Physics Letters B 772 (2017) 464–470

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Emergent geometry, thermal CFT and surface/state correspondence

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ARTICLE INFO

Article history:
Received 17 April 2017
Received in revised form 2 July 2017
Accepted 5 July 2017
Available online 12 July 2017
Editor: N. Lambert

Keywords:
AdS/CFT correspondence
cMERA
Holography
BTZ black hole

ABSTRACT

We study a conjectured correspondence between any codimension-two convex surface and a quantum state (SS-duality for short). By applying thermofield double formalism to the SS-duality, we show that thermal geometries naturally emerge as a result of hidden quantum entanglement between two boundary CFTs. We therefore propose a general framework to emerge the thermal geometry from CFT at finite temperature, without knowing many details about the thermal CFT. As an example, the case of 2*d* CFT is considered. We calculate its information metric and show that it is either BTZ black hole or thermal AdS as expected.

1. Introduction

The fascinating idea that spacetimes might emerge from more fundamental degrees of freedom has attracted more and more attention in the past few years. This idea was revived recently by the discovery of the AdS/CFT correspondence [1–3]. Even though it has lead tremendous progresses in the past few years, fundamental mechanism of the AdS/CFT correspondence still remains a mystery. The situation became better not until the discovery of Ryu–Takayanagi formula [4–6], which states that the entanglement entropy of a subregion A of a $d+1$ dimensional CFT on the boundary of $d+2$ dimensional AdS is proportional to the area of a certain codimension-two extremal surface in the bulk:

$$S_A = \frac{Area(\gamma_A)}{4G_N^{d+2}}$$

where γ_A is the minimal surface whose boundary coincides the boundary of A: ∂A.

A recent step for our understanding of holography is made by Miyaji et al. in [7,8] where they proposed a duality called surface/state correspondence (SS-duality). It claims that any codimension two convex surface is dual to a quantum state of a QFT. With the help of the SS-duality, one can, in principle, find out the equivalent description of any spacetimes described by Einstein's gravity. In this way, we might encode the information of the boundary QFTs into the bulk geometry, and vice versa.

There is a very different way in mapping states and operators in the boundary Hilbert space to those in the bulk. This is known as the tensor networks. One specific example which is of particular significance is the multi-scale entanglement renormalization ansatz (MERA) [9–11] (see e.g. [12] for an introduction), and MERA of the ground state of a lattice model at critical point is naturally related to CFT [13]. The connection between the AdS/CFT and the MERA was first pointed out by Swingle in [14], where he noticed that the renormalization direction along the graph can be viewed as an emergent (discrete) radial direction of the AdS space (see e.g. [15–18] for further discussion on the resemblance between MERA and AdS geometry). This elegant method was latter generalized to continuous version (cMERA), which makes entanglement renormalization available for quantum fields in real space [19]. Equipped with this toolkit, the holographic (smooth) geometry can naturally emerge from QFTs [20].

Though it is very successful, the full investigation of AdS/cMERA is still very limited. Most past works paid their attention to zero-temperature systems. In this paper, we take a step forward and investigate how to emerge thermal spacetimes from boundary CFT at finite temperature, by making use of cMERA and SS-duality. At first glance the generalization is trivial and one can achieve this as long as the boundary CFT is replaced by a thermal one. However, there are two obstacles that prevent us from this generalization: First of all, the appearance of black hole (BH) horizon leads to a closed and topologically nontrivial surface in the bulk. This implies, according to the SS-duality, that the dual state in the boundary

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http://dx.doi.org/10.1016/j.physletb.2017.07.011
0370-2693/

QFT is no longer a pure state. All calculations must be replaced by thermal mixed states. Secondly, for finite-temperature CFT, turning on a temperature introduces a scale which screens long-range correlations and the state have thermal correlations in addition to entanglement. One important effect is that the thermal correlations become more relevant as one runs the MERA. The MERA, therefore, truncates at a certain level, which is suggestive of a BH horizon [14]. We often call it the truncated MERA [21]. In our previous work [22], we discussed the emergent thermal geometry by generalizing the truncated MERA to continuous one.

An alternative way which is more natural is based on the thermofield double formalism [23] and the emergent tensor network is often called doubled MERA. This proposal [24] states that the eternal black hole is dual to two copies of the CFT, in the thermofield double state $|TFD\rangle$. Each asymptotic boundary of AdS is a copy of the original dual CFT. With the help of the SS-duality, we will find by this formulation that the thermal spacetimes naturally appear as a result of hidden quantum entanglement between two boundary CFTs. The road map is the following: we first propose a TFD-like state in CFT which is dual to bulk locally excited state in thermal spacetime background and then write down two TFD-like states which differ by infinitesimal parameters. The bulk coordinates can be naturally recognized as these parameters in CFT. Then similar to the proposal in [8], the Fisher information metric distance between two nearby TFD-like states which we propose to be dual to bulk local excitations is identified to the emergent bulk metric distance. Once the bulk metric is obtained, we get the emergent spacetime. In this way, we formulate a general framework by which thermal geometries emerge from dual CFTs, without knowing the details of the thermal correlations of the CFT.

