2.19 Schottky Noise and Beam Transfer Functions

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2.19.1 Introduction

Beam transfer functions (BTF)s encapsulate the stability properties of charged particle beams [1,2]. In general one excites the beam with a sinusoidal signal and measures the amplitude and phase of the beam response. Most systems are very nearly linear and one can use various Fourier techniques to reduce the number of measurements and/or simulations needed to fully characterize the response. Schottky noise is associated with the finite number of particles in the beam. This signal is always present. Since the Schottky current drives wakefields, the measured Schottky signal is influenced by parasitic impedances.

2.19.2 Beam Transfer Functions

BTFs can be for longitudinal or transverse motion, for a single bunch or multiple bunches. In principle they are always the same with a sinusoidal drive yielding the phase and amplitude of the response. When there are no collective effects the BTF is relatively straightforward to calculate. One simply needs to get the response of a single particle and do appropriate averaging of the initial synchrotron and betatron coordinates. When space charge and or wakefields are present things get more interesting. There is a profound difference between coasting and bunched beams. Longitudinal coasting beam BTFs are effectively a homework problem. Transverse BTFs of coasting beams can be solved within the approximation of transverse forces that vary linearly [3]. This covers the important case of arbitrary wall wakes and space charge with a transverse KV distribution. The inclusion of wall induced frequency spread from octupolar fields is straightforward to include but the effects of nonlinear space charge forces have only been addressed in crude approximation. Of course one can always do simulations but the difficulties associated with numerical convergence can be significant.

Transfer functions of bunched beams with collective effects are difficult to calculate. The author knows of no closed form solutions. Various ways to numerically solve for appropriate moments of the Vlasov equation have been developed but generally it seems that numerical simulations give the quickest, most reliable results, at least for bunches that are short compared to the circumference of the synchrotron [4]. In this case it is possible to use a relatively straightforward Fourier technique to obtain the BTF spanning an entire revolution line with only 2 independent simulations. The idea is quite simple. Suppose you have a bunch that is short compared to the radius of the accelerator so that the a signal at the revolution frequency, f_{rev}, has a small phase advance along the bunch. Then along the bunch an excitation at frequency f looks much like an excitation at frequency $f+f_{rev}$ except for a slip in phase from turn to turn. So, one just calculates the impulse response function of the bunch from single turn kicks of $\sin(2\pi ft)$ and $\cos(2\pi ft)$ and employs linearity. Figure 1 shows a simulation of the transverse beam transfer function [4].

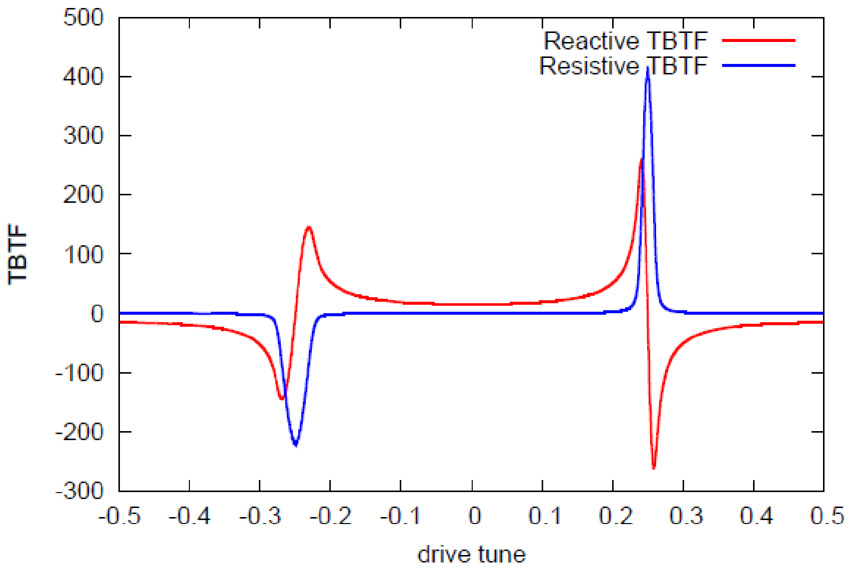

Figure 1: Transverse beam transfer function obtained from two simulations. While calculated for a bunched beam the resolution is intentionally low to suppress the synchrobetatron structure.

