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Earth matter effect on active-sterile neutrino oscillations

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Abstract. Oscillations between active and sterile neutrinos remain as an open possibility to explain some experimental observations. In a four-neutrino mixing scheme, we use the Magnus expansion of the evolution operator to study the evolution of neutrino flavor amplitudes within the Earth. We apply this formalism to calculate the transition probabilities from active to sterile neutrinos taking into account the matter effect for a varying terrestrial density.

1. Introduction

Experimental results from solar, atmospheric, reactor and accelerator neutrino experiments, have proved the existence of neutrino mixing, showing also that neutrinos are massive particles. Although the dominant behavior of neutrino oscillations is well described by a three–neutrino picture with two mass splitting parameters and three mixing angles (plus a CP–violating phase), some anomalous experimental results can be interpreted as an indication of the existence of an additional (sterile) neutrino.

It has been proved that the Magnus expansion [1] is an useful tool to obtain an approximate analytical description of matter effects on neutrino oscillations in the framework of two or three neutrinos [2, 3, 4, 5]. Here we apply this formalism to describe the propagation of neutrinos through the Earth, in the case of four neutrinos: the three standard active neutrinos plus and additional sterile neutrino.

2. Evolution of the four neutrino system

The evolution of the four-neutrino system can be written as

$$\Psi(t) = \mathcal{U}(t, t_0)\Psi(t_0), \tag{1}$$

where $\Psi(t)$ is the neutrino state vector at time t and $\mathcal{U}(t, t_0)$ is the evolution operator, which satisfies the Schrödinger-like equation ($\hbar = c = 1$)

$$i\,\frac{d\mathcal{U}}{dt}(t, t_0) = H(t)\,\mathcal{U}(t, t_0), \tag{2}$$

with the initial condition $\mathcal{U}(t_0, t_0) = I$. In the flavor basis $\{|\nu_\alpha\rangle, \alpha = e, \mu, \tau, s\}$

$$H(t) = UH_0U^\dagger + V(t), \tag{3}$$

where $H_0 = \text{diag}(0, 0, \Delta_{31}, \Delta_{41})$, $V(t) = \text{diag}(V_{CC}(t), 0, 0, -V_{NC}(t))$, and U is the mixing matrix that relates the neutrino flavor states to the mass states: $|\nu_\alpha\rangle = \sum_{i=1}^4 U_{\alpha i}^* |\nu_i\rangle$. Here, $\Delta_{ij} = \Delta m_{ij}^2/2E = (m_i^2 - m_j^2)/2E$, and V_{CC} and V_{NC} are the charge current and neutral current contributions to the matter effective potential for the active neutrinos. Since we are interested in neutrinos with an energy E of a few GeV and higher, the effect of the Δm_{21}^2 mass splitting can be safely discarded and we put $\Delta m_{21}^2 = 0$. We also subtracted a common term $V_{NC}(t)$, which has no observable effect on the oscillations.

Hereafter, we assume that neutrinos are of Dirac type and parametrize the mixing matrix as follows:

$$U = R_{23}\Gamma_3 R_{34}\Gamma_3^* R_{14}\Gamma_2 R_{24}\Gamma_2^*\Gamma_1 R_{13}\Gamma_1^* R_{12}, \tag{4}$$

which helps to simplify the calculations. In this formula, R_{ij} are 4×4 rotation matrices expressed in terms of the vacuum mixing angles θ_{ij} and the diagonal matrices Γ_i include the three CP-violating phases $\delta_i\,(i = 1, 2, 3)$ that appear in the present scheme. For $\theta_{i4} = 0$, U reduces to the standard 3-neutrino mixing matrix [6].

It is convenient to introduce a new vector $\hat\Psi \equiv \hat U^\dagger \Psi$, with $\hat U = R_{23}\Gamma_3 R_{34}\Gamma_3^* R_{14}$. Its evolution is driven by the Hamiltonian $\hat H = \hat U^\dagger H \hat U$ which, under the assumption $\Delta m_{41}^2 \gg \Delta m_{31}^2, V_{CC}, V_{NC}$, becomes

$$\hat H(t) \cong \begin{pmatrix} D_1 & 0 & Z_1^* & 0 \\ 0 & D_2 & 0 & Z_2^* \\ Z_1 & 0 & D_3 & 0 \\ 0 & Z_2 & 0 & D_4 \end{pmatrix}, \tag{5}$$

with

$$\begin{aligned} D_1 &= \Delta_{31}s_{13}^2 + V_{CC}c_{14}^2 - V_{NC}s_{14}^2 c_{34}^2, & D_2 &= \Delta_{41}s_{24}^2, \\ D_4 &= \Delta_{41}c_{24}^2 + V_{CC}s_{14}^2 - V_{NC}c_{14}^2 c_{34}^2, & D_3 &= \Delta_{31}c_{13}^2 - V_{NC}s_{34}^2, \\ Z_1 &= \Delta_{31}c_{13}s_{13}e^{i\delta_1} - V_{NC}s_{14}c_{34}s_{34}e^{i\delta_3}, & Z_2 &= \Delta_{41}c_{24}s_{24}e^{i\delta_2}. \end{aligned}$$

For brevity we defined $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$.