2. Thermofield dynamics and surface/state correspondence

2.1. Thermofield double formalism and doubled cMERA

We start by introducing a new QFT H_{tot} which is two copies of the original QFT (with Hilbert spaces H_1 and H_2 respectively). The thermofield double formalism treat the thermal, mixed state $\rho = e^{-\beta H_i}$ $(i = 1, 2)$ as a pure state in the new double system $H_{tot} = H_1 \bigotimes H_2$. Thermofield double state (or Hartle–Hawking state in the dual bulk) in this doubled system is defined as

$$|TFD\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\beta E_n/2} |n\rangle_1 |n\rangle_2, \tag{1}$$

where $|n\rangle_1, |n\rangle_2$ are energy eigenstates of the two copies of QFT respectively. This is a particular (entangled) pure state in the doubled system. The density matrix of the doubled QFT in this state is

$$\rho_{tot} = |TFD\rangle\langle TFD|. \tag{2}$$

The thermofield double formalism can be applied to the case of the AdS eternal black hole. The Penrose diagram of an eternal black hole separates the whole spacetime into two asymptotically AdS regions as depicted in Fig. 1. Each asymptotic boundary of AdS is a copy of the original dual CFT. It is convenient to denote these two identical, non-interacting copies of CFT by CFT_1 and CFT_2, respectively. According to Maldacena [24], this eternal black hole which is described by the Hartle–Hawking state $|HH\rangle$, is dual to two copies of the CFT in the thermofield double state $|TFD\rangle$.

Due to the presence of the horizon, observers in one of those two asymptotically AdS regions (say, region I) cannot come in contact with the other one directly.[1] From the viewpoint of the dual CFTs, for CFT_1, information from CFT_2 must be traced out. As a consequence, the CFT_1 is in a thermal state described by

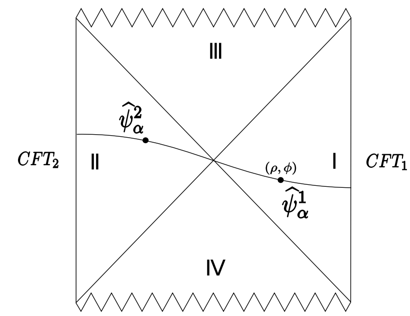

Fig. 1. Penrose diagram for an eternal black hole. There are two asymptotically AdS regions which are dual to two copies of CFT. The bulk quantized fields $\hat{\psi}^1_\alpha$ and $\hat{\psi}^2_\alpha$ are put in the bulk points (ρ, ϕ) in the two asymptotically AdS time slices, respectively.

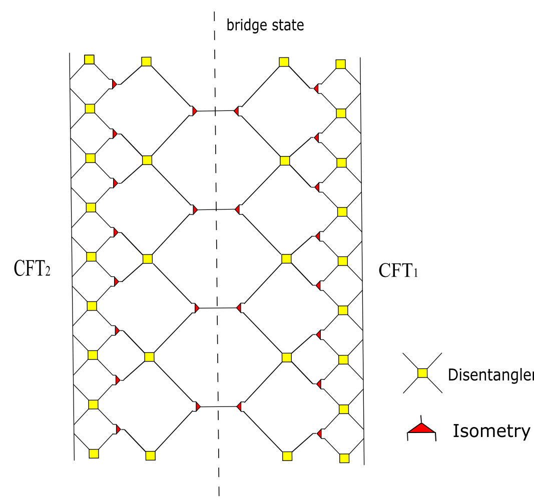

Fig. 2. Doubled MERA network. At the center there is a bridge state which glues two copies of the standard MERA. This state is usually viewed as a black hole horizon.

$$\rho_1 = \text{Tr}_2 \rho_{tot} = e^{-\beta H_1}. \tag{3}$$

The above picture nicely agrees with the MERA at finite temperature as proposed in [25–28], which is known as the doubled MERA network. It is composed of two copies of the standard MERA for a pure state which are gluing together at infrared points by a "bridge" state. Fig. 2 shows a schematic representation of the doubled MERA network. The continuous version of MERA (cMERA) at finite temperature has already been considered in [29], where the authors found that, similar to the MERA, finite-temperature cMERA can be constructed by doubling two copies of the standard cMERA.

2.2. SS-duality description of thermofield dynamics

Now let us generalize the above picture to a description in terms of the SS-duality. The SS-duality argues a correspondence between any codimension-two convex surface Σ and a quantum state of a quantum theory which is dual to the Einstein's gravity. It can be applied to any spacetimes described by Einstein's gravity and therefore can be viewed as a generalization of the AdS/CFT

[1] However, they connect with each other indirectly through hidden quantum entanglement. The hidden quantum entanglement entropy of the thermal CFT can be viewed as the black hole entropy.

correspondence. More specifically, this duality states that a closed topological trivial convex surface is dual to a pure quantum state $|\Phi(\Sigma)\rangle$, while a closed topological non-trivial convex surface[2] Σ corresponds to a mixed quantum state $\rho(\Sigma)$, such as the surface which wraps a black hole. In particular, the zero-size closed surface (i.e. a point) is dual to a boundary state $|B\rangle$ [7,30,31]. When Σ_1 and Σ_2 are related by a smooth deformation which preserves convexity, the deformation can be expressed by a unitary transformation

$$\rho(\Sigma_1) = U(s_1, s_2)\rho(\Sigma_2)U^{-1}(s_1, s_2), \tag{4}$$

where $U = \mathcal{P}\exp\{-i\int_{u_2}^{u_1}\hat{M}(s)\,ds\}$ with $\mathcal{P}$ the path-ordering and $\hat{M}(s)$ a Hermitian operator.

