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INDIRECT COLLIDER TESTS FOR LARGE EXTRA DIMENSIONS

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New physics signatures arising from different sources may be confused when first observed at future colliders. Thus it is important to examine how various scenarios may be differentiated given the availability of only limited information. Arkani-Hamed, Dimopoulous, and Dvali have proposed a model (ADD) of low-scale quantum gravity featuring large extra dimensions. In this model, the exchange of Kaluza-Klein towers of gravitons can manifest themselves through deviations of the observables from the Standard Model predictions. Here, we assess the expected "identification reach" on the ADD model of gravity in large compactified extra dimensions, against the compositeness-inspired four-fermion contact interaction. As basic observables we take the differential cross sections for fermion-pair production at a 0.5–1 TeV electron-positron linear collider with both beams longitudinally polarized. For the four-fermion contact interaction we assume a general linear combination of the individual models with definite chiralities, with arbitrary coupling constants. In this sense, the estimated identification reach on the ADD model can be considered as "model-independent". In the analysis, we give estimates also for the expected "discovery reaches" on the various scenarios. We emphasize the substantial rôle of beams polarization in enhancing the sensitivity to the contactlike interactions under consideration.

1 Introduction

Numerous New Physics (NP) scenarios are described by local, contactlike, effective interactions between the Standard Model (SM) particles. This is the typical case of interactions mediated by exchanges of quanta that are constrained, by either conceptual or phenomenological considerations, to have a mass, we generically denote as Λ, in the multi-TeV range. These states may be beyond the kinematical reach of the collider and therefore could not appear as final products of the studied reactions. Accordingly, the existence of such nonstandard scenarios can be verified only through their *indirect* effects, represented by deviations of the measured observables from the SM predictions. The effective interaction framework leads to the expansion of the deviations caused by these novel interactions in powers of the corresponding small ratios $E_{\rm C.M.}/\Lambda \ll 1$, multiplied by matrix elements of local operators between initial and final states. Generally, the dominance of the leading power is taken as a reasonable assumption.

Referring to experiments at planned high energy colliders and their sensitivity to NP, one can define for the individual contactlike effective interactions the expected *discovery reach*, as the maximum value of the relevant Λ for which deviations from the SM predictions can be detected within the foreseen experimental accuracy. This limit can be assessed by a comparison of theoretical deviations, functions of Λ, and expected experimental uncertainties by assuming that no such deviations are observed.

Conversely, one can envisage a situation where corrections to the SM predictions are observed, and found compatible with one of the effective interactions for a certain value of the relevant Λ. In this case, one should consider that, in principle, different contactlike interactions can cause similar corrections. Therefore, it should be desirable to attempt the identification of the source of the observed deviations among the various possible scenarios. To this purpose, one can define the expected *identification reach* on an individual contact interaction model, as the maximum value of the corresponding Λ for which not only it can cause observable deviations but, also, can be discriminated as the source of such corrections, were they observed, against the other effective interactions for any value of their characteristic Λs. Obviously, the identification reach can only be smaller than the discovery reach.

Here, we consider as basic observables the differential cross sections for the fermion pair production processes

$$e^+ + e^- \to \bar{f} + f, \qquad f = e, \mu, \tau, c, b, \tag{1}$$

at the International Linear Collider (ILC) with longitudinally polarized electron and positron beams [1]. This option is considered with great interest in the project for this collider, and its impact on the physics programme has been reviewed recently in Ref. [2].

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As a significant example, we focus on the identification reach on the ADD model of gravity in large, compactified, extra spatial dimensions [3–5], with respect to the compositeness-inspired four-fermion contact interactions [6, 7]. Also, we insist on the rôle played by the longitudinal polarization of the e^+ and e^- beams in enhancing the identification power of processes (1) on this scenario, at the planned ILC energies and luminosities.

2 Polarized differential observables

The expression of the polarized differential cross section for the process $e^+e^- \to f\bar{f}$ with $f \neq e, t$ and in approximation where $m_f \ll \sqrt{s}$ can be expressed as [8]:

$$\begin{aligned}\frac{d\sigma(P^-,P^+)}{dz} &= \frac{D}{4}\left[(1-P_{\rm eff})\left(\frac{d\sigma_{\rm LL}}{dz}+\frac{d\sigma_{\rm LR}}{dz}\right)\right.\\ &\left.+(1+P_{\rm eff})\left(\frac{d\sigma_{\rm RR}}{dz}+\frac{d\sigma_{\rm RL}}{dz}\right)\right]. \end{aligned} \tag{2}$$

In Eq. (2), $z = \cos\theta$ with θ the angle between initial and final fermions in the C.M. frame, and the subscripts L, R denote the respective helicities. Furthermore, with P^- and P^+ denoting the degrees of longitudinal polarization of the e^- and e^+ beams, respectively, one has

$$D = 1 - P^-P^+\,, \qquad P_{\rm eff} = \frac{P^- - P^+}{1 - P^-P^+}. \tag{3}$$

The SM amplitudes for these processes are determined by γ and Z exchanges in the s-channel.

