Theory of Heavy Ion Single and Double Charge Exchange Reactions

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Abstract
Peripheral heavy ion single and double charge reactions are described by fully quantum mechanical distorted wave methods. A special class of nuclear double charge exchange (DCE) reactions proceeding as a one-step reaction through a two-body process are shown to proceed by nuclear matrix elements of a diagrammatic structure as found also in $0\nu2\beta$ decay. These hadronic Majorana-type DCE reactions (MDCE) have to be distinguished from second order DCE reactions, given by double single charge exchange (DSCE) processes, resembling $2\nu2\beta$ decay. The theoretical concepts of MDCE are discussed. First results show that ion-ion DCE reactions are the ideal testing grounds for investigations of rare second order nuclear processes, giving insight into nuclear in-medium two-body correlation.

1 Introduction

To a large extent, nuclear phenomena are well explained by the mean-field dynamics of nucleons, competing with residual interactions of much smaller strength. Because of the central role played by the independent (quasi-)particle model of nuclei, it is tempting to search for processes by which the limits of the shell model are tested. Promising candidates are rare processes which are suppressed by selection rules or because they can proceed only through higher order correlations. Such processes may reveal details of nuclear dynamics which otherwise are hidden behind dominating leading order effects. Interestingly, already in the 1930ties Marie Goeppert-Mayer formulated such ideas in her work on atomic double-gamma and nuclear double-beta decay [1], in the latter case even before the discovery of the nuclear shell model. Nuclear double-beta decay is still an example of highest actuality. Of special interest is neutrino-less nuclear double beta-decay ($0\nu2\beta$), heavily searched for but still waiting to be detected. There is broad consensus that $0\nu2\beta$ decay will be a highly promising gateway to physics beyond the standard model of elementary particle physics. Once observed, it will give direct evidence on the Majorana-nature of neutrinos with far reaching implications for neutrino masses, neutrino-matter interactions and flavour mixing up to the question of the matter-antimatter asymmetry in the universe [2–6]. Such a signal has to be distinguished from the two-neutrino beta-decay ($2\nu2\beta$) [1, 7] which is fully compatible with the standard model. Although both decays correspond to second order nuclear processes, they are dynamically distinct. Double beta-decay with neutrino emission is a sequential decay process where the leptons are emitted subsequently in an uncorrelated manner. A few nuclei are known to decay by this already rather rare process, as discussed e.g. in Ref. [8, 9]. While the matrix elements are accessible by the observed $2\nu2\beta$ transitions such a check against data does not yet exist for $0\nu2\beta$ processes. Thus, estimates of life times and transition probabilities are relying on theoretical investigations, notoriously showing an uncomfortably large spread of values. Independent tests of the nuclear structure input under controllable dynamical conditions are highly necessary, allowing to evaluate and gauge the theoretical results by an independent process. The field will profit tremendously if a surrogate process could be identified which is technically and physically easily accessible. While single charge exchange (SCE) reactions with light and heavy ions have been studied intensively, including our own work [10–12], close to nothing is known about double charge exchange reactions. Only very recently, the NUMEN project has been initiated [13], using heavy ion reactions to explore that unknown territory, also aiming at establishing the relation to double beta decay. Clearly, to establish that connection requires additional efforts in our theoretical understanding of nuclear multi-step reactions. Hence, in section 2 we start with a brief introduction into the

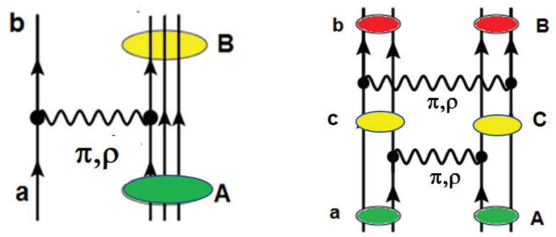

Fig. 1: Diagrammatic structure of nuclear charge exchange reactions by the exchange of $\pi^{\pm}$ and $\rho^{\pm}$ mesons. SCE reactions (left) can be studied by nucleon-induced and light and heavy ion reactions while the sequential DSCE process (right) requires interactions of two complex nuclei.

theoretical background for single and double single charge exchange reactions. In section 3 the physical concept of Majorana DCE reactions is briefly introduced. First results are discussed in section 4 and in section 5 an outlook will be given.