To proceed, let us turn to a cMERA description of the CFT state. From [8], we learn that a CFT ground state can be expressed in terms of cMERA as follows:

$$|0\rangle_{CFT} = \mathcal{P}\exp\left(-i\int_{-\infty}^{0} du\hat{K}(u)\right)|I_0\rangle, \tag{5}$$

where $\hat{K}(u)$ is the disentangling operator of cMERA at scale u, and $|I_0\rangle$ is the Ishibashi state. The cMERA flow can be adjusted by a conformal transformation. Specifically, for the case of $1+1$ dimensions, it was shown in [8] that one has a transformation $g(\rho,\phi)$ which takes the origin $\rho = 0$ to any point (ρ,ϕ). After acting $g(\rho,\phi)$ transformation, Eq. (5) can be rewritten as

$$|0\rangle_{CFT} = \mathcal{P}\exp\left(-i\int_{-\infty}^{0} du\hat{K}_{(\rho,\phi)}(u)\right)|I_0\rangle \equiv U(\rho,\phi)|I_0\rangle, \tag{6}$$

where

$$\hat{K}_{(\rho,\phi)}(u) = g(\rho,\phi)\hat{K}(u)g(\rho,\phi)^{-1}.$$

Similarly, the CFT excited states $|\Psi_\alpha(\rho,\phi)\rangle_{CFT}$ can be expressed in terms of Ishibashi states $|I_\alpha\rangle$ for primary field Ψ_α

$$|\Psi_\alpha(\rho,\phi)\rangle_{CFT} = U_{(\rho,\phi)}|I_\alpha\rangle. \tag{7}$$

According to surface/state correspondence, there are dualities between quantum states in the CFT and states in the bulk gravity. In particular,

$$\begin{aligned} &|0\rangle_{CFT} \;\Leftrightarrow\; |0\rangle_{bulk}, \\ &|\Psi_\alpha(\rho,\phi)\rangle_{CFT} \;\Leftrightarrow\; |\Psi_\alpha(\rho,\phi)\rangle_{bulk} \equiv \hat{\psi}_\alpha(\rho,\phi)|0\rangle_{bulk}, \end{aligned} \tag{8}$$

where $|0\rangle_{bulk} \in \mathcal{H}_{bulk}$ is the vacuum state of the bulk gravity, and $|\Psi_\alpha(\rho,\phi)\rangle_{bulk} \in \mathcal{H}_{bulk}$ denotes the locally excited state in the bulk. The proposal [8] is to find the dual state of $|\Psi_\alpha\rangle_{bulk} \equiv |\Psi_\alpha(0,0)\rangle_{bulk}$ by noting that the $SL(2,R)$ subgroup of $SL(2,R)\times SL(2,R)$ (whose generators are (L_1, L_0, L_{-1}) and $(\tilde{L}_1, \tilde{L}_0, \tilde{L}_{-1})$) which preserves the point $\rho = t = 0$ impose constraints on $|\Psi_\alpha\rangle_{bulk}$. This is equivalent to impose the same constraints on $|\Psi_\alpha\rangle \equiv |\Psi_\alpha(0,0)\rangle_{CFT}$ in the dual CFT, i.e.,

$$(L_0 - \tilde{L}_0)|\Psi_\alpha\rangle = (L_1 + \tilde{L}_{-1})|\Psi_\alpha\rangle = (L_{-1} + \tilde{L}_1)|\Psi_\alpha\rangle = 0. \tag{9}$$

In the same footing, as we try to generalize it to the thermal case, we firstly insert a locally excited field $\hat{\psi}_\alpha(\rho,\phi)$ on the time slice of the thermal spacetimes, which is the Hartle–Hawking vacuum $|HH\rangle$[3] for an eternal black hole as shown in Fig. 1. This implies that the following dual relation which is similar to (8) holds

$$|\Psi_\alpha^\beta(\rho,\phi)\rangle_{CFT} \;\Leftrightarrow\; |\Psi_\alpha(\rho,\phi)\rangle_{bulk} \equiv \hat{\psi}_\alpha^1(\rho,\phi)\hat{\psi}_\alpha^2(\rho,\phi)|HH\rangle, \tag{10}$$

where two equivalent local bulk quantized fields $\hat{\psi}_\alpha^1$ and $\hat{\psi}_\alpha^2$ act on CFT_1 and CFT_2, respectively, and superscript β introduced in $|\Psi_\alpha^\beta(\rho,\phi)\rangle$ to distinguish them from the one at zero temperature. And the bulk coordinate (ρ,ϕ) can be naturally recognized as parameters in CFT.

As the second step, we would like to learn what is the explicit form of the states $|\Psi_\alpha^\beta(\rho,\phi)\rangle_{CFT}$. Without loss of the generality, we only need to focus on $|\Psi_\alpha^\beta\rangle_{CFT} \equiv |\Psi_\alpha^\beta(0,0)\rangle_{CFT}$. Other cases can be obtained by using $|\Psi_\alpha^\beta(\rho,\phi)\rangle_{CFT} = g(\rho,\phi)|\Psi_\alpha^\beta\rangle_{CFT}$. The dual relation (10) implies that $|\Psi_\alpha^\beta\rangle_{CFT}$ should satisfy the following conditions:

(i) The duality (10) strongly favors that these are TFD-like states;

(ii) $|\Psi_\alpha^\beta\rangle_{CFT}$ should be the energy eigenstates of the dual CFT;

(iii) Just like the zero-temperature case, constraints imposed by the generators which preserves the point $\rho = t = 0$ in the bulk will, through the duality, impose the same constraints on $|\Psi_\alpha^\beta\rangle_{CFT}$ in the dual doubled CFTs, equally. Each of them is similar to (9).

