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APPLICATIONS OF MUELLER THEORY TO TWO-PARTICLE INCLUSIVE PROCESSES*

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Abstract

Assuming factorization, and extracting inclusive Reggeon vertices from one-particle inclusive data, we predict Regge corrections to scaling for several two-particle inclusive processes. The data on $p + p \rightarrow \pi^- + \pi^- + X$, $K^+ + p \rightarrow \pi^- + \pi^- + X$, $K^- + p \rightarrow \pi^+ + \pi^- + X$, and $\pi^+ + p \rightarrow \pi^- + \pi^- + X$ agree with our predictions.

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According to the Mueller analysis[1,2] of inclusive processes using Regge theory and unitarity, the leading corrections to hadronic scaling come from exchanging the leading meson Regge trajectories (ρ, f, ω, A_2). These trajectories have intercepts $\sim \frac{1}{2}$, implying[2] that scaling is approached as $s^{-\frac{1}{2}}$ and giving correlations of length ~ 2 in rapidity. Experimental evidence has been presented for the $s^{-\frac{1}{2}}$ approach to scaling,[3] and for the factorization of both the Pomeron[4] and meson trajectories[5] in one-particle inclusive processes $a + b \to c + X$.

In this paper,[6] we make a phenomenological analysis of some two-particle inclusive processes $a + b \to c + d + X$, using Mueller theory. If factorization is correct, the dominant corrections to scaling in the limit $s \to \infty$, $\delta y_c = y_c - y_{min} = y_c - \ell n\left(\frac{m_c}{m_a}\right)$ fixed, $\delta y_d = y_{max} - y_d = y_b - \ell n\left(\frac{m_d}{m_b}\right) - y_d$ fixed, (which limit we denote by $(a \to c|d \leftarrow b)$) are:

$$\left(\frac{s}{s_o}\right)^{-\frac{1}{2}} \sum_{i=\rho, f, \omega, A_2} \tau_i \, F_i^{a\to c}\left(\delta y_c, p_\perp^c\right) F_i^{b \to d}\left(\delta y_d, p_\perp^d\right) \qquad (1)$$

The τ_i are Regge signatures: the $F_i^{a\to c}\left(\delta y_c, p_\perp^c\right)$ are the inclusive Reggeon vertices controlling the approach to scaling in one-particle inclusive processes $a + b \to c + X$ (denoted $(a \to c|b)$):

$$\left(\frac{s}{s_o}\right)^{-\frac{1}{2}} \sum_{i=\rho, f, \omega, A_2} \tau_i \, \beta_i^b \, F_i^{a\to c}\left(\delta y_c, p_\perp^c\right) \qquad (2)$$

In Eq. (2), the β_i^b are ordinary two-particle Regge residues: Eq. (1) and (2) are indicated graphically in Fig. 1 and 2. We will use single-particle inclusive data to estimate the inclusive Reggeon vertices $F_i^{a\to c}$ of Eq. (2) and use them in Eq. (1) to make predictions for two-particle inclusive processes in the limit $(a \to c|d \leftarrow b)$.

We find that, because of approximate exchange degeneracies, Regge corrections to the following processes should be small:

$$\left.\begin{array}{ll} (p \to \pi^- | \pi^- \leftarrow \pi^+) & (p \to \pi^- | \pi^- \leftarrow K^+) \\ (p \to \pi^- | \pi^- \leftarrow p) & (p \to \pi^- | \pi^+ \leftarrow p) \\ (p \to \pi^+ | \pi^- \leftarrow K^+) & \end{array}\right\} \quad \text{(A)}$$

On the other hand, some Regge corrections should be large:

$$\left.\begin{array}{ll} \dfrac{(p \to \pi^- | \pi^+ \leftarrow K^-)}{(p \to \pi^- | \pi^+ \leftarrow K^-)_{s=\infty}} & \approx 1 + 12\left(\dfrac{s}{s_o}\right)^{-\frac{1}{2}} \\ \dfrac{(p \to \pi^- | \pi^+ \leftarrow \bar{p})}{(p \to \pi^- | \pi^+ \leftarrow \bar{p})_{s=\infty}} & \approx 1 + 12\left(\dfrac{s}{s_o}\right)^{-\frac{1}{2}} \end{array}\right\} \quad \text{(B)}$$