2 Single Charge Exchange and Double Single Charge Exchange Reactions

Single charge exchange reactions (SCE) have become a widely used tool for studying the spin-isospin response of nuclei. The discovery of the giant Gamow-Teller resonance (GTR) by the pioneering experiments at IUCF [14] initiated widespread experimental and theoretical research activities, continuing with even increasing intensity until today. Over the years, a wealth of data has been accumulated as reviewed e.g. in [15–19]. Beyond using nucleonic probes, light ion reactions as e.g. $(^3He, ^3H)$ have become another workhorse of the field, now reaching accuracies allowing to investigate subtle details of spectral distributions in both the τ_+ and the τ_- branches. Soon after the first light ion studies, also heavy ions were used in charge exchange studies as in [11, 20]. It was recognized that peripheral heavy ion collisions, leading to direct reactions, are as useful for spectral studies as light ion scattering. An especially appealing aspects is the broad range of projectile-target combinations which, for example, allow to project out selectively specific features, e.g. spin flip and non-spin flip transitions [10]. Nuclear spin-dynamics and the population of continuum states were central aspects of the $(^7Be, ^7Li)$ reactions considered in [21, 22]. In Fig, 1 the diagrams contributing to a SCE reaction is indicated. Also shown is the DSCE reactions, given by a two-step reaction of sequential SCE processes.

As discussed in detail in Ref. [23], the SCE reaction amplitudes are expressed as DWBA matrix elements of the nucleon-nucleon T-matrix with spin-isospin elements of tensorial rank 0 and form factors $V^{(C)}_{ST}$, and rank 2 with corresponding form factors $V^{(Tn)}_{ST}$. They are connecting the initial channel $\alpha = a + A$ and the final channel $\beta = b + B$. The SCE reaction kernel is given by products of nuclear form factors

$$K^{(ST)}_{\alpha\beta}(\mathbf{p}) = (4\pi)^2 (V^{(C)}_{ST}(p^2) F^{(ab)\dagger}_{ST}(\mathbf{p}) \cdot F^{(AB)}_{ST}(\mathbf{p}) + \delta_{S1}\sqrt{\frac{24\pi}{5}} V^{(Tn)}_{ST}(p^2) Y^*_2(\hat{\mathbf{p}}) \cdot \left[F^{(ab)\dagger}_{ST}(\mathbf{p}) \otimes F^{(AB)}_{ST}(\mathbf{p})\right]_2) \tag{1}$$

where the rank-2 tensorial coupling relates to the spin degrees of freedom only. Through the form factors $F^{(ab),(AB)}_{ST}$, the kernels contain the spectroscopic information on the nuclear transitions, and the dynamics by the interaction form factors $V^{(C),(Tn)}_{ST}$. In the central interaction part, the scalar product indicates the contraction of the projectile and target form factor with respect to the spin and isospin degrees of freedom. The isospin degrees of freedom are of course projected by the nuclear transitions to the proper combination of $\tau_{\pm}$ operators. In terms of the reaction kernels, the SCE transition potential is found

$$\mathcal{U}_{\alpha\beta}(\mathbf{p}) = \sum_{ST} K^{(ST)}_{\alpha\beta}(\mathbf{p}). \tag{2}$$

A caveat of heavy ion scattering is the dominant role played by initial (ISI) and final (FSI) state ion-ion interactions. Hence, the question arises whether under such conditions it is still possible to deduce spectroscopic information from data. From our former work, we conclude that peripheral heavy ion reactions indeed show a clear correlation of cross sections and nuclear spectroscopy. However, in order to understand the subtleties of that connection a detailed study of the reaction mechanism will be helpful, as done recently for single charge exchange reactions in Ref. [23]. Here, we only indicate the approach allowing to separate formally reaction and nuclear structure effects.

The ISI/FSI effects contained in the distorted waves $\chi^{(\pm)}_{\alpha,\beta}$ are well described by optical potentials. The strategy is to factorize the reaction amplitude into a plane wave form factor part, i.e. the Fourier transform of the nuclear transition currents and densities, and an amplitude containing the elastic ion-ion interactions. As discussed in [23] a momentum space representation allows to perform such a separation. The one-step SCE reaction amplitude is obtained

$$M_{\alpha\beta}(\mathbf{k}_\alpha, \mathbf{k}_\beta) = \int d^3p N_{\alpha\beta}(\mathbf{p})\mathcal{U}_{\alpha\beta}(\mathbf{p}). \tag{3}$$

