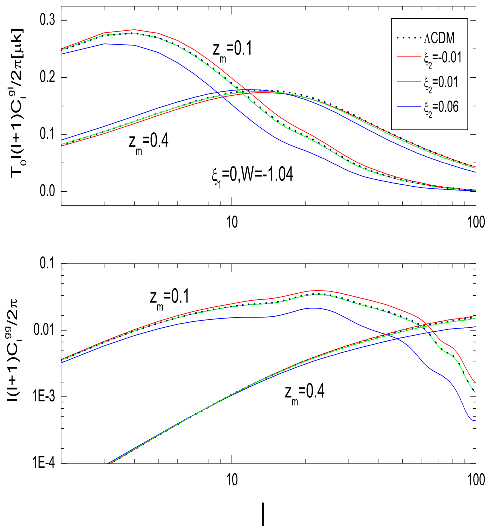
$$\Pi(z) = \frac{3}{2}\frac{z^2}{z_0^3} \exp\left[-(\frac{z}{z_0})^{3/2}\right] \tag{17}$$

which is normalized to unity and peaks near the median redshift $z_m = 1.4z_0$. For illustrative purpose,we choose $z_m = 0.1$ and $z_m = 0.4$ throughout our analysis. The first choice resembles a shallow survey like 2MASS and the second one resembles a survey similar to SDSS photo-z galaxy samples.

When the coupling is proportional to the energy density of DE and the EoS is larger than $-1 (w > -1)$, the cross power spectra and auto correlation power spectra of galaxies are shown



(a)



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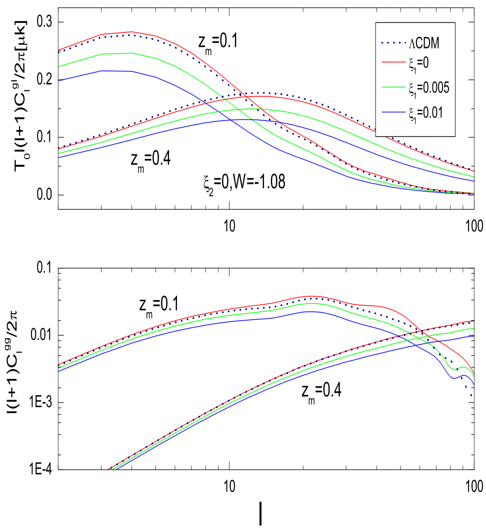
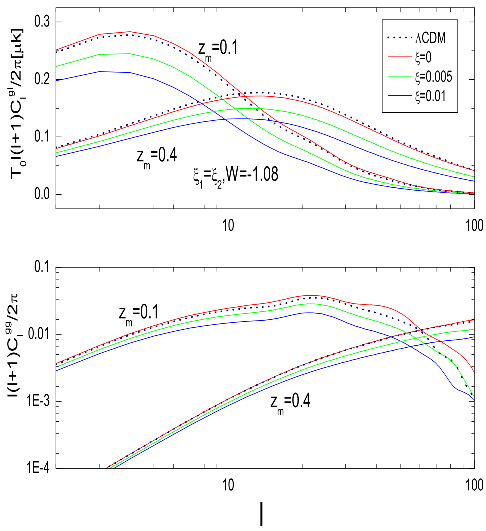


(c)



(d)

Figure 2. The upper panel shows the cross-spectra and the lower panel shows the galaxies auto power spectra.

in Fig 2a. For lower redshift galaxies survey $z_m = 0.1$, comparing with the LCDM model we see that the couplings significantly change both the cross spectra and the auto spectra. The negative couplings enhance the power of the correlation while the positive couplings hinder such correlation. When the EoS is smaller than -1, we find very similar results as shown in Fig 2b. However, for deeper redshift galaxies survey $z_m = 0.4$, the couplings do not imprint significantly on the cross power spectra and auto correlation power spectra of galaxies as compared with $z_m = 0.1$.

