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RESEARCH ARTICLE

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A note on the generalized Friedmann equations for a thick brane

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Abstract Within our thick brane approach previously used to obtain the cosmological evolution equations on a thick brane embedded in a five-dimensional Schwarzschild Anti-de Sitter spacetime it is explicitly shown that the consistency of these equations with the energy conservation equation requires that, in general, the thickness of the brane evolves in time. This varying brane thickness entails the possibility that both Newton's gravitational constant G and the effective cosmological constant Λ_4 are time dependent.

Keywords Brane cosmology, Variable brane thickness, Variable physical constants

1 Introduction

Cosmological models in which the observed universe is realized as a four-dimensional thin brane embedded in a higher dimensional spacetime (bulk) are of current interest. In these theories the cosmological evolution on the brane is described by the effective Friedmann equations that incorporates non-trivially the effects of the bulk. The most important feature that distinguishes brane cosmology from the standard scenario is the fact that at high energies Friedmann equation modifies by an extra term quadratic in the energy density of matter on the brane [**?**]. Several features and variations of this scenario including thick brane configurations, in particular, in the cosmological context have been considered. In the approach used in Ref. [**?**] to derive generalized Friedmann equations, the four-dimensional effective brane quantities are obtained by integrating the corresponding five-dimensional ones along the extra-dimension over the brane thickness. These cosmological equations describing a brane of finite thickness interpolate

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between the case of an infinitely thick brane corresponding to the familiar Kaluza–Klein picture and the opposite limit of an infinitely thin brane giving the unconventional Friedmann equation, where the energy density enters quadratically. The latter case is then made compatible with the conventional cosmology at late times by introducing and fine tuning a negative cosmological constant in the bulk and an intrinsic positive tension in the brane [**? ?**]. Navarro and Santiago [**?**] considered a thick codimension 1 brane including a matter pressure component along the extra dimension in the energy–momentum tensor of the brane. By integrating the 5D Einstein equations along the fifth dimension, while neglecting the parallel derivatives of the metric in comparison with the transverse ones, they wrote the equations relating the values of the first derivatives of the metric at the brane boundary with the integrated componentes of the brane energy–momentum tensor. These, so called matching conditions were then used to obtain the cosmological evolution of the brane which is of a non-standard type, leading to an accelerating universe for special values of the model parameters. On the other hand the cosmological implications of brane world scenario in which fundamental physical constants are variable functions of time have been investigated [**? ?**]. For instance, in Ref. [**?**] was shown that the introduction of a scalar function varying with time in the brane-world theory yields a number of cosmological models which do not admit constant values for the Newton's gravitational coupling G and the effective cosmological term Λ_4.

We are interested in the effects arising from taking into account the thickness of the brane. In this way, we have presented a general equation to study the dynamics of codimension one brane of finite thickness immersed in an arbitrary bulk spacetime. That was obtained in a general setting by imposing the Darmois junction conditions on the brane boundaries with the two embedding spacetimes and then using an expansion scheme for the extrinsic curvature tensor at the brane boundaries in terms of the proper thickness of the brane. Using this approach we have derived the generalized Friedmann equations written up to the first order of the brane proper thickness governing the cosmological evolution of a thick brane embedded in a five-dimensional Schwarzschild Anti-de Sitter spacetime with a Z_2 symmetry [**?**].

The aim of this note is to address the presence of the brane with a time dependent thickness as a general feature of our thick brane cosmological solutions.

In Sect. 2 we give a brief review of the generalized cosmological equations as obtained in Ref. [**?**]. In Sect. 3 we derive an evolution equation for the brane thickness as required from the self-consistency of our equations. In Sect. 4 conclusion is presented.

2 Generalized cosmological equations

We consider a thick brane embedded in a five-dimensional Schwarzschild Anti-de Sitter spacetime with the metric

$$ds^2 = -f(r)dT^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_k^2, \tag{1}$$

with

$$f(r) = k - \frac{\Lambda}{6} r^2 - \frac{C}{r^2}, \tag{2}$$

where $d\Omega_k^2$ is the metric of the 3D hypersurfaces Σ of constant curvature that is parameterized by $k = 0, \pm 1$; Λ is the bulk cosmological constant, and constant C is identified with the mass of a black hole located at $r = 0$. We then take the following ansatz for the metric of the brane being written in a Gaussian normal coordinate system in the vicinity of the core of the thick brane situated at $y = 0$

$$ds^2 = -n^2(t,y)dt^2 + dy^2 + a^2(t,y)d\Omega_k^2, \tag{3}$$

where $n^2(t,y)$ and $a^2(t,y)$ are some unknown functions, and y is the normal coordinate of the extra dimension. Compatibility with the cosmological symmetries requires that the energy–momentum tensor of the matter content in the brane takes the simple form

$$T^{\mu}_{\nu} = (-\rho, P_L, P_L, P_L, P_T), \tag{4}$$

where the energy density ρ, the longitudinal pressure P_L, and the transverse pressure P_T are functions of t and y.

