STUDY OF CPT VIOLATION IN B SYSTEMS

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Abstract

The thesis deals with one particular type of physics beyond the Standard Model (SM), *viz.* possible violation of the discrete symmetry CPT. Recently the issue of CPT violation (CPTV) has received a lot of attention due to the growing phenomenological importance of CPT violating scenarios in neutrino physics and in cosmology. It is also necessary to find some observables that will clearly discriminate CPT violating signals from CPT conserving ones. The combined discrete symmetry CPT, taken in any order, is an exact symmetry of any axiomatic quantum field theory (QFT), and local field theories must violate Lorentz symmetry in order to be CPT violating.

Right now, there is no signature of CPT violation, or for that matter any type of new physics, in the width difference of $B_d - \overline{B}_d$ and decay channels of B_d. The width difference for the B_d system, $\Delta\Gamma_d$, is too small yet to be measured experimentally, and the bound is compatible with the Standard Model (SM). On the other hand, it is expected that the width difference $\Delta\Gamma_s$ would be significant for the B_s system, but at the same time we know that the theoretical uncertainties are quite significant. If there is some new physics (NP) that does not contribute to the absorptive part of the $B_s - \overline{B_s}$ box, the width difference can only go down, while there are models where this conclusion may not be true.

We first formulate the way to incorporate CPT violation in neutral $B_q - \overline{B_q}$ $(q = d, s)$ mixing, and show its possible ramifications in various observables. In particular, we pay special attention to the anomalous like-sign dimuon asymmetry recently observed in Tevatron. Next, we discuss how to incorporate CPTV in the B decay amplitudes. As an illustrative example, we consider the decay $B_s \to D_s^{\pm} K^{\mp}$. We also show how CPTV in mixing can be disentangled from CPT conserving new physics, and propose a few observables. This proposal is under consideration by the LHCb Collaboration.

Acknowledgement

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Finally, I would like to thank my family for always remaining by my side. One of them left us forever recently. She remains in our thoughts.

Sunando Kumar Patra

List of Publications

List of Publications in refereed journals included in this thesis

1. Probing CPT Violation in B Systems
   Anirban Kundu, Soumitra Nandi and Sunando Kumar Patra
   Physical Review **D81**, *076010 (2010).*

2. Reconciling Anomalous Measurements in $B_s - \bar{B}_s$ Mixing: The Role of CPT Conserving and CPT Violating New Physics
   Amol Dighe, Diptimoy Ghosh, Anirban Kundu and Sunando Kumar Patra
   Physical Review **D84**, *056008 (2011).*

3. $B_s \to D_s K$ as a Probe of CPT Violation
   Anirban Kundu, Soumitra Nandi, Sunando Kumar Patra, and Amarjit Soni
   Physical Review **D87**, *016005 (2013).*

4. CPT violation and triple-product correlations in B decays
   Sunando Kumar Patra and Anirban Kundu
   Physical Review **D87**, *116005 (2013).*

Contents

Chapter 1

CP Violation in Standard Model and B Physics

"A slight asymmetry inspired instant fondness."

- Nicholson Baker

Weak interaction is known to violate parity P and the charge conjugation symmetry C maximally, because of its Lorentz structure: the $V - A$ current changes to $V + A$. In 1964, Cronin *et al.* found that even the combined symmetry CP is violated [2], albeit by a very small amount, in the kaon system; the mass eigenstates of neutral kaons are not the CP eigenstates. A possible mechanism was suggested by by Kobayashi and Maskawa [3] in 1972, who extended the quark mixing mechanism of Cabibbo to three generations. Over the last decade, the Cabibbo-Kobayashi-Maskawa (CKM) paradigm for CP violation has been vindicated in several experiments, notably the B-factories. At the same time, the amount of CP violation needed to explain the baryon asymmetry of the universe is about 9 orders of magnitude larger than that provided by the CKM mechanism, so there must be some new sources of CP violation.

The third discrete symmetry that we will talk about is the time reversal, T, which flips the temporal coordinates. It has been shown by Lüders, Pauli, and Bell that the combined symmetry CPT taken in any order, must be an exact symmetry of any local axiomatic quantum field theory (QFT) [4]. This is known as the CPT theorem and is one of the cornerstones of QFT. By the CPT theorem, we can say that the weak interaction also violates T, and T violating quantities like the electric dipole moment of electron or neutron is also a signature of CP

violation.

1.1 Discrete symmetries: C, P and T

Symmetries have always played a crucial role in physics. The connection between continuous symmetries and conserved quantities has been formulated through Noether's theorems. Apparently complicated and chaotic atomic and nuclear spectra could be understood through an analysis of underlying symmetry groups, even approximate ones. Symmetries and symmetry breaking in local gauge theories hold an essential role in constructing fully relativistic quantum field theories that are both non-trivial and renormalizable[5].

Similarly, discrete symmetries, transformations relating to which cannot be viewed as the continuous change of a variable, have also formed an important part in our understanding of the physical world - as in crystallography and chemistry. They appear as permutation symmetries in quantum theory through Bose-Einstein and Fermi-Dirac statistics. Our main focus is on three of the discrete symmetries that are of general and fundamental relevance for physics:

- parity P
- charge conjugation C
- time reversal T
- the combined transformation of CP;
- the combined transformation of CPT.

We have learnt that nature is largely, but not completely, invariant under the first three transformations. Once it was realized that these were not only violated, but violated maximally - it was noted with considerable relief that CP was apparently still conserved. It had been suggested T was microscopically invariant following from Mach's Principle and as CPT holds naturally for all local axiomatic quantum field theories, it by default meant CP conservation.

1.1.1 Expressions for C, P and T

Let us consider the Dirac equation of a particle with charge e in an electro-magnetic field,

$$\left(i\gamma^\mu \frac{\partial}{\partial x^\mu} - \gamma^\mu e A_\mu - m \right) \psi(\vec{x}, t) = 0 \ ,$$
$$\Rightarrow \left(\gamma^0 \left[i\frac{\partial}{\partial t} - e\phi(\vec{x}, t) \right] - \gamma^i \left[i\frac{\partial}{\partial x_i} - eA_i(\vec{x}, t) \right] - m \right) \psi(\vec{x}, t) = 0 \ , \qquad (1.1)$$

where $\psi(\vec{x}, t)$ is a four component spinor.

After parity transformation and some remodeling to keep the form of the equation intact, eq.(1.1) becomes:

$$\left(\gamma^0 \left[i\frac{\partial}{\partial t} - e\phi(-\vec{x}, t) \right] - \gamma^i \left[i\frac{\partial}{\partial x_i} - eA_i(-\vec{x}, t) \right] - m \right) \gamma^0 \psi(-\vec{x}, t) = 0 \ . \qquad (1.2)$$

So, we can conclude that,

$$\boxed{\psi(\vec{x}, t) \xrightarrow{P} \psi_P(\vec{x}, t) = \gamma^0 \psi(-\vec{x}, t) = P\psi(\vec{x}, t)} \qquad (1.3)$$

Of course also $e^{i\phi}\gamma^0\psi(-\vec{x}, t)$, with ϕ an arbitrary real phase, would provide a valid solution.

Following the same procedure and changing $e \to -e$, utilizing the arbitrariness of the multiplicative phase and recognizing the fact that $\psi_C(\vec{x}, t)$ must satisfy

$$\left(\gamma^0 \left[i\frac{\partial}{\partial t} + e\phi(\vec{x}, t) \right] - \gamma^i \left[i\frac{\partial}{\partial x_i} + eA_i(\vec{x}, t) \right] - m \right) \psi_C(\vec{x}, t) = 0 \ , \qquad (1.4)$$

we find

$$\boxed{\psi(\vec{x}, t) \xrightarrow{C} \psi_C(\vec{x}, t) = i\gamma^2 \psi^*(\vec{x}, t) = i\gamma^2\gamma^0 \overline{\psi}^T(\vec{x}, t) = C\overline{\psi}^T(\vec{x}, t)} \ . \qquad (1.5)$$

For time reversal, again using the arbitrary phase to give the factor i we get,

$$\boxed{\psi(\vec{x}, t) \xrightarrow{T} \psi_T(\vec{x}, t) = i\gamma^1\gamma^3 \psi^*(\vec{x}, -t) = T\psi^*(\vec{x}, -t)} \ . \qquad (1.6)$$

Field		P	C
Scalar Field	$\phi(\vec{x},\ t)$	$\phi(-\vec{x},\ t)$	$\phi^\dagger(\vec{x},\ t)$
Dirac Spinor	$\psi(\vec{x},\ t)$	$\gamma^0\psi(\vec{x},\ t)$	$i\gamma^2\gamma^0\bar{\psi}^T(\vec{x},\ t)$
	$\bar{\psi}(\vec{x},\ t)$	$\bar{\psi}(-\vec{x},\ t)\gamma^0$	$-\psi^T(\vec{x},\ t)C^{-1}$
Axial Vector Field	$A_\mu(\vec{x},\ t)$	$-A_\mu(-\vec{x},\ t)$	$A_\mu^\dagger(\vec{x},\ t)$

Table 1.1: C and P transforms of fields. $\mu = 0, 1, 2, 3,\ A^k = -A_k$ and $A^0 = A_0$.

