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Generalized Quasispin Formalism and Fermion Bases of the Model of Interactable Bosons

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Abstract

A relatively simple, detailed microscopic calculation of IBM parameters based on the generalized quasi-spin formalism is proposed, by including in the theory of the double tensor, acting in the spaces, both angular moments and generalized quasispins. The method is applied to the study of the structure of the collective state of even isotopes $^{118-130}Xe$. The spectrum and probabilities of electromagnetic transitions of nuclei are calculated and they are compared with the available experimental data.

I. Introduction

In the presented work, we propose a fermionic-microscopic calculation of model of interacting bosons [IBM] [1, 2]. IBM parameters based on the generalized quasi-spin formalism (GQF), in which the model SD-pair shell space is described by the quantum number of the generalized senoriti and with given inter-nucleon forces [3-5].

The proposed method the shortcomings of the generalized quasi-spin formalism are eliminated by the introduction of new generalized quasi-spin interaction operators that more adequately describe the real properties of existing nuclei. These operators include values that reflect the uneven distribution of nucleons in different no degenerate $j-$ shells.

The use of the method of generalized quasispin formalism (GQF), gave the opportunity to more accurately resolve the many-particle problem for fermionic systems with a certain number of particles and with the specified nucleon forces. The introduction of the concept of generalized quasispin allows us to distinguish in the full fermionic space SD-pair collective and quasi-its areas. This, in turn, facilitates the calculation of matrix elements of the interaction operators through the nucleon pair forces.

II. Generalized quasispin method

It is known that the particle birth operator a^{+}_{jm} is a spherical tensor operator of half-integer rank j – in the angular momentum space. Then tensor products of these operators are defined as: $A^{+}(j_1 j_2; JM)$ $\tilde{A}(j_1 j_2; JM)$ $U(j_1 j_2; JM)$,

where, $A^{+}\left(j_1 j_2; jM\right) = \left(1+\delta_{j_1 j_2}\right)^{-1/2}\left[a^{+}_{j_1 m_1} a^{+}_{j_2 m_2}\right]^{J)}_{M}$, (2.1)

These operators record all kinds of two-part interactions:

$$V = \sum_{j_1 j_2 j_3 j_4} \sqrt{2J+1} G_j\left(j_1 j_2 j_3 j_4\right)\left[A^{+}\left(j_1 j_2 J\right)\tilde{A}\left(j_3 j_4 J\right)\right]^{0)}_{0}, \quad (2.2)$$

where, $G_J\left(j_1 j_2 j_3 j_4\right) = \left(1+\delta_{j_1 j_2}\right)\left(1+\delta_{j_3 j_4}\right)/4 \cdot \left\langle j_1 j_2 J \middle| V \middle| j_3 j_4 J \right\rangle$ (2.3)

On the other hand, the three tensor operators form a generalized quasispin group:

$$S_{+} = \sum_j a_j S^{+}_j; S_{-} = \sum_j \frac{1}{a_j} S^{-}_j; S_0 = S^0_j = \frac{N-\Omega}{2}, \quad (2.4)$$

$$\left[S_{+}, S_{-}\right] = 2S_0, \left[S_0, S_{\pm}\right] = \pm S_{\pm}, \quad (2.5)$$

where, a_j – some constant values that determine the amplitudes of the probabilities of orbiting $N = \sum_j N_j, \Omega = \sum_j a_j, \Omega_j = j + 1/2$. These three generalized quasispin operators $S_0, S_{\pm}$ also have the switching properties of ordinary quasispin operators and are generators of the Lie algebra. As can be seen from (2.5), the generalized quasispin operators consist of ordinary quasispin operators $S^{\pm}_j, S^0_j$.

$$S^{+}_j = \sqrt{\Omega} A^{+}\left(j_1 j_2; 00\right);\ S^{-}_j = \sqrt{\Omega}\tilde{A}\left(j_1 j_2; 00\right), S^0_j = \frac{1}{2}\left(N^j - \Omega^j\right) \quad (2.6)$$

Where $N^j = \sum_m a^{+}_{jm} a_{jm} = \sum (-)^{j-m} a^{+}_{jm} a_{jm}$.