This transfer function was obtained for a bunched beam below transition with a fractional betatron tune of 0.25 and a negative chromaticity. This broadens the sideband near -0.25 and leads to enhanced Landau damping of the unstable modes with n-Q_β >0. While this figure was made for a bunched beam the individual synchrobetatron lines have been smoothed over. When individual synchrobetatron lines are resolved the data become quite rich. Figure 2 shows the resistive part of the BTF measured at low frequency in the presence of a step wake potential (like long stripline beam position monitors) for various chromaticity values [4].

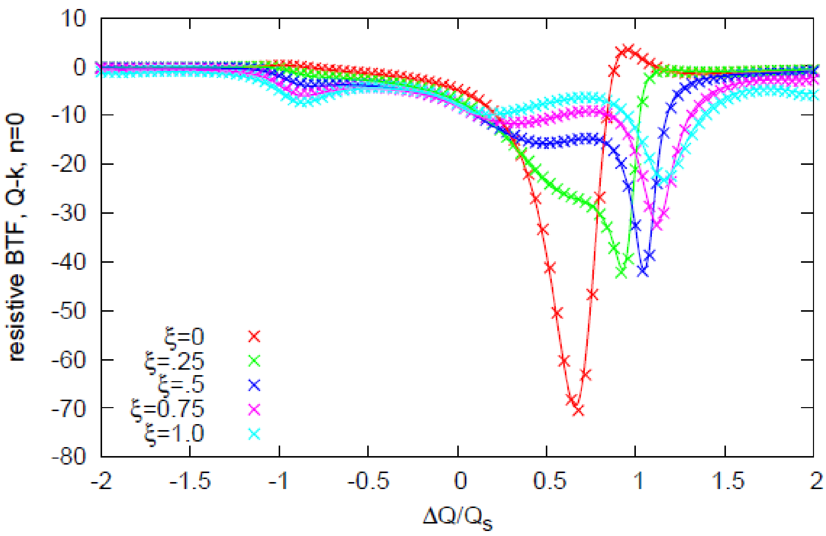

Figure 2: Low frequency transverse beam transfer function obtained from two simulations. This is a high resolution image showing the detailed synchrobetatron structure.

The solid lines in the figure are from simulations while the crosses are from a solution of the Vlasov equation. The near perfect agreement suggests both techniques are accurate. It is clear there is a rich behavior waiting to be measured and in fact many measurements have been done. See [5] for a good example. It is hoped that the new methods of calculation will allow for more and better understanding of parasitic impedances.

2.19.3 **Schottky Signals**

Schottky signals have been used to measure the broad band longitudinal impedance in RHIC. The technique is straightforward. In a stable beam the broad band impedance creates a potential well distortion that modifies the synchrotron frequency. By measuring the synchrotron frequency as a function of intensity one gets the broad band impedance. There is one subtlety in this technique. Any measurement one makes is, by necessity, a measurement of a collective mode of the beam. This includes the self excited Schottky modes. If the center frequency of the Schottky signal is too low the coherent tune shift can be quite different from the estimated incoherent response. A toy model will illustrate the idea. Suppose we have N particles in the bunch and approximate the equation of motion for particle j as

$$\ddot{z}_j + \omega^2 z_j = \frac{2\omega\Delta}{N}\sum_{k=1}^{N} z_j - z_k \tag{1}$$

where ω is the unperturbed synchrotron frequency, z_j is the longitudinal position of particle j, and Δ is the small, coherent frequency shift. For an actual wake field the sum on the right would be over $W(z_j\text{-}z_k)$ with the highly nonlinear wake W. Such a wake is easily used in simulations, but not analytically tractable. Solving Eq. (1) yields

$$z_j = \bar{z}\cos(\omega t + \varphi) + (a_j - \bar{a})\cos[(\omega - \Delta)t] + (b_j - \bar{b})\sin[(\omega - \Delta)t] \tag{2}$$

where the overbars denote arithmetic means. Measurements that are dominated by the a_js and b_js are sensitive to the impedance. The current from the bunch is

$$I(t) = \frac{q}{T}\sum_{k=1}^{N}\sum_{m}\exp[im(z_k - ct)/R] \tag{3}$$

where $2\pi R=cT$ for our relativistic beam. Inserting Eq. (2) in Eq. (3) and defining $\phi=z/R$, the current for a single value of m is

$$I_m(t) = \exp(im\bar{\phi}\cos(\omega t))\sum_{k=1}^{N}\sum_{p=-\infty}^{\infty} J_p(m\hat{\phi}_k)\exp(ip[(\omega - \Delta)t + \psi_k]) \tag{4}$$