	Bilinear	P	C	T	CP	CPT
scalar	$\bar{\psi}_1\psi_2$	$\bar{\psi}_1\psi_2$	$\bar{\psi}_2\psi_1$	$\bar{\psi}_1\psi_2$	$\bar{\psi}_2\psi_1$	$\bar{\psi}_2\psi_1$
pseudo scalar	$\bar{\psi}_1\gamma_5\psi_2$	$-\bar{\psi}_1\gamma_5\psi_2$	$\bar{\psi}_2\gamma_5\psi_1$	$-\bar{\psi}_1\gamma_5\psi_2$	$-\bar{\psi}_2\gamma_5\psi_1$	$\bar{\psi}_2\gamma_5\psi_1$
vector	$\bar{\psi}_1\gamma_\mu\psi_2$	$\bar{\psi}_1\gamma^\mu\psi_2$	$-\bar{\psi}_2\gamma_\mu\psi_1$	$\bar{\psi}_1\gamma^\mu\psi_2$	$-\bar{\psi}_2\gamma^\mu\psi_1$	$-\bar{\psi}_2\gamma_\mu\psi_1$
axial vector	$\bar{\psi}_1\gamma_\mu\gamma_5\psi_2$	$-\bar{\psi}_1\gamma^\mu\gamma_5\psi_2$	$\bar{\psi}_2\gamma_\mu\gamma_5\psi_1$	$\bar{\psi}_1\gamma^\mu\gamma_5\psi_2$	$-\bar{\psi}_2\gamma^\mu\gamma_5\psi_1$	$-\bar{\psi}_2\gamma_\mu\gamma_5\psi_1$
tensor	$\bar{\psi}_1\sigma_{\mu\nu}\psi_2$	$\bar{\psi}_1\sigma^{\mu\nu}\psi_2$	$-\bar{\psi}_2\sigma_{\mu\nu}\psi_1$	$-\bar{\psi}_1\sigma^{\mu\nu}\psi_2$	$-\bar{\psi}_2\sigma^{\mu\nu}\psi_1$	$\bar{\psi}_2\sigma_{\mu\nu}\psi_1$

Table 1.2: C and P transforms of bilinears.

For the combined operations we have:

$$CP\psi(\vec{x},t) = ie^{i\phi}\gamma^2\gamma^0\psi^*(-\vec{x},t) \tag{1.7}$$

$$CPT\psi(\vec{x},t) = e^{i\phi}\gamma^5\psi(-\vec{x},-t) \tag{1.8}$$

where $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$.

For completeness the transformation properties of different wave-functions and bi-linear forms are listed in tables (1.1) and (1.2).

1.1.2 CP violation and the SM lagrangian

Yukawa couplings and the origin of quark mixing

The full Standard Model Lagrangian consists of three parts:

$$\mathcal{L}_{SM} = \mathcal{L}_{kinetic} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}.$$

After spontaneous symmetry breaking,

$$\phi(x) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \xrightarrow{\text{sym. breaking}} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix},$$

the following mass terms for the fermion fields arise:

$$
\begin{aligned}
-\mathcal{L}^{quarks}_{Yukawa} &= Y^d_{ij}\overline{Q^I_{Li}}\phi d^I_{Rj} + Y^u_{ij}\overline{Q^I_{Li}}\tilde{\phi} u^I_{Rj} + h.c. \\
&= Y^d_{ij}\overline{d^I_{Li}}\frac{v}{\sqrt{2}}d^I_{Rj} + Y^u_{ij}\overline{u^I_{Li}}\frac{v}{\sqrt{2}}u^I_{Rj} + h.c. + \text{fermion to Higgs interaction terms} \\
&= M^d_{ij}\overline{d^I_{Li}}d^I_{Rj} + M^u_{ij}\overline{u^I_{Li}}u^I_{Rj} + h.c. + \text{interaction terms}
\end{aligned}
$$

To obtain proper mass terms, the matrices M^d and M^u should be diagonalized. We do this with unitary matrices V_d as follows:

$$
\begin{aligned}
M^d_{diag} &= V^d_L M^d V^{d\dagger}_R \\
M^u_{diag} &= V^u_L M^d V^{u\dagger}_R \; .
\end{aligned}
$$

Using the requirement that the matrices V are unitary ($V^{d\dagger}_L V^d_L = \mathbb{1}$), the Lagrangian can now be expressed as follows:

$$
-\mathcal{L}^{quarks}_{Yukawa} = \overline{d_{Li}}\left(M^d_{ij}\right)_{diag} d_{Rj} + \overline{u_{Li}}\left(M^u_{ij}\right)_{diag} u_{Rj} + h.c. + ...
$$

where the matrices V are absorbed in the quark states, resulting in the following quark mass eigenstates:

$$
\begin{aligned}
d_{Li} &= \left(V^d_L\right)_{ij} d^I_{Lj} \; ; \qquad & d_{Ri} &= \left(V^d_R\right)_{ij} d^I_{Rj} \; ; \\
u_{Li} &= \left(V^u_L\right)_{ij} u^I_{Lj} \; ; \qquad & u_{Ri} &= \left(V^u_R\right)_{ij} u^I_{Rj} \; .
\end{aligned}
$$

If we now express the Lagrangian in terms of the quark mass eigenstates d, u instead of the weak interaction eigenstates d^I , u^I, the price to pay is that the quark mixing between families (i.e. the off-diagonal elements) appears in the charged current interaction:

$$
\begin{aligned}
\mathcal{L}_{kinetic,cc}\left(Q_L\right) &= \frac{g}{\sqrt{2}}\overline{u^I_\mu}\gamma_\mu W^{-\mu} d^I_\mu + \frac{g}{\sqrt{2}}\overline{d^I_\mu}\gamma_\mu W^{+\mu} u^I_\mu + ... \\
&= \frac{g}{\sqrt{2}}\overline{u_\mu}\left(V^u_L V^{d\dagger}_L\right)_{ij}\gamma_\mu W^{-\mu} d_\mu - \frac{g}{\sqrt{2}}\overline{d_\mu}\left(V^d_L V^{u\dagger}_L\right)_{ij}\gamma_\mu W^{+\mu} u_\mu + ...
\end{aligned}
$$

The unitary 3×3 matrix

$$
V_{CKM} = \left(V^u_L V^{d\dagger}_L\right)_{ij} \tag{1.9}
$$

is the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix [3].

By convention, the interaction eigenstates and the mass eigenstates are chosen to be equal for the up-type quarks, whereas the down-type quarks are chosen to be rotated, going from the interaction basis to the mass basis:

$$\begin{aligned} u_i^I &= u_j \\ d_i^I &= V_{CKM} d_j \end{aligned}$$

or explicitly:

$$\begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{td} \end{pmatrix} \cdot \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{1.10}$$

CP violation in SM

If we examine the Yukawa part of the Lagrangian:

$$\begin{aligned} -\mathcal{L}_{Yukawa} &= Y_{ij}\overline{\psi_{Li}}\phi\psi_{Rj} + h.c. \\ &= Y_{ij}\overline{\psi_{Li}}\phi\psi_{Rj} + Y_{ij}^*\overline{\psi_{Rj}}\phi^\dagger\psi_{Li} \ . \end{aligned}$$

As the CP operation transforms the spinor fields as follows:

$$CP\left(\overline{\psi_{Li}}\phi\psi_{Rj}\right) = \overline{\psi_{Rj}}\phi^\dagger\psi_{Li} \ ,$$

$\mathcal{L}_{Yukawa}$ remains unchanged under the CP operation if $Y_{ij} = Y_{ij}^*$.

Similarly, if we look at the charged current coupling in the basis of quark mass eigenstates and the CP-transformed expression, then we can conclude that the Lagrangian is unchanged if $V_{ij} = V_{ij}^*$. Thus, we can introduce CP violation in Standard Model by making the CKM matrix complex.

1.2 The Cabibbo-Kobayashi-Maskawa matrix and CP violation

The CKM-mechanism is the origin of CP violation, and earned Kobayashi and Maskawa the Nobel price in 2008, *for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature.*

A general $n \times n$ complex matrix has n^2 complex elements, and thus $2n^2$ real parameters. Unitarity $\left(V^{\dagger}V = \mathbb{1}\right)$ implies n^2 constraints - *a)* n unitary conditions (unity of the diagonal elements); *b)* $n^2 - n$ orthogonality relations (vanishing off-diagonal elements). The phases of the quarks can be rotated freely: $u_{Li} \rightarrow e^{i\phi_i^u} u_{Li}$ and $d_{Lj} \rightarrow e^{i\phi_i^d} d_{Lj}$. Since the overall phase is irrelevant, $2n-1$ relative quark phases can be removed. Summarizing, the CKM-matrix describing the flavor couplings of n generations of up and down type quarks has $2n^2 - n^2 - (2n-1) = (n-1)^2$ free parameters. Subsequently, we can divide these free parameters into Euler angles and phases: *a)* A general $n \times n$ orthogonal matrix can be constructed from $\frac{1}{2}n(n-1)$ angles describing the rotations among the n dimensions. *b)* The remaining free parameters are the phases: $(n-1)^2 - \frac{1}{2}n(n-1) = \frac{1}{2}(n-1)(n-2)$. For the Standard Model with three generations we find three Euler angles and one complex phase.