Then in quasispecular introduced generalized formalism it is also possible to introduce a complete generalized statement of quasispin S:

$$S^2 = S_{+} S_{-} + S^2_0 - S_0 \quad (2.7)$$

Such generalized operators satisfy similar commutation relations as the angular momentum operators obey. The state vectors are now characterized by new quantum numbers s and $s_0, = -s, s+1, ...s$ and which determine the eigenvalues of the full quasispin operators s and its projections s_0. These numbers form a

generalized quasispin space whose wave functions are denoted by $|s, s_0, q\rangle$. Using the switching rules (2.5), we obtain:

$$S_-|s, s_0, 1, q\rangle = const|s, s_0 - 1, q\rangle, S_+|s, s_0, 1, q\rangle = const|s, s_0 + 1, q\rangle \quad (2.8)$$

The values of the quantum numbers s_0 and s are also determined by the alternative numbers of the generalized senerity v and the total number of nucleons N using the relations:

$$s = \frac{1}{2}(\Omega - v), s_0 = \frac{1}{2}(N - \Omega) \quad (2.9)$$

The interaction pairing Hamiltonian

$$H_s = \varepsilon N - GS_+S_- \quad (2.10)$$

also diagonal in the representation of the generalized quasispin as in the usual quasispin space.

Wave functions of excited states with JM:

$$|s, s_0 JM\rangle = K_{n,v}^{\Omega}(S_+)^n \left| J, J_0^{'} = -J, JM \right\rangle, \quad (2.11)$$

where $n = \frac{1}{2}(N - v)$ number of paired particles and $K_{n,v}^{\Omega} -$ normalization constants.

Although, in General, the $a_j \neq 1$ operator H_s of non-hermites, but its eigenvalues are real, in form formally coincide with the eigenvalues H_s in ordinary quasispin space. And all physical quantities are determined by means of constants a_j, describing the probability of distribution of particles in non-degenerate States.

III. A problem with a complete Hamiltonian in the representation of a generalized quasispin

We consider a multiparticle problem in double spaces of a generalized quasispin with an arbitrary pair interaction operator. The complete Hamiltonian in this case is conveniently split into two parts by isolating the pairing interaction from it: $H = H_s + W$, where $W -$ the operator expressing the rest of the particle interaction but diagonal in the representation of the generalized quasispin s

$$W = \sum_{j_1 j_2 j_3 j_4} \left\langle j_1 j_2 \left| V^{'} \right| j_3 j_4 \right\rangle A_+(j_1 j_2 JM) A_-(j_3 j_4 JM) \quad (3.1)$$

Then the eigenvalue problem of the full Hamiltonian H diagonal in $s -$ representation is reduced to the solution of the equation

$$H|s, s_0, q\rangle = E(n = s + s_0, v = \Omega - 2s, q)|s, s_0, q\rangle. \quad (3.2)$$

The total energy of the system is also divided into two parts:

$$E(n, v, q) = E_s(N = 2n + v, v)E^{'}(n, v, q), \quad (3.3)$$

where $E_s -$ eigenvalues of the pairing part of the Hamiltonian H_s.

We find the conditions under which the full Hamiltonian H will be diagonalized in the representation of the generalized quasispin. It is sufficient for the functions (2.11) to be the operator's own functions W:

$$W\left|s,s_0,q\right\rangle E^{'}\left(n,v,q\right)\left|s,s_0,q\right\rangle . \tag{3.4}$$

This equation can be reduced to several easily solved, independent of n equations. For this purpose, we use the commutator:

$$[W,S_+]2\sum\left\langle j_1j_2\left|V^{'}\right|j_3j_4\right\rangle A_+\left(j_1j_2JM\right)\left\{\frac{\sqrt{\Omega^{js}}aj_3\delta^{00}_{JM}}{\sqrt{2}}-(-)^{J-M}aj_3T^J_{-M}+(-)^{j_s+j_4-M}aj_4T^J_{-M}\left(j_3j_4\right)\right\}, \tag{3.5}$$

$$\text{where } T^J_M\left(jj^{'}\right)=\left(2\sqrt{1+\delta_{jj^{'}}}\right)^2\sum\nolimits_{mm^{'}}\left(jj^{'}m-m^{'}/JM\right)a^+_{jm}\tilde{a}_{j^{'}m^{'}} \tag{3.6}$$

This operator breaks a pair of particles in the state $S_+\left|0\right\rangle$ and transfers them to the excited state $A_+\left(j_1j_2JM\right)$. In addition, we introduce the operator of the birth of v unpaired particles with a total angular momentum J:

$$Q^+\left(v,JM\right)\left|0\right\rangle\sum\nolimits_j\gamma^{v,J}_jQ^+\left(j^v,JM\right)\left|0\right\rangle . \tag{3.7}$$

