GINGER - A terrestrial experiment to verify the Lense-Thirring effect or possible deviations from General Relativity

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After reviewing the signal to be expected from a ring laser of convenient size, located on earth, the project of a three-dimensional array of ring lasers named GINGER is presented. The sensitivity analysis is discussed, stressing that the available techniques for research lasers do allow for the detection of general relativistic effects originated by the mass and the angular momentum of the earth. The project is under development at the Gran Sasso National Laboratories of the INFN in Italy. Two intermediate instruments in the road towards the full GINGER have been built, one in Pisa (GP2) and one at the Gran Sasso (GINGERino). They are being used to validate the dynamical control of the geometry and to characterize the site allotted to the experiment.

1 Introduction

General Relativity (GR) is a most powerful and elegant theory, whose successes in describing the evolution of the universe and the behaviour of matter in strong gravitational fields are well established. Though there are reasons not to leave it as unchallenged and unperfectable. First there is the fundamental incompatibility with quantum mechanics, but, besides that, there is the requirement to allow for dark matter and dark energy in order to account for the behaviour of the universe on large enough scales. These remarks tell us that it is worthwhile to pursue the experimental verification of all predicted consequences of GR. The high energy or strong field domain pertains to cosmology and astrophysics, but the theory has also peculiar predictions in the domain of ultra-low energies. In particular it is useful and important to explore the relativistic effects of the motion, and especially the proper rotation, of masses producing gravity. Those effects are often qualified, altogether, as *gravito-magnetic.* The number of direct experiments in that area is limited and they have all been performed in space, facing various problems of averaging along the orbits and necessary knowledge of the gravito-electric (Newtonian) component of the gravitational field. [1,2,3,4,5,6]

Here I present a terrestrial experiment, based on ring laser technology, aimed to the detection of the GR effects associated with the rotation of the earth. The advantages of a ground based experiment are: lower cost with respect to experiments in space; hands on setting, allowing for immediate and direct intervention to amend configuration faults or to allow for real time changes of strategy; fixed position in the gravitational field of the earth allowing for point measurements, without averaging over different field configurations.

2 Small effects in General relativity

Considering a freely falling test particle, the equation of its geodetic space-time trajectory, in terms of the velocity four-vector u^μ and the Christoffel symbols $\Gamma^\alpha_{\mu\nu}$, may be written as

$$\frac{du^\alpha}{ds} + \Gamma^\alpha_{00}(u^0)^2 + 2\Gamma^\alpha_{0i}u^0u^i + \Gamma^\alpha_{ij}u^iu^j = 0 \tag{1}$$

Two terms in the above formula depend on the space velocity of the test particle. It can be safely assumed that in most cases the velocity is much smaller than c. In other words, we may assume $u^0 \sim 1$ and $u^i \ll 1$. In a terrestrial non-rotating reference frame, the ratio of practical speeds to the speed of light is typically in the order of 10^{-6}. With such values, we may neglect the terms quadratic in the space velocities with respect to the linear ones. Finally we see that in non-relativistic (in the sense of 'low' velocity) approximation the four acceleration of a freely falling test particle is composed by a dominant term, independent from the velocity of the body, plus terms linearly depending on that velocity. The former term depends on the gravitational potential: we may say it to be due to the *gravito-electric* field. The latter terms are the analog of the Lorentz force of electromagnetism, and they are ascribed to the *gravito-magnetic* field.

As we have seen, the relevance of the gravito-magnetic effects is controlled by the Christoffels. The next step is to consider a static gravitational field, which is quite reasonable in the case of the earth. If so, all time derivatives in the Christoffels go to zero and we end with the explicit gravito-magnetic form

$$\Gamma^\alpha_{0i} = \frac{1}{2}g^{\alpha\beta}(\frac{\partial g_{0\beta}}{\partial x^i} - \frac{\partial g_{0i}}{\partial x^\beta}) = \frac{1}{2}g^{\alpha 0}\frac{\partial g_{00}}{\partial x^i} + \frac{1}{2}g^{\alpha j}(\frac{\partial g_{0j}}{\partial x^i} - \frac{\partial g_{0i}}{\partial x^j}) \tag{2}$$

The last terms in the round brackets looks like the k-th component of a three-dimensional curl. Interpreting the g_{0i}'s as the components of a three-dimensional vector potential, the bracket identifies the components of the corresponding gravito-magnetic axial field vector.

3 Asymmetric propagation in the field of a rotating mass

When the source of the gravitational field is a steadily rotating mass, the space-time assumes a chiral symmetry about the time axis of any reference frame centered on the main body. The line element may be written as:

$$ds^2 = g_{00}c^2dt^2 + 2g_{0i}cdtdx^i + g_{ij}dx^idx^j \tag{3}$$

Considering light, the interval is identically zero, so that it is possible to solve eq. (3) for dt:

$$dt = -\frac{g_{0i}}{cg_{00}}dx^i + \frac{1}{cg_{00}}\sqrt{(g_{0i}dx^i)^2 - g_{00}g_{ij}dx^idx^j} \tag{4}$$

The sign in front of the square root could also be $-$ but we have chosen $+$ because we are interested in propagation towards the future.