Before the third family was known, Cabibbo suggested in 1963 the mixing between d and s quarks, by introducing the Cabibbo mixing angle Θ_C. This is the only free parameter for a 2×2 unitary matrix, and the mixing matrix is a pure real matrix. To allow for CP violation the mixing matrix has to contain complex elements, satisfying $V_{ij} \neq V_{ij}^*$. This requires at least three families. After the discovery of CP violation in Kaon systems by Cronin *et al.*in 1964, Kobayashi and Maskawa suggested in 1973 the possibility that the existence of a third family could explain the CP violation within the Standard Model. The 4th quark, the charm quark was only discovered a year later, in 1974, in the form of the J/ψ resonance. The bottom and the top quark were discovered in 1977 and 1994 respectively.

1.2.1 Unitarity Triangle(s)

The unitarity condition for the CKM-matrix, $V^{\dagger}V = VV^{\dagger} = \mathbb{1}$ leads to the following unitary relations:

$$V_{pd}V_{pd}^* + V_{ps}V_{ps}^* + V_{pb}V_{pb}^* = 1 \quad \text{for} \quad p = u,\ c,\ \text{or}\ t\ . \tag{1.11}$$

These relations express the so-called *weak universality*, because it shows that the squared sum of the coupling strengths of u to d, s and b is equal to the overall charged coupling of c(and t). In addition, we see that this sum adds up to 1, meaning that there is no probability remaining to couple to a 4th down-type quark. Obviously, this relation deserves continuous experimental scrutiny.

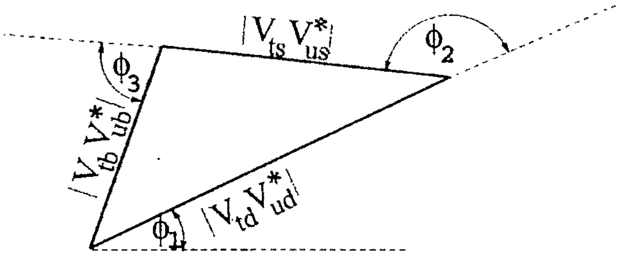

Figure 1.1: One of the six unitarity triangles. $V_{td}V^*_{ud} = |V_{td}V^*_{ud}|\, e^{i\phi_1}$, $V_{ts}V^*_{us} = |V_{ts}V^*_{us}|\, e^{i\phi_2}$ and $V_{tb}V^*_{ub} = |V_{tb}V^*_{ub}|\, e^{i\phi_3}$

The remaining relations:

$$\begin{aligned} V_{ud}V^*_{cd} + V_{us}V^*_{cs} + V_{ub}V^*_{cb} &= 0 \\ V_{ud}V^*_{td} + V_{us}V^*_{ts} + V_{ub}V^*_{tb} &= 0 \\ V_{td}V^*_{cd} + V_{ts}V^*_{cs} + V_{tb}V^*_{cb} &= 0 \end{aligned} \tag{1.12}$$

and their three complex conjugate versions are known as the orthogonality conditions. An additional three interesting equations arise from the unitarity relation $V^\dagger V = \mathbb{1}$, along with their complex conjugate versions:

$$\begin{aligned} V_{ud}V^*_{us} + V_{cd}V^*_{cs} + V_{td}V^*_{ts} &= 0 \\ V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} &= 0 \\ V_{us}V^*_{ub} + V_{cs}V^*_{cb} + V_{ts}V^*_{tb} &= 0\ . \end{aligned} \tag{1.13}$$

Equations (1.12 - 1.13) give relations in which the complex phase is present. As these are sums of three complex numbers that must yield zero they can be viewed as a triangle in the complex plane, see for example Fig.(1.1).

1.2.2 Accurate and approximate parametrizations of CKM matrix

In the literature there are many different parameterizations of the CKM matrix. A convenient representation uses the Euler angles θ_{ij} with i, j denoting the generation labels. With the notation $c_{ij} = cos\theta_{ij}$ and $s_{ij} = sin\theta_{ij}$ the following parametrization was introduced by Chau and Keung [7], and has been adopted by the Particle Data Group [9]:

$$V_{CKM} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$
$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \quad (1.14)$$

The phase δ is necessary for CP violation and can be made to appear in many elements, and is chosen here to appear in the matrix describing the relation between the 1st and 3rd family. c_{ij} and s_{ij} can all be chosen to be positive and δ may vary in the range $0 \leq \delta \leq 2\pi$. However, the measurements of CP violation in K decays force δ to be in the range $0 < \delta < \pi$ [8].

1.3 Neutral meson oscillations

The phenomenon of neutral meson oscillations is important for various reasons. Firstly, in many measurements of CKM-parameters, the oscillations play a crucial role in providing a second transition amplitude from the initial state to a given final state. This second amplitude is needed to determine the relative phase difference between two amplitudes. Secondly, the observation of two K0 particles with largely varying lifetimes and the resulting discovery of CP violation is of historical importance and is described in terms of a superposition of $|Ki\rangle$-states and its quantum mechanical evolution. The formalism described in this section is valid for all weakly decaying neutral mesons: K^0, D^0, B_d and B_s - although the difference in mass (and thus available phase space for the final state) and coupling strength (CKM-elements) results in dramatically different phenomenology.

1.3.1 The mass and decay matrix

The states $|P^0\rangle$ and $|\bar{P}^0\rangle$ are eigenstates of the strong and electromagnetic interactions with common mass m0 and opposite flavor content. We consider an arbitrary superposition of the P^0 and $\bar{P}^0$ states, which has time-dependent coefficients $a(t)$ and $b(t)$ respectively:

$$\psi(t) = a(t)|P^0\rangle + b(t)|\bar{P}^0\rangle$$

We can write $\psi(t)$ in the subspace of P^0 and $\bar{P}^0$ as follows

$$\psi(t) = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

The effective Hamiltonian that governs the time evolution is a sum of the strong, electromagnetic and weak Hamiltonians.

$$H = H_{st} + H_{em} + H_{wk}$$

The wavefunction ψ must then obey

$$i\frac{\partial\psi}{\partial t} = H\psi$$

The Hamiltonian can then, in the (P^0, $\bar{P}^0$) basis, be written as 2×2 complex matrix:

$$H = M - \frac{i}{2}\Gamma$$

where both M and Γ are Hermitian matrices. M will provide a mass term and due to the $-i$, Γ will provide the exponential decay. H is not hermitian reflected in the property that the probability to observe either P^0 or $\bar{P}^0$ is not conserved, but goes down with time. If the weak part of the Hamiltonian did not exist the P system would be stable and so H would reduce to a diagonal matrix with mass terms for P^0 and $\bar{P}^0$. With the weak interaction responsible for the decay we get additional Γ terms in H. If we now allow for the transitions $P^0 \to \bar{P}^0$, the off-diagonal elements are introduced:

$$i\frac{\partial}{\partial t}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \left[\begin{pmatrix} M_0 & M_{12} \\ M_{12}^* & M_0 \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_0 & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_0 \end{pmatrix}\right] \cdot \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \quad (1.15)$$

The off-diagonal elements consist of two parts, M_{12} and $\frac{1}{2}\Gamma_{12}$, which describe different ways of the $P^0 \to \bar{P}^0$ transition. M_{12} quantifies the short-distance contribution from the (calculable) box diagram and Γ_{12} is a measure of the contribution from the virtual, intermediate, decays to a state f, see 1.2.

If we now assume that CPT is valid then it follows that $M_{11} = M_{22}$, $M_{21} = M_{12}^*$ and $\Gamma_{11} = \Gamma_{22}$, $\Gamma_{21} = \Gamma_{12}^*$ meaning that mass and total decay width of particle and antiparticle are identical. In general there can be a relative phase between the on-shell (or dispersive) and off-shell (or absorptive) transition, i.e. between

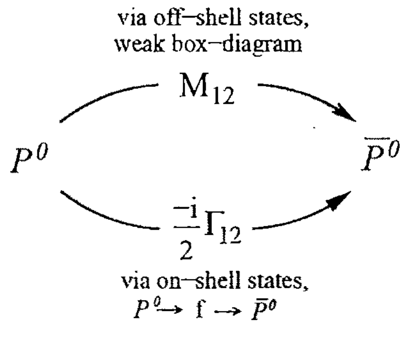

Figure 1.2: The neutral meson oscillation consists of two contributions, namely through off-shell states and on-shell states.

Γ_{12} and M_{12} [17]:

$$\begin{aligned} \phi &= \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right) \\ \Rightarrow\ \Delta m &= 2\left|M_{12}\right| \\ \text{and } \Delta\Gamma &= 2\left|\Gamma_{12}\right|\cos\phi \end{aligned} \tag{1.16}$$

If T is conserved then it follows that $\Gamma^*_{12}/\Gamma_{12} = M^*_{12}/M_{12}$ so that by introducing a free phase we can make Γ_{12} and M_{12} real.

