COUPLING COEFFICIENT AND TUNE SHIFT

MEASUREMENTS IN HIGH-ENERGY ACCELERATORS

Karabekov I.P., Tsakanov V.M.
Yerevan Physics Institute, Armenia, USSR

Summary

The results of theoretical and experimental investigations of a method of undisturbing measurement of the coupling coefficient, Q, and of the tune shift, $\Delta\nu$, in high-energy accelerators and storage rings, using the geometrical properties of the synchrotron radiation, are presented. The calculations show that the tune shift and coupling coefficient can be measured with accuracies to 10^{-4} and 10^{-2} respectively.

Introduction

The most important parameters of accelerators and storage rings, such as luminosity, quantum lifetime, ultimate number of particles in bunch, brightness of the synchrotron radiation, extracted beam space-angular behaviours, are determined by the transverse size of the beam, which depends on the proximity to the resonance lines and the coupling value. This proximity is given by the design value of betatron numbers ν_x, ν_y, which exist only to an accuracy of $\Delta\nu_x$, $\Delta\nu_y$ defined in their turn by the stability of the focusing field parameters.

Thus, the development of a method for continuous and non-disturbing measurement of the tune shift $\Delta\nu_x$, $\Delta\nu_y$ and of coupling coefficient, Q, with a view of introduction of the corresponding control systems, will permit to increase quantitative characteristics and the efficiency of using the accelerators and storage rings. These non-disturbing methods of measurement are very essential especially for high-energy accelerators, as the particle losses in them may be of great danger for the machines and for the environment. As is shown below, for these measurements only the geometrical properties of synchrotron radiation (SR) are essential and the proposed method is useful for both the electron and the proton machines[1].

Tune shift measurement

The method is based on the tune shift dependence of bumped orbit parameters at the constant distribution of guide field disturbances. The closed orbit distortion for an arbitrary azimuthal field errors $g(s)$ distribution is given by the integral [2]

$$x_p(s)=\frac{\sqrt{\beta(s)}}{2\sin(\pi\nu_x)}\oint g(\bar{s})\sqrt{\beta(\bar{s})}\cos(\phi(s)-\phi(\bar{s})-\pi\nu_x)\,d\bar{s} \qquad (1)$$

where $$\phi(s)=\int_0^s\frac{d\bar{s}}{\beta(\bar{s})} .$$

At given values ν_x, ν_y it is easy to obtain the necessary distribution of disturbances, $g_i(s)$, which allows to create completely compensated local deviation of the equilibrium orbit. In particular, the phase differences between two equal plane disturbances is

$$\Delta\phi=\phi(s_2')-\phi(s_1')=\pi \qquad (2)$$

In this case the orbit parameters $x_p(s)$, $x_p'(s)$ in bumped part are defined by the expressions:

$$x_p(s)=g\,\Delta s\sqrt{\beta(s)\beta(s_1)}\,\sin\phi(s),$$
$$x_p'(s)=\frac{\beta'(s)}{2\beta(s)}x_p(s)+\Delta s\cdot g\left(\frac{\beta(s_1)}{\beta(s)}\right)^{1/2}\cdot\cos\phi(s). \qquad (3)$$

Let the field disturbances make completely compensated bump for the chosen values ν_x, ν_y and do not change the working parameters of the machine. The SR beam formed by the slit located tangently at the distance L from an arbitrary point S_0 in the bumped part (it may be the point where $x'(s)=0$) is observed at the screen spaced at the distance L_2 from the collimator (see Fig.1).

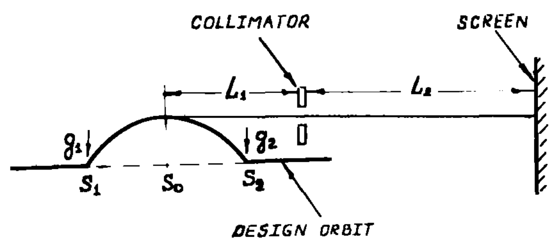


FIG. 1

The beam position on the screen for designed value ν_x may be easily obtained from simple geometrical assumptions. The tune shift leads to the change of orbit parameters $x(s_0)$ and $x'(s_0)$ at S_0 and the corresponding displacement of SR beam position on the screen. Neglecting the displacement of SR beam formation centre from S_0 we can find for small $\Delta\nu_x$ the corresponding displacement of the beam position by means of the formula:

$$\Delta X=\frac{L_2}{L_1}\frac{dx_p}{d\nu_x}(s_0)\,\Delta\nu_x \qquad (4)$$

The value of $dx_p/d\nu_x$ may be easily calculated if the derivative $d\beta/d\nu$ is known. As the amplitude functions are periodical, this derivative may be approximated by simple analytical expressions useful for preliminary calculations and estimations:

$$\frac{d\beta}{d\nu}(S_0)=-\frac{2\pi[\beta_{MAX}\cos(\mu-2\phi_F)-\beta_{MIN}\cos(\mu-2\phi_D)]}{N\sin(\mu)(\beta_{MAX}+\beta_{MIN})}, \quad (5)$$

where N is the number of cells, μ is the phase difference per cell, ϕ_F , ϕ_D are phase differences from S_0 to the nearest quadrupole lenses. To estimate the sensitivity of the method, the values of $\Delta\nu_x$ corresponding to the displacement ΔX =0.1mm of SR beam on the screen were calculated for some working and constructed facilities. The ratio L_2/L_1 for large machines (R > 100m) was taken to be 5, and for small ones (R < 100 m) to be 2. The results of calculations presented in Table 1 show sufficient sensitivity of the method that can be increased with the corresponding choice of the values of L_1, L_2 and the application of special optics. For the measurement of $\Delta\nu_y$ it is required to produce the compensated bump in the vertical plane.