The polarized differential cross section for the Bhabha process $e^+e^- \to e^+e^-$, where γ and Z can be exchanged also in the t-channel, can be conveniently written as [9–11]:

$$\begin{aligned}\frac{\mathrm{d}\sigma(P^-,P^+)}{\mathrm{d}z} &= \frac{(1+P^-)\,(1-P^+)}{4}\,\frac{\mathrm{d}\sigma_{\rm R}}{\mathrm{d}z}+\frac{(1-P^-)\,(1+P^+)}{4}\,\frac{\mathrm{d}\sigma_{\rm L}}{\mathrm{d}z}\\ &+\frac{(1+P^-)\,(1+P^+)}{4}\,\frac{\mathrm{d}\sigma_{{\rm RL},t}}{\mathrm{d}z}+\frac{(1-P^-)\,(1-P^+)}{4}\,\frac{\mathrm{d}\sigma_{{\rm LR},t}}{\mathrm{d}z}, \end{aligned} \tag{4}$$

with the decomposition

$$\frac{\mathrm{d}\sigma_{\rm L}}{\mathrm{d}z} = \frac{\mathrm{d}\sigma_{\rm LL}}{\mathrm{d}z}+\frac{\mathrm{d}\sigma_{{\rm LR},s}}{\mathrm{d}z}, \qquad \frac{\mathrm{d}\sigma_{\rm R}}{\mathrm{d}z} = \frac{\mathrm{d}\sigma_{\rm RR}}{\mathrm{d}z}+\frac{\mathrm{d}\sigma_{{\rm RL},s}}{\mathrm{d}z}. \tag{5}$$

In Eqs. (4) and (5), the subscripts t and s denote helicity cross sections with SM γ and Z exchanges in the corresponding channels. In terms of helicity amplitudes:

$$\begin{aligned}&\frac{\mathrm{d}\sigma_{{\rm LR},t}}{\mathrm{d}z} = \frac{\mathrm{d}\sigma_{{\rm RL},t}}{\mathrm{d}z} = \frac{2\pi\alpha^2_{\rm e.m.}\left|G_{{\rm LR},t}\right|^2}{s}, \quad \frac{\mathrm{d}\sigma_{{\rm LR},s}}{\mathrm{d}z} = \frac{\mathrm{d}\sigma_{{\rm RL},s}}{\mathrm{d}z} = \frac{2\pi\alpha^2_{\rm e.m.}\left|G_{{\rm LR},s}\right|^2}{s},\\ &\frac{\mathrm{d}\sigma_{\rm LL}}{\mathrm{d}z} = \frac{2\pi\alpha^2_{\rm e.m.}\left|G_{{\rm LL},s}+G_{{\rm LL},t}\right|^2}{s}, \quad \frac{\mathrm{d}\sigma_{\rm RR}}{\mathrm{d}z} = \frac{2\pi\alpha^2_{\rm e.m.}\left|G_{{\rm RR},s}+G_{{\rm RR},t}\right|^2}{s}. \end{aligned} \tag{6}$$

The polarized differential cross section (2) for the leptonic channels $e^+e^- \to l^+l^-$ with $l = \mu, \tau$ can be obtained directly from Eq. (4), basically by dropping the t-channel poles. The same is true, after some obvious adjustments, for the $\bar{c}c$ and $\bar{b}b$ final states.

According to the previous considerations the amplitudes $G_{\alpha\beta,i}$, with $\alpha, \beta = {\rm L}, {\rm R}$ and $i = s, t$, are given by the sum of the SM γ, Z exchanges plus deviations representing the effect of the novel, contactlike, effective interactions:

$$\begin{aligned}G_{{\rm LL},s} &= u\left[\frac{1}{s}+\frac{g^2_{\rm L}}{s-M^2_Z}+\Delta_{{\rm LL},s}\right],\ G_{{\rm LL},t} = u\left[\frac{1}{t}+\frac{g^2_{\rm L}}{t-M^2_Z}+\Delta_{{\rm LL},t}\right],\\ G_{{\rm RR},s} &= u\left[\frac{1}{s}+\frac{g^2_{\rm R}}{s-M^2_Z}+\Delta_{{\rm RR},s}\right],\ G_{{\rm RR},t} = u\left[\frac{1}{t}+\frac{g^2_{\rm R}}{t-M^2_Z}+\Delta_{{\rm RR},t}\right],\\ G_{{\rm LR},s} &= t\left[\frac{1}{s}+\frac{g_{\rm R}\,g_{\rm L}}{s-M^2_Z}+\Delta_{{\rm LR},s}\right],\ G_{{\rm LR},t} = s\left[\frac{1}{t}+\frac{g_{\rm R}\,g_{\rm L}}{t-M^2_Z}+\Delta_{{\rm LR},t}\right]. \end{aligned} \tag{7}$$

Here $u, t = -s(1 \pm z)/2$, $g_{\rm R} = \tan\theta_W$ and $g_{\rm L} = -\cot 2\theta_W$ with θ_W the electroweak mixing angle. The deviations $\Delta_{\alpha\beta,i}$ caused by the models of interest here have been tabulated in earlier references, see for example Refs. [10, 12, 13]. However, for convenience, we report their explicit expressions and briefly comment on their properties in the next section.

