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Nuclear physics with an effective field theory around the unitarity limit

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Summary. — I outline the first steps in the process of describing nuclear ground states with an expansion around two-body unitarity. At leading order, the effective field theory displays discrete scale invariance with a single dimensionful parameter, which determines all ground-state and low-lying excitation energies. Sub-leading corrections are smaller and perturbative. Results for light nuclei, large bosonic clusters at unitarity, and nuclear matter are reported.

1. – Introduction

San Domenico is credited with the quote that "We must sow the seed, not hoard it". Next door to his resting place, I indulged in my favorite proselytism, that of effective field theories (EFTs).

The picture that EFT offers of nature is more akin to neo-impressionism than to the painting, which I suppose depicted San Domenico, under which we spoke at this conference. At a resolution scale $M_{\rm lo}^{-1}$, we do not need, and do not depict, details at much shorter range $M_{\rm hi}^{-1} \ll M_{\rm lo}^{-1}$. It is enough that we consider the most general dynamics of the appropriate degrees of freedom at $M_{\rm lo}^{-1}$, constrained only by the known symmetries. Dynamics at distances shorter than $M_{\rm hi}^{-1}$ is encapsulated in the interaction strengths. The S matrix—from which observables can be extracted—is obtained as a systematic expansion in the small ratio $M_{\rm lo}/M_{\rm hi}$. This paradigm is useful not only when probing deviations from the Standard Model of particle physics, where the shorter-distance theory is unknown, but also when the underlying theory is difficult to solve, such as the QCD behind nuclear physics.

EFT has been applied to nuclear physics for more than 25 years now—for a review, see Ref. [1]. Most work has revolved around a particular EFT, Chiral EFT. "Chiral potentials" collecting some Feynman diagrams have been used as input to "*ab initio*" methods for the solution of the Schrödinger equation, with considerable phenomenological success. Unfortunately, this success has usually relied on high orders in the expansion, where the large number of parameters obscures the physics that should have emerged cleanly at leading order (LO). Moreover, most calculations compromise one of the pillars that sustains EFT, renormalization-group (RG) invariance.

RG invariance is necessary to free a hadronic theory from extraneous assumptions about the QCD dynamics. It was the effort to understand the challenges of renormalizing Chiral EFT that originally motivated a simpler EFT, Pionless EFT. Chiral EFT is formulated in terms of pions and the lowest nucleon excitations, in addition to nucleons—all other degrees of freedom are "integrated out". In Pionless EFT, we focus on an $M_{\rm lo}$ small enough for even pions to be integrated out, contact interactions among nonrelativistic nucleons being all that remains. For over two decades now, Pionless EFT has been applied to nuclear and atomic systems with great success *and* renormalizability.

From the get-go it was understood that the usefulness of Pionless EFT stemmed from the existence of shallow bound states such as the deuteron. "Shallow" here means a state with size—measured in the two-body case by the scattering length—much larger than the range of the underlying potential. The new development, which I elaborate upon below, is the realization [2] that a good starting point for the EFT expansion is the unitarity limit of an infinite scattering length. In this case, at LO the two-body system is invariant under scale transformations, while properties of larger systems are determined predominantly by a *single* parameter associated with *discrete* scale invariance. We are starting to explore the extent to which nuclear physics can be described in a first approximation by this one parameter, with everything else accounted for perturbatively.

2. – Pionless EFT around unitarity

For distances $M_{\rm lo}^{-1}$ much larger than the range of the interaction—set by the inverse of the pion mass and itself much larger than the inverse nucleon mass m_N—the only relevant degrees of freedom are nucleons, encoded in a color-singlet, isospin-doublet, two-component spinor field. Lorentz invariance can be implemented through an $M_{\rm lo}/m_N$ expansion, expected to converge faster than the $M_{\rm lo}/M_{\rm hi}$ expansion. Gauge invariance under the $U_{\rm e}(1)$ of electromagnetism is incorporated in the usual way, namely by using electromagnetic covariant derivatives and field strength. Violations of parity, time reversal, and baryon number amount to additional, very quickly convergent $M_{\rm lo}/M_{\rm vi}$ expansions, where $M_{\rm vi}$ represents the high-energy scales associated with the breaking of these symmetries.