Initial and final state interactions are now described by the distortion coefficient [23, 24]

$$N_{\alpha\beta}(\mathbf{p}) = \frac{1}{(2\pi)^3}\langle\chi^{(-)}_\beta|e^{-i\mathbf{p}\cdot\mathbf{r}}|\chi^{(+)}_\alpha\rangle. \tag{4}$$

As discussed in detail in [23], the distortion coefficient $N_{\alpha\beta}$ is closely related to the elastic scattering amplitude: For $p \to 0$ and $k_\beta \to k_\alpha$ the definition of the elastic S-matrix is indeed recovered. Thus, in leading order, the above equation corresponds to the folding of the nuclear transition form factors with the ion-ion elastic scattering amplitude. Becasue of the strong absorption, the distortion coefficient acts mainly as a scaling factor, typically reducing the forward cross section by several orders of magnitudes compared to the plane wave limit. Only at momentum transfers exceeding 100 MeV/c $N_{\alpha\beta}$ leads to modifications of the momentum structure of cross sections.

If we consider, on the other hand, the effective operator underlying the conventional double-SCE two-step reaction mechanism, we find

$$V^{(DSCE)}(\mathbf{13}, \mathbf{24}) \sim \sum_{cC} T_{NN}(\mathbf{3}, \mathbf{4})\mathcal{G}_{cC}(\mathbf{2}-\mathbf{4}, \mathbf{1}-\mathbf{3})T_{NN}(\mathbf{2}, \mathbf{1}) \tag{5}$$

where T_{NN} is the isovector nucleon-nucleon T-matrix and $\mathcal{G}_{cC}$ denotes the (full many-body) propagator of the intermediate nuclei reached in the first SCE reaction step. Using distorted waves, we find in momentum representation

$$\mathcal{G}_{cC}(\omega) = \int \frac{d^3k}{(2\pi)^3}\frac{|\chi^{(+)}_\gamma cC\rangle\langle cC\tilde{\chi}^{(+)}_\gamma|}{\omega - \omega_k - E_c - E_C - i0+} \tag{6}$$

where the biorthogonality of the optical waves has been taken into account by $\tilde{\chi}^{(+)}_\gamma$. Hence, the DSCE reaction amplitude is given effectively by a second order distorted wave expression

$$M^{(DSCE)} = \langle\chi^{(-)}_\beta bB|V^{(DSCE)}|aA\chi^{(+)}_\alpha\rangle. \tag{7}$$

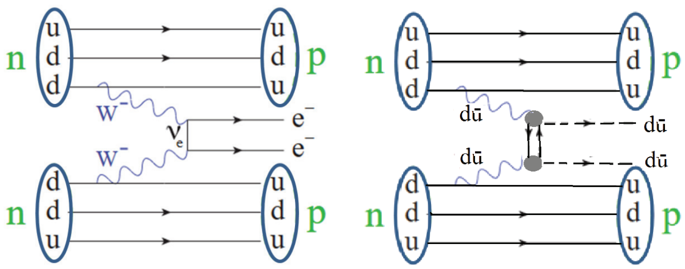

Fig. 2: The elementary weak interaction process mediating nuclear $0\nu2\beta$ decay (left) and a corresponding strong interaction process (right) are depicted schematically. The QCD counterpart is given by the simultaneous emission of two $[d\bar{u}]$ pairs in an isovector-vector, e.g. 1^-, configuration (wavy lines), decaying into a pionic $[d\bar{u}]$ configuration and a charge-neutral $q\bar{q}$ pair.

3 Majorana Double Charge Exchange Reactions

Second order quantal processes like heavy ion double charge exchange reactions are of genuine reaction theoretical interest. First of all, until now heavy ion DCE reactions have not been studied, neither experimentally nor theoretically. Some attempts were made on (π^+, π^-) reactions but the notoriously bad energy definition of the incoming pion beams is unfavorable for spectroscopic work. Thus, double charge exchange reactions with heavy ions are much better suited for explorations of weakly populated transitions. Here, we consider collisional charge exchange processes given by elementary interactions between target and projectile nucleons. In accordance with explicit calculations, the mean-field driven transfer contributions are neglected because they are at least of 4th order for DCE reactions considered here [25]. Thus, only processes with changes of the charge partitions but leaving the projectile-target mass partition unaltered will be discussed.