We now proceed to a derivation of these results, and show that the predictions for $(p \to \pi^- | \pi^- \leftarrow p)$, $(p \to \pi^- | \pi^- \leftarrow K^+)$, and $(p \to \pi^- | \pi^+ \leftarrow K^-)$ agree with experiment. Our prediction for $(p \to \pi^- | \pi^- \leftarrow \pi^+)$ is also consistent with experimental data.

To extract the inclusive Reggeon vertices $F_i^{a \to c}$ from one-particle inclusive data, we need values for the two-particle residues β_i^b: we take these from total cross-section data, assuming the normal exchange degeneracy patterns:

$$\begin{aligned} &\beta_{I\!P}^{p} = 6.1, \quad \beta_{I\!P}^{\pi^+} = 3.6, \quad \beta_{I\!P}^{K^+} = 2.9, \\ &\beta_f^p = \beta_\omega^p = 6.3, \quad \beta_\rho^p = \beta_{A_2}^p = 1.4, \\ &\beta_\rho^{\pi^+} = \beta_f^{\pi^+} = 2.9, \\ &\beta_\rho^{K^+} = \beta_{A_2}^{K^+} = \beta_f^{K^+} = \beta_\omega^{K^+} = 1.5 \end{aligned} \qquad (3)$$

We now consider data on various fragmentations $(a \to c)$: because of the vagaries of the data, we always consider data integrated over $p_\perp$, and take the ratios $F_i^{a \to c}/F_{I\!P}^{a \to c}$ to be independent of δy_c. This is a good approximation for the data cited below when $\delta y_c \lesssim 2$.

I. $(p \rightarrow \pi^-)$

Alston-Garnjost et al.[3] have found that $(p \rightarrow \pi^- | \pi^+)$ scales early which, together with (3), implies

$$F_f^{p\rightarrow\pi^-} \approx F_\rho^{p\rightarrow\pi^-} \tag{4}$$

Data presented by Stroynowski[7] indicate that at 16 GeV/c

$$\frac{(p \rightarrow \pi^- | \pi^-)}{(p \rightarrow \pi^- | \pi^+)} \approx 1.5 \tag{5}$$

Using Eq. (2), (3), and (4), this implies

$$\frac{F_f^{p\rightarrow\pi^-}}{F_{\mathbb{P}}^{p\rightarrow\pi^-}} \approx 1.7 \tag{6}$$

The apparent early scaling[4] of $(p \rightarrow \pi^- | p)$ and $(p \rightarrow \pi^- | K^+)$ indicates that also

$$F_f^{p\rightarrow\pi^-} \approx F_\omega^{p\rightarrow\pi^-} \quad \text{and} \quad F_\rho^{p\rightarrow\pi^-} \approx F_{A_2}^{p\rightarrow\pi^-} \tag{7}$$

so that all four $(p \rightarrow \pi^-)$ residues are equal.

II. $(K^+ \rightarrow \pi^-)$

If we assume that the following processes scale early: $(K^+ \rightarrow \pi^- | K^+)$, $(K^+ \rightarrow \pi^- | \pi^+)$, and $(K^+ \rightarrow \pi^- | p)$, which is consistent with some early scaling hypotheses,[8] then

$$F_f^{K^+\rightarrow\pi^-} \approx F_\rho^{K^+\rightarrow\pi^-} \approx F_\omega^{K^+\rightarrow\pi^-} \approx F_{A_2}^{K^+\rightarrow\pi^-} \tag{8}$$

At 9 GeV/c, Foster et al.[9] find

$$\frac{(K^- \rightarrow \pi^+ | p)}{(K^+ \rightarrow \pi^- | p)} \approx 2 \tag{9}$$