The generalized first Friedmann equation written up to first order of the brane thickness takes the following form within our formalism [**?**]

$$\sqrt{f_0 + \dot{a}_0^2} = \frac{w}{a_0}\left(\frac{-\Lambda}{3}a_0^2 + \frac{\kappa^2}{3}\rho_0 a_0^2\right), \tag{5}$$

where the subscript "0" means evaluation at the center of the thick brane and the dot stands for the derivative with respect to the proper time τ on the brane center. Taking the square of Eq. (5), substituting the expression (2) and rearranging, we arrive at the following equation

$$H_0^2 + \frac{k}{a_0^2} = \frac{8\pi G}{3}\rho + \frac{\kappa^4}{36}\rho^2 + \frac{\Lambda_4}{3} + \frac{C}{a_0^4}, \tag{6}$$

where $H_0 = \frac{\dot{a}_0}{a_0}$, and the effective four-dimensional energy density ρ associated to the five-dimensional energy density ρ has been defined as

$$\rho = \int_{-w}^{w} \rho\, dy \simeq 2w\rho_0 + O(w^2), \tag{7}$$

and the following identifications have been made

$$\frac{\Lambda_4}{3} = \frac{\Lambda}{6} + \frac{w^2\Lambda^2}{9}, \qquad 8\pi G = \frac{\kappa^2 w(-\Lambda)}{3}. \tag{8}$$

We see that there is a linear in addition to a quadratic term in the matter density, due to the non-vanishing of the thickness w. There is no need of introducing an ad hoc tension for the brane, and splitting it from the matter density on the brane.

The generalized second Friedmann equation written up to first order of the brane thickness takes the following form within our formalism [**?**]

$$\frac{\frac{\ddot{a}_0}{a_0}-\frac{\Lambda}{6}+\frac{C}{a_0^4}}{\sqrt{f_0+\dot{a}_0^2}}=\frac{-w}{a_0}\left(\frac{2\kappa^2}{3}\left(\rho_0+\frac{3}{2}P_L^0-P_T^0+\frac{3}{a_0^4}\tilde{P_T}\right)+\frac{\Lambda}{6}+\frac{3E}{a_0^4}+\frac{3C}{a_0^4}\right), \tag{9}$$

where $E>0$ is an integration constant, $P_L^0=P_L(t,y=0)$, and $P_T^0=P_T(t,y=0)$, and we have also defined $\tilde{P_T}\equiv\int_0^\tau P_T^0 a_0^4 H_0 d\tau$. Note now that the time component of the covariant derivative of the brane energy–momentum tensor (4), using the metric (3), leads to the familiar energy conservation condition on the core of the thick brane:

$$\dot{\rho}_0+3H_0(\rho_0+P_L^0)=0. \tag{10}$$

Defining the four-dimensional effective quantities associated to the five-dimensional longitudinal and transverse pressures P_L and P_T in the form

$$p_L=\int_{-w}^{+w}P_L\,dy\simeq 2wP_L^0+O(w^2), \tag{11}$$

$$p_T=\int_{-w}^{+w}P_T\,dy\simeq 2wP_T^0+O(w^2), \tag{12}$$

Let us assume the arbitrary effective equations of state of the form

$$p_L=\omega_L\rho, \qquad p_T=\omega_T\rho, \tag{13}$$

with constants ω_L and ω_T. The conservation equation (10) can then be integrated with the result as usual

$$\rho_0=\rho_i a_0^{-3(1+\omega_L)}, \tag{14}$$

where ρ_i is a constant. Then $\tilde{P_T}$ can be computed as

$$\tilde{P_T}=\frac{\rho_i\omega_T}{1-3\omega_L}a_0^{1-3\omega_L}. \tag{15}$$

3 Evolution of brane thickness

Let us now write down the time derivative of the first Fridmann equation (5) while allowing the brane thickness w to evolve in time. We get

$$\frac{\frac{\ddot{a}_0}{a_0}-\frac{\Lambda}{6}+\frac{C}{a_0^4}}{\sqrt{f_0+\dot{a}_0^2}}=\frac{-w}{a_0}\left(\frac{2\kappa^2}{3}\left(\rho_0+\frac{3}{2}P_L^0\right)+\frac{\Lambda}{3}\right)+\frac{\dot{w}}{\dot{a}_0}\left(-\frac{\Lambda}{3}+\frac{\kappa^2}{3}\rho_0\right), \tag{16}$$

where the energy conservation equation (10) has been used. Comparing this with the second Fridmann equation (9) we infer

$$\frac{\dot{w}}{w} = \frac{-H_0}{\frac{-\Lambda}{3} + \frac{\kappa^2}{3}\rho_0} \left(\frac{2\kappa^2\rho_0}{3} \left(\frac{3\omega_T}{1-3\omega_L} - \omega_T \right) + \left(\frac{3E}{a_0^4} + \frac{3C}{a_0^4} + \frac{-\Lambda}{6} \right) \right), \qquad (17)$$

where we have used Eqs. (7), (11)–(15). This is an evolution equation for the brane thickness indicating, in general, the presence of the brane with a time dependent thickness. Depending on the magnitude and sign of the different terms within the bracket in Eq. (17), the brane thickness may increase or decrease with time. For instance, we observe that for $\omega_T = 0$, the brane thickness extended along the extra dimension always decreases with time while its ordinary spacial dimensions expand if $k = 0$, or -1. On the other hand for $\omega_T < 0$ leading to an accelerating brane cosmology at late time [**?**], from Eq. (17) one can see that it is possible to have an increasing brane thickness with time. Furthermore, as the equation (17) poses, the parameters within the bracket must be finely tuned to yield a time constant brane thickness.

4 Conclusion

In this note we showed that the requirement of compatibility our thick brane cosmological equations written up to the first order of the brane thickness with the familiar energy conservation equation leads to an evolution equation for the brane thickness. In the absence of the pressure along the extra dimension in the brane energy–momentum tensor the thickness of the brane decreases with time while at late time a negative transverse pressure can lead to an increase in the brane thickness. We have realized that our two generalized Friedmann equations (5) and (9) together with the energy conservation equation (10) form the three independent equations to determine the evolution of the three unknown functions a_0, ρ_0, and w once one gets the equations of state (13) for brane matter. According to the identifications made in (8) a time dependent brane thickness induces time variations in the Newton's gravitational coupling G and the effective cosmological term Λ_4. This suggests our thick brane cosmological model as a general framework in which one enables to study the simultaneous variation of G and Λ_4.

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