Eigenvalues(-vectors) of mass-decay Matrix

Given the Schrödinger equation (1.15) we find the eigenvalues of the mass-decay matrix, by solving the determinant equation:

$$\begin{vmatrix} M - \frac{\Gamma}{2} - \lambda & M_{12} - \frac{i\Gamma}{2} \\ M^*_{12} - \frac{i\Gamma^*_{12}}{2} & M - \frac{\Gamma}{2} - \lambda \end{vmatrix} = 0$$

If we express the eigenstates P_1 and P_2 as:

$$\begin{aligned} |P_1\rangle &= p|P^0\rangle - q|\bar{P}^0\rangle \\ |P_2\rangle &= p|P^0\rangle + q|\bar{P}^0\rangle \end{aligned}$$

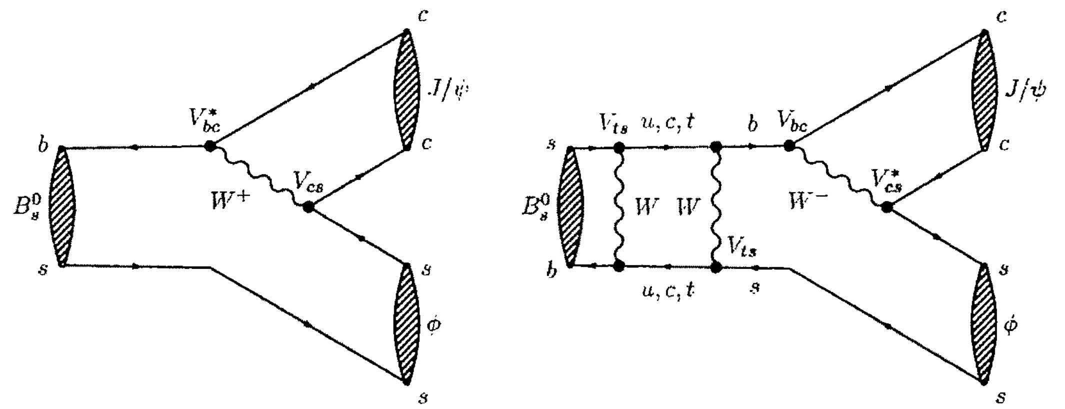

Figure 1.3: The two interfering diagrams of the decay $B_s \to J/\psi\ \phi$, with phase difference $2\beta_s$. The second diagram involves the mixing box.

yielding:

$$\frac{q}{p} = \pm\sqrt{\frac{M^*_{12} - \frac{i}{2}\Gamma^*12}{M_{12} - \frac{i}{2}\Gamma 12}}$$

We can also relate q/p to the mixing phase as introduced in eq.(1.16) [17]:

$$\frac{|\Gamma_{12}|}{|M_{12}|} = \frac{\Delta\Gamma}{\Delta m}\tan\phi = 2\left(1 - \frac{q}{p}\right) . \tag{1.17}$$

Time evolution

We define the two mass eigenstates of the neutral mesons as[1]:

$$\begin{aligned} |P_H\rangle &= p\,|P^0\rangle + q\,|\bar{P}^0\rangle \\ |P_L\rangle &= p\,|P^0\rangle - q\,|\bar{P}^0\rangle \end{aligned} \tag{1.18}$$

The states $|P_H\rangle$ and $|P_L\rangle$ are mass eigenstates and from the Schrödinger equation (with diagonal Hamiltonian) the usual time dependent wave functions are

[1]There are some subtleties concerning the sign (or phase) convention. Let us assume CP symmetry, $|q/p| = 1$. We can *choose* $q/p = \pm 1$ and $CP|P^0\rangle = \pm|\bar{P}^0\rangle$. Once the sign of q/p is fixed, experiment decides if P_H is the state that is (more) even or odd, which fixes $CP|P^0\rangle = \pm|\bar{P}^0\rangle$. In principle this can be different for K^0, B_d and B_s . We choose the sign convention $\Delta m_K > 0$ and $CP|K^0\rangle = -|\bar{K}^0\rangle$ such that $CP|K_L\rangle = \pm|K_L\rangle$ (or $\Delta\Gamma_K = \Gamma_S - \Gamma_L > 0$) according to experiment. This leads to the sign convention in eq.(1.18), and implies $\Delta m_K = m_L - m_S$. Also in the B-system the heavier mass eigenstate B_H is (more) CP odd, and the CP-even state in the B_s-system can decay to the final state $D^+_s\ D^-_s$, and has therefore a slightly shorter lifetime.

obtained:

$$
\begin{aligned}
|P^0(t)\rangle &= g_+(t)\,|P^0\rangle + \left(\frac{q}{p}\right) g_-(t)\,|\bar{P}^0\rangle \\
|\bar{P}^0(t)\rangle &= g_-(t)\left(\frac{p}{q}\right)|P^0\rangle + g_+(t)\,|\bar{P}^0\rangle
\end{aligned} \tag{1.19}
$$

where we define the functions

$$
\begin{aligned}
g_+(t) &= \frac{1}{2}e^{-iMt}\left(e^{-\frac{i}{2}\Delta mt-\frac{1}{2}\Gamma_H t} + e^{\frac{i}{2}\Delta mt-\frac{1}{2}\Gamma_H t}\right) \\
g_-(t) &= \frac{1}{2}e^{-iMt}\left(e^{-\frac{i}{2}\Delta mt-\frac{1}{2}\Gamma_H t} - e^{\frac{i}{2}\Delta mt-\frac{1}{2}\Gamma_H t}\right)
\end{aligned}
$$

where $M = (m_H + m_L)/2$ and $\Delta m = m_H - m_L$.

If we start from a pure sample of $|P^0\rangle$ particles (e.g. produced by the strong interaction) then we can calculate the probability of measuring the state $|\bar{P}0\rangle$ at time t:

$$
|\langle\bar{P}^0|P^0(t)\rangle|^2 = |g_-(t)|^2\left|\frac{p}{q}\right|^2
$$

with

$$
|g_\pm(t)|^2 = \frac{e^{-\Gamma t}}{2}\left(\cosh\frac{1}{2}\Delta\Gamma t \pm \cos\Delta mt\right)
$$

where $\Gamma = (\Gamma_L + \Gamma_H)/2$ and $\Delta\Gamma = \Gamma_H - \Gamma_L$. Here we see that Γ fulfills the natural role of decay constant, $\Gamma = 1/t$, justifying the choice of $\frac{1}{2}$ in the hamiltonian. We could, and in later chapters would, define $\Delta\Gamma = \Gamma_L - \Gamma_H$, which is okay, as though the sign of Δm is by definition positive, but the sign of $\Delta\Gamma$ has to be determined experimentally.

1.3.2 Meson decays

Extending the formalism of neutral meson oscillations, and including the subsequent decay of the meson to a final state f, we consider the following four decay amplitudes:

$$
\begin{aligned}
A(f) &= \langle f|T|P^0\rangle, \quad \bar{A}(f) = \langle f|T|\bar{P}^0\rangle, \\
A(\bar{f}) &= \langle\bar{f}|T|P^0\rangle, \quad \bar{A}(\bar{f}) = \langle\bar{f}|T|\bar{P}^0\rangle
\end{aligned}
$$

and define the complex parameter λ_f (not the Wolfenstein parameter λ):

$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f}, \quad \lambda_{\bar{f}} = \frac{q}{p}\frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}}, \quad \bar{\lambda}_f = \frac{1}{\lambda_f}, \quad \bar{\lambda}_{\bar{f}} = \frac{1}{\lambda_{\bar{f}}} \tag{1.20}$$

The general expression for the time dependent decay rates, $\Gamma_f(t) = |\langle f|T|P0(t)\rangle|^2$, give us the probability that the state P^0 at $t = 0$ decays to the final state f at time t, and can now be constructed as follows, using eq.(1.19):

$$\begin{aligned}
\Gamma_f(t) &= |A_f|^2 \left(|g_+(t)|^2 + |\lambda_f|^2|g_-(t)|^2 + 2\mathrm{Re}\left[\lambda_f g_+^*(t)g_-(t)\right]\right) \\
\Gamma_{\bar{f}}(t) &= |\bar{A}_{\bar{f}}|^2 \left|\frac{q}{p}\right|^2 \left(|g_-(t)|^2 + |\bar{\lambda}_{\bar{f}}|^2|g_+(t)|^2 + 2\mathrm{Re}\left[\bar{\lambda}_{\bar{f}} g_+(t)g_-^*(t)\right]\right) \\
\bar{\Gamma}_f(t) &= |A_f|^2 \left|\frac{p}{q}\right|^2 \left(|g_-(t)|^2 + |\lambda_f|^2|g_+(t)|^2 + 2\mathrm{Re}\left[\lambda_f g_+(t)g_-^*(t)\right]\right) \\
\bar{\Gamma}_{\bar{f}}(t) &= |\bar{A}_{\bar{f}}|^2 \left(|g_+(t)|^2 + |\bar{\lambda}_{\bar{f}}|^2|g_-(t)|^2 + 2\mathrm{Re}\left[\bar{\lambda}_{\bar{f}} g_+^*(t)g_-(t)\right]\right)
\end{aligned} \tag{1.21}$$

with

$$\begin{aligned}
|g_\pm(t)|^2 &= \frac{e^{-\Gamma t}}{2}\left(\cosh\frac{1}{2}\Delta\Gamma t \pm \cos\Delta m t\right) \\
g_+^*(t)g_-(t) &= \frac{e^{-\Gamma t}}{2}\left(\sinh\frac{1}{2}\Delta\Gamma t + i\ \sin\Delta m t\right) \\
g_+(t)g_-^*(t) &= \frac{e^{-\Gamma t}}{2}\left(\sinh\frac{1}{2}\Delta\Gamma t - i\ \sin\Delta m t\right)
\end{aligned} \tag{1.22}$$

The terms proportional $|A|^2$ are associated with decays that occurred without oscillation, whereas the terms proportional to $|A|^2(q/p)^2$ or $|A|^2(p/q)^2$ are associated with decays following a net oscillation. The third terms, proportional to $\mathrm{Re}(g^*g)$, are associated to the interference between the two cases.