Under charge conjugation, $F_{\rho}^{K^+\to\pi^-}$ and $F_{\omega}^{K^+\to\pi^-}$ change sign, but $F_{f}^{K^+\to\pi^-}$ and $F_{A_2}^{K^+\to\pi^-}$ do not. Using these relations and Eq. (8) and (9), we find

$$\frac{F_{f}^{K^+\to\pi^-}}{F_{\mathbb{P}}^{K^+\to\pi^-}} \approx 1.7 \tag{10}$$

III. $(\pi^+ \to \pi^-)$

If $(\pi^+ \to \pi^- | \pi^+)$ scales early, then

$$F_{f}^{\pi^+\to\pi^-} \approx F_{\rho}^{\pi^+\to\pi^-} \tag{11}$$

Estimating these vertices from experiment is difficult, but the fact[7] that $(\pi^- \to \pi^+ | p) > (\pi^+ \to \pi^- | p)$ at 16 GeV/c suggests that

$$F_{\rho}^{\pi^+\to\pi^-} > 0 . \tag{12}$$

IV. $(p \to \pi^+)$

Data presented by Lander[10] suggest that $(p \to \pi^+ | p)$ and $(p \to \pi^+ | K^+)$ scale early, implying

$$F_{f}^{p\to\pi^+} \approx F_{\omega}^{p\to\pi^+} , \quad F_{\rho}^{p\to\pi^+} \approx F_{A_2}^{p\to\pi^+} \tag{13}$$

We now turn to the combination of these vertices $F_i^{a\to c}$ in the two-particle distributions, using Eq. (1). It is easy to check[6] that the signature factors τ_i and the exchange degeneracies (4), (7), (8), (11), and (13) conspire to predict that the processes (A) should have no net $s^{-\frac{1}{2}}$ Regge corrections. On the other hand, the contributions of the four meson trajectories to the four processes (B) add constructively to give

$$\frac{(p \to \pi^- | \pi^+ \leftarrow K^-)}{(p \to \pi^- | \pi^+ \leftarrow K^-)_{s=\infty}} = 1 + 4\left(\frac{s}{s_o}\right)^{-\frac{1}{2}} \frac{F_M^{p\to\pi^-}}{F_{\mathbb{P}}^{p\to\pi^-}} \frac{F_M^{K^+\to\pi^-}}{F_{\mathbb{P}}^{K^+\to\pi^-}}$$

$$\approx 1 + 12\left(\frac{s}{s_o}\right)^{-\frac{1}{2}}$$

using the estimates (6) and (10), and a similar result for $(p \to \pi^- | \pi^+ \leftarrow \bar{p})$.

There are data available on $(p \to \pi^- | \pi^- \leftarrow p)$,[11] $(p \to \pi^- | \pi^- \leftarrow K^+)$,[12] $(p \to \pi^- | \pi^+ \leftarrow K^-)$,[13] and $(p \to \pi^- | \pi^- \leftarrow \pi^+)$,[14] which we can compare with the predictions (A) and (B). In Fig. 3, we compare $(p \to \pi^- | \pi^- \leftarrow p)$ at 21 GeV/c,[11] with $\frac{(p | p \to \pi^-)(p \to \pi^- | p)}{\sigma_{tot}(pp)}$. Pomeron factorization and energy independence of σ_{tot} (pp) and $(p \to \pi^- | p)$ suggest the two should be the same if $(p \to \pi^- | \pi^- \leftarrow p)$ indeed scales early. The data are consistent with this prediction, especially if one recalls the magnitudes of other expected Regge corrections (B).

Data on $K^+ + p \to \pi^- + \pi^- + X$ at 12 GeV/c[12] indicate that for $0.4 \le |y_c - y_d| \le 4$. the correlation integrated over $(y_c + y_d)$ is between 10% and -50%, consistent with our prediction of early scaling for $(p \to \pi^- | \pi^- \leftarrow K^+)$.