Because relatively little specific to QCD remains in this EFT, essentially the same theory applies to neutral atoms at distances beyond the so-called "Van der Waals length", which sets the scale of the Van der Waals potential for a particular atomic species. A spinless atom such as ^{4}He is represented by a scalar field, but otherwise the structure of the theory is very close to Pionless EFT in channels where nucleons are not restricted by the exclusion principle. With some abuse of language, I refer to both theories as "Pionless EFT" and to either nucleons or bosonic atoms as "particles" of mass m, represented by a field ψ of canonical dimension $3/2$.

The most general Lagrangian contains, in addition to particle kinetic terms, all local interactions with an even number of particle fields. Particle-antiparticle loops are short-range effects that can be absorbed in interaction strengths, so that the theory splits into sectors of definite particle number A. Interactions with $2A$ fields give rise to A-body potentials represented by Dirac delta functions and its derivatives. The S matrix can be obtained by solving the Schrödinger equation, or one of its many-body variants, with these potentials.

The organizational principle to order interactions according to their importance to observables—"power counting"—depends on how the interaction strengths scale with $M_{\rm lo}$ and $M_{\rm hi}$. When all interaction strengths scale with inverse powers of $M_{\rm hi}$ according to their canonical dimension, the EFT is fully perturbative and thus contains no bound states within its regime of validity set by $M_{\rm hi}$. In particular, few-body forces come from operators with larger number of fields and thus higher canonical dimension, so they are highly suppressed in comparison with two-body forces. In contrast, in a situation like nuclear physics where there are bound states and resonances at low energies, interaction strengths must scale with some inverse powers of $M_{\rm lo}$. One of the surprises is the importance of a three-body force. A thorough discussion of Pionless EFT with original references can be found Ref. [1].

For nuclei as well as ^{4}He atomic clusters, the $A = 3, 4$ ground states are considerably deeper than the $A = 2$ ground state, while still larger than the range of the potential. We have thus proposed [2] that as far as the physics of nuclear ground states is concerned, we should take the $A = 2$ system at LO to be in the so-called unitarity limit, where the two-body binding energy $B_2 = 0$ or, alternatively, the inverse two-body scattering length $a_2^{-1} = 0$. For some atomic systems, this limit can be achieved by dialing an external magnetic field—a "Feshbach resonance". A possible explanation for the large scattering lengths in the nuclear case is a Feshbach-like resonance generated by variation of the quark masses [3]. Incidentally, Pionless EFT can be matched to present-day lattice QCD simulations of light systems at unphysically large quark masses, and then used with *ab initio* methods to predict larger systems [4]—but that is a topic for another talk.

3. – Potential, regularization, and renormalization

Around unitarity there must exist an LO two-body contact interaction with just enough attraction to bring a bound state to the verge of existence. There are no important two-body parameters, which appear only at sub-leading orders. The scale $M_{\rm lo}$ is identified with the typical nucleon binding momentum in $A > 2$ ground states, and must arise through a few-body force. What additional interactions appear at leading and higher orders is not immediately clear, because power counting in the presence of the fine tuning necessary to produce shallow states goes beyond dimensional analysis. Renormalization provides a useful tool to infer the minimum acceptable power of $M_{\rm lo}^{-1}$ for a given interaction.

The quantum nature of the problem materializes in diagrams with loops, which in Pionless EFT are all generated by the solution of the Schrödinger equation. Loop diagrams are sensitive to high momenta and require regularization. An additional dimensionful quantity, the regulator Λ, cuts off virtual momenta. Before renormalization, it appears explicitly through powers of $\Lambda/M_{\rm lo}$, which threaten the $M_{\rm lo}/M_{\rm hi}$ expansion. The regulator represents an infinite number of short-range interactions correlated by the same parameter Λ. In a model, such a correlation is just part of the dynamical assumptions and usually no attempt is made to remove Λ from observables. The regulator is interpreted as a physical form factor.

In contrast, in an EFT, all allowed interactions, which build up form factors and everything else, are already present. For these interactions to be model independent, their strengths must be correlated by the underlying physics, not the *ad hoc* Λ. Renormalization is the procedure that removes the dependence on the arbitrary regulator from observables and guarantees that parameters determined by the underlying dynamics are present instead. A power counting must ensure RG invariance at each order in the

$M_{\rm lo}/M_{\rm hi}$ expansion, up to terms of size comparable to higher-order terms. This can be achieved by taking $\Lambda \gtrsim M_{\rm hi}$ since after renormalization only *negative* powers of $\Lambda/M_{\rm lo}$ remain in *observables*. The regulator determines the "model space" where the EFT calculation is performed. Just as in standard many-body *ab initio* calculations, we want the limit of a large model space to be well defined so that when the cutoff is large enough results are "converged".