The contactlike nonstandard interactions considered in the sequel are listed below:

a) The ADD, compactified large extra dimensions, scenario [3–5], motivated by the gauge hierarchy problem. In this scenario, only gravity can propagate in the full multidimensional space. Correspondingly, a tower of graviton KK states with equally-spaced spectrum is exchanged in the ordinary four-dimensional space, and induces indirect corrections to the SM γ and Z exchanges. The relevant Feynman rules have been derived in Refs. [14, 15]. In the parameterization of Ref. [16], the exchange of such a KK tower is represented by the effective interaction:

$$\mathcal{L} = i\frac{4\lambda}{\Lambda_H^4}T^{\mu\nu}T_{\mu\nu}, \qquad \lambda = \pm 1. \tag{8}$$

In Eq. (8), $T_{\mu\nu}$ denotes the energy-momentum tensor of the SM particles and Λ_H is an ultraviolet cut-off on the summation over the KK spectrum, expected in the (multi) TeV range. The corresponding corrections to the SM amplitudes for Bhabha scattering, see Eq. (7), read:

$$\begin{aligned}
\Delta_{\mathrm{LL},s} = \Delta_{\mathrm{RR},s} &= \frac{\lambda(u+3s/4)}{\pi\alpha_{\mathrm{e.m.}}\Lambda_H^4}, \quad & \Delta_{\mathrm{LL},t} = \Delta_{\mathrm{RR},t} &= \frac{\lambda(u+3t/4)}{\pi\alpha_{\mathrm{e.m.}}\Lambda_H^4},\\
\Delta_{\mathrm{LR},s} &= -\frac{\lambda(t+3s/4)}{\pi\alpha_{\mathrm{e.m.}}\Lambda_H^4}, & \Delta_{\mathrm{LR},t} &= -\frac{\lambda(s+3t/4)}{\pi\alpha_{\mathrm{e.m.}}\Lambda_H^4}.
\end{aligned} \tag{9}$$

As observed in the previous section, the deviations for the other processes in Eq. (1) can easily be obtained from Eqs. (9). One can remark, also, that the effective interaction (8) has dimension-8, which explains the high negative power of the characteristic mass scale Λ_H.

b) The dimension-6 four-fermion contact interaction (CI) scenario [6, 7]. With $\Lambda_{\alpha\beta}$ ($\alpha, \beta = \mathrm{L}, \mathrm{R}$) the "compositeness" mass scales, and δ_{ef} =1 (0) for $f = e$ ($f \neq e$):

$$\mathcal{L} = \frac{4\pi}{1+\delta_{ef}}\sum_{\alpha,\beta}\frac{\eta_{\alpha\beta}}{\Lambda_{\alpha\beta}^2}\left(\bar{e}_\alpha\gamma_\mu e_\alpha\right)\left(\bar{f}_\beta\gamma^\mu f_\beta\right), \qquad \eta_{\alpha\beta} = \pm 1, 0. \tag{10}$$

The induced deviations in Eq. (7) are:

$$\Delta_{\alpha\beta,s} = \Delta_{\alpha\beta,t} = \frac{1}{\alpha_{\mathrm{e.m.}}}\frac{\eta_{\alpha\beta}}{\Lambda_{\alpha\beta}^2}. \tag{11}$$

Rather generally, this kind of effective interactions applies to the cases of very massive virtual exchanges, such as heavy Z's, leptoquarks, *etc.*

Current experimental lower bounds on Λs are mostly derived from nonobservation of deviations at LEP and Tevatron colliders. At the 95% C.L., they are: $\Lambda_H > 1.3\,\mathrm{TeV}$ [17] and, generically, $\Lambda_{\alpha\beta} > 10 - 15\,\mathrm{TeV}$, depending on the processes measured and the type of analysis performed [18].

c) In models with TeV^{-1}-scale extra dimensions, the SM gauge bosons may propagate also in the additional dimensions, and the new, contact-like, effective interaction relevant to the processes of interest here is generated by the exchange of γ and Z KK excitations [19, 20]. For one additional dimension, and with $M_C \gg M_{W,Z}$ the inverse of the compactification radius, for $e^+e^- \to \bar{f}f$ it can be written as

$$\begin{aligned}
\mathcal{L}^{\mathrm{TeV}} &= -\frac{\pi^2}{3M_C^2}[Q_eQ_f(\bar{e}\gamma_\mu e)(\bar{f}\gamma^\mu f)\\
&+ (g_{\mathrm{L}}^e\bar{e}_{\mathrm{L}}\gamma_\mu e_{\mathrm{L}} + g_{\mathrm{R}}^e\bar{e}_{\mathrm{R}}\gamma_\mu e_{\mathrm{R}})(g_{\mathrm{L}}^f\bar{f}\gamma^\mu f_{\mathrm{L}} + g_{\mathrm{R}}^f\bar{f}_{\mathrm{R}}\gamma^\mu f_{\mathrm{R}})].
\end{aligned} \tag{12}$$

