METHOD OF COMPUTATION OF DRIFT TUBE SHAPES

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During the investigation of possible geometries for the radiofrequency system of the Brookhaven linear accelerator, one of the schemes considered by the writer was the travelling-wave, cavity accelerator (see fig. 1) in which resonant cavities are arranged in sequence, mutually coupled through small holes in their common walls. Investigation of the losses in such resonant cavities showed that, as the ratio of drift-tube diameter to tank diameter was decreased, the losses also decreased in spite of the fact that the magnetic field intensity increased sharply in the region 3 (fig. 1) in the center of the cylindrical part of the drift tube. The decrease in losses is due to the decrease in gap capacity and hence in the currents necessary to charge that capacity. Therefore it appeard possible to decrease the losses still further by shaping the drift tube as shown in fig. 2 so that the gap capacity is low, but the diameter at the center is increased so that the field is less concentrated in that region. Two problems arise in this approach: first, solution of the field equations is extremely difficult and, second, there is no assurance that any particular assumed oval shape presents the optimum shunt impedance for a given tank diameter, and gap to pitch ratio.

In view of the above, a quite different procedure was followed to solve the problem. Instead of assuming a shape and then solving the field equations, it was assumed that the solution is the sum of a *finite* number of cylindrical harmonics; after determining the coefficients of these harmonics from the boundary conditions, the shape is determined by numerical integration of the equation

$$\frac{dr}{dz} = -\frac{E_z}{E_r} \tag{1}$$

where r and z are cylindrical coordinates and E_r and E_z are the radial and axial components of the electric field.

The following quantities are considered to be given:

b = radius of the tank
L = Length of the unit cell
g = gap between drift tubes
k = $2\pi/\lambda$ (λ is the free space wavelength).

Because of the capacitative load of the drift tubes, the tank radius must be less than the radius given by the first zero of the Bessel (J_0) function. Consequently, to make E_z zero at the outer radius it is necessary to include the Neumann function which becomes infinite on the axis. Hence different solutions must be used for the region around the axis and the region remote from the axis. These solutions will be matched at an intermediate radius a. In these two regions the axial component of electric field will be assumed to have the forms:
for $a < r < b$

$$E_{1z} = c_1 J_0(kr) - c_2 Y_0(kr) + \sum_{n=1}^{n=m} \alpha_n K_0(k_n r) \cos(n k_L z) \tag{2}$$

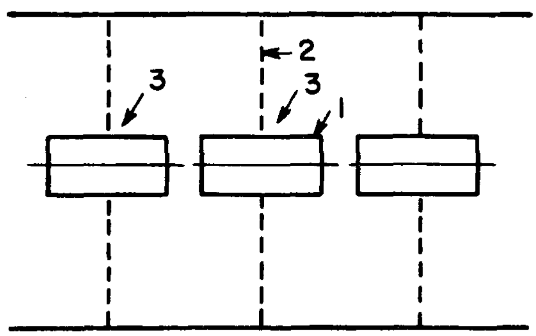

Fig. 1.

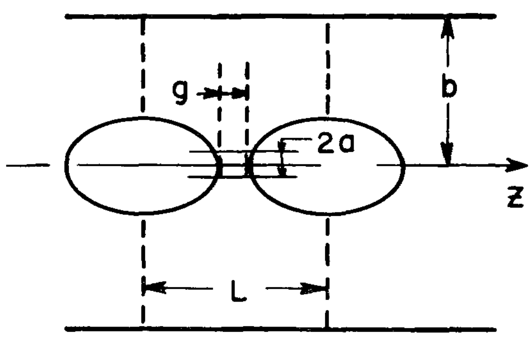

Fig. 2.

for $0 < r < a$

$$E_{2z} = \beta_0 J_0 (kr) + \sum_{n=1}^{n=m} \beta_n I_0 (k_n r) \cos (nk_L z) \tag{3}$$

where J_0 and Y_0 are the Bessel and Neuman functions respectively

$$\begin{aligned} K_0 (kr) &= i(\pi/2) H_0^{(1)} (ikr) \\ I_0 (kr) &= J_0 (ikr) \\ k_L &= 2\pi/L \\ k_n^2 &= (nk_L)^2 - k^2. \end{aligned}$$