A central question is whether we can identify on the elementary level a correspondences between strong and weak interaction processes. The answer is yes, as illustrated in Fig. 2. Under nuclear structure aspects, the $0\nu2\beta$ decay of a nucleus is nothing but special class of two-body correlation, sustained by the exchange of a (pair of) Majorana neutrino(s) between two nucleons where the interaction vertices are given by the emission of virtual $W^{\pm}$ gauge bosons. The strong interaction counterpart is a two-nucleon correlation built up by the exchange of a virtual charge-neutral quark-antiquark ($q\bar{q}$) pair accompanied by the emission of a charged $q\bar{q}$ component, thus changing at the same time the nucleonic charges. Similar to the weak process, the strong vertices are originating from gauge bosons, here given by the initial emission of gluons which materialize into two $q\bar{q}$ pairs. At the end, the highly off-shell $q\bar{q}$ compounds will decay into mesons, preferentially into pions but also multi-pion configurations like the scalar and vector mesons.

Such interaction process may occur frequently in nuclei, both on the weak as well as on the strong interaction scale. They remain unobserved if the emitted electrons or charged mesons are reabsorbed by the same nucleon. This will lead to vertex and propagator correction and, as such, contributes to the nucleon in-medium mass operator of a, however, negligibly small strength. Nevertheless, it is worthwhile to keep in mind that we are dealing here with phenomena belonging to the large class of nuclear ground state correlations beyond the commonly studied mean-field sector [26–29]. Short-range correlations are known to modify nuclear momentum contributions on the level of up to 20%.

Both processes become of interest if they reveal their existence and nature in observable signals. In this respect, we encounter a fundamental difference between $0\nu2\beta$ decay and the hadronic process: Only the former may occur in an isolated nucleus while the latter one is inhibited by energy conservation. Thus, in order to observe the double-meson emission by a nucleon pair a partner nucleus is required which takes

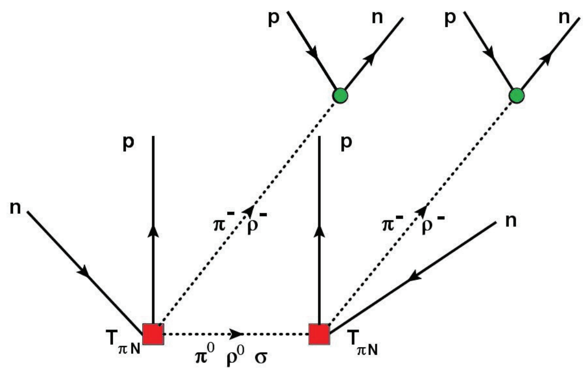

Fig. 3: Generic diagram illustrating the hadronic surrogate process for $0\nu 2\beta$ decay. A virtual $nn \to pp\pi^-\pi^-$ scattering process, causing the $\Delta Z = +2$ target transition $A \to B$, is accompanied by $nnp^{-1}p^{-1}$ double-CC excitation in the projectile. Crossed diagrams are not displayed.

care of the virtuality of the process by absorbing the two charged virtual mesons. For that purpose, heavy ion double charge exchange reactions are the ideal tool. The diagrammatic structure of such a reaction is indicated in Fig. 3. The target undergoes a correlated double-meson pair decay $nn \to pp\pi^-\pi^-$ (in fact a $2p2n^{-1}$ 2p-2h transition with coherent emission of a pion pair) and the projectile absorbs the pions by the simultaneous excitation of two np^{-1} -type configurations. Other mesons will contribute as well.

The whole reaction will proceed as a one-step reaction via a special kind of two-body interaction generated by the correlation diagram. Denoting the (in-medium) pion-nucleon T-matrix by $T_{\pi N,\pi' N'}$, the target-part of the interaction is in a somewhat symbolic notation

$$V^{(MDCE)}(\mathbf{13},\mathbf{24}) \sim T_{\pi^- p,\pi^0 n}(\mathbf{1},\mathbf{3}) D_{\pi^0}(\mathbf{1}-\mathbf{2}) T_{\pi^0 n,\pi^- p}(\mathbf{2},\mathbf{4}) \tag{8}$$