The matrix in Eq. (5) can be diagonalized by a time-dependent unitary transformation

$$V_m = \Gamma_2 R_{24}(\theta_{24}^m)\Gamma_2^*\Gamma_1 R_{13}(\theta_{13}^m)\Gamma_1^* R_{12}(\pi/2), \tag{6}$$

such that $V_m \hat H\, V_m^\dagger = H_D = \text{diag}(\lambda_1, \lambda_2, \lambda_3, \lambda_4)$, with the energy eigenvalues in matter $\lambda_k(t)$ ordered in such a way that $\lambda_1 < \lambda_2 < \lambda_3 < \lambda_4$. The mixing angles in matter are given by formulas similar to the familiar ones for two neutrinos:

$$\cos 2\theta_{13}^m = \frac{D_3 - D_1}{\sqrt{(D_3 - D_1)^2 + 4|Z_1|^2}}, \tag{7}$$

$$\cos 2\theta_{24}^m = \frac{D_4 - D_2}{\sqrt{(D_4 - D_2)^2 + 4|Z_2|^2}}. \tag{8}$$

The left panel of Figure 1 shows the mixing angles in matter and the energy eigenvalues λ_i as functions of the potential. We see that λ_2 and λ_3 almost cross and the angle θ_{13}^m exhibit the characteristic resonant behavior for the values of the terrestrial density.

The evolution operator in the flavor basis can be expressed as $\mathcal{U}(t, t_0) = U_m(t)\mathcal{U}_A(t, t_0)U_m^\dagger(t_0)$, where $U_m(t) = \hat U V_m(t)$ is the mixing matrix that relates the flavor states with the set $\{|\nu_k^m(t)\rangle\,, k = 1, \ldots, 4\}$ of the (instantaneous) energy eigenstates in matter. The latter define the adiabatic basis and in it the evolution operator $\mathcal{U}_A(t, t_0)$ obeys Eq. (2) with the Hamiltonian

$$H_A(t) = H_D(t) - iV_m^\dagger(t)\,\dot V_m(t), \tag{9}$$

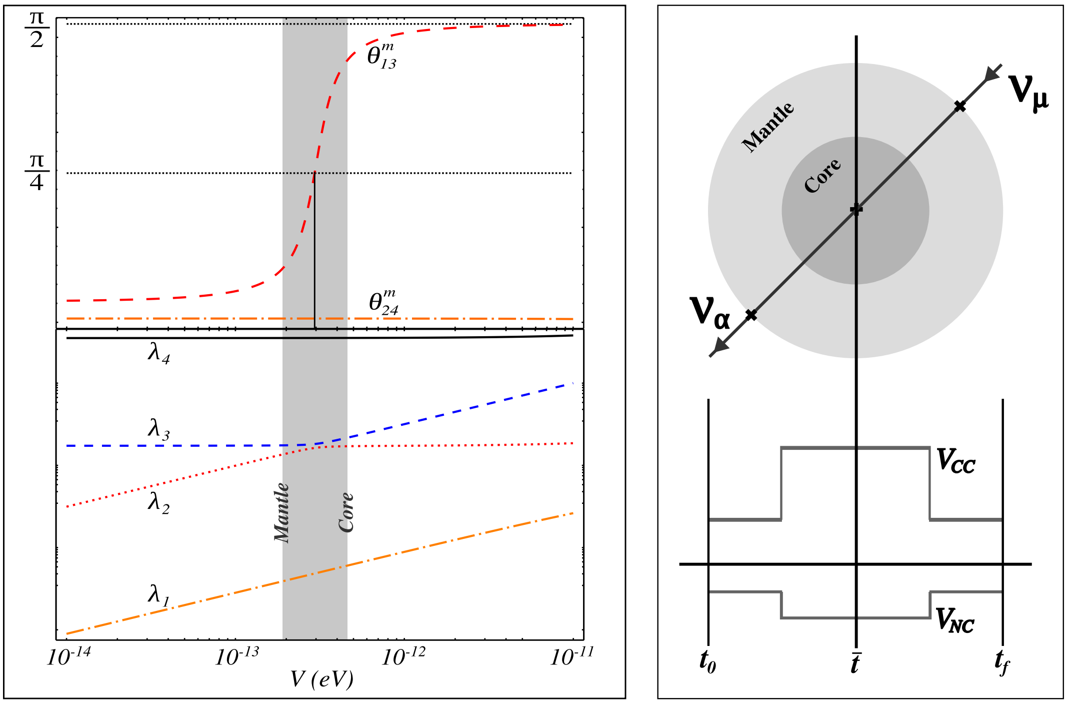

Figure 1. *Left panel.* Evolution of the matter-mixing angles and the (approximated) eigenvalues of the Hamiltonian vs. the potential. *Right panel.* Mantle–core–mantle potential modelling the Earth's interior.