In principle the matching radius, a, is arbitrarily chosen but in practice it is found that its value must lie between rather close limits. At the matching radius it is possible to match exactly two components: the radial component of the electric field and the azimuthal component of the magnetic field. Then the average value of the other component (the axial electric field) across the gap is matched thus:

$$\int_0^{g/2} E_{1z} dz = \int_0^{g/2} E_{2z} dz \tag{4}$$

As a result of matching only the average value of the axial component, the electric field will be discontinuous at the drift-tube surface. Since the drift-tube surface is determined by numerical integration of equation (1), it is obvious that a physical discontinuity will appear on the drift tube. To avoid such a discontinuity we require that $E_r = 0$ at $r = a$, $z = g/2$. Then the slope dr/dz become infinite at that point and the physical discontinuity is removed.

The field components can now be tabulated as follows: for $a < r < b$

$$E_z = \alpha (g/L) C_0 (kr) + \sum_{n=1}^{n=m} \alpha_n K_0 (k_n r) \cos (nk_L z) \tag{5}$$

$$E_r = - \sum_{n=1}^{n=m} \alpha_n (nk_L/k_n) K_1 (k_n r) \sin (nk_L z) \tag{6}$$

$$H_\varphi = \alpha (g/L) C_1 (kr) - \sum_{n=1}^{n=m} \alpha_n (k/k_n) K_1 (k_n r) \cos (nk_L z) \tag{7}$$

for $0 < r < a$

$$E_z = \beta_0 J_0 (kr) + \sum_{n=1}^{n=m} \beta_n I_0 (k_n r) \cos (nk_L z) \tag{8}$$

$$E_r = \sum_{n=1}^{n=m} \beta_n (nk_L/k_n) I_1 (k_n r) \sin (nk_L z) \tag{9}$$

$$H_\varphi = \beta_0 J_1 (kr) - \sum_{n=1}^{n=m} \beta_n (k/k_n) I_1 (k_n r) \cos nk_L z \tag{10}$$

$$\text{where } C_0 (kr) = \frac{Y_0 (kb) J_0 (kr) - J_0 (kb) Y_0 (kr)}{Y_0 (kb) J_0 (ka) - J_0 (kb) Y_0 (ka)} \tag{11}$$

$$C_1 (kr) = \frac{Y_0 (kb) J_1 (kr) - J_0 (kb) Y_1 (kr)}{Y_0 (kb) J_0 (ka) - J_0 (kb) Y_0 (ka)} \tag{12}$$

From the requirement that E_r and H_φ are to be identical functions of z at the matching radius a, we determine the constants

$$\beta_0 = \alpha \frac{g}{L} \frac{C_1 (ka)}{J_1 (ka)} \tag{13}$$

$$\beta_n = -\alpha_n \frac{K_1 (k_n a)}{I_1 (k_n a)} \tag{14}$$

Three conditions remain to be satisfied

$$\int_0^{g/2} E_{1z} dz = 1 \tag{15}$$

$$\int_0^{g/2} E_{2z} dz = 1 \tag{16}$$

$$E_r = 0 \text{ at } r = a, \quad z = g/2 \tag{17}$$

The equations (15) - (17) can define three coefficients. Consequently we can choose $m = 2$ in equations (5) to (10). We now have α, α_1, and α_2 determined by equations (15) - (17) and the β's determined by equations (13) and (14).

Actually, the harmonics involving the K_0-functions should have had the form

$$\alpha_n K_0 (k_n r) - \gamma_n I_0 (k_n r)$$

with coefficients chosen so that the combination is zero at the tank wall. But it turns out that, at the tank wall,

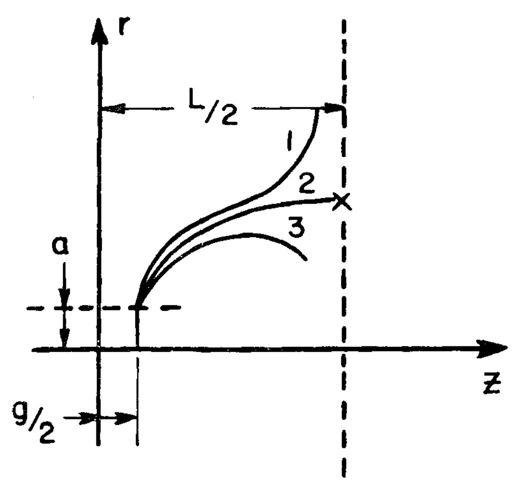

Fig. 3.

the K_0-functions are several orders of magnitude smaller than the C_0-functions and, consequently, the γ's can be neglected.