The corresponding deviation can be written as

$$\Delta_{\alpha\beta,s} = \Delta_{\alpha\beta,t} = -(Q_eQ_f + g_\alpha^e\,g_\beta^f)\,\frac{\pi^2}{3\,M_C^2} \tag{13}$$

For the TeV^{-1}-scale extra dimension scenario the current limit, mostly determined by LEP data, is $M_C > 6.8\,\mathrm{TeV}$ [17].

It may be worth noticing that in cases **b)** and **c)**, Eqs. (11) and (13), the deviations are z-independent and the appropriate helicity cross sections have the same angular structure as in the case of the SM. Conversely, in case **a)**, Eq. (9), the deviations introduce extra z-dependencies in the angular distributions. In turns out that, as a consequence, the ADD model contribution to the integrated cross sections for the annihilation channels in Eq. (1) is quite small, due to the vanishing interference with the SM amplitudes after integration over the full angular range $-1 \leq z \leq 1$. This suppresses the possibility of identifying the ADD interaction effects in the total cross sections for these processes. In these cases, specifically defined integrated asymmetries with polarized initial beams may be expected to be more efficient contactlike interaction analyzers [8, 21]. In the next section we discuss the rôle of polarized angular differential distributions themselves, in selecting signatures of ADD effective interactions at ILC.

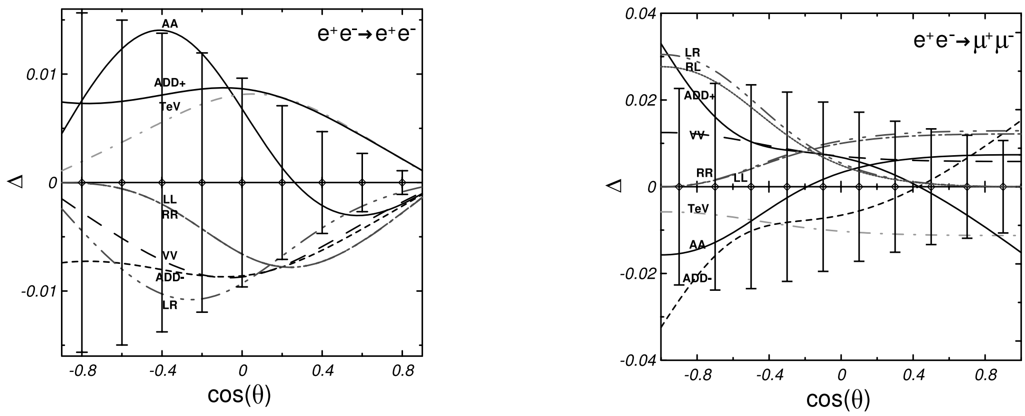

Figure 1. Left panel: relative deviations of the unpolarized Bhabha differential cross section from the SM prediction as a function of $\cos\theta$ at $\sqrt{s} = 0.5$ TeV for the CI models: AA ($\Lambda^+_{\rm AA}$=48 TeV), VV ($\Lambda^+_{\rm VV}$=76 TeV), LL ($\Lambda^+_{\rm LL}$=37 TeV), RR ($\Lambda^+_{\rm RR}$=36 TeV), LR ($\Lambda^+_{\rm LR}$=60 TeV); for the TeV^{-1} model (M_C=12 TeV) and the ADD$\pm$ models (Λ_H=4 TeV). The vertical bars represent the statistical uncertainty in each bin for $\mathcal{L}_{\rm int} = 100\,{\rm fb}^{-1}$. **Right panel:** same as in left panel but for $e^+e^- \to \mu^+\mu^-$, for the CI models: AA ($\Lambda^+_{\rm AA}$=80 TeV), VV ($\Lambda^+_{\rm VV}$=90 TeV), LL ($\Lambda^+_{\rm LL}$=45 TeV), RR ($\Lambda^+_{\rm RR}$=42 TeV), LR ($\Lambda^+_{\rm LR}$=41 TeV), RL ($\Lambda^+_{\rm RL}$=43 TeV); for the TeV^{-1} model (M_C=17 TeV) and the ADD$\pm$ models (Λ_H=2.8 TeV).