where dot means differentiation with respect to time. Discarding the second term in Eq. (9) corresponds to solving the problem in the adiabatic approximation. In any case, the time dependence generated by H_D can be integrated exactly by a change of the representation accomplished by means of the unitary transformation $\mathcal{P}(t) = \exp[-\int_{t_0}^{t} dt' H_D(t')]$.

For the remaining part of Eq. (9), in general it is not possible to find an analytical solution in a close form and one has to rest on some approximation to determine it. As mentioned in the introduction, an appropriate procedure, which preserves unitarity, is based in the exponential expansion of the evolution operator. Accordingly, next we write $\mathcal{U}_A(t,t_0) = \mathcal{P}(t)\exp\Omega(t,t_0)$ and evaluate Ω in terms of the first two terms of its Magnus expansion: $\Omega \cong \Omega_1 + \Omega_2$. Proceeding in this manner, after some algebraic manipulations we arrive at:

$$\mathcal{U}_A(t_f,t_0) = \mathcal{I}^*_{32} \begin{pmatrix} \mathcal{I}_{32}e^{-i\alpha_1} & 0 & 0 & 0 \\ 0 & (c_\xi - i s_\xi \frac{\xi_{(2)}}{\xi})e^{i\phi_{\bar{t}\to t_f}} & i e^{i\delta_1} s_\xi \frac{\xi_{(1)}}{\xi} & 0 \\ 0 & i e^{-i\delta_1} s_\xi \frac{\xi_{(1)}}{\xi} & (c_\xi + i s_\xi \frac{\xi_{(2)}}{\xi})e^{-i\phi_{\bar{t}\to t_f}} & 0 \\ 0 & 0 & 0 & \mathcal{I}_{32}e^{-i\alpha_4} \end{pmatrix} \tag{10}$$

where

$$\mathcal{I}_{32} = \exp\Big\{-\frac{i}{2}\int_a^b dt' \big[\lambda_3(t') + \lambda_2(t')\big]\Big\}, \tag{11}$$

$$\phi_{a\to b} = \int_a^b dt' \big[\lambda_3(t') - \lambda_2(t')\big], \tag{12}$$

and we have introduced the notations $c_\xi = \cos\xi$, $s_\xi = \sin\xi$, and $\xi = \sqrt{\xi^2_{(1)} + \xi^2_{(2)}}$. The