Table 1

Machine	R (m)	ν_x	ν_y	$\lvert\Delta\nu_x\rvert$
PETRA	367	25.2	23.3	0.0051
PEP	350	18.25	18.75	0.0036
CESR	121	9 - 11	9 - 11	0.0022
KEK	50	6.25	5.25	0.0026
ARUS	35	5.38	5.38	0.0025
SRS	15.3	3.25	2.25	0.0012

Measurement of Coupling Coefficient

The stationary particle distribution for an electron beam in a storage ring in the presence of linear coupling between the transverse x , y oscillations may be written as

$$\psi(\bar{x})=\psi_0 \exp\Big(-\frac{1}{2}\sum_{i,j=1}^{4} A_{ij}x_i x_j\Big), \quad (6)$$

with (x_1 , x_2 , x_3 , x_4) denoting the canonical variables x , x' , y , y' and A_{ij} being the elements of the matrix inverse to the correlation matrix σ_{ij} . This matrix can be obtained by solving the Fokker-Plank equation for the given magnetic system, the properties of which are described by the diffusion integral over all the bending magnets

$$H_{ij}=\frac{C\cdot\gamma^5}{2\pi R}\oint\frac{ds}{\rho^3\sqrt{\beta_i\beta_j}}\Big[\eta_i\eta_j+(\beta_i\eta_i'-\tfrac{1}{2}\beta_i'\eta_i)(\beta_j\eta_j'-\tfrac{1}{2}\beta_j'\eta_j)\Big], \quad (7)$$

where (i , j)=(x , y), β , β', η , η' are the amplitude functions of the system, ρ is the bending radius, R is the average radius of machine, γ is the beam energy in units of rest energy and C =2.16·10^{-19}m^3/sec.

The expressions for correlation matrix elements in the form useful for calculations are given in [3,4]. In particular, the expressions for the parameters of distribution responsible for coupling σ_{xy}, $\sigma_{x'y}$ $\sigma_{x'y'}$ in the case of $\eta_y(s)$ =0 are:

$$\sigma_{xy}=\frac{\alpha_y Q_1 \Delta\nu\, H_{xx}}{2[(\alpha_x+\alpha_y)^2|Q|^2+\alpha_x\alpha_y\Delta\nu^2]}\sqrt{\beta_x\beta_y},$$

$$\sigma_{x'y}=\frac{\beta_x'}{2\beta_x}\sigma_{xy}, \quad (8)$$

$$\sigma_{x'y'}=\Big(1-\frac{1}{4}\beta_x'\beta_y'\Big)\sigma_{xy},$$

where α_x , α_y are the radiation damping constants, $\Delta\nu$, $\Delta\nu=\nu_x-\nu_y-m$ is the distance from the nearest coupling resonance and m is an integer, Q is the coupling coefficient at a reference point in the lattice, calculated by the integration over the coupling elements [5]:

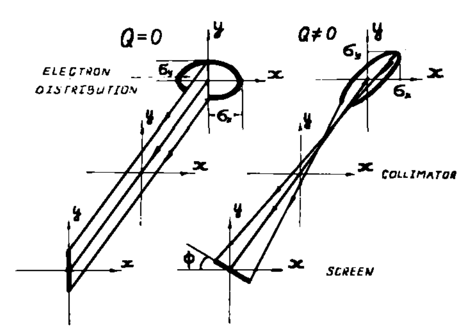
$$Q=Q_1+iQ_2=\oint_{\theta_0}^{\theta_0+2\pi}d\theta\Big\{\exp\Big[i\int_{\theta_0}^{\theta}d\theta\Big(\frac{R}{\beta_x}-\frac{R}{\beta_y}-\Delta\nu\Big)\Big]\Big\}\cdot$$
$$\cdot\frac{\sqrt{\beta_x\beta_y}}{4\pi R}\Big[S_Q-\frac{S_m R}{4}\Big(\frac{\beta_x'}{\beta_x}-\frac{\beta_y'}{\beta_y}\Big)-\frac{i}{2}S_m R\Big(\frac{1}{\beta_x}+\frac{1}{\beta_y}\Big)\Big]. \quad (9)$$

In this formula $S_Q=\frac{R^2}{B\rho}\frac{\partial B_x}{\partial x}$ is the strength of the skew quadrupole field, $S_m=\frac{R}{B\rho}B_z$ is the strength of solenoid field and $B\rho$ is the particle rigidity.