3 Discovery and identification reaches

We here briefly outline the derivation of the expected "discovery reaches" on the New Physics scenarios introduced in the previous section. The basic objects are the relative deviations of observables from the SM predictions due to the NP:

$$\Delta(\mathcal{O}) = \frac{\mathcal{O}({\rm SM+NP}) - \mathcal{O}({\rm SM})}{\mathcal{O}({\rm SM})}, \tag{14}$$

and, as anticipated, we concentrate on the polarized differential cross section, $\mathcal{O} \equiv {\rm d}\sigma/{\rm d}\cos\theta$. To get an illustration of the effects induced by the individual NP models, we show in Fig. 1 the angular behaviour of the relative deviations (14) for the two leptonic processes under consideration (with unpolarized beams), for c.m. energy $\sqrt{s} = 0.5$ TeV and selected values of the relevant mass scale parameters close to their "discovery reaches" (unpolarized cross sections). The superscript "+" on the CI mass scales $\Lambda_{\alpha\beta}$ denotes the choice $\eta_{\alpha\beta} = 1$ in Eq. (10), while the notation ADD$\pm$ corresponds to $\lambda = \pm 1$ in Eq. (8). Vertical bars represent the statistical uncertainty in each angular bin, for an integrated luminosity $\mathcal{L}_{\rm int} = 100\,{\rm fb}^{-1}$. The comparison of deviations with statistical uncertainties is an indicator of the sensitivity of an observable to the individual non-standard effective interaction models.

In this figure, the numerical value chosen for Λ_H is such that the interference of the graviton-exchange with the SM dominates the deviations of the differential cross sections, so that the ADD+ and ADD- models give corrections of the same size and opposite sign. Moreover, due to the chosen values $\Lambda^+_{\rm LL} \simeq \Lambda^+_{\rm RR}$, the corresponding CI models generate almost equal deviations of the differential cross sections because, in the (dominant) interferences with the SM, numerically $g^2_{\rm L} \simeq g^2_{\rm R}$ [see Eq. (7)].

To derive the constraints on the models, one has to compare the theoretical deviations from the SM predictions, that are functions of Λs, to the foreseen experimental uncertainties on the differential cross sections. To this purpose, taking the polarized angular distributions as basic observables for the analysis, $\mathcal{O} = {\rm d}\sigma(P^-, P^+)/{\rm d}z$, we introduce χ^2:

$$\chi^2(\mathcal{O}) = \sum_{\{P^-,\ P^+\}} \sum_{\rm bins} \left(\frac{\Delta(\mathcal{O})^{\rm bin}}{\delta\mathcal{O}^{\rm bin}}\right)^2. \tag{15}$$

Here, for the individual processes, the cross sections for the different initial polarization configurations are combined in the χ^2, and $\delta\mathcal{O}$ denotes the expected experimental relative uncertainty (statistical plus systematic one). As indicated in Eq. (15), we divide the angular range into bins. For Bhabha scattering, the cut angular range $|\cos\theta| < 0.90$ is divided into ten equal-size bins. Similarly, for annihilation into muon, tau and quark pairs we consider the analogous binning of the cut angular range $|\cos\theta| < 0.98$.

For the Bhabha process, we combine the cross sections with the following initial electron and positron longitudinal polarizations:

$$(P^-, P^+) = (|P^-|, -|P^+|); \quad (-|P^-|, |P^+|); \quad (|P^-|, |P^+|); \quad (-|P^-|, -|P^+|).$$

For the "annihilation" processes in Eq. (1), with $f \neq e, t$, we limit to combining the $(P^-, P^+) = (|P^-|, -|P^+|)$ and $(-|P^-|, |P^+|)$ polarization configurations. Numerically, we take the "standard" envisaged values $|P^-| = 0.8$ and $|P^+| = 0.6$.

Regarding the ILC energy and time-integrated luminosity, for simplicity we assume the latter to be equally distributed among the different polarization configurations defined above. The explicit numerical results will refer to C.M. energy $\sqrt{s} = 0.5$ TeV with time-integrated luminosity $\mathcal{L}_{\rm int} = 100\ fb^{-1}$, and to $\sqrt{s} = 1$ TeV with $\mathcal{L}_{\rm int} = 1000\ fb^{-1}$. The assumed reconstruction efficiencies, that determine the expected statistical uncertainties, are 100% for e^+e^- final pairs; 95% for final l^+l^- events ($l = \mu, \tau$); 35% and 60% for $c\bar{c}$ and $b\bar{b}$, respectively. The major systematic uncertainties are found to originate from uncertainties on beams polarizations and on the time-integrated luminosity: we assume $\delta P^-/P^- = \delta P^+/P^+ = 0.2\%$ and $\delta\mathcal{L}_{\rm int}/\mathcal{L}_{\rm int} = 0.5\%$, respectively.