When the coupling is proportional to the energy density of DM or total dark sectors, we find from Fig 2c,Fig 2d that the cross power spectra are more sensitive to the couplings at lower l part than that of higher l part when the median redshift around 0.1. This feature is different from that shown in Fig 2a and Fig 2b when the interaction is proportional to the energy density of DE, where it was found that at small l when ISW effect amplified, the ISW-LSS cross-correlation is not so much different due to the coupling. While in the other way, for higher redshift survey $z_m = 0.4$, we find less such effect on the cross and auto spectra.

The qualitative behaviors presented here show that the 2MASS survey has more possibility in discriminating the interaction between dark sectors than that of SDSS survey. It is expected in the future that galaxy surveys with photo-z measurements or even spec-z measurements, along with better CMB measurements, could provide better ISW-LSS cross-correlation measurements at each redshift bin and provides more precise signature of the interaction between dark sectors.

5. Global fitting and coincidence problem

In this section we confront the interacting model with observational data. We take the parameter space as

$$P = (h, \omega_b, \omega_{cdm}, \tau, \ln[10^{10}A_s], n_s, \xi_1, \xi_2, w)$$

where h is the hubble constant, $\omega_b = \Omega_b h^2, \omega_{cdm} = \Omega_{cdm}h^2$, A_s is the amplitude of the primordial curvature perturbation, n_s is the scalar spectral index, ξ_1 and ξ_2 are coupling constants proportional to the energy density of DM and DE respectively, w is the EoS of DE. We choose the flat universe with $\Omega_k = 0$ and our work is based on CMBEASY code[28].

In the global fitting, we have used CMB data coming from WMAP5 temperature and polarization power spectra. We used Gibbs sampling routine provided by WMAP team for the likelihood calculation. In the small scale CMB measurements, we included BOOMERanG[29], CBI [30], VSA[31] and ACBAR[32] data. In order to get better constraint on the background evolution, we have added SNIa[33] data and marginalized over the nuisance parameters. We also incorporated the data from large scale luminous red galaxies(LRGs) survey, we used SDSS[34] data as powerful constraint on real-space power spectrum $P(k)$ at redshift $z \sim 0.1$.

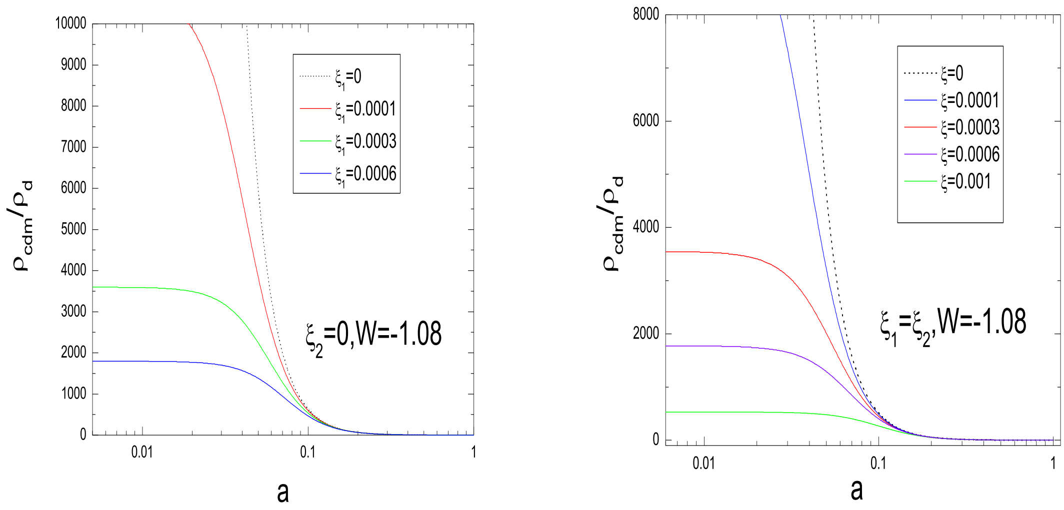
The global fitting results for different forms of interaction between dark sectors are shown in Table 1. When the coupling proportional to the energy density of DM($\xi_1 \neq 0, \xi_2 = 0$) and total dark sectors ($\xi_1 = \xi_2 = \xi$), we constrain the coupling constant to be positive to avoid the zero crossing problem of the energy density of DE at the early time of the universe. From the fitting results, we find that in 1σ range the coupling naturally is positive. Thus with this kind of interaction the coincidence problem can be overcome(see, Fig 3a,b).