By now many different $A \leq 4$ calculations—for a review, see Ref. [5]—have indicated that the LO potential at two-body unitarity is

$$V^{(0)} = \frac{4\pi}{m}\left[C_0^{(0)}(\Lambda)\sum_{\{ij\}}\delta_\Lambda^{(3)}(\vec{r}_{ij}) + 4\pi D_0^{(0)}(\Lambda)\sum_{\{ijk\}}\delta_\Lambda^{(3)}(\vec{r}_{ij})\delta_\Lambda^{(3)}(\vec{r}_{jk})\right], \tag{1}$$

where $\{ij\}$ and $\{ijk\}$ denote doublets and triplets, and $\delta_\Lambda^{(3)}(\vec{r})$ is a regularization of the three-dimensional delta function, that is, a smearing over distances $r \lesssim \Lambda^{-1}$ with $\lim_{\Lambda\to\infty}\delta_\Lambda^{(3)}(\vec{r}) = \delta^{(3)}(\vec{r})$. The interaction strengths $C_0^{(0)}(\Lambda)$ and $D_0^{(0)}(\Lambda)$ are functions of the cutoff Λ that ensure the $A = 2, 3$ binding energies are regulator independent [6,7]:

$$C_0^{(0)}(\Lambda) \propto -\frac{1}{\Lambda}, \qquad D_0^{(0)}(\Lambda) \propto \frac{1}{\Lambda^4}\,\frac{\sin\big(s_0\ln(\Lambda/\Lambda_\star) - \tan^{-1}(s_0^{-1})\big)}{\sin\big(s_0\ln(\Lambda/\Lambda_\star) + \tan^{-1}(s_0^{-1})\big)}. \tag{2}$$

where the (dimensionless) proportionality factors depend on the chosen regularization and $s_0 \simeq 1.00624$. For the two-body binding energy to vanish at LO, $B_2^{(0)} = 0$, $C_0^{(0)}$ must contain no physical parameter. In contrast, $D_0^{(0)}$ is characterized by the dimensionful parameter $\Lambda_\star$, which determines the $A = 3$ ground-state binding energy, B_3. The three-body force with form (2) is needed for renormalization [7], that is, to prevent a collapse under the attractive two-body contact when Λ increases [8].

At next-to-leading order (NLO), there is a two-body potential with two derivatives that accounts for energy dependence, that is, the effective range. The most important electromagnetic interaction for nucleons is the Coulomb potential, which is nonperturbative only at momenta $\lesssim \alpha m_N/2$ (where α is the fine-structure constant). Since this quantity is numerically not very different from $|a_2^{-1}|$ in the two-nucleon S waves, in ground states Coulomb can also be treated as a sub-leading effect [9]. Conservatively, we can assign a_2^{-1} and Coulomb effects to NLO. Quark-mass difference effects do not appear before next-to-next-to-leading order ($\mathrm{N^2LO}$). The first momentum-dependent three-body potential enters at $\mathrm{N^2LO}$ as well. It is currently unknown at which sub-leading order the four-body potential appears for the first time. In order to retain RG invariance, sub-leading potentials must be treated in distorted-wave perturbation theory.

4. – Discrete scale invariance

The interaction strengths $C_0^{(0)}(\Lambda)$ and $D_0^{(0)}(\Lambda)$ represent an RG non-trivial fixed point [6] and limit cycle [7], respectively. It is not difficult to show (see, for example, Ref. [5]) that at LO the theory possesses a discrete scale invariance (DSI). The $A = 2$ system is in fact invariant under continuous scale transformations where position, time, regulator and field transform as, respectively,

$$r \to \alpha r, \qquad t/m \to \alpha^2 t/m, \qquad \Lambda \to \alpha^{-1}\Lambda, \qquad \psi \to \alpha^{-3/2}\psi, \tag{3}$$

with a real parameter $\alpha > 0$. However, the appearance for $A \geq 3$ of $\Lambda_\star$ breaks continuous scale invariance down to a discrete subgroup with $\alpha \to \alpha_n \equiv \exp(n\pi/s_0)$, n an integer. At sub-leading orders DSI is broken explicitly. Approximate DSI has two main consequences:

1. The LO ground-state energy of the A-body system is determined by $\Lambda_\star$. Eliminating the latter in favor of B_3, we can write

(4) $$\frac{B_A^{(0)}}{A} = \kappa_A \frac{B_3}{3},$$

   where κ_A is a universal number—it depends on whether the particle is a boson or a multi-state fermion, but different species of bosons or fermions taking on at least three states have the same κ_As. By construction $\kappa_2 = 0$ and $\kappa_3 = 1$.