The above conditions force us to propose the following form of $|\Psi_\alpha^\beta\rangle_{CFT}$

$$|\Psi_\alpha^\beta\rangle_{CFT} \equiv |TFD-like\rangle_{CFT} = \frac{1}{\sqrt{Z(\beta)}}\sum_\alpha e^{-\beta\Delta_\alpha/2}|\tilde{\Psi}_\alpha\rangle_1|\tilde{\Psi}_\alpha\rangle_2,$$

where $|\tilde{\Psi}_\alpha\rangle_i$ are eigenstates of the dual CFTs and as well, should be solutions of the constraints similar to (9), which implies they should be generated by some primary states $|\alpha\rangle$ (and their descendants) with conformal dimensions Δ_α. In the next section we will show that in the case of BTZ black hole, $|\tilde{\Psi}_\alpha\rangle_i$ are nothing but $|\Psi_\alpha\rangle_{CFT}$ in (8) i.e. the zero-temperature solution satisfying the constraints (9).

The density matrix of the double CFTs which is dual to bulk locally excited state is given by

$$\rho_{tot} = |\Psi_\alpha^\beta\rangle_{CFT}\langle\Psi_\alpha^\beta| \equiv |TFD-like\rangle_{CFT}\langle TFD-like|. \tag{11}$$

Note that the TFD-like state though is not the usual TFD states, they are very similar. It can be viewed as a perturbative version of the usual TFD state. The reason is the following: the duality (8) shows that the vacuum $|0\rangle_{CFT}$ corresponds to a pure AdS configuration in the bulk, and the excitations $|\Psi_\alpha\rangle_{CFT}$ are dual to perturbative bulk states, which are perturbative configurations deviated slightly from the pure AdS. The TFD-like state here is a generalization of the above picture to the eternal black hole case, by putting two equivalent local bulk quantized fields into the bulk. The induced bulk configuration is a deviation from the unperturbed geometry.

Similar to the thermofield double, the thermal density matrix in one of the copies of the CFT is obtained by tracing out the contributions of the other copy of the CFT, as shown in (3). Explicitly, for density matrix of the double CFT given by (11), the reduced density matrix of one of the CFT is

$$\begin{aligned} \rho_{CFT_1} &= \mathrm{Tr}_{CFT_2}\,\rho_{tot} \\ &= \frac{1}{Z(\beta)}\sum_\alpha e^{-\beta\Delta_\alpha}|\tilde{\Psi}_\alpha(\rho,\phi)\rangle_{CFT_1}\langle\tilde{\Psi}_\alpha(\rho,\phi)|, \end{aligned} \tag{12}$$

where $Z(\beta) = \mathrm{Tr}\sum_\alpha e^{-\beta\Delta_\alpha}|\tilde{\Psi}_\alpha(\rho,\phi)\rangle\langle\tilde{\Psi}_\alpha(\rho,\phi)|$ is the partition function.

[2] Topologically trivial surface is the surface which can be smoothly deformed into a point, as a contrast, topologically non-trivial one fails to do so.

[3] To make it more readable, we use $|HH\rangle$ to denote the bulk TFD state, so as to distinguish it from TFD state in the CFT.

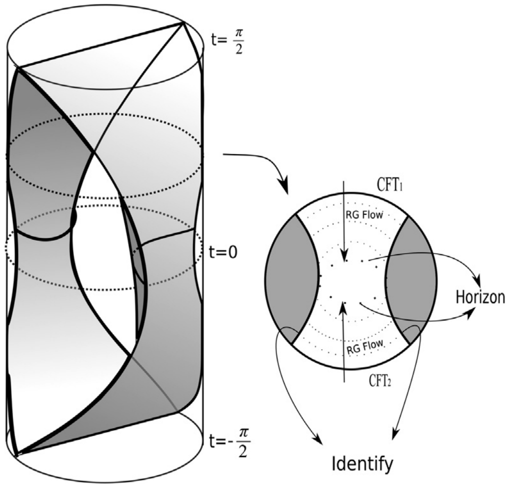

Fig. 3. The 2+1 dimensional (spinless) BTZ solution in coordinates (t, ρ, ϕ). All points inside the cylinder belong to anti-de Sitter space, its surface ($\rho = 1$) representing spatial infinity. The BTZ spacetime lies between the two surfaces inside the cylinder which are identified under an isometry generated by (16)–(18). The RG flow and the horizon (bridge states) are indicated by the dashed lines in the constant time slices to the right.

To proceed, let us employ the idea of Fisher information metric

$$ds^2 = \mathcal{D}_B = 1 - \mathrm{Tr}\sqrt{\rho_1^{1/2}\rho_2\rho_1^{1/2}}, \tag{13}$$

where

$$\rho_1 \equiv \rho_{CFT}(\lambda) = \frac{1}{Z}\sum_{\alpha} e^{-\beta\Delta_\alpha}|\tilde{\Psi}_\alpha(\lambda)\rangle_{CFT}\langle\tilde{\Psi}_\alpha(\lambda)|, \tag{14}$$

$$\begin{aligned}\rho_2 &\equiv \rho_{CFT}(\lambda+\delta\lambda)\\ &= \frac{1}{Z}\sum_{\alpha'} e^{-\beta\Delta_{\alpha'}}|\tilde{\Psi}_{\alpha'}(\lambda+\delta\lambda)\rangle_{CFT}\langle\tilde{\Psi}_{\alpha'}(\lambda+\delta\lambda)|,\end{aligned} \tag{15}$$

and it measures the distance between two infinitesimally close states ρ_1 and ρ_2 which parameterized by $\lambda = (\rho, \phi)$. As usual, we identify this information metric with the metric of the time slice of the emergent spacetime. In this way, the dual geometry of the eternal black hole can be obtained, according to the SS-duality, by considering the distance between two local excitations, which is given by (13).