where the $n \to p$ target transitions are denoted by $\mathbf{1}$ and $\mathbf{2}$, respectively. The coordinates $\mathbf{3}$ and $\mathbf{4}$ indicate the outgoing charged pions, inducing the complementary transitions in the projectile. The correlation built up by the neutral pion is described by the propagator D_{π^0}. A decomposition into irreducible tensors gives rise in particular to an effective rank-2 iso-tensor projectile-target interaction of operator structure $[\boldsymbol{\tau}_{1A} \otimes \boldsymbol{\tau}_{2A}]_2 \cdot [\boldsymbol{\tau}_{3a} \otimes \boldsymbol{\tau}_{4a}]_2$. We recognize immediately the similarity to the nuclear matrix element of $0\nu 2\beta$ decay, justifying the name Majorana-DCE. At present, the strengths of the nucleon-nucleon and the pion-nucleon T-matrices are taken from data. More refined description will be scrutinized in future work by an effective field theoretical description [30] and by referring to the data base available for free space $nn \to pp\pi^-\pi^-$ reactions. Previously, the charge-conjugated reaction $pp \to nn\pi^+\pi^+$ reaction and other double-pion production channels were investigated at CELSIUS and COSY [31–35] and more recent also at HADES [36]. Theoretical studies of the on-shell reaction combining meson exchange and resonance excitation are found in [37, 38].

4 Heavy Ion DCE Reactions and Data

The full DCE reaction amplitude is given by the coherent sum of the MDCE and the DSCE amplitudes:

$$M_{\alpha\beta} \sim \langle \chi_\beta^{(-)\dagger}, bB | V^{(MDCE)} + V^{(DSCE)} | aA, \chi_\alpha^{(+)} \rangle = M_{\alpha\beta}^{(MDCE)} + M_{\alpha\beta}^{(DSCE)} \tag{9}$$

The results discussed below were obtained fully quantum mechanically by one-step DWBA calculations for the MDCE amplitudes and second order DWBA calculations for the DSCE amplitudes, respectively. Thus, ion-ion interactions are treated to all orders. As discussed in Ref. [23] the strong absorption

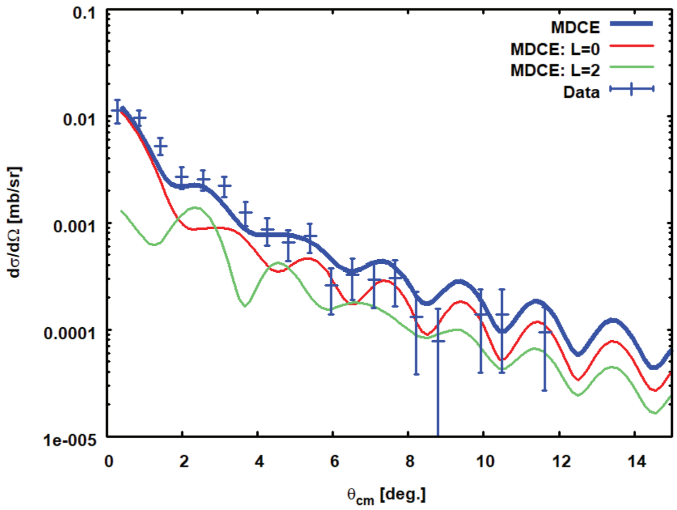

Fig. 4: Angular distribution of the DCE reaction $^{18}O + ^{40}Ca \to ^{18}Ne + ^{40}Ar$ at $T_{lab} = 270$ MeV. Theoretical MDCE results are compared to the data of Ref. [39]. In addition to the full MDCE cross section also the partial contributions with $L = 0, S = 1$ and $L = 2, S = 2$, respectively, are shown separately. The theoretical results have been normalized to the data at the smallest scattering angle.

in grazing ion-ion collisions allows to evaluate the distortion coefficient in black disk approximation. As mentioned before, under such conditions the ISI/FSI effects are resulting effectively in a scaling factor, allowing to relate at forward angles the cross sections to the corresponding plane wave cross sections. Thus, in principle spectroscopic information can be extracted from the data, provided the elastic interactions are known to the necessary precision. In the present calculations, double folding potentials have been used.

First results of a DCE calculation along the line discussed above are shown in Fig. 4 and compared to recent NUMEN data [39] for the reaction $^{18}O + ^{40}Ca \to ^{18}Ne + ^{40}Ar$ at $T_{lab} = 15$ AMeV. The reaction leads from the 0^+ ground states of the initial to the 0^+ ground states of the final nuclei, constraining the total angular momentum transfer to $J^P = 0^+$. The transition strengths are taken from QRPA calculations, see e.g. Ref. [23]. For these exploratory investigations MDCE form factors and interactions were treated schematically by approximating the complex off-shell momentum structure by the on-shell strength. Only the pionic contributions were included. This leaves open an overall scaling factor which was fixed by normalizing the MDCE cross section to the data point at the smallest scattering angle. The forward peak of the angular distribution is dominated, in fact, by the ($L = 0$, $S = 1$) MDCE component. However, as seen in Fig. 4, the ($L = 2$, $S = 2$) components are of comparable importance at the larger scattering angles. Moreover, they are essential for the description of the data. Overall, the shape of the measured angular distribution is described decently well in view of the exploratory character of the calculations.