Then the equation (3,4) can be rewritten as:

$$W\left(S_+\right)^nQ^+\left(v,JM\right)\left|0\right\rangle=E^{'}\left(n,v,J\right)\left(S_+\right)^nQ^+\left(v,JM\right)\left|0\right\rangle . \tag{3.8}$$

This equation takes place only if executed equality:

$$\left[\left[W\left(S_+\right)\right],Q^+\left(v,JM\right)\left|0\right\rangle\right]=\lambda\left(v,JM\right)S_+Q^+\left(v,JM\right)$$

$$E^{'}\left(n,v,J\right)=E^{'}\left(0,v,J\right)+n\lambda\left(v,J\right)$$

As a result, the diagonalization conditions of the full Hamiltonian $H=H_s+W$ in $s-$ representation will be reduced to solving a system of equations

$$\begin{aligned}&HS_+\left|0\right\rangle=E_0S_+\left|0\right\rangle,\left[\left[H,S_+\right],S_+\right]=2G\left(S_+\right)^2\\&HQ^+\left(v,JM\right)\left|0\right\rangle=E\left(v,J\right)Q^+\left(v,JM\right)\left|0\right\rangle,\\&\left[\left[H,S_+\right],Q^+\left(v,JM\right)\right]=\left(vG+\lambda\left(v,J\right)\right)S_+Q^+\left(v,JM\right),\end{aligned} \tag{3.9}$$

where, $E_0=E_s\left(N=2,v=0\right)2\varepsilon-G\Omega.$

The total energy of the system is determined by equality (3.3).Thus, the solution of the problem with the full Hamiltonian H leads to the removal of degeneration of states by angular momentum *J* in multiplets characterized by the quantum number of the generalized senority *v*, whose positions depend linearly on the number of pairs in the system.

As mentioned, the generalized quasispin method allows expressing multiparticle matrix elements through two-particle. To this end, the operators a^+_{jm} and $\tilde{a}_{jm}$ and are recorded as components of double spherical and irreducible tensor ranks $1/2$ and J in spaces such as corner points and quasispin [8-10].

Then the operators consisting of pieces a^+ and a can express the introduced dual tensor T. For example, for the case of $k-$even numbers we have

$$\begin{aligned}&T^{(1.k)}_{1q}(jj)=A^+(jjkq),\ T^{(1.k)}_{-1q}(jj)=\tilde{A}(jjkq)\\&T^{(1.k)}_{1q}(jj)=-\left\{U\left(jjkq\right)+\sqrt{\frac{\Omega}{2}}\delta\left(k,0\right)\right\}.\end{aligned} \tag{3.10}$$

Any one-particle vector operator $\sum_i f_i^{(k)}$ is proportional to a double rank tensor $k-$ in normal and first rank in quasi-spaces $T^{(1.k)}(jj)$.

For the vector potential:

$$\left\langle j^n vaJ \left\| \sum_i f_i^{(k)} \right\| j^n v' a' J \right\rangle = \frac{f_{10}(n)}{f_{10}(v)} \left\langle j^v vaJ \right\rangle \left\| T_0^{(1,k)} \right\| j^n v' a' J, \tag{3.11}$$

where $v' = v, v+2$, $f_{10}(n) = \left((\Omega - v)/2, 1, (n-\Omega)/2, 0 \middle| (\Omega - v)/2, (n-\Omega)/2\right)$

$f_{10}-$the ratio of the Clebsch-Gordan.

In a similar way, two-particle matrix elements can be computed using double tensors. Such formulas are written for scalar two-particle pair interaction operators in the form:

$$V' = -\sum_J \sqrt{2J + G_J} \left[T_{(jj)}^{1,J}\right] \times T_{(jj')}^{1,J} \left(111-1 \middle| \lambda 0\right), \tag{3.12}$$

where $G_J = \left\langle j^2 J(V) j^2 J \right\rangle, \lambda = 0,1,2$ determine the ranks of tensors in space.