With the caveat, corresponding to subtracting the arithmetic means in Eq. (2),

$$\sum_k \hat{\phi}_k \exp(i\psi_k) = 0 \tag{5}$$

The p=0 term in the sum of Eq. (4) has frequency ω and is the coherent mode in the toy model. Hence we need p and m to be large enough so that Eq. (5) has little effect. We also need the arguments of the Bessel functions to be significant. The data shown in Fig. 3 are the center frequencies of synchrotron lines measured in the yellow RHIC ring as a function of the central curvature of the current pulse [6]. The linear correlation, corresponding to the variation in synchrotron tune with beam current, is clear. Figure 4 shows the broad band impedance obtained from the slopes of the lines in Fig. 3. For p>2 there is a nearly constant value. The larger values at p=1 and 2 are ascribed to low lying collective modes, similar to the p=0 mode in the toy model. The blue results in Fig. 6 assume the accelerator was stable during data acquisition. The green lines allow for a linear drift of the synchrotron tune with time. The differences are comparable to the error bars.

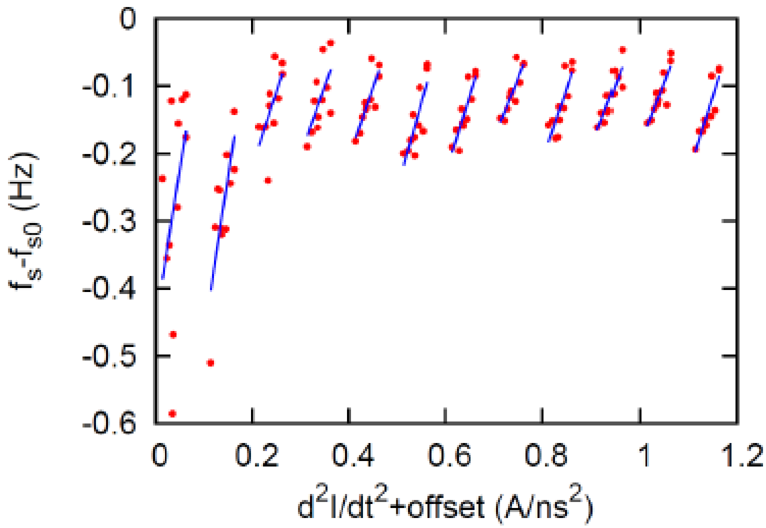

Figure 3: Measured synchrotron frequency shifts as a function of intensity for 12 synchrotron sidebands.

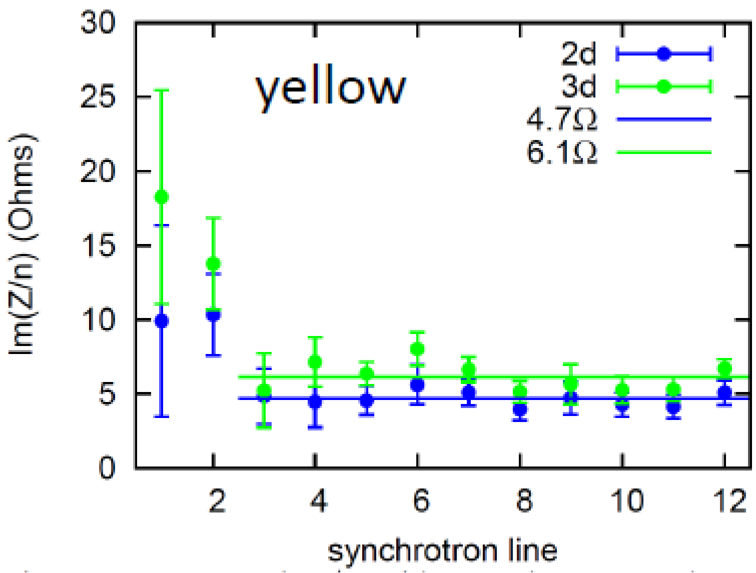

Figure 4: Broad band impedance needed to produce the slopes observed in Fig. 3.

2.19.4 **Conclusions**

Beam Transfer Functions and Schottky signals are useful to constrain both machine impedance and beam dynamics. BTFs can be simulated quite well allowing for a detailed comparison between model and measurement. Conversely, Schottky signals can be used to study the fields present when there are no large collective oscillations, greatly simplifying the analysis. Additionally, these measurements are made with stable beams allowing for adequate set up time and minimal beam loss.

2.19.5 **References**

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