Combining eqs.(4.10) and (1.22) results in the following expressions for the decay rates for neutral mesons, also known as the master equations:

$$\begin{aligned}
\Gamma_f(t) &= |A_f|^2 \left(1 + |\lambda_f|^2\right) \frac{e^{-\Gamma t}}{2}\left(\cosh\frac{\Delta\Gamma t}{2} + D_f \sinh\frac{\Delta\Gamma t}{2} + C_f \cos\Delta m t - S_f \sin\Delta m t\right) \\
\bar{\Gamma}_f(t) &= |A_f|^2 \left|\frac{p}{q}\right|^2 \left(1 + |\lambda_f|^2\right) \frac{e^{-\Gamma t}}{2}\left(\cosh\frac{\Delta\Gamma t}{2} + D_f \sinh\frac{\Delta\Gamma t}{2} - C_f \cos\Delta m t + S_f \sin\Delta m t\right)
\end{aligned} \tag{1.23}$$

with

$$D_f = \frac{2\mathrm{Re}(\lambda_f)}{1+|\lambda_f|^2}, \quad C_f = \frac{1+|\lambda_f|^2}{1+|\lambda_f|^2}, \quad S_f = \frac{2\mathrm{Im}(\lambda_f)}{1+|\lambda_f|^2} \tag{1.24}$$

For a given final state f we therefore only have to find the expression for λ_f to fully describe the decay of the (oscillating) mesons.

In the next chapter we will concentrate on specifically the $B_s - \overline{B_s}$ mixing. An effort to reconcile some anomalous results from different experiments would eventually give rise to a quantitative model independent way to determine the type of New Physics (NP) which can be held responsible for those anomalies.

Chapter 2

CPT-conserving NP and anomaly in $B_S - \overline{B_S}$ mixing:

"Mixing one's wines may be a mistake,
but old and new wisdom mix admirably."
- Bertolt Brecht,
The Caucasian Chalk Circle (1944)

The Cabibbo-Kobayashi-Maskawa (CKM) paradigm of quark mixing in the standard model (SM) has been verified in the $B_d - \overline{B}_d$ system to such an extent that we know that the NP effects therein are subdominant at most. However, that is not yet the case with the $B_s - \overline{B_s}$ mixing. It is quite possible that the NP can affect the $B_s - \overline{B_s}$ system while keeping the $B_d - \overline{B}_d$ system untouched. Indeed, for most of the flavor-dependent NP models, the couplings relevant for the second and third generations of SM fermions are much less constrained than those for the first generation fermions, allowing the NP to play a significant role in the $B_s - \overline{B_s}$ mixing, in principle.

Over the last few years before the publication of LHC results, the Tevatron experiments CDF and DØ , and to a smaller extent the B factories Belle and BaBar, have provided a lot of data on the B_s meson, most of which are consistent with the SM. There are some measurements, though, which show a significant deviation from the SM expectations, and hence point towards new physics (NP). The major ones among these are the following. (i) Measurements in the decay mode $B_s \to J/\psi\phi$ yield a large CP-violating phase $\beta_s^{J/\psi\phi}$ [18]. In addition, though the difference $\Delta\Gamma_s$ between the decay widths of the mass eigenstates measured in this decay is consistent with the SM, it allows $\Delta\Gamma_s$ values that are almost twice the SM prediction [19]. (ii) The like-sign dimuon asymmetry A^b_{sl} in the combined B data at DØ [20] is more than 3σ away from the SM expectation.

The resolutions of the above anomalies, separately or simultaneously, have been discussed in the context of specific NP models: a scalar leptoquark model [21, 22], models with an extra flavor-changing neutral gauge boson Z' or R-parity violating supersymmetry [23], two-Higgs doublet model [24], models with a fourth generation of fermions [25], supersymmetric grand unified models [26], supersymmetric models with split sfermion generations [27] or models with a very light spin-1 particle [28]. Possible four-fermion effective interactions that are consistent with the data have been analyzed by [29] and the results are consistent with [22]. A similar study, based on the minimal flavor violating (MFV) models, was carried out in [30]. A solution for the like-sign dimuon asymmetry using a type-III two-Higgs doublet model has been proposed [31], which could also explain the W plus 2 jets excess near the di-jet invariant mass of 140 GeV, as observed by CDF [32].

In this chapter, we try to determine, in a model-independent way, which kind of NP would be able to account for both the above anomalies simultaneously. We take a somewhat different approach than the references cited above. Rather than confining ourselves to specific models, we assume that the NP responsible for the anomalies contributes entirely through the $B_s - \overline{B_s}$ mixing, and parameterize it in a model independent manner through the effective Hamiltonian for the $B_s - \overline{B_s}$ mixing. This effective Hamiltonian $\mathcal{H}$ is a 2×2 matrix in the flavor basis, and the relevant NP contribution appears in its off-diagonal elements. The NP can then be parameterized by using four parameters: the magnitudes and phases of the dispersive part and the absorptive part of the NP contribution to $\mathcal{H}$. We perform a χ^2 fit to the $B_s - \overline{B_s}$ mixing observables and obtain a quantitative measure for which kind of NP is preferred by the data. This would lead us to short-list specific NP models that have the desired properties, which can give testable predictions for other experiments. It is found that the NP needs to contribute to both the dispersive as well as absorptive part of the Hamiltonian in order to avoid any tension with the data[48].

2.1 The effective Hamiltonian

The evolution of a $B_s - \overline{B_s}$ state can be described by the effective Hamiltonian

$$\mathcal{H} = \begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{22} \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{22} \end{pmatrix} \tag{2.1}$$

in the flavor basis, where M_{ij} and Γ_{ij} are its dispersive and absorptive parts, respectively. When CPT is conserved, $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$. The eigenstates of this Hamiltonian are B_{sH} and B_{sL}, with masses M_{sH} and M_{sL} respectively, and decay widths Γ_{sH} and Γ_{sL} respectively. The difference in the masses and decay widths can be written in terms of the elements of the Hamiltonian as

$$\begin{aligned} \Delta M_s &\equiv M_{sH} - M_{sL} \approx 2|M_{12}| \; , \\ \Delta \Gamma_s &\equiv \Gamma_{sL} - \Gamma_{sH} \approx 2|\Gamma_{12}| \cos[\mathrm{Arg}(-M_{12}/\Gamma_{12})] \, . \end{aligned} \tag{2.2}$$

The above expressions are valid as long as $\Delta\Gamma_s \ll M_s$, which is indeed the case here.

Since CPT is conserved, the effect of NP can be felt only through the off-diagonal elements of $\mathcal{H}$. We separate the SM and NP contributions to these terms via

$$\begin{aligned} M_{12} &= M_{12}^{\mathrm{SM}} + M_{12}^{\mathrm{NP}} \; , \\ \Gamma_{12} &= \Gamma_{12}^{\mathrm{SM}} + \Gamma_{12}^{\mathrm{NP}} \; . \end{aligned} \tag{2.3}$$

The NP can then be completely parameterized in terms of four real numbers: $|M_{12}^{\mathrm{NP}}|$, $\mathrm{Arg}(M_{12}^{\mathrm{NP}})$, $|\Gamma_{12}^{\mathrm{NP}}|$ and $\mathrm{Arg}(\Gamma_{12}^{\mathrm{NP}})$. We take the phases $\mathrm{Arg}(M_{12}^{\mathrm{NP}})$ and $\mathrm{Arg}(\Gamma_{12}^{\mathrm{NP}})$ to lie in the range 0-2π.

In a large class of models, including the Minimal Flavor Violation (MFV) models, the NP contribution has no absorptive part, i.e. $\Gamma_{12} = \Gamma_{12}^{\mathrm{SM}}$. This is the case when NP does not give rise to any new intermediate light states to which B_s or $\overline{B}_s$ can decay. For such models, eq. (2.2) implies that $\Delta\Gamma_s \lesssim \Delta\Gamma_s(\mathrm{SM}) \approx 2|\Gamma_{12}^{\mathrm{SM}}|$, i.e. the value of $\Delta\Gamma_s$ is always less than its SM prediction [34]. In such models, the NP is parameterized by only two parameters: $|M_{12}^{\mathrm{NP}}|$ and $\mathrm{Arg}(M_{12}^{\mathrm{NP}})$. An analysis restricted to this class of models was performed in [35].

However there exists a complementary class of viable models where the NP contributes to Γ_{12} substantially. These include models with leptoquarks, R-parity violating supersymmetry, a light gauge boson, etc. It has been pointed out in [21] that such a nonzero absorptive part that arises naturally in these class of models can enhance $\Delta\Gamma_s$ significantly above its SM value, contrary to the popular expectations based on [34]. As we shall see later in this paper, such models are favored by the data.

2.2 The measurements

The $B_s - \overline{B_s}$ oscillation and CP violation therein can be quantified by four observables, viz. the mass difference ΔM_s, the decay width difference $\Delta\Gamma_s$, the CP-violating phase $\beta_s^{J/\psi\phi}$, and the semileptonic asymmetry $a_{\rm sl}^s$.