2. For each $A \geq 3$, excited states form at least one geometric tower above the ground state with a spacing factor $\exp(-2n\pi/s_0) \simeq 1/515$. This is because DSI can be satisfied if, as a consequence of Eq. (3), two levels n and $n + l$ are related by $B_{A;n+l} = \alpha_l^{-2} B_{A;n}$. The spectrum is cut off from below by $M_{\rm hi}$ and from above by a_2^{-1} effects, which become important at low energies. For $A = 3$ there is one tower of the famous Efimov states [10].

Thus, all states within the regime of validity of the EFT are determined at LO by $\Lambda_\star$, with everything else smaller, perturbative corrections. How far can we go with this?

5. – $A \leq 4$ systems

At LO we can solve the $A = 3$ Faddeev equation for the potential (1) with a sharp momentum cutoff, while the NLO potential is treated in first-order distorted-wave approximation [2]. Triton and helion are degenerate at LO. Using the binding energy of the former, B_t, to fix $\Lambda_\star$, we find for helion at NLO

(5) $$B_h^{(1)} - B_t = -(0.92 \pm 0.18)\ \text{MeV},$$

to be compared with the experimental value, -0.764 MeV. There is no excited triton state which is bound, but we have recently shown using EFT that virtual states in nd scattering become the triton excited states as the deuteron binding energy is decreased [11]. The three-nucleon spectrum carries the footprints of DSI, and its perturbative breaking improves agreement with experimental data.

Calculations for bosons at unitarity have shown that each Efimov state spawns two $A = 4$ states [12]: one very near the $A = 3$ binding energy, the other about 4.6 times deeper [13]—which translates to $\kappa_4 \simeq 3.5$ in Eq. (4). The same happens for four nucleons at unitarity. Equation (4) means that there is an approximately linear correlation between alpha-particle and triton binding energies as $\Lambda_\star$ is varied. And, indeed, such correlation has long been known for phenomenological potentials, the "Tjon line" [14]. Sub-leading effects introduce small departures from linearity. Solving the Faddeev-Yakubovsky equation at LO with a Gaussian regulator and incorporating a_2^{-1} effects in perturbation theory at an incomplete NLO, we obtain [2]

(6) $$B_\alpha^{(0)} = 39 \pm 12, \qquad B_\alpha^{(1,\text{inc})} = 29.5 \pm 8.7\ \text{MeV},$$

TABLE I. – *Binding energy per particle for A bosons at unitarity relative to the three-boson energy per particle, κ_A defined in Eq. (4)* [20].

A	κ_A	A	κ_A	A	κ_A
4	3.5(1)	10	18.3(5)	30	38.8(1.0)
5	6.3(2)	12	21.9(6)	40	43.5(1.1)
6	8.9(2)	14	24.9(7)	50	46.8(1.2)
7	11.6(3)	16	27.4(7)	60	49.4(1.3)
8	14.2(4)	18	29.7(8)		
9	16.3(4)	20	31.6(8)		

to be compared with the experimental value, 28.4 MeV. The observed first-excited alpha state is also recovered near the triton-nucleon threshold. An expansion around unitarity clearly captures the basic elements of the four-nucleon system as well.

6. – $A \geq 5$ systems

It is remarkable that light nuclei, at least, can be described by a systematic theory with one parameter at LO. To check how well this works for larger systems we need to calculate $\kappa_{A\geq 5}$ in Eq. (4), as well as the corresponding excited spectra.

The characteristic features of DSI have indeed been found in systems of $A = 5, 6$ *bosons*:

- Ground states satisfy the "generalized Tjon lines" embodied in Eq. (4) [15].
- Excited spectra exhibit a doubling each time A increases by 1 [16-19].