Data on $(p \to \pi^- | \pi^+ \leftarrow K^-)$ at 9 GeV/c[13] suggest that for data points with $-0.4 < x_{\pi^-} < -0.2$ and $0.4 > x_{\pi^+} > 0.2$,

$$1 \le \frac{(p \to \pi^- | \pi^+ \leftarrow K^-)}{\left[\frac{(p \to \pi^- | K^-)(p | \pi^+ \leftarrow K^-)}{\sigma_{tot}(K^- p)}\right]} \le 1.5 \; .$$

At this energy

$$\frac{(p \to \pi^- | K^-)}{(p \to \pi^- | K^+)} \approx \frac{(p | \pi^+ \leftarrow K^-)}{(p | \pi^- \leftarrow K^+)} \approx 2$$

and

$$\sigma_{tot}(K^-p) \approx \frac{5}{4}\,\sigma_{tot}(K^-p)_{s=\infty}$$

Assuming factorization and early scaling for $K^+ + p \to \pi^- + X$, and using charge conjugation invariance of the Pomeron, this indicates that if at this energy and in this kinematic range

$$\frac{(p \to \pi^- | \pi^+ \leftarrow K^-)}{(p \to \pi^- | \pi^+ \leftarrow K^-)_{s=\infty}} \approx 1 + \lambda\left(\frac{s}{s_o}\right)^{-\frac{1}{2}}$$

is a valid approximation, then $9 \leq \lambda \leq 14$, in agreement with our prediction (B).

Data on $(p \to \pi^- | \pi^- \leftarrow \pi^+)$ at 18.5 GeV/c[14] indicate a negative correlation in the relevant kinematic range: this is the sign we expect on the basis of our prediction (A), the early scaling of $(p \to \pi^- | \pi^+)$, and the expected sign (13) of $F_\rho^{\pi^+ \to \pi^-}$.

We have two main conclusions from our work. The Mueller phenomenology of two-particle inclusive processes has considerable predictive power see (A), (B)] even considering the incomplete state of data on one-particle inclusive prosesses. Also several of these predictions are confirmed by existing data, showing that Regge factorization continues to be approximately valid.[6]

As more single-particle inclusive data become available (e.g., on $\bar{p}$-induced reactions, and on the fragmentation $(p \to \pi^+)$), the range of two-particle Mueller phenomenology will be extended. More precise data will permit a refinement of our predictions and our comparisons with two-particle cross sections.

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Footnotes

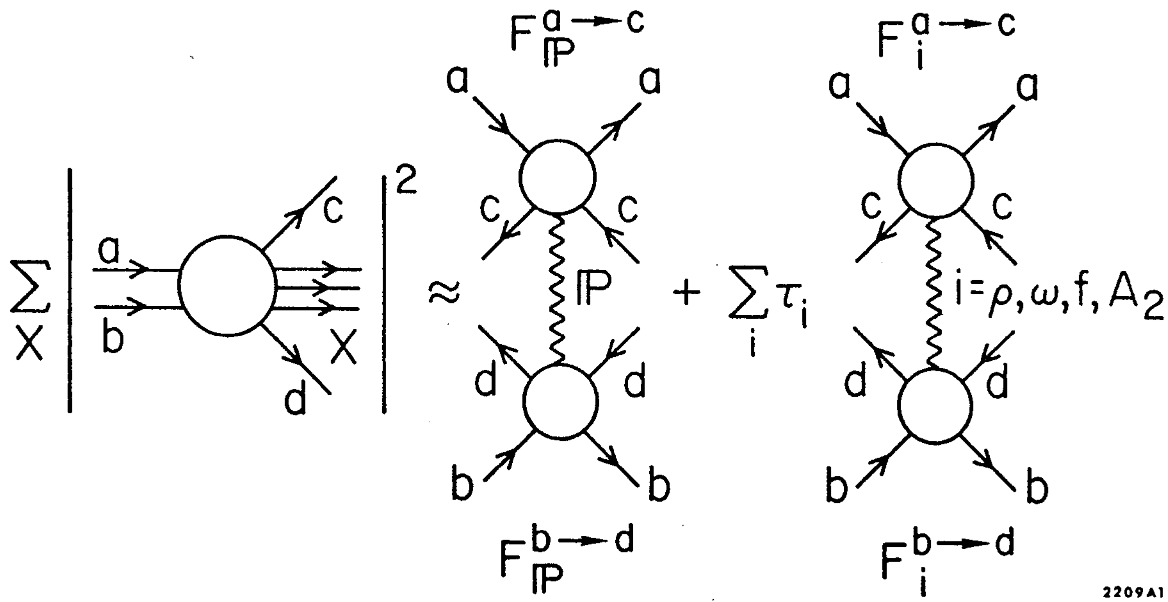
*We denote particle rapidities y_a, etc., transverse moments $p_\perp^c$, etc., Feynman variables x_a, etc., and normalize Regge corrections using $s_o = 1$ GeV^2.