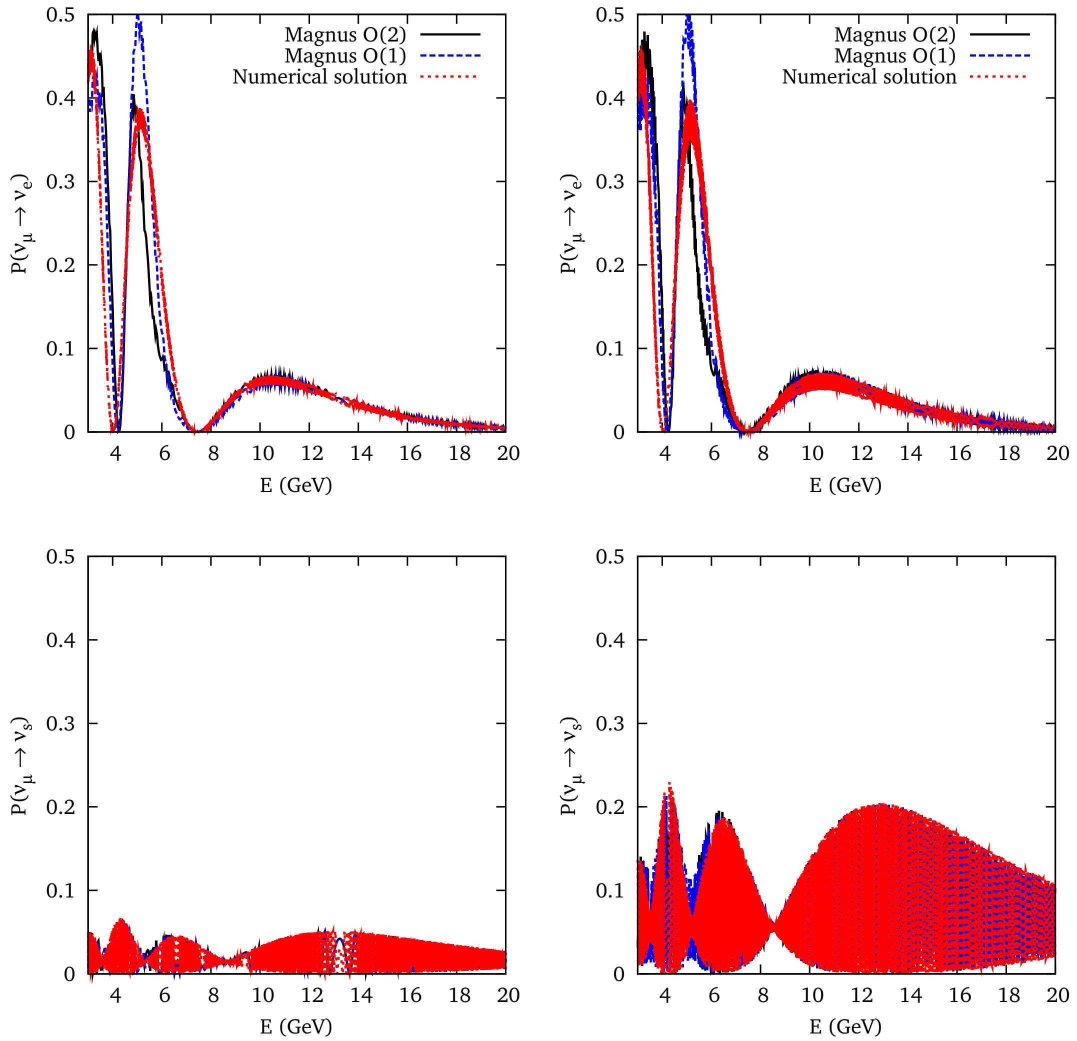

Figure 2. $P(\nu_\mu \to \nu_{e,s})$ as a function of the energy for a neutrino crossing the center of the Earth. Parameters: Common: $\Delta m^2_{31} = 2.5 \times 10^{-3}$ eV^2, $\Delta m^2_{41} = 1.0$ eV^2, $\theta_{12} = 34°$, $\theta_{13} = 7°$, $\theta_{23} = 45°$. Changed: *Left:* $\theta_{i4} = 5°$; *Right:* $\theta_{14} = 5°$, $\theta_{24} = \theta_{34} = 10°$.

quantities

$$\xi_{(1)} = 2\int_{\bar{t}}^{t_f} dt' \dot{\theta}^m_{13}(t') \sin\phi_{\bar{t}\to t'}, \tag{13}$$

$$\xi_{(2)} = \int_{t_0}^{t_f} dt' \int_{t_0}^{t'} dt''\, \dot{\theta}^m_{13}(t')\dot{\theta}^m_{13}(t'') \sin\phi_{t'\to t''}, \tag{14}$$

come from $\Omega_1(t_f, t_0)$ and $\Omega_2(t_f, t_0)$, respectively [4]. In the above expressions we take into account that the potential is symmetric with respect to the middle point of the neutrino trajectory $\bar{t} = (t_f + t_0)/2$ (Fig. 1).

In Figure 2, we show the behavior of the transition probabilities in terms of the neutrino energy for different values of the parameters involved in the analysis. We consider a simplified

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model for the electron density inside the Earth, the so-called mantle-core-mantle [7] (see Fig. 1). One can see that the Magnus approximation agrees very well with the numerical calculation for $E \gtrsim 7$ GeV. As the plots shows, the effect of the inclusion of the sterile neutrino generates noticeable changes on the $\nu_\mu \to \nu_\alpha$ ($\alpha = e, \mu, \tau$) transition probabilities. Moreover, the $\nu_\mu \to \nu_s$ transition remains as an open possibility. These effects increase for larger values of the three additional mixing angles θ_{i4} ($i = 1, 2, 3$). This is particularly relevant for θ_{24} and θ_{34}, on which no stringent limits have been placed directly, contrary to the case of θ_{14} that has been constrained by the Bugey results [8].

3. Conclusions

We used the Magnus expansion to calculate the flavor transition of a system composed by the three active neutrinos plus a sterile neutrino under the effect of the terrestrial potential. This approximation coincides very well with the numerical computations for high neutrino energies ($E \gtrsim 7$ GeV). The inclusion of the fourth neutrino has a significative effect on $P(\nu_\mu \to \nu_\alpha)$ which depends strongly on θ_{i4} and exhibits a rapid oscillatory behavior superimposed to the standard oscillations for the active neutrinos.

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