The ground-state energies for up to $A = 60$ bosons at unitarity have now been calculated by solving the Schrödinger equation for the potential (1) with variational and diffusion Monte Carlo methods and Gaussian regulators [20]. Selected values are shown in Table I. It is seen that the κ_As increase approximately linearly at small A [16], that is, $\kappa_A \simeq 3(A-2)^2/A$ [15]. However, the growth tapers off, a behavior well fitted by a "liquid-drop" formula,

$$\kappa_{A\gg 4} = \kappa \left(1 + \eta A^{-1/3} + \dots\right), \tag{7}$$

with $\kappa = 90 \pm 10$ and $\eta = -1.7 \pm 0.3$, respectively, the dimensionless "volume" and "surface" terms. The factor of $\simeq 90$ is large but still well below the $\simeq 515$ that provides an upper bound for the EFT breakdown scale. Calculated radii confirm this saturation. Qualitatively the same physics takes place for ^{4}He atoms but the corresponding κ_As are larger, with an asymptotic value twice as large as at unitarity [21]. The apparent importance of sub-leading effects [22] suggests that the ^{4}He liquid might be outside the EFT.

Unfortunately there are no similar calculations for nucleons. A hand-waving argument suggests that they should display saturation as well. Since the number of triplets grows faster than the number of doublets, the three-body force prevents the collapse for $A \geq 3$. For nucleons, the exclusion principle reduces the effects of the two-body S-wave attraction for $A \geq 5$, so we might expect smaller values for $\kappa_{A\geq 5}$. In fact, calculations of $A = 16, 40$ nucleons *without* the unitarity expansion [23, 24] suggest that Pionless EFT might generate, if anything, too much saturation. Still, it is tantalizing that at least for bosons the potential (1) gives rise to saturation that is qualitatively similar to the one seen for nuclei.

7. – Nuclear matter

The previous results give us hope, but by no means certainty, that nuclear matter around the saturation point is within the regime of validity of Pionless EFT. Results from closely related models [25] are also encouraging.

We are thus led to speculate that DSI might be relevant, in which case the form of the equation of state at LO must be [5, 26]

$$\lim_{A\to\infty} \frac{B_A^{(0)}(\rho)}{A} = -\left(\frac{3\pi^2}{2}\right)^{2/3} \frac{\rho^{2/3}}{m}\, \Upsilon\left(\frac{s_0}{3}\ln\left(\frac{3\pi^2\rho}{2\Lambda_\star^3}\right)\right), \tag{8}$$

where ρ is the particle density and Υ is a (real) periodic function with period π. The minimum of Eq. (8) describes a straight line in $\rho^{2/3}$ under variation of $\Lambda_\star$, just as the empirical "Coester line" [27]. The energy around the saturation density obtained in models can also be recovered, if the derivatives of Υ at saturation are of $\mathcal{O}(1)$ as expected from the absence of dimensionful parameters other than $\Lambda_\star$.

We can conclude that DSI is not inconsistent with the known properties of nuclear matter at saturation. Stronger statements require an *ab initio* calculation of Υ.

8. – Conclusion

A nuclear EFT, when properly renormalized, represents the low-energy limit of QCD. All interactions allowed by QCD symmetries must be included, with their importance to observables ordered by a power counting. Interaction strengths can in principle be matched to the results from lattice QCD, but for physical quark masses we presently must rely on experimental input. The power counting ensures that at each order only a few low-energy data are needed as input while the rest is predicted. In a successful EFT, the leading order captures the essential physics, such as the existence of nuclear ground states and saturation, with systematic improvements at higher orders.

Although Chiral EFT has received most attention in the nuclear community, the "poster theory" is Pionless EFT. A consistent power counting has been developed and led to a successful description of $A \leq 40$ systems, although in some cases only LO and a resummed NLO, which destroys RG invariance, have been examined. I reported here on an elaboration of Pionless EFT designed for ground states, where the departure from two-body unitarity is taken as a sub-leading effect. Two of the advantages of this starting point are that at leading order: *i)* there is a single dimensionful parameter, which determines all ground-states energies; and *ii)* discrete scale invariance is exact, and determines the excitation spectrum.

We have seen evidence that $A \leq 4$ nuclear systems are *perturbatively* close to the unitarity limit: the ground and first-excited states are qualitatively reproduced at LO and quantitatively already at NLO. Larger systems of unitary bosons display the telltale signs of discrete scale invariance. Moreover, thanks to the three-body force, they saturate similarly to ^{4}He clusters and nuclei. I have also speculated on possible implications of discrete scale invariance to nuclear matter. All results are encouraging, but still, this is just the seed...

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