The mass difference is measured to be

$$\Delta M_s = (17.77 \pm 0.10 \pm 0.07)\ \mathrm{ps}^{-1}\ , \tag{2.4}$$

which is consistent with the SM expectation [95]

$$\Delta M_s(\mathrm{SM}) = (17.3 \pm 2.6)\ \mathrm{ps}^{-1}\ . \tag{2.5}$$

However measurements in the $B_s \to J/\psi\phi$ decay mode show a hint of some deviation from the SM. The CP-violating phase $\beta_s^{J/\psi\phi}$ in this decay is

$$\beta_s^{J/\psi\phi} = \frac{1}{2}\mathrm{Arg}\left(-\frac{(V_{cb}V_{cs}^*)^2}{M_{12}}\right)\ , \tag{2.6}$$

whose average value measured at the Tevatron experiments [18] is

$$\beta_s^{J/\psi\phi} = (0.41^{+0.18}_{-0.15}) \cup (1.16^{+0.15}_{-0.18})\ . \tag{2.7}$$

In the SM,

$$\beta_s^{J/\psi\phi}(\mathrm{SM}) = \mathrm{Arg}\left(-\frac{V_{cb}V_{cs}^*}{V_{tb}V_{ts}^*}\right) \approx 0.019 \pm 0.001\ . \tag{2.8}$$

Thus, the measured value of $\beta_s^{J/\psi\phi}$ is more than 2σ away from the SM expectation. On the other hand, the difference in the decay widths of the mass eigenstates B_H and B_L is measured to be [18]

$$\Delta\Gamma_s = \pm(0.154^{+0.054}_{-0.070})\ \mathrm{ps}^{-1}\ , \tag{2.9}$$

while the SM expectation is [95]

$$\Delta\Gamma_s(\mathrm{SM}) = (0.087 \pm 0.021)\ \mathrm{ps}^{-1}\,. \tag{2.10}$$

The measurement is consistent with the SM expectation to $\sim 1\sigma$, however it allows for $\Delta\Gamma_s$ values that are almost twice the SM prediction. Note that the sign of $\Delta\Gamma_s$ is undetermined experimentally and this gives us more room to play with the NP parameters.

Newer results from CDF, based on 5.2 fb^{-1} of data [36]:

$$\begin{aligned} \Delta\Gamma_s &= (0.075 \pm 0.035 \pm 0.010)\ \mathrm{ps}^{-1}\,, \\ \beta_s^{J/\psi\phi} &= (0.02 - 0.52) \cup (1.08 - 1.55) \end{aligned} \tag{2.11}$$

to 68% C.L.. While we note that the results are consistent with the SM, it should be mentioned that instead of the final Tevatron averages, we used the values in eq. (2.9) in our analysis.

The other anomalous measurement is the like-sign dimuon asymmetry. The average of the DØ [20] and CDF [37] measurements gives

$$A^b_{\mathrm{sl}} = -(8.5 \pm 2.8) \times 10^{-3}\,, \tag{2.12}$$

which differs by more than 3σ from its SM prediction

$$A^b_{\mathrm{sl}}(\mathrm{SM}) = (-0.23^{+0.05}_{-0.06}) \times 10^{-3}\,. \tag{2.13}$$

Note that for A^b_{sl}, CDF has a poorer statistics than DØ and therefore the average value is dominated by the DØ data.

Even in the presence of new physics, the SM relationship holds:

$$A^b_{\mathrm{sl}} = (0.506 \pm 0.043)a^d_{\mathrm{sl}} + (0.494 \mp 0.043)a^s_{\mathrm{sl}}\,, \tag{2.14}$$

where a^s_{sl} and a^d_{sl} are the semileptonic asymmetries for the $B_s - \overline{B_s}$ and the $B_d - \overline{B}_d$ systems, respectively. The former is related to the $B_s - \overline{B_s}$ mixing observables through

$$a^s_{\mathrm{sl}} = \frac{\Delta\Gamma_s}{\Delta M_s}\tan\phi_s \tag{2.15}$$

where $\phi_s \equiv \mathrm{Arg}(-M_{12}/\Gamma_{12})$. The latter is defined analogously. The coefficients in eq. (2.14) are experimentally measured, and contain information about $\Delta M_{d(s)}$,

$\Delta\Gamma_{d(s)}$, and production fractions of B_d and B_s mesons. Using $a_{\rm sl}^d = -(4.7 \pm 4.6) \times 10^{-3}$ [19], this leads to

$$a_{\rm sl}^s = -0.012 \pm 0.007 \,, \tag{2.16}$$

which is about 2σ away from the SM prediction

$$a_{\rm sl}^s({\rm SM}) = (2.06 \pm 0.57) \times 10^{-5} \,. \tag{2.17}$$

The value of $a_{\rm sl}^d$ depends on $\Delta M_d, \Delta\Gamma_d$ and ϕ_d, the parameters in the B_d sector analogous to those in eq. (2.15). These parameters depend on the NP in the B_d sector, which is independent of the NP parameters in the B_s sector that we are considering. We therefore do not consider the measured values of $a_{\rm sl}^d$ as a direct constraint, but express it in terms of ΔM_d, $\Delta\Gamma_d$, and ϕ_d, whose experimental values are taken as inputs.

In the SM, we have $\phi_s({\rm SM}) = 0.0041 \pm 0.0007$ [95]. Note that if the dominating contribution to Γ_{12s} were from a pair of intermediate c quarks, $\phi_s({\rm SM})$ would have been equal to $-2\beta_s^{J/\psi\phi}$. Since the intermediate $u-c$ and $u-u$ quark states give comparable contributions to Γ_{12s}, we have $\phi_s({\rm SM}) \neq -2\beta_s^{J/\psi\phi}({\rm SM})$ [38].

2.3 The statistical analysis

We perform a χ^2 fit to the observed quantities $\Delta M_s, \Delta\Gamma_s, \beta_s^{J/\psi\phi}$ and $a_{\rm sl}^s$, using the NP parameters $|M_{12}^{\rm NP}|$, ${\rm Arg}(M_{12}^{\rm NP})$, $|\Gamma_{12}^{\rm NP}|$ and ${\rm Arg}(\Gamma_{12}^{\rm NP})$. We assume all the measurements to be independent for simplicity, though the measurements of $\Delta\Gamma_s$ and $\beta_s^{J/\psi\phi}$ are somewhat correlated. The values of all the observables and their SM values are as given in Sec. 2.2. In order to express them in terms of $M_{12}, M_{12}^{\rm SM}, \Gamma_{12}$ and $\Gamma_{12}^{\rm SM}$, one has to use eq. (2.2) in addition. In order to take into account the errors on the SM parameters, we add the theoretical and experimental errors on our observed quantities in quadrature.

Note that since we have four observable quantities and four parameters, it is not surprising that we obtain the global minimum value of χ^2 as $\chi^2_{min} = 0$ when all the NP parameters are allowed to vary. The questions we address here are (i) what the preferred values of the NP parameters are, and (ii) to what confidence level (C.L.) a given set of NP parameters (or SM, which is a special case of NP with $M_{12}^{\rm NP} = \Gamma_{12}^{\rm NP} = 0$) is allowed. The latter is obtained assuming all errors to be Gaussian. Here we give our results in terms of the goodness-of-fit contours for the joint estimations of two parameters at a time. The $(1\sigma, 2\sigma, 3\sigma, 4\sigma)$ contours, that

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are equivalent to p-values of $(0.3173, 0.0455, 0.0027, 0.0001)$, or confidence levels of $(68.27\%, 95.45\%, 99.73\%, 99.99\%)$, correspond to $\chi^2 = (2.295, 6.18, 11.83, 19.35)$, respectively.

In Fig. 2.1, we show the $1\sigma, 2\sigma, 3\sigma, 4\sigma$ contours in the $|M_{12}| - \mathrm{Arg}(M_{12})$ plane, where the other NP parameters are marginalized over. Clearly, we see a preference towards nonzero $|M_{12}^{NP}|$ as well as nonzero $\mathrm{Arg}(M_{12}^{\mathrm{NP}})$ values. There are two best-fit points with $\chi^2 = 0$, one at $M_{12}^{\mathrm{NP}} \approx 6.5 \exp(2.0\ i)\ \mathrm{ps}^{-1}$ and the other at $M_{12}^{\mathrm{NP}} \approx 16.4 \exp(2.8\ i)\ \mathrm{ps}^{-1}$, shown with crosses in Fig. 2.1. Actually, each of these crosses is a superimposed double, with two values of $\Gamma_{12}^{\mathrm{NP}}$, as shown in Fig. 2.2. The points correspond to the constructive and destructive interference between the SM and NP amplitudes in order to give the measured central values of ΔM_s. The region with $M_{12}^{\mathrm{NP}} = 0$, i.e. the x-axis, is outside the 2σ region, indicating that it will be rather difficult to fit the current data without some NP contribution to the dispersive part of the $B_s - \overline{B_s}$ mixing. The contours also imply that $|M_{12}^{\mathrm{NP}}| \lesssim 21.1$ ps^{-1} to 3σ.