As theoretical inputs, for the SM amplitudes we use the effective Born approximation [22] with $m_{\rm top} =$ 175 GeV and $m_{\rm H} = 120$ GeV. Concerning the $\mathcal{O}(\alpha)$ QED corrections, the (numerically dominant) effects from initial-state radiation for Bhabha scattering and the annihilation processes in (1) are accounted for by a structure function approach including both hard and soft photon emission [23], and by a flux factor method [24], respectively. Effects of radiative flux return to the s-channel Z exchange are minimized by the cut $\Delta \equiv E_\gamma/E_{\rm beam} < 1 - M_Z^2/s$ on the radiated photon energy, with $\Delta = 0.9$. In this way, only interactions that occur close to the nominal collider energy are included in the analysis and, accordingly, the sensitivity to the manifestations of the searched for nonstandard physics can be optimized. By a calculation based on the ZFITTER code [25], other QED effects such as final-state and initial-final state emission are found, in processes $e^+e^- \to l^+l^-$ and $e^+e^- \to \bar{q}q$ ($q = c, b$), to be numerically unimportant for the chosen kinematical cuts. Finally, correlations between the different polarized cross sections (but not between the individual angular bins) are taken into account in the derivation of the numerical results presented below.

The expected discovery reaches on the contactlike effective interactions are assessed by assuming a situation where no deviation from the SM predictions is observed within the experimental uncertainty. Accordingly, the corresponding upper limits on the accessible values of Λs are determined by the condition $\chi^2(\mathcal{O}) \leq \chi^2_{\rm CL}$, and we take $\chi^2_{\rm CL} = 3.84$ for a 95% C.L.

In Table 1, we present the numerical results from the processes listed in the caption, at an ILC with $\sqrt{s} = 0.5$ TeV, $\mathcal{L}_{\rm int} = 100\ fb^{-1}$, and with $\sqrt{s} = 1$ TeV, $\mathcal{L}_{\rm int} = 1000\ fb^{-1}$. Here, l^+l^- denotes the combination of $\mu^+\mu^-$ and $\tau^+\tau^-$ final states, and $\mu-\tau$ universality has been assumed for the limits on the CI mass scales. In this table, only the results for positive interference between SM amplitudes and nonstandard contributions are reported, i.e., the cases $\lambda = 1$ for the ADD model of Eq. (8) and $\eta_{\alpha\beta} = 1$ for the CI models of Eq. (10). The sensitivity reach for negative interference turns out to be practically the same. Indeed, the angular dependence of the corrections to the SM predictions induced by NP is found to be almost symmetric under reversing the sign of the interference terms, see for example Ref. [10]. Therefore, the interference terms turn out to numerically dominate over the pure, quadratic, NP contributions.

The results in Table 1 clearly show the enhancement in sensitivity to the considered effective interactions allowed, for given C.M. energy and luminosity, by beams polarization. This effect is particularly substantial in the case of the CI models (10), for which the limits on the relevant Λs are quite high compared to the current ones.

Continuing the previous χ^2-based analysis, we now assume that deviations has been observed and are consistent with the ADD scenario (8) for some value of Λ_H. To assess the level at which the ADD model can be discriminated from the general CI model as the source of the deviations or, equivalently, to determine the 'model-independent' identification reach on the effective interaction (8), we introduce in analogy with Eq. (15) the relative deviations $\tilde{\Delta}$ and the corresponding $\tilde{\chi}^2$:

$$\tilde{\Delta}(\mathcal{O}) = \frac{\mathcal{O}(\mathrm{CI}) - \mathcal{O}(\mathrm{ADD})}{\mathcal{O}(\mathrm{ADD})}; \quad \tilde{\chi}^2(\mathcal{O}) = \sum_{\{P^-,\ P^+\}} \sum_{\rm bins} \left(\frac{\tilde{\Delta}(\mathcal{O})^{\rm bin}}{\tilde{\delta}\mathcal{O}^{\rm bin}} \right)^2 . \tag{16}$$

In Eq. (16), $\tilde{\Delta}(\mathcal{O})$ depends on all Λs, and somehow represents the 'distance' between the ADD and the CI model in the parameter space $(\Lambda_H, \Lambda_{\alpha\beta})$. Moreover, $\tilde{\delta}\mathcal{O}^{\rm bin}$ is the expected relative uncertainty referred to the cross sections that include the ADD model contributions: its statistical component is therefore determined from helicity amplitudes with the deviations (9) predicted for the given value of Λ_H. In turn, the CI contributions to the cross sections bring in the dependence of Eq. (16) on the parameters $\Lambda_{\alpha\beta}$ of Eq. (11), now considered as *all* independent. Therefore, for each of processes (1), $\tilde{\chi}^2$ is a function of λ/Λ_H^4 and in general, simultaneously of the four CI couplings $\eta_{\alpha\beta}/(\Lambda^{ef}_{\alpha\beta})^2$.

In this situation we can determine *confusion regions* in the parameter space, where the CI model can be considered as consistent with the ADD model, in the sense that it can mimic the differential cross sections of the individual processes (1) determined by the latter one. At a given C.L., these confusion regions are determined by the condition

$$\tilde{\chi}^2 \leq \chi^2_{\rm CL}. \tag{17}$$

Table 1. 95% C.L. discovery reaches (in TeV). Left and right entries in each column refer to the polarizations $(|P^-|, |P^+|)$=(0,0) and (0.8,0.6), respectively.