3. Emergent BTZ black hole

As an explicit example, in this section we would like to employ our proposal to $2d$ CFT and to see how the expected BTZ black hole is emergent. The quantum distance (13) plays a significant role in the derivation of the geometry. The key to the derivation is to find out the form of the states $|\tilde{\Psi}_\alpha(\rho,\phi)\rangle_{CFT} = g(\rho,\phi)|\tilde{\Psi}_\alpha\rangle_{CFT}$ which is the building blocks of our proposal $|\Psi_\alpha^\beta\rangle_{CFT}$. This can be achieved by imposing conditions (ii) and (iii) as mentioned in the last section.

Let us start with a brief review on how to make a BTZ black hole (the AdS black hole in $2+1$ dimensions [32,33]). Fig. 3 shows a sketch of the way in obtaining a BTZ black hole. Roughly speaking, we first find out the "identification surfaces" in AdS. These are hypersurfaces that divide the whole spacetime into several regions, some of them have the timelike or null Killing vectors. These regions must be cut out from anti-de Sitter space to make the identifications permissible. This means that they should lie entirely within the region where the Killing vector field is space-like. Identifying corresponding points on these surfaces gives us the BTZ black hole. Before identification, it is a patch of the whole AdS space, whose isometry is given by $SL(2,R) \times SL(2,R)$ generated by the (global) Virasoro generators (L_1, L_0, L_{-1}) and $(\tilde{L}_1, \tilde{L}_0, \tilde{L}_{-1})$ of the dual $2d$ CFT. In the global coordinate, they are

$$L_0 = i\partial_+ = i\frac{\partial}{\partial x^+}, \quad \tilde{L}_0 = i\partial_- = i\frac{\partial}{\partial x^-}, \tag{16}$$

$$L_{\pm1} = ie^{\pm ix^+}[\frac{\cosh 2\rho}{\sinh 2\rho}\partial_+ - \frac{1}{\sinh 2\rho}\partial_- \mp \frac{i}{2}\partial\rho], \tag{17}$$

$$\tilde{L}_{\pm1} = ie^{\pm ix^-}[\frac{\cosh 2\rho}{\sinh 2\rho}\partial_- - \frac{1}{\sinh 2\rho}\partial_+ \mp \frac{i}{2}\partial\rho], \tag{18}$$

where $x^\pm = t \pm \phi$. The identification breaks the symmetry group from $SL(2,R)\times SL(2,R)$ to $SL(2,R)\times U(1)$, however, the BTZ black hole (and its higher dimensional generalization [34]) remains locally AdS. Our procedures of deriving the thermal spacetimes in the last section only need local information, it is therefore safe enough to start with (16)–(18). For the same sake, the fact that the BTZ geometry has the same local isometry as the AdS_3 suggests the same constraints should be imposed on $|\tilde{\Psi}_\alpha\rangle_{CFT}$ as the one given in (9), which implies $|\tilde{\Psi}_\alpha\rangle_{CFT}$ has exactly the same solution as the one for AdS, i.e., $|\tilde{\Psi}_\alpha\rangle_{CFT} = |\Psi_\alpha\rangle_{CFT}$.

Following [8], the excited state $|\Psi_\alpha(\rho,\phi)\rangle_{CFT}$ can be obtained by acting the conformal transformation $g(\rho,\phi)$ to $|\Psi_\alpha\rangle_{CFT} \equiv |\Psi_\alpha(0,0)\rangle_{CFT}$, that is

$$|\Psi_\alpha(\rho,\phi)\rangle_{CFT} = g(\rho,\phi)|\Psi_\alpha\rangle_{CFT}, \tag{19}$$

where $g(\rho,\phi) = e^{i\phi l_0}e^{\frac{\rho}{2}(l_1 - l_{-1})}$ with $l_0 = L_0 - \tilde{L}_0$, $l_{-1} = \tilde{L}_1 - L_{-1}$, $l_1 = \tilde{L}_{-1} - L_1$. We will see later that the CFT parameters (ρ,ϕ) can be recognized as bulk coordinates. The state $|\Psi_\alpha\rangle_{CFT}$ turns out to be of the following form

$$|\Psi_\alpha\rangle_{CFT} \propto e^{-\delta(L_0+\tilde{L}_0)}e^{i\frac{\pi}{2}(L_0+\tilde{L}_0)}|J_\alpha\rangle, \tag{20}$$

where $\delta \sim 1/c$ is a UV cut off, $|J_\alpha\rangle = \sum_{k=0}^{\infty}|k\rangle_L \otimes |k\rangle_R$ are boundary states, and $|k\rangle_L \propto (L_{-1}^k)|\alpha\rangle$, $|k\rangle_R \propto (\tilde{L}_{-1}^k)|\alpha\rangle$ are descendants of the primary states $|\alpha\rangle$. In the following manuscript, we just denote $|\Psi_\alpha\rangle_{CFT}$ as $|\Psi_\alpha\rangle$ to simplify notation.