A competing reaction mechanism is the two-step DSCE process. In Fig. 5, DSCE and MDCE cross sections are displayed and compared to data as a function of the momentum transfer. Remarkably, the measured angular range covers a momentum range of more than 400 MeV/c. The DSCE cross section was normalized to the large angle region because higher order reactions typically prevail at larger momentum transfers. That conjecture is confirmed by the DSCE angular distribution: Aside from the typical L=0 forward structure the main body of the angular distribution oscillates around a mean-value of a few times $10^{-4} mb/sr$. Cross sections of igher multipolarities are carrying less strengths and are of flatter shape. With all caution, we may conclude that the data are in favor of the one-step MDCE

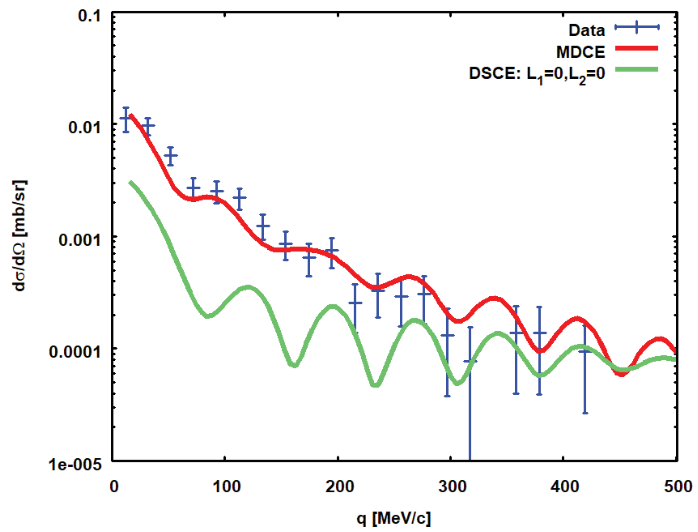

Fig. 5: Angular distribution of the DCE reaction ${}^{18}O + {}^{40}Ca \to {}^{18}Ne + {}^{40}Ar$ at $T_{lab} = 270$ MeV. The one-step MDCE and the two-step DSCE cross sections are shown separately in comparison to the data of Ref. [39].

angular distribution. Even if the DSCE cross section would be scaled to the measured forward angle cross section, its shape would not match the observed angular distribution, an observation giving further evidence to the dominance of the MDCE one-step reaction mechanism.

5 Outlook

A new theoretical scenario for heavy ion double charge exchange reactions was introduced. At the diagrammatic level, structures similar to $0\nu2\beta$ matrix elements have been identified. The hadronic Majorana-DCE process is accessible only by reactions of composite nuclei. We have discussed explicitly the case of a DCE reaction with medium mass ions at relatively low incident energy. ISI and FSI ion-ion interactions were taken into account and the quantum mechanical coherence of the MDCE and the DSCE reaction mechanism was treated properly. The strongly forward peaked measured angular distributions indicate a direct mechanism which indeed is confirmed by the calculations. These first results are very promising by indicating a new way of accessing second order nuclear matrix elements of charge changing interactions. Together with the much better studied SCE reactions and their established usefulness for spectroscopic work, heavy ion DCE reactions are opening a new window to high-precision spectroscopy. Although it will not be possible to insert the extracted matrix elements directly into a $0\nu2\beta$ analysis, DCE reactions provide an unique way to validate nuclear structure models under controllable laboratory conditions by comparison to data on processes of comparable physical content. New impact on theoretical investigations in both reaction and nuclear structure theory is demanded for a quantitative understanding of these special reactions. Although the present calculations do not yet include the full spectrum of contributions, they are establishing the hadronic Majorana-DCE reaction mechanism. The refinements may lead to changes in detail but will not alter the overall picture. An exciting and encouraging result is that the MDCE process is clearly visible, even dominating the cross section at extreme forward angles.

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