Model	Process			
	$e^+e^- \to e^+e^-$	$e^+e^- \to l^+l^-$	$e^+e^- \to \bar{b}b$	$e^+e^- \to \bar{c}c$
	$\sqrt{s} = 0.5$ TeV, $\mathcal{L}_{\rm int} = 100 fb^{-1}$			
Λ_H	4.1; 4.3	3.0; 3.2	3.0; 3.4	3.0; 3.2
Λ^{ef}_{VV}	76.2; 86.4	89.7; 99.4	76.1; 96.4	84.0; 94.1
Λ^{ef}_{AA}	47.4; 69.1	80.1; 88.9	76.7; 98.2	76.5; 85.9
Λ^{ef}_{LL}	37.3; 52.5	53.4; 68.3	63.6; 72.7	54.5; 66.1
Λ^{ef}_{RR}	36.0; 52.2	51.3; 68.3	42.5; 71.2	46.3; 66.8
Λ^{ef}_{LR}	59.3; 69.1	48.5; 62.8	51.3; 68.7	37.0; 57.7
Λ^{ef}_{RL}	$\Lambda^{ee}_{RL} = \Lambda^{ee}_{LR}$	48.7; 63.6	46.8; 60.1	52.2; 60.7
	$\sqrt{s} = 1$ TeV, $\mathcal{L}_{\rm int} = 1000 fb^{-1}$			
Λ_H	8.7; 9.4	6.7; 7.0	6.7; 7.5	6.7; 7.1
Λ^{ef}_{VV}	173.6; 205.1	218.8; 244.3	185.6; 238.2	206.2; 232.3
Λ^{ef}_{AA}	109.9; 166.1	194.7; 217.9	186.; 242.7	186.4; 210.8
Λ^{ef}_{LL}	83.7; 122.8	128.3; 165.5	154.5; 175.8	131.3; 159.6
Λ^{ef}_{RR}	80.5; 122.1	123.4; 166.1	103.5; 176.9	111.8; 164.1
Λ^{ef}_{LR}	136.6; 166.8	120.5; 156.6	124.9; 170.2	92.7; 144.6
Λ^{ef}_{RL}	$\Lambda^{ee}_{RL} = \Lambda^{ee}_{LR}$	120.8; 158.3	120.1; 151.9	129.6; 151.1

Table 2. 95% C.L. identification reach on the ADD model parameter Λ_H obtained from $e^+e^- \to \bar{f}f$ at $\sqrt{s} = 0.5$ (1) TeV and $\mathcal{L}_{\rm int} = 10^2$ (10^3) fb^{-1} with polarizations $(|P^-|,|P^+|)$=(0,0) and (0.8, 0.6), respectively.

$\sqrt{s}$	Process			
	$e^+e^- \to e^+e^-$	$e^+e^- \to l^+l^-$	$e^+e^- \to \bar{b}b$	$e^+e^- \to \bar{c}c$
0.5 TeV	2.2; 2.9	2.3; 2.3	2.6; 2.9	2.3; 2.4
1.0 TeV	5.0; 6.4	4.9; 5.1	5.8; 6.2	5.1; 5.3

According to the number of independent CI couplings active in the different processes, for 95% C.L. we choose $\chi^2_{\rm CL} = 7.82$ for Bhabha scattering and $\chi^2_{\rm CL} = 9.49$ for lepton ($\mu^+\mu^-$, $\tau^+\tau^-$) and quark ($\bar{c}c$, $\bar{b}b$) pair production processes.

The simple χ^2 procedure outlined above is clearly 'CI model-independent', and we represent graphically some examples of the numerical results from Bhabha scattering at $\sqrt{s} = 0.5\,$TeV and $\mathcal{L}_{\rm int} = 100\,fb^{-1}$. For this process, Eq. (17) defines a four-dimensional surface enclosing a volume in the (λ/Λ^4_H, $\eta_{\rm LL}/\Lambda^2_{\rm LL}$, $\eta_{\rm RR}/\Lambda^2_{\rm RR}$, $\eta_{\rm LR}/\Lambda^2_{\rm LR}$) parameter space. In Fig. 2, we show the planar surfaces that are obtained by projecting the 95% C.L. four-dimensional surface, hence the corresponding confusion region that results from the condition $\tilde{\chi}^2 = \chi^2_{\rm CL}$, onto the two planes ($\eta_{\rm LL}/\Lambda^2_{\rm LL}$, λ/Λ^4_H) and ($\eta_{\rm LR}/\Lambda^2_{\rm LR}$, λ/Λ^4_H) (we limit our graphical examples to these pairs of parameters).