For later convenience, we first calculate the following inner product,

$$\begin{aligned}&|\langle\Psi_\alpha(\rho,\phi)|\Psi_{\alpha'}(\rho+d\rho,\phi+d\phi)\rangle|^2\\ =&[1-\frac{1}{8}(d\rho^2+\sinh^2\rho d\phi^2)\langle\Psi_\alpha|l_{-1}l_1+l_1l_{-1}|\Psi_{\alpha'}\rangle]^2\\ =&[1-\frac{1}{8\delta^2}(d\rho^2+\sinh^2\rho d\phi^2)\delta_{\alpha\alpha'}]^2,\end{aligned} \tag{21}$$

where in the second line the following relations have been employed [8]

$$\begin{aligned}&|\langle\Psi_\alpha(\rho,\phi)|\Psi_\alpha(\rho+d\rho,\phi+d\phi)\rangle|\\ =&1-\frac{1}{8}(d\rho^2+\sinh^2\rho d\phi^2)\langle\Psi_\alpha|l_{-1}l_1+l_1l_{-1}|\Psi_\alpha\rangle,\end{aligned} \tag{22}$$

and in the third line, we used (see Appendix A for more detail).

$$\begin{aligned}&{}_{CFT}\langle\Psi_\alpha|l_{-1}l_1+l_1l_{-1}|\Psi_{\alpha'}\rangle_{CFT}\\ =&{}_{CFT}\langle\Psi_\alpha|l_{-1}l_1+l_1l_{-1}|\Psi_\alpha\rangle_{CFT}\delta_{\alpha\alpha'} \simeq \frac{1}{\delta^2}\delta_{\alpha\alpha'}.\end{aligned} \tag{23}$$

With the above preparation, we are now in the situation to derive the Fisher information metric. In the limit $\beta^3\delta\lambda^3 \ll 1$, we have (see Appendix B for more detail)

$$\begin{aligned} ds^2 = \mathcal{D}_B &= 1 - \mathrm{Tr}\sqrt{\rho_1^{1/2}\rho_2\rho_1^{1/2}} \\ &\simeq 1 - \mathrm{Tr}\sqrt{\rho_1\rho_2} \\ &= 1 - \sqrt{\sum_{k,j}\langle k|\rho_1|j\rangle\langle j|\rho_2|k\rangle} \\ &= 1 - \sqrt{\sum_k \langle k|\rho_1\rho_2|k\rangle}. \end{aligned} \tag{24}$$

After substituting (19) and (20) into (12) and making use of (13)–(15) and (21) and (24), it becomes

$$\begin{aligned} ds^2 &= 1 - \sqrt{\frac{1}{Z}\frac{1}{Z(\rho+d\rho,\phi+d\phi)}\sum_{\alpha,\alpha'} e^{-\beta(\Delta_\alpha+\Delta_{\alpha'})}|\langle\Psi_\alpha(\rho,\phi)|\Psi_{\alpha'}(\rho+d\rho,\phi+d\phi)\rangle|^2} \\ &= 1 - \sqrt{\frac{1}{Z}\frac{1}{Z(\rho+d\rho,\phi+d\phi)}\sum_\alpha e^{-2\beta\Delta_\alpha}[1-\frac{1}{8\delta^2}(d\rho^2+\sinh^2\rho d\phi^2)]^2} \\ &= 1 - f(\beta)[1-\frac{1}{8\delta^2}(d\rho^2+\sinh^2\rho d\phi^2)], \end{aligned} \tag{25}$$

where

$$f(\beta) = \sqrt{\frac{1}{Z}\frac{1}{Z(\rho+d\rho,\phi+d\phi)}\sum_\alpha e^{-2\beta\Delta_\alpha}}. \tag{26}$$

If we treat $(\delta\lambda)^2 \equiv d\rho^2 + \sinh^2\rho d\phi^2$ as small perturbations in the parameters $\lambda = (\rho,\phi)$, then $\mathcal{F}(\beta,\lambda_1,\lambda_2) = \mathrm{Tr}\sqrt{\rho_1^{1/2}\rho_2\rho_1^{1/2}}$ by definition is the quantum fidelity of the field [35]. The explicit expression of $f(\beta)$ can be calculated in this way. Noticing that the metric is given by $ds^2 = 1 - \mathcal{F}(\beta,\lambda_1,\lambda_2)$, Eq. (B.5) implies that

$$\begin{aligned} \mathcal{F}(\beta,\lambda_1,\lambda_2) \sim e^{-\frac{\beta}{8}(\delta\lambda)^2\chi} &\simeq 1 - \frac{\beta}{8}(d\rho^2+\sinh^2\rho d\phi^2)\chi \\ &= 1 - \frac{\beta}{8\delta^2}(d\rho^2+\sinh^2\rho d\phi^2), \end{aligned} \tag{27}$$

where $\chi = 1/\delta^2$ has been used. This implies that

$$f(\beta) \sim 1 + \frac{1-\beta}{8\delta^2}(d\rho^2+\sinh^2\rho d\phi^2), \tag{28}$$

after plugging it into (25), the metric of time slice turns out to be[4] (keeping only quadratic terms)

$$ds^2 \simeq \frac{\beta}{8\delta^2}(d\rho^2+\sinh^2\rho d\phi^2). \tag{29}$$

This is the spatial part of the AdS metric in global coordinate up to a constant factor, but now it involves temperature through the parameter β, and possibly can be viewed as thermal AdS for low temperature (large β). However, for temperature larger than its critical value the Hawking–Page transition [36] will be induced, and it becomes a BTZ black hole. In this case, we define a set of new coordinates

$$r = r_+\cosh\rho,\ \hat{t} = \frac{i\sqrt{\beta}\phi}{2\sqrt{2}r_+\delta},\ \theta = \frac{\sqrt{\beta}t}{2\sqrt{2}r_+\delta}, \tag{30}$$

the full thermal metric (29) after adding g_{tt} (which is a prior given by assumption) can be recast as

$$ds^2 = -\left(r^2 - r_+^2\right)d\hat{t}^2 + \frac{\beta}{8\delta^2}\frac{dr^2}{r^2-r_+^2} + r^2 d\theta^2. \tag{31}$$

This is exactly the BTZ metric as expected. With the help of the thermofield double formalism, above procedures allow us knowing little information about the thermal CFT, which is one of the main merits of the proposal.