In Fig. 2.2, we show the goodness-of-fit contours in the $|\Gamma_{12}| - \mathrm{Arg}(\Gamma_{12})$ plane, marginalizing over other two NP parameters. As the measurements do not determine the sign of $\Delta\Gamma_s$, for any particular value of $|\Delta\Gamma_s|$, we perform the χ^2 fit for both positive and negative values, and keep the minimum χ^2 of the two. This doubles the number of best-fit solutions, and the two best-fit points of Fig. 2.1 now split into four. For $|M_{12}^{\mathrm{NP}}| = 6.5$, the solutions are $\Gamma_{12}^{\mathrm{NP}} = 0.20 \exp(6.1\ i)$ or $0.18 \exp(5.2\ i)$, and for $|M_{12}^{\mathrm{NP}}| = 16.4$, the corresponding solutions are $\Gamma_{12}^{\mathrm{NP}} = 0.20 \exp(0.3\ i)$ or $0.18 \exp(1.1\ i)$ (both M_{12}^{NP} and $\Gamma_{12}^{\mathrm{NP}}$ are in ps^{-1}, here, and also later where not mentioned explicitly). Note that there is a reflection symmetry about $\mathrm{Arg}(\Gamma_{12}^{\mathrm{NP}}) = \pi$. Again, a preference for nonzero values of $|\Gamma_{12}^{\mathrm{NP}}|$ is indicated, though $\mathrm{Arg}(\Gamma_{12}^{\mathrm{NP}})$ may vanish. The region with $\Gamma_{12}^{\mathrm{NP}} = 0$, i.e. the x-axis, is outside the 3σ allowed region, indicating that NP contribution to the absorptive part of the effective Hamiltonian is highly favored. The contours also imply that $|\Gamma_{12}^{\mathrm{NP}}| \lesssim 0.42$ at 3σ.

Fig. 2.3 displays the contours in the $|M_{12}^{\mathrm{NP}}| - |\Gamma_{12}^{\mathrm{NP}}|$ plane, and the two NP phases are marginalized over. Not only does it show a preference for nonzero values of M_{12}^{NP} and $\Gamma_{12}^{\mathrm{NP}}$, but the $M_{12}^{\mathrm{NP}} = 0$ axis is outside the 2σ allowed region and the $\Gamma_{12}^{\mathrm{NP}} = 0$ axis is outside the 3σ allowed region. The best fit points are again superimposed doubles, whose values can be read off from the discussion above. The origin in this figure is the SM, which has $\chi^2_{\mathrm{SM}} = 20.75$, and lies even outside the 4σ allowed region. This dramatically quantifies the failure of the SM

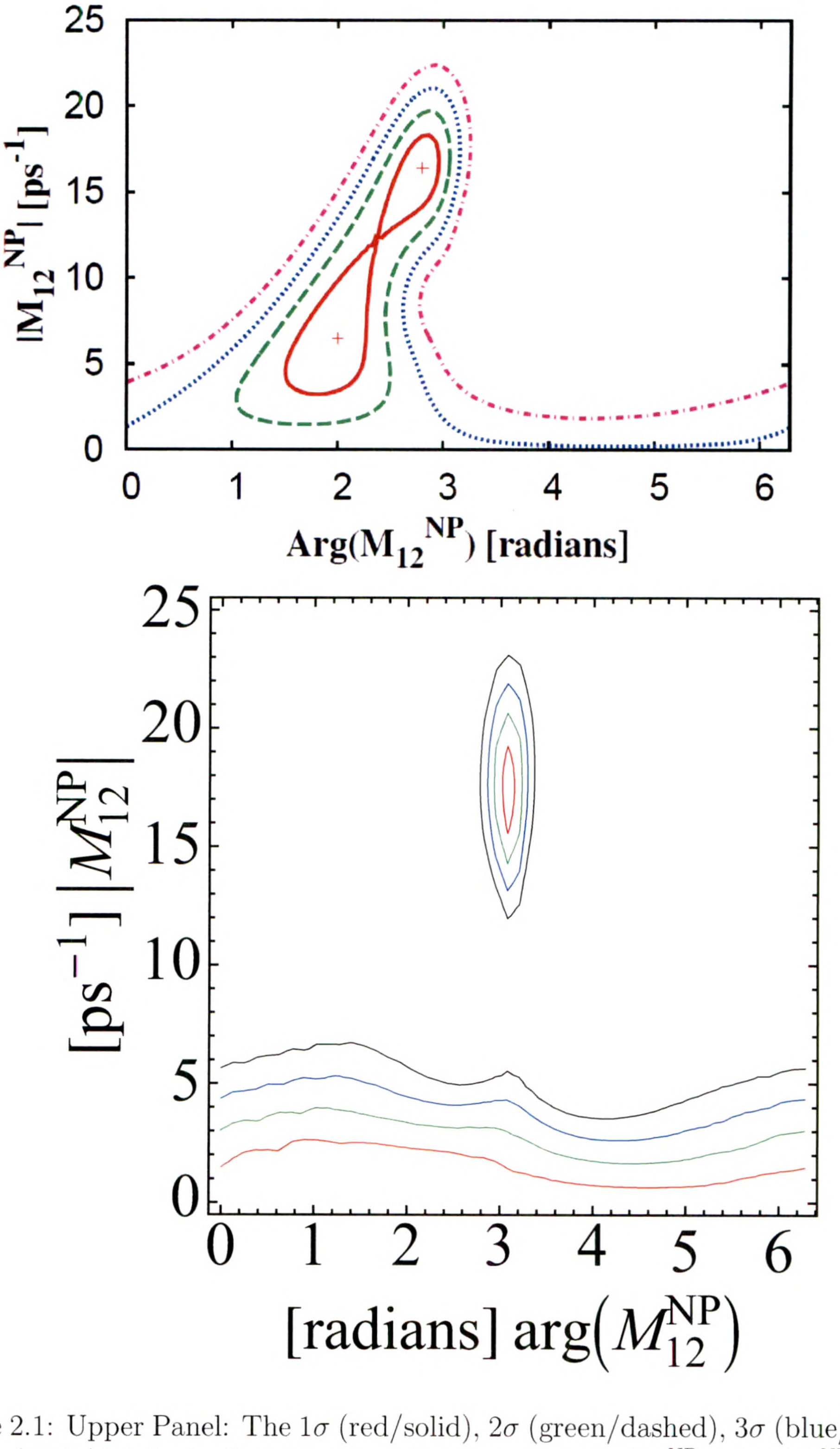

Figure 2.1: Upper Panel: The 1σ (red/solid), 2σ (green/dashed), 3σ (blue/dotted) and 4σ (pink/dot-dashed) goodness-of-fit contours in the $|M_{12}^{\rm NP}|$–Arg$(M_{12}^{\rm NP})$ plane, where the other NP parameters are marginalized over. The best-fit points, with $\chi^2 = 0$, are denoted by crosses. Lower Panel: Updated plot with the new LHCb data as shown in Appendix.