The operator of the multipole-multipole interaction is often written in phenomenological form:

$$V_\lambda (2\lambda + 1) K^\lambda \sum_{i<j} \left(U_i^{(\lambda)} \cdot U_j^{(\lambda)}\right) \tag{3.13}$$

in which $K^\lambda = const$ and $U_i^{(\lambda)} -$ a single spherical tensor. If the number λ odd, then $V_\lambda -$ quasienergy a scalar, therefore, it is expressed through $T^{(0,\lambda)}$:

$$V_\lambda = \frac{1}{2}(2\lambda + 1)^{-3/2} \cdot K^\lambda \cdot (-)^\lambda \left[T^{(0,\lambda)} \times T^{(0,\lambda)}\right]^{(0,0)} - \frac{N}{4\Omega} K^\lambda (2\lambda + 1)$$

Then the matrix element of this operator can be easily expressed linearly by the quantum number n:

$$\left\langle j^n vaJ \middle| V \middle| j^n va' J \right\rangle = \delta_{vv'} \left[\frac{1}{2}(n - v) G_0 \delta_{aa'}\right] + \left\langle j^v vaJ \middle| V \middle| j^v va' J \right\rangle \tag{3.14}$$

In the case of even λ, the multipole interaction operator contains quasispin scalar and tensor parts. The potential of paired nucleon-nucleon interaction is chosen in the simplest form:

$$V(1,2) = (U_\omega + U_s \pi_s + U_\tau S_{12}) f(r, r_0) + U_0, \tag{3.15}$$

in which $(U_\omega, U_s, U_\tau) -$the parameters of Wigner, singlet and tensor forces, and singlet and tensor design operators; $f(r, r_0) -$ radial dependence of nuclear forces, chosen as Gauss potential, $U_0 -$ Coulomb potential. The pair potential of nucleon interactions consists of three parts: $V = V_{pp} + V_{nn} + V_{np}$, interactions of nucleons of the same name and the neutron-proton part.

IV. Application of the method to study the structure of even isotopes of $^{118-130}Xe$

The method will be applied to the even-numbered isotopes $^{118-130}Xe$. The core properties in this area are interpreted by IBM [11]. In our approach, the lower

states of nuclei are taken as single-particle energy $^{133}_{51}Sb, ^{131}_{50}Sn$ taken from [11,12], which are given in Table 1.

Table 1.Single-particle (hole) energies of protons and neutrons in nuclei $^{133}_{51}Sb, ^{131}_{50}Sn$ MeV.

ε_p	$g_{7/2}-0$	$d_{5/2}-0.96$	$d_{3/2}-2.69$	$h_{11/2}-2.76$	$s_{1/2}-2.99$
ε_n	$d_{3/2}-0$	$h_{11/2}-0.24$	$s_{1/2}-0.33$	$d_{5/2}-1.66$	$g_{7/2}-2.34$

Parameters of pair interactions of nucleons were determined from the description of experimental spectra of even isotopes of the nucleus Xe. The depth of the proton-proton interaction V_{pp} should change slowly over the isotopes of the nuclei, but will not differ much from each other. In this work, we choose them the same for all isotopes and equal to $U_\omega^p = -26MeV$, $U_s^p = -18MeV$. The amplitude of the tensor interaction was considered negligible. The selected parameters for nn and np interactions are listed in Table-2.

Table 2. The parameters V_{nn} and V_{np} for isotopes Xe (MeV).

nucleus	U_ω^n	U_s^n	U_ω^{np}	U_s^{np}
^{118}Xe	-24	-18	-32	-20
^{120}Xe	-22	-16	-30	-18
^{122}Xe	-21	-13	-27	-15
^{124}Xe	-18	-12	-23	-13
^{126}Xe	-17	-13	-22	-12

These values turned out to be close to the values of the corresponding potential parameters for heavy nuclei obtained in earlier works [13,14]. They vary with the number of neutrons monotonously and slowly.

Next, we consider the properties of wave functions in the spectrum calculated by the microscopic method. Unfortunately, there are very few experimental data on the absolute values of the reduced probabilities of transitions between states. Nevertheless, we made quite a satisfactory comparison of the calculated relative values of B (E2) between various transitions with $J_+ \to J_i$ their experimental values, which are collected in Table 3.

As can be seen from this table3, the above trends in the contributions of matrix elements of pair interaction operators are confirmed here, and the belonging of the States of O(6)–symmetry of phenomenological U(6)–group consideration is confirmed. This agreement is approved, for transitions within a homogeneous bands. For transitions between states of heterogeneous bands, this trend is also slightly broken.

Table-3. Isotope states Xe.