Figure Captions

1. Graphical representation of Eq. (1) for the two-particle inclusive cross section in the Mueller description.

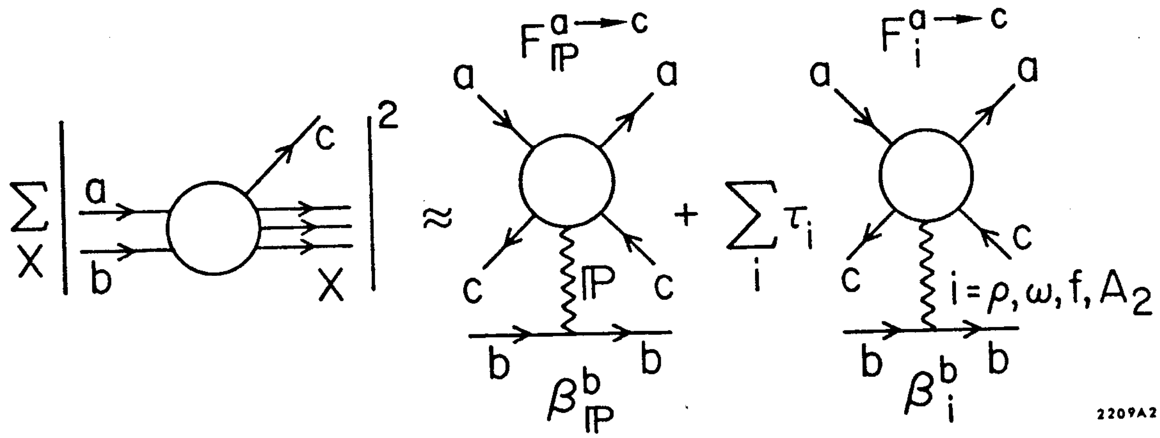
2. Graphical representation of Eq. (2) for the single-particle inclusive cross section in the Mueller description.

3. Comparison of the invariant cross sections $(p \rightarrow \pi^- | \pi^- \leftarrow p)$ and $(p \rightarrow \pi^- | p)(p | \pi^- \leftarrow p)/\sigma(pp)$, using data[11, 15] at 21 GeV/c. The predictions of this paper apply to the region $y_2 < -1$. Figure based on ref. 15.



$\sum_X$
a
b
c
d
X
2
≈
$F_{IP}^{a\to c}$
a
a
c
c
IP
d
d
b
b
$F_{IP}^{b\to d}$
+ $\sum_i \tau_i$
$F_i^{a\to c}$
a
a
c
c
i=ρ,ω,f,A2
d
d
b
b
$F_i^{b\to d}$
2209A1

Fig. 1



$\sum_X$
a
b
c
X
2
≈
$F_{IP}^{a\to c}$
a
a
c
c
IP
b
b
β_{IP}^{b}
+ $\sum_i \tau_i$
$F_i^{a\to c}$
a
a
c
c
i = ρ, ω, f, A_2
b
b
β_i^b
2209A2