We would like to make a comment here. The fact that the BTZ metric relates to the thermal AdS merely through a coordinate transformation (30) is based on an observation that BTZ geometry is locally equivalent to AdS_3 [32,33]. However, this does not mean the BTZ black hole is trivially the same as the AdS spacetime—they have different topology [32,33] which can be influenced by temperature. Therefore, there is finite-temperature phase transition between AdS and BTZ black hole which is called Hawking–Page phase transition [36]. Our method in this paper is to construct bulk local excited sates to probe bulk metric. Thus, we can only access to local information. The global topology, however, cannot be obtained in the present approach. To probe global properties, e.g. Hawking–Page phase transition, we need non-local objects such as operator product expansion (OPE) block constructed by Czech et al. [37], which is a new challenge and will be discussed in the future work.

4. Conclusions

In this paper we have studied emergent geometries from CFT at finite temperature in the setup of the surface/state correspondence. We propose a general framework through which thermal geometries emerge from boundary CFTs. Instead of introducing a truncated level to the MERA tensor network, our proposal is realized by applying the thermofield double formalism to the SS-duality, and the thermal correlations are read off by tracing over one of the copies of the CFT. The main advantage of this framework is that the details of the thermal correlations of the CFT are not required. As an explicit example, we computed the information metric for a locally excited mixed state of two dimensional CFT at finite temperature, and showed that the emergent spacetimes are either thermal AdS or BTZ black hole as expected.

In the present framework, the information metric only depends on the behavior of two nearby states. This implies, according to the SS-duality, emergent bulk metric can be obtained by merely knowing the local information. This is one of the advantages of this framework. However, the other side of the coin is that nonlocal information is missing and the global symmetry cannot emerge from local operators. Although it is important, it is a tough difficulty and is out of the scope of the present paper. One possible clue is to resort to the entwinement (long geodesics which is no longer the one in the Ryu–Takayanagi prescription in BTZ case) which extracts non-local information as shown in [38,39], where the authors developed tools for constructing bulk curves in spacetimes beyond pure AdS, such as conical geometry and BTZ black hole, further, entwinement can be described well by kinematic space, and together with entanglement, can be used to reconstruct bulk geometry [37,40,41].

Another important future problem which is of close correlation is to find which factor determines the emergent spacetime to be thermal AdS or BTZ black hole, or equivalently, the Hawking–Page phase transition [36]. That is to say, how to determine the critical temperature of the transition? In our previous work [22] we have found a cMERA description of the Hawking–Page phase transition in the framework of the truncated MERA. In the present framework, however, its solution obviously depends on the details of the global behavior, implying that the entwinement can be a candidate. Nevertheless, the full picture is far from being achieved

[4] This is correct only when $\beta^3\delta\lambda^3 \ll 1$ as explained in Appendix B.

currently.[5] We hope in the future work we can find its description in this framework.

Acknowledgements

We are very grateful to Bartlomiej Czech and Tadashi Takayanagi for careful reading of the first version of this paper and giving us valuable comments. We also thank Xi Dong, Kanato Goto, Yuting Hu, Yi Ling, Masamichi Miyaji, Xiao-Liang Qi, Edward Witten and Shao-Feng Wu for useful conversations. We are grateful for stimulating discussions to participants of "Holographic duality for condensed matter physics" held in KITPC at CAS and the conference "Strings 2016" held in YMSC, Tsinghua. This work was supported in part by the National Natural Science Foundation of China under Grant No. 11465012, the Natural Science Foundation of Jiangxi Province under Grant No. 20142BAB202007 and the 555 talent project of Jiangxi Province.

Appendix A. Orthogonal relation of the fidelity susceptibility

In our main letter we show that $\langle\Psi_\alpha|l_{-1}l_1+l_1l_{-1}|\Psi_{\alpha'}\rangle$ can be interpreted as fidelity susceptibility between two boundary states associated with two different primary states $|\alpha\rangle$ and $|\alpha'\rangle$. Following [8], we have

$$\langle\Psi_\alpha|l_{-1}l_1+l_1l_{-1}|\Psi_{\alpha'}\rangle = 2\langle\Psi_\alpha|l_{-1}l_1|\Psi_{\alpha'}\rangle$$
$$= 2(2+e^{2\delta}+e^{-2\delta})\langle\Psi_\alpha|L_{-1}L_1|\Psi_{\alpha'}\rangle \simeq \frac{2}{\delta}\langle\Psi_\alpha|L_0|\Psi_{\alpha'}\rangle, \quad (A.1)$$

where

$$\langle\Psi_\alpha|L_0|\Psi_{\alpha'}\rangle$$
$$= \langle J_\alpha|e^{-i\frac{\pi}{2}(L_0+\tilde{L}_0)}e^{-\delta(L_0+\tilde{L}_0)}L_0e^{-\delta(L_0+\tilde{L}_0)}e^{i\frac{\pi}{2}(L_0+\tilde{L}_0)}|J_{\alpha'}\rangle$$
$$= \langle J_\alpha|e^{-2\delta(L_0+\tilde{L}_0)}L_0|J_{\alpha'}\rangle. \quad (A.2)$$