Ядра / J^{π}	^{118}Xe		^{120}Xe		^{122}Xe		^{124}Xe		^{126}Xe	
	Exp.	Theor.	Exp.	Theor.	Exp.	Theor.	Exp.	Theor.	Exp.	Theor.
0_1^+	0	0	0	0	0	0	0	0	0	0
2_1^+	0.34	0.33	0.32	0.33	0.33	0.34	0.35	0.36	0.39	0.41
4_1^+	0.81	0.79	0.80	0.78	0.83	0.85	0,88	0,89	0.94	0.96
6_1^+	1.40	1.41	1.41	1.42	1.47	1.49	1.55	1.57	1.64	1.67
8_1^+	2.07	2.04	2.10	2.12	2.22	2.24	2.33	2.39	2.44	2.48
10_1^+	2.82	2.86	2.87	2.92	3.04	3.12	3.17	3.24	3.32	3.41
2_2^+	0.93	0.96	0.88	0.94	0.84	0.95	0.85	0.97	0.88	0.94
4_2^+	1.44	1.48	1.40	1.49	1.40	1.51	1.44	1.51	1.49	1.54
6_2^+	2.00	2.12	1.99	2.14	2.06	2.13	2.14	2.21	2.21	2.25
8_2^+	2.63	2.73	2.65	2.83	2.80	2.91	2.91	2.99	-	3.08

These values were close to the values of the corresponding parameters of the potential for heavy nuclei obtained in earlier works [13,14]. They change with the change in the number of neutrons monotonously and slowly.

Using the obtained wave state functions, the reduced probabilities $E2-$ of transitions between the levels, as well as the ratios of probability transitions between different States are calculated.

Table 4. Relationships $(J_i \to J_5)/\left(J_i^{'} \to J_5^{'}\right)$ of transition probabilities in isotopes Xe.

nucleus / $J_i \to J_5$	^{118}Xe		^{120}Xe		^{122}Xe		^{124}Xe		^{126}Xe	
	Exp.	Theor.	Exp.	Theor.	Exp.	Theor.	Exp.	Theor.	Exp.	Theor.
$\frac{2_2 \to 2_1}{2_2 \to 0_1}$	25	30	31	35	28	38	22	28	62 ± 12	49
$\frac{0_2 \to 2_2}{0_2 \to 2_1}$	-	18	-	21	-	24	-	14	10.4 ± 1.6	16.2
$\frac{4_2 \to 2_2}{4_2 \to 4_1}$	4.3	5.6	1.7	2.7	0.5	0.28	0.35	0.48	0.27	0.35
$\frac{3_1 \to 2_2}{3_1 \to 2_1}$	21	14	30	18	42	21	9.4	16.4	67	31.5
$\frac{5_1 \to 3_1}{5_1 \to 4_1}$	-	21	27 ± 10	16	-	17	83	52	25	12.6
$\frac{3_1 \to 4_1}{3_1 \to 2_1}$	-	17	6,6	11	6.7	8.4	-	12.0	-	18.7
$\frac{2_3 \to 2_2}{2_3 \to 4_1}$	1.28	3.1	-	5.4	-	7.6	-	9.2	-	11.4

The results of the calculations and their experimental values are given in tables 4, the transition probabilities are calculated in the approximation, in which the single-particle radial integrals are replaced by the expression $3R^{\lambda}/(\lambda+3)$.
The tables show a satisfactory agreement between the calculated and experimental values of these values. The ratios of transition probabilities within the same bands (irast, β and γ) are within the same order, whereas they differ in two orders of magnitude between the levels of different bands. These are, for example, relations $3_1 \to 4_1 / 3_1 \to 2_1$, $8_2 \to 6_1 / 8_2 \to 6_1$, $5_1 \to 4_2 / 5_1 \to 4_1$. Such simple calculations, in General, well convey a sharp drop $B(E2)$ from the nucleus to the nucleus, which is a consequence of configuration mixing of wave functions with small components.

Since the experimental data for isotopes ^{128}Xe and ^{130}Xe are very small, the data for them is excluded from Table 4. Experimental [15, 16] and theoretical values of the energies of states generally agree well with each other. Especially, as you can see, it is good for irast band states. But for the β and $\gamma-$ bands there are some discrepancies. These divergences grow for higher levels. This is explained not only by the choice of intensity $pp, nn, np-$ of interactions, but also by taking into account only S and D pairs in the calculations. In addition, we completely excluded from consideration the contribution of the tensor part of the pair interaction.

V. Conclusion

The proposed relatively simple, at the same time detailed microscopic calculation of IBM parameters on the basis of generalized quasi-spin formalism, which takes into account the model $SD-$ pair shell space. The theory quite satisfactorily describes the properties of the lower collective States of the nuclei of medium and heavy atomic weights. Parameters simple pair interactions vary from one nucleus to another smoothly. At the same time, the operator of the interaction between the nucleons of a different nature V_{np} strongly mixes states with different quasi-spins that affects the process of collectivisation states especially those with large spin. This requires taking into account the contributions of high multipolar pairs.

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