Fig. 2

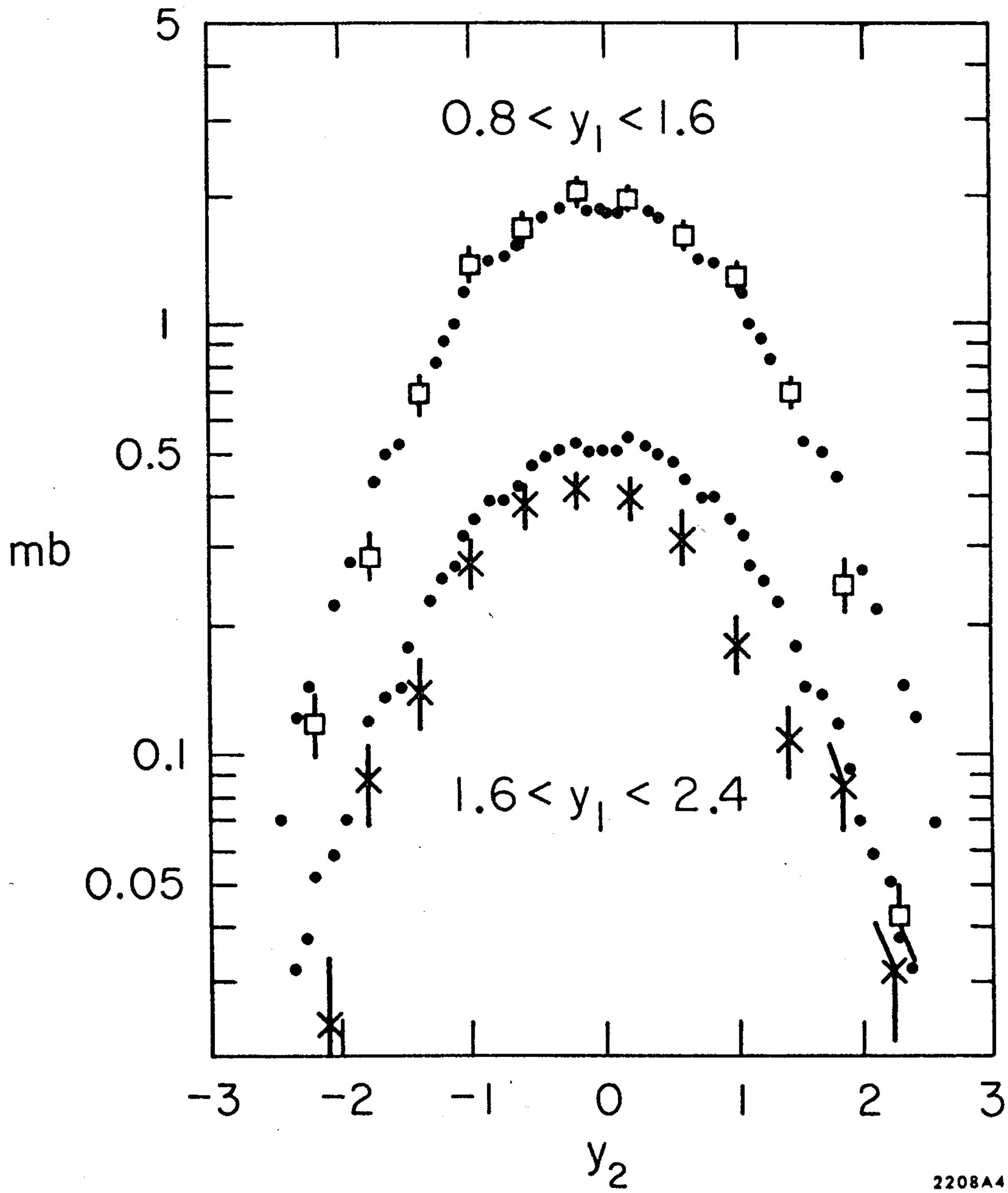


Fig. 3