6. Conclusions and discussions

We have reviewed the formalism of the perturbation theory when there is interaction between DE and DM. Based upon this formalism we have studied the signature of the interaction between dark sectors on the structure formation and the large scale temperature CMB fluctuations. We found that the effect of the interaction between dark sectors overwhelms that of the DE perturbation on the growth function of DM perturbation. When the DE EoS w is in the vicinity of -1 somewhere around the best fitted value at the moment, the DE perturbation is

Table 1. The best-fitted results for coupling models

h	$\Omega_b h^2$	$\Omega_{cdm} h^2$	n_s	$\xi_1 = 0, \xi_2 \neq 0$	$1 + w > 0$
$0.687^{+0.013}_{-0.013}$	$0.0225^{+0.0006}_{-0.0005}$	$0.107^{+0.007}_{-0.009}$	$0.963^{+0.013}_{-0.013}$	$-0.028^{+0.023}_{-0.032}$	< 0.052
h	$\Omega_b h^2$	$\Omega_{cdm} h^2$	n_s	$\xi_1 = 0, \xi_2 \neq 0$	$1 + w < 0$
$0.700^{+0.012}_{-0.011}$	$0.0223^{+0.0006}_{-0.0005}$	$0.119^{+0.009}_{-0.006}$	$0.956^{+0.013}_{-0.014}$	$-0.010^{+0.025}_{-0.020}$	$-0.051^{+0.051}_{-0.043}$
h	$\Omega_b h^2$	$\Omega_{cdm} h^2$	n_s	$\xi_1 > 0, \xi_2 = 0$	$1 + w < 0$
$0.690^{+0.015}_{-0.015}$	$0.0224^{+0.0005}_{-0.0005}$	$0.121^{+0.003}_{-0.003}$	$0.953^{+0.013}_{-0.013}$	$0.0007^{+0.0006}_{-0.0006}$	$-0.072^{+0.072}_{-0.053}$
h	$\Omega_b h^2$	$\Omega_{cdm} h^2$	n_s	$\xi = \xi_1 = \xi_2 > 0$	$1 + w < 0$
$0.690^{+0.014}_{-0.014}$	$0.0224^{+0.0006}_{-0.0006}$	$0.121^{+0.004}_{-0.003}$	$0.955^{+0.014}_{-0.014}$	$0.0006^{+0.0006}_{-0.0005}$	$-0.065^{+0.065}_{-0.054}$



(a) (b)

Figure 3. The evolution of the ratio ρ_{cdm}/ρ_d for different coupling models.

suppressed, however, when the interaction presents, the growth index can differ from the value without interaction by a big amount up to the observational sensibility.

We also found that in addition to disclosing the DE EoS, sound speed, the late ISW effect is a promising tool to measure the coupling between dark sectors. When the interaction between DE and DM takes the form proportional to the energy density of DM or of the total dark sectors, because in these cases DE and DM started to chase each other since early time, the interaction not only presented in the late ISW source term but also left imprint in the SW and early ISW effects. These properties provide a possible way to examine the interaction between DE and DM even from smaller scale in CMB observations.

We have performed the global fitting by using the CMB power spectrum data including WMAP5 data and balloon observational data together with SNIa and SDSS data to constrain the interaction between DE and DM. When the interaction between DE and DM takes the form proportional to the energy density of DM or of the total dark sectors, the coupling can be constrained in a very precise range. In 1σ the coupling is positive indicating that there is energy transfer from DE to DM. This kind of energy transfer can help to alleviate the coincidence problem compared to the noninteracting case.

It is of great interest to extend our study to a field theory description of the interaction between DE and DM and examine its signature in observations. A possible field theory model was proposed in [35] and further investigation in this direction is called for.

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