As suggested by Fig. 2, the contour of the confusion region turns out to identify a maximal value of $|\lambda/\Lambda^4_H|$ (equivalently, a minimum value of Λ_H), for which the CI scenario can be excluded at the 95 % C.L. for any value of $\eta/\Lambda^2_{\alpha\beta}$. This value, $\Lambda^{\rm ID}_H$, is the identification reach on the ADD scenario, namely, for $\Lambda_H < \Lambda^{\rm ID}_H$ the CI scenario can be excluded as explanation of deviations from SM predictions attributed to the ADD interaction, and the latter can therefore be *identified*.

Fig. 2 shows the dramatic rôle of initial beams polarization in obtaining a restricted region of confusion in the parameter space or, in other words, in enhancing the identification sensitivity of the differential angular distributions to $\Lambda^{\rm ID}_H$. Table 2 shows the numerical results for the foreseeable 'model-independent' identification reaches on Λ_H, for the two choices of C.M. energy and luminosity.

4 Concluding remarks

We have presented a simple, χ^2-based, estimate of the power for discovering and for distinguishing signatures of the spin-2 graviton exchange envisaged by the ADD model, that is foreseeable at the polarized ILC with $\sqrt{s} = 0.5$-1 TeV. The basic observables in the analysis are the polarized differential cross sections for fermion-pair production processes. The compositeness-inspired four-fermion contact interaction, from which the ADD

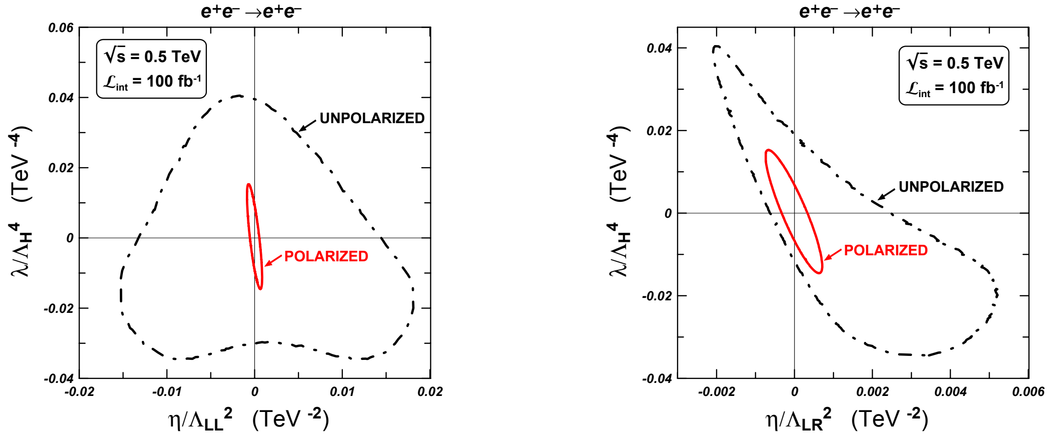

Figure 2. Two-dimensional projection of the 95% C.L. confusion region onto the planes $(\eta_{\rm LL}/\Lambda^2_{\rm LL},\ \lambda/\Lambda^4_H)$ (left panel) and $(\eta_{\rm LR}/\Lambda^2_{\rm LR},\ \lambda/\Lambda^4_H)$ (right panel) obtained from Bhabha scattering with unpolarized beams (dot-dashed curve) and with both beams polarized (solid curve).

model should be discriminated in case of observation of corrections to the SM predictions, has been assumed to be of the general form, i.e., a linear combination of the individual contact interaction operators with definite chiralities. The coefficients of such a combination have been taken into account simultaneously as independent, and potentially nonvanishing, constants.

The discovery reaches, as well as the identification reaches, are quite high compared to the current bounds, and depend on energy and luminosity as shown in Table 1 and in Table 2, respectively. In particular, Table 2 shows that, of the four considered e^+e^- processes, Bhabha scattering and $\bar{b}b$ pair production definitely have the best identification sensitivity on the mass scale Λ_H characterizing the ADD model for gravity in 'large' compactified extra dimensions. The substantial rôle of beams polarization is exemplified by Fig. 2 (where the confusion region between the considered models is dramatically reduced), and by the discovery reaches on the models shown in Table 1.

The enhancement of the estimated identification sensitivity on the ADD effective interaction is quite considerable: as exemplified by the entries of Table 2, in the polarized case the identification reach on Λ_H ranges from 2.9 TeV to 6.4 TeV, depending on energy, luminosity and degree of longitudinal polarization. Although unavoidably somewhat depressed by the penalty due to the general multi-parameter expression assumed for the CI scenario (that implies taking large values of the $\chi^2_{\rm CL}$), these 'model-independent' identification values of Λ_H are still much higher than the current limits. In fact, we find that they are only moderately lower (by some 10-20%) than the 'model-dependent' ones obtained in Ref. [10] by assuming only one nonzero CI coupling at a time. These nice features reflect in part the small values assumed for the relative uncertainties on electron and positron beams polarization in the previous section, and call for very high precision on polarimetry measurements at the ILC.

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