The collimated by the slit with horizontal size h located at the tangent distance L_1 , the SR beam is formed by an orbit element ΔS , which is determined in the first approximation by means of the relation:

$$\Delta S=\frac{2h}{L_1}\rho, \quad (10)$$

To obtain the distribution of photons passing through the slit, one must transport the distribution (6) from S_0 to the collimator plane and integrate it over ($-h$, h). As the slit boundary has zero thickness, its image on the screen according to (10) is formed by photons emitted from a point of orbit.
The slit boundaries (x=0) extract the subsystem of canonical variables (x',y,y') and permit to investigate the coupling in this phase space. The presence in the distribution (6) of the elements, defining the dependence x'(y,y'), leads to the sloping of the vertical boundary of the slit, as is seen in Fig.2.



a) $x'\neq S(y,y')$ b) $x'=S(y,y')$

Fig. 2

The slope angle ϕ of the slit boundary image with respect to the orbit plane is determined by means of the following relation:

$$tg2\phi = \frac{L_2 \sigma^{\kappa}_{x'y'} + L_2^2 \sigma^{\kappa}_{x'y'}}{L_2^2 \sigma^{\kappa}_{x'x'} - \sigma^{\kappa}_{yy} - 2L_2 \sigma^{\kappa}_{yy'} - L_2^2 \sigma^{\kappa}_{y'y'}}, \quad (11)$$

where the index κ has the meaning of intersection of the distribution (6) with the slit boundary image inversely transported to the phase space at the point S_0.

The described method of continuous measurement of the coupling coefficient Q was experimentally investigated on the SR beam of Yerevan Physics Institute Accelerator. According to (8) and (11), the angle ϕ is a function of Q and $\Delta\nu$. As the values of ν_x and ν_y for this accelerator are nearly equal, so in the present work ϕ has been investigated depending on the values of $\Delta\nu = \nu_x - \nu_y$. The shifts of ν_x and ν_y were obtained by means of quadrupole lenses arranged on the ring for the extraction of the electron beam. The measurement data on the boundary image slope with respect to the vertical axis $\varphi = \frac{\pi}{2} - \phi$ in dependence of $\Delta\nu$ and the corresponding current in quadrupole lenses are given in Fig.3.

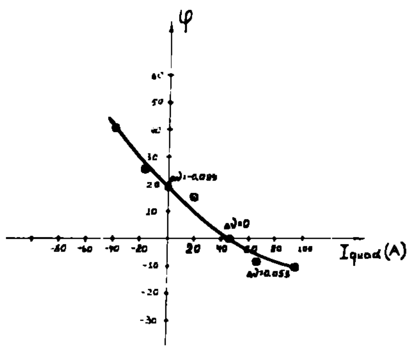
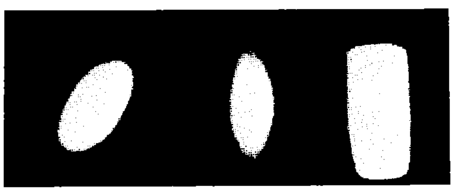


Fig. 3

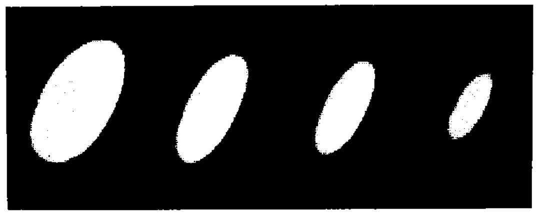
One can see from the diagram that the slope angle φ is zero when the horizontal and vertical betatron numbers ν_x and ν_y are equal. The transformation of slit image, connected with the change of the value of $\Delta\nu$, is shown in Fig.4.



$\Delta\nu = -0.099$ $\Delta\nu = 0$ $\Delta\nu = 0.053$

FIG. 4

The images of the collimated beam for different values of the slit size h are shown in Fig.5.



$h = 1mm$ $0.5mm$ $0.25mm$ $0.125mm$

FIG. 5

The angle ϕ is seen from the figures to be independent of the slit width. This can be considered as an experimental proof of the fact that the slope angle is not an integral characteristic of radiation and in the first approximation it is defined only by the distribution of electrons in the given azimuthal point.

Acknowledgements

The authors are thankful to Doctors D.L.Eghikyan, V.Ts.Nikogosyan and R.A.Mikaelyan for their help during the measurements.

References

1. R.Coisson, "Observing high-energy proton beams by synchrotron radiation in non-uniform magnetic fields", Proc. VIIth All-Union Conf. on Charged Particle Accelerators, Dubna, 14-16 Oct.,1980, Vol.2, Dubna, 1981.
2. M.Sands, "The Physics of Electron Storage Rings", SLAC Report, SLAC-121(1970)
3. A.W.Chao and M.J.Lee, Journal of Applied Physics, Vol.47, No.10, Oct.1976.
4. A.W.Chao and M.J.Lee, "Vertical beam size due to orbit and alignment errors", SLAC-PUB-1915, PEP-244, March 1977 (A).
5. G.Guignard, "The General Theory of All Sum and Difference Resonances in a Three Dimensional Magnetic Field in a Synchrotron", CERN-ISR-MA/75-23 (1975).