Now let us assume that light follows a closed path in space. The parametric equation of the path, if its proper length is P, would be: $x^i(l) = x^i(l+P)$. The coordinated time interval (4) along the trajectory becomes

$$dt = -\frac{g_{0i}}{cg_{00}}\frac{dx^i}{dl}dl + \frac{1}{cg_{00}}\sqrt{(g_{0i}\frac{dx^i}{dl})^2 - g_{00}g_{ij}\frac{dx^i}{dl}\frac{dx^j}{dl}}|dl| \tag{5}$$

It is now possible to sum (integrate) along the loop, both to the right ($dl > 0$) and to the left ($dl < 0$), in order to obtain the time of flight for the round trip, getting two different results. Subtracting one result from the other and expressing the difference in terms of proper time τ of the laboratory, we get

$$\Delta\tau = \tau_{+} - \tau_{-} = -\frac{2}{c}\sqrt{g_{00}} \oint \frac{g_{0i}}{g_{00}} \frac{dx^i}{dl} dl \tag{6}$$

The subscripts + and − stand for co- and respectively counter-rotating with the central mass.

$\Delta\tau$ differs from zero whenever it is $g_{0i} \neq 0$, which is the case for the space-time surrounding a rotating mass.

3.1 Ring lasers

Physically, a closed loop for light may be obtained using an optical fiber or three or more mirrors. In both solutions the source of light is the active cavity of a LASER so that the whole thing is called a ringlaser. The loop forms a resonating cavity within which standing light waves form. The stationarity condition converts the times of flight difference into a frequency difference for the right- and left-handed beams. The frequency difference, in turn, being the beams superposed, leads to a beat. Finally the beat frequency f_b is related to the proper time of flight difference by the formula

$$f_b = \frac{\Delta f}{2} = \frac{c^2}{2\lambda P}\Delta\tau \tag{7}$$

P is the proper length of the contour and λ is the wavelength of the laser.

When explicitly introducing the metric around the earth and expressing it to the lowest order of approximation in the angular momentum of the planet $J_{\otimes}$, in a reference frame originating in the center of the earth and rotating with it, the expected signal from a ring laser fixed to the ground will be [7]:

$$f_b = 2\frac{A}{\lambda P}[\vec{\Omega} - 2\frac{GM}{c^2 R}\Omega sin\theta\, \widehat{u}_\theta + \frac{GJ_{\otimes}}{c^2 R^3}(2cos\theta\, \widehat{u}_r + sin\theta\, \widehat{u}_\theta] \cdot \widehat{u}_n \tag{8}$$

A is the area enclosed by the loop; the factors multiplying the square bracket, altogether, are the *scale factor* of the ring; θ is the colatitude of the laboratory; the $\widehat{u}$'s are unit vectors along the radial direction, along the local meridian, from the North to the South pole, along the perpendicular to the enclosed area of the ring. The angular velocity Ω coincides with the rotation speed of the earth.

The first term into the square brackets accounts for the kinematical Sagnac effect; the second term is the de Sitter precession, due to the coupling of the motion of the laboratory with the local gravito-electric field; the last term accounts for the gravito-magnetic frame dragging, originating from the angular momentum of the planet. The last two terms represent the GR effects, that should be measured.

4 GINGER

For an earth based laboratory, the two GR terms of (8) are approximately nine orders of magnitude smaller than the Sagnac effect; the de Sitter (or geodetic) term is a bit (less than one order of magnitude) smaller than the frame dragging (or Lense-Thirring) term. In order to perform an experiment, extremely stable and sensible instruments are needed, with a conveniently big scale factor. In order to see the LT term, a sensitivity better than 1 $prad/s$ is required. The best existing research ring laser is G (Groß Ring), located in Wettzell, Bavaria [8]. G is a square ring (4 m side) whose sensitivity is now within one order of magnitude from the threshold of detectability of the GR effects.

We are presenting here GINGER (Gyroscopes IN General Relativity) [9], designed to be a three-dimensional array made of three square mutually perpendicular rings (6 m side or more) assembled to form an octahedron (alternatively it could be a cube carrying six rings on its faces). The planned geometry control will be achieved dynamically using Fabry-Pérot cavities along the main diagonal. The Fabry-Pérot will pilot piezoelectric actuators controlling the mirrors at the corners of the loops. The aim is to measure the GR terms and in particular the LT effect with an accuracy better than 1% (one year integration time). The location of GINGER will be the Gran Sasso National Laboratories in Italy, under more than 1400 m of rock. The underground location is needed in order to screen the apparatus from all rotational disturbances present on the surface of the earth (originating from wind, rain, pressure and temperature changes, moving masses in the vicinity etc.).

5 Intermediate steps on the GINGER roadmap

The full implementation of GINGER will require two or three years [9]. Meanwhile a couple of intermediate steps have been made. These are two rings already built and working. One is GP2, located in Pisa, destined to the test and calibration of the dynamic control process of the geometry using the main diagonals. GP2 is a square ring (1.6 m side) mounted on a granite table, oriented perpendicularly to the rotation axis of the earth, in order to maximize the Sagnac signal.

The second ring is GINGERino, located in a side tunnel of the Gran Sasso Laboratories. GINGERino is a square ring, 3.6 m side. The support is a granite cross, laid horizontally. GINGERino is used to characterise rotationally the underground site of the GS laboratories and to test various solutions for the control of the temperature and environmental conditions of the lab.

Work is steadily progressing, so that the goal of revealing general relativistic rotational effects on earth is now within reach in a reasonably short time.

The collaboration

The principal investigator of GINGER is Angela Di Virgilio of the INFN section in Pisa. The institutions wherefrom the members of the collaboration come, are: INFN-Pisa, Pisa University, Siena University, Padua University, LNL-INFN-Padua, INFN-Naples, CNR-SPIN Naples, Politecnico di Torino, INFN-Torino, National Gran Sasso Laboratory. Out of Italy, the collaboration includes the Technische Universität München (DE) and the University of Canterbury in Christchurch (NZ).

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