Using the commutation relation $[L_0, L_{-1}] = L_{-1}, [\tilde{L}_0, \tilde{L}_{-1}] = \tilde{L}_{-1}$ repeatedly, we have

$$L_0|J_{\alpha'}\rangle = \sum_k (k+\Delta_{\alpha'})(L_{-1})^k|\alpha'\rangle(\tilde{L}_{-1})^k|\alpha'\rangle, \quad (A.3)$$

and

$$e^{-2\delta(L_0+\tilde{L}_0)}L_0|J_{\alpha'}\rangle$$
$$= \sum_k (k+\Delta_{\alpha'})e^{-4\delta\Delta_{\alpha'}}\left(\frac{1}{e^{2\delta}+1}\right)^{2k}|k\rangle_{\alpha'}|k\rangle_{\alpha'}. \quad (A.4)$$

We therefore get

$$\langle J_\alpha|e^{-2\delta(L_0+\tilde{L}_0)}L_0|J_{\alpha'}\rangle$$
$$= \sum_{k,k'}\langle k|_\alpha\langle k|_\alpha(k'+\Delta_{\alpha'})e^{-4\delta\Delta_{\alpha'}}\left(\frac{1}{e^{2\delta}+1}\right)^{2k'}|k'\rangle_{\alpha'}|k'\rangle_{\alpha'}$$
$$= \sum_{k,k'}\langle k|_\alpha\langle k|_\alpha(k'+\Delta_\alpha)e^{-4\delta\Delta_\alpha}\left(\frac{1}{e^{2\delta}+1}\right)^{2k'}|k'\rangle_\alpha|k'\rangle_\alpha\delta_{\alpha\alpha'}$$
$$= \langle J_\alpha|e^{-2\delta(L_0+\tilde{L}_0)}L_0|J_\alpha\rangle\delta_{\alpha\alpha'}, \quad (A.5)$$

where we have used the orthogonality between the highest weight states $\langle\alpha|\alpha'\rangle = \delta_{\alpha\alpha'}$. In the end, one has

$$\langle\Psi_\alpha|l_{-1}l_1+l_1l_{-1}|\Psi_{\alpha'}\rangle = \langle\Psi_\alpha|l_{-1}l_1+l_1l_{-1}|\Psi_\alpha\rangle\delta_{\alpha\alpha'} = \frac{1}{\delta^2}\delta_{\alpha\alpha'}, \quad (A.6)$$

which is the formula (23) in our main letter.

Appendix B. Simplification of noncommutative density matrix

In this section, we would like to give a simplification of $\mathrm{Tr}\sqrt{\rho_1^{1/2}\rho_2\rho_1^{1/2}}$ (physically this is field fidelity) under a general perturbation ($\delta\lambda$) in the parameter space. Generally speaking, ρ_1 and ρ_2 do not commute. Formally, we can write ρ_1 and ρ_2 in terms of Hamiltonian

$$\rho_1 = \frac{e^{-\beta H(\lambda_1)}}{Z(\beta,\lambda_1)}, \rho_2 = \frac{e^{-\beta H(\lambda_2)}}{Z(\beta,\lambda_2)} \quad (B.1)$$

where λ_i $(i=1,2)$ denotes the parameters with $\lambda_2 = \lambda_1+\delta\lambda$. The Trotter–Suzuki formula [42] can be used to give an approximation

$$\left|\left|\rho_1^{1/2}\rho_2\rho_1^{1/2} - \frac{e^{-\beta H(\lambda_1)+H(\lambda_2)}}{Z(\beta,\lambda_1)Z(\beta,\lambda_2)}\right|\right|$$
$$< \beta^3\Delta_2[H(\lambda_1), H(\lambda_2)]e^{\beta\|H(\lambda_1)\|+\|H(\lambda_2)\|} \sim (\beta\delta\lambda)^3, \quad (B.2)$$

where

$$\Delta_2[H(\lambda_1), H(\lambda_2)] = \frac{1}{12}(\|\,[[H(\lambda_1), H(\lambda_2)], H(\lambda_2)]\,\|$$
$$+\|\,[H(\lambda_1), H(\lambda_2)], H(\lambda_1)]\,\|). \quad (B.3)$$

If $\beta^3\delta\lambda^3 \ll 1$, then the fidelity becomes

$$\mathcal{F}(\beta,\lambda_1,\lambda_2) \equiv \mathrm{Tr}\sqrt{\rho_1^{1/2}\rho_2\rho_1^{1/2}}$$
$$\approx \mathrm{Tr}\sqrt{\frac{e^{-\beta H(\lambda_1)+H(\lambda_2)}}{Z(\beta,\lambda_1)Z(\beta,\lambda_2)}} = \mathrm{Tr}\sqrt{\rho_1\rho_2}. \quad (B.4)$$

It was shown in [35] that in this limit, the fidelity has the following behavior

$$\mathcal{F}(\beta,\lambda_1,\lambda_2) \approx e^{-\frac{\beta(\delta\lambda^2)\chi}{8}}, \quad (B.5)$$

where χ is the fidelity susceptibility.

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