When the coefficients have been determined as described above, and E_z and E_r are known, it is possible to proceed with numerical integration of equation (1). The integration starts from the point 0 (fig. 4) where $r = a$, $z = g/2$. The result of the integration is usually one of the curves indicated by 1 or 3 in fig. 3. Curve 3 turning downward offers no solution and indicates that the matching radius selected was too small. A higher value of matching radius is selected and the procedure is repeated. A limiting solution is indicated by the curve 2 in fig. 3. At $z = L/2$ and at the radius of the drift tube, both E_z and E_r are zero. In this case, the drift-tube surface is continuous with the surface of the drift tube in the adjacent unit cell and the wall (2, fig. 1) can be removed. The system has now become the Berkeley system, but with new drift-tube shapes. This particular solution is the one which has been choosen for use in the Brookhaven machine.

With the additional requirement that $E_z = E_r = 0$ on the drift-tube surface at $z = L/2$, it is possible to find a solution for only one value of the matching radius a. Consequently, in order to vary and minimize the shunt impedance, one more harmonic was introduced and the constant α is now arbitrarily selected. α is now varied to find the solution which gives optimum shunt impedance.

The results of the analysis thus far are as follows :

1. A constant tank diameter can be used throughout the length of the linear accelerator from 0.75 Mev to 50 Mev. Some difficulty was experienced in the high energy range where it has been found necessary to add one more harmonic. The constant of this harmonic is determined by the additional requirement that

$$(g/2)E_z \text{ (at } z = 0,\ r = a) = \int_0^{g/2} E_z dz.$$

2. With a tank diameter of 92 cm. the diameter of the drift tubes varies from 25 cm. at 0.75 Mev to 18 cm. at 50 Mev. This large drift-tube diameter is more than adequate to contain the strong focusing quadruploes plus an efficient water-cooling so that the drift-tube temperature perturbations will be held down to about a tenth of a degree centigrade. Due to this efficient watercooling (the temperature difference between cooling water and drift-tube surface will not exceed .2 or .3 °C), it is possible to construct a Berkeley-type machine 110 ft long with a flat mode. It can be shown that the first harmonic of the temperature perturbation along the tank should be kept within .5 °C in order to maintain a mode flatness of 5%. With the water-cooling now possible, both the drift tubes and the tank can be maintained within these temperature limits; consequently automatic tuning devices to maintain the mode flatness are not necessary.

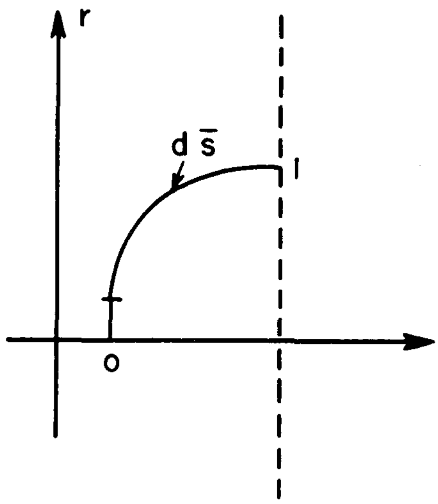

Fig. 4.

3. The overall power consumption, estimated from computation of five drift tubes along the machine, appears to be less than 2.5 megawatts.

A prototype cavity including a drift-tube shape computed by the above method has been constructed and tested. The measured resonant frequency agreed with that predicted within one tenth of one percent.

The problem of computing the necessary 124 drift-tube shapes is now (April, 1956) being programmed on the UNIVAC. The UNIVAC program also includes computation and optimization of shunt impedance for about ten drift tubes distributed throughout the range. This should be sufficient to establish the optimum parameters for the intermediate drift tubes by interpolation.

A typical drift tube computed by the above methods is shown in fig. 6 of the paper by J. P. Blewett. (See p. 159).