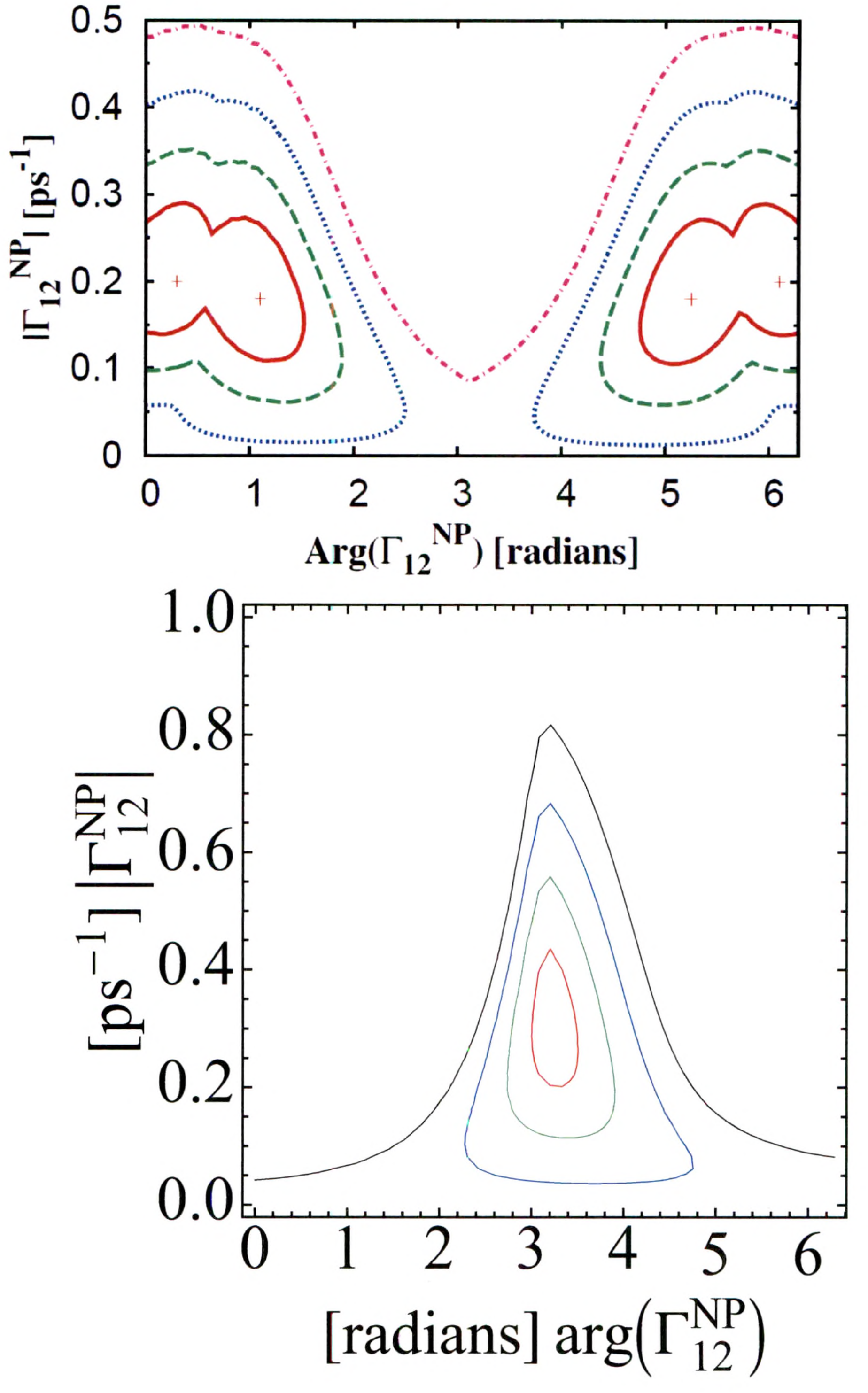

Figure 2.2: The 1σ (red/solid), 2σ (green/dashed), 3σ (blue/dotted) and 4σ (pink/dot-dashed) goodness-of-fit contours in the $|\Gamma_{12}^{NP}| - \mathrm{Arg}(\Gamma_{12}^{NP})$ plane, where the other NP parameters are marginalized over. The best-fit points, with $\chi^2 = 0$, are denoted by crosses. Lower Panel: Updated plot with the new LHCb data as shown in Appendix.

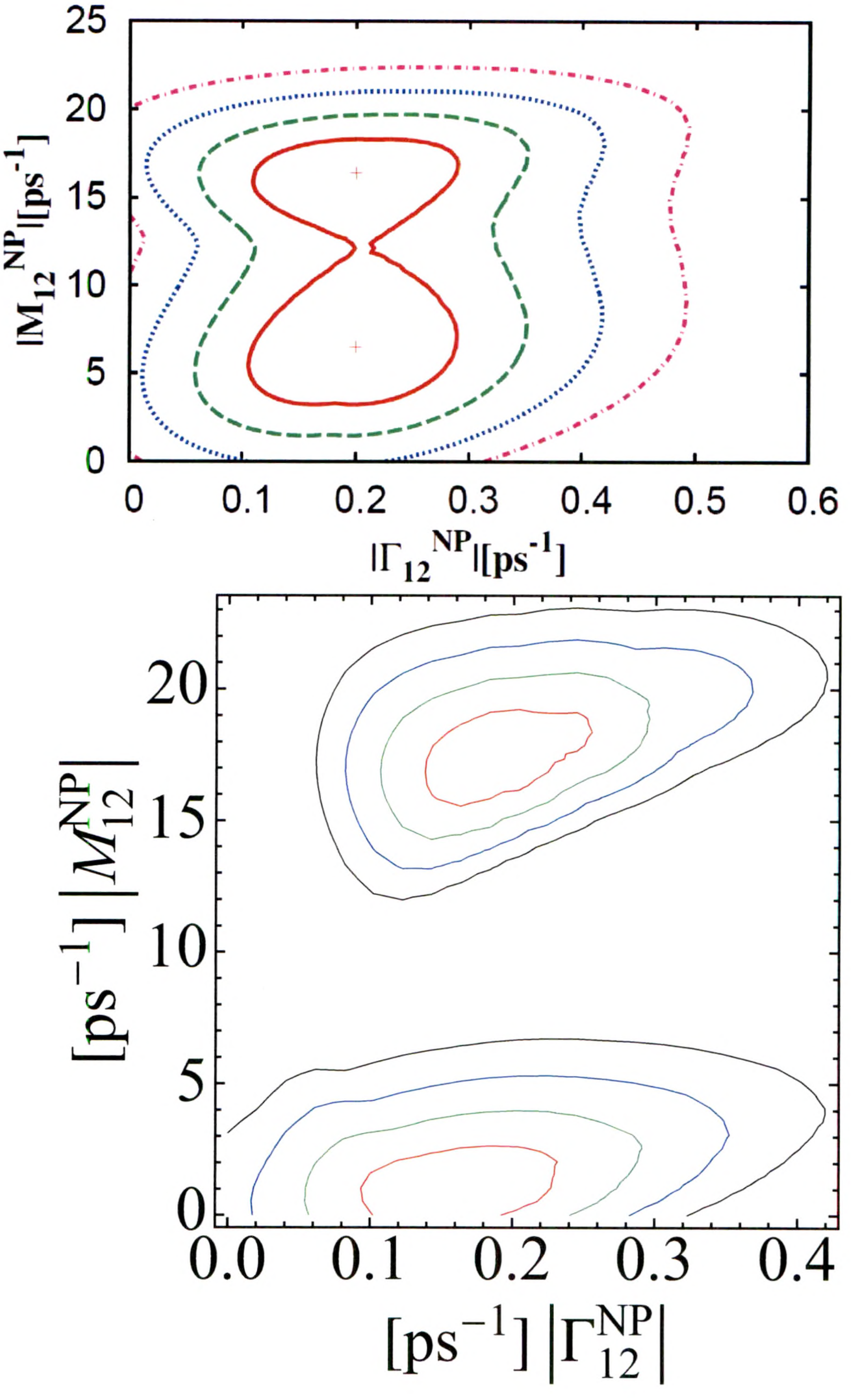

Figure 2.3: The 1σ (red/solid), 2σ (green/dashed), 3σ (blue/dotted) and 4σ (pink/dot-dashed) goodness-of-fit contours in the $|M_{12}^{\rm NP}| - |\Gamma_{12}^{\rm NP}|$ plane, where the other NP parameters are marginalized over. The best-fit points, with $\chi^2 = 0$, are denoted by crosses. Lower Panel: Updated plot with the new LHCb data as shown in Appendix.

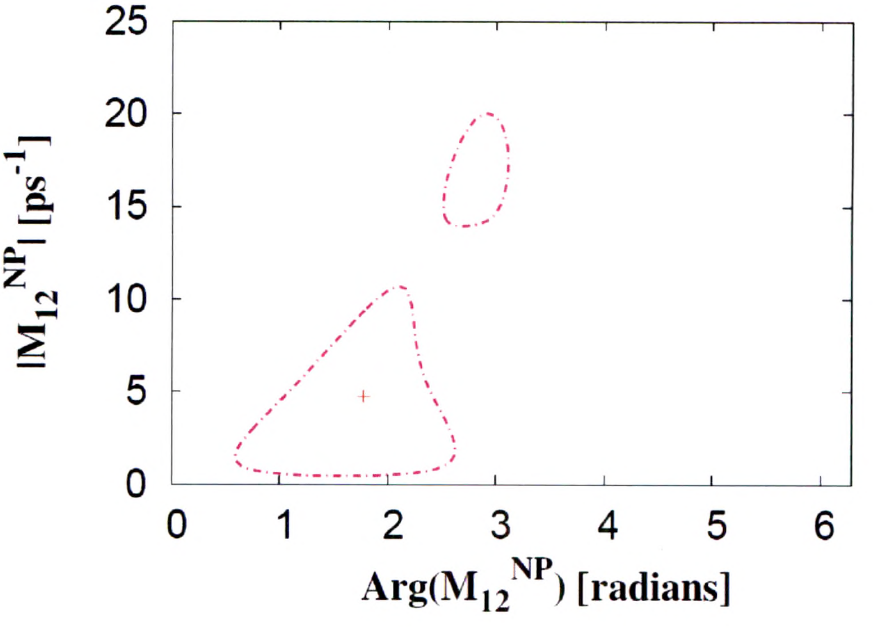

Figure 2.4: The 4σ (pink/dot-dashed) goodness-of-fit contour in the $|M_{12}^{\rm NP}| - {\rm Arg}(M_{12}^{\rm NP})$ plane, when $\Gamma_{12}^{\rm NP} = 0$, i.e. NP does not contribute to the absorptive part of the effective hamiltonian. There are no points that are allowed to within 3σ. The best-fit point, with $\chi^2 = 13.55$, is denoted by a cross.

to accommodate the current data. The reason is evident from eqs. (2.7) and (2.15); while $B_s \to J/\psi\phi$ prefers $\beta_s^{J/\psi\phi}$ close to $\pi/8$ or $3\pi/8$, with a probability minimum near $\beta_s^{J/\psi\phi} \approx \pi/4$, the measurement of $A_{\rm sl}^b$, and hence that of $a_{\rm sl}^s$, prefers large $\tan\phi_s$, forcing $\beta_s^{J/\psi\phi}$ close to $\pi/4$. This creates the tension between these two measurements.

Fig. 2.3 also tells us that the models for which $\Gamma_{12}^{\rm NP} = 0$, like R-parity conserving supersymmetry, universal extra dimension, and extra scalars, fermions, or gauge bosons, cannot bring the tension even down to the 3σ range, unless the data moves towards the SM expectations (and unless the new bosons are flavor-changing so as to generate a nonzero $\Gamma_{12}^{\rm NP}$). The best fit point with $\Gamma_{12}^{\rm NP} = 0$ has $\chi^2 = 13.55$ and corresponds to $M_{12}^{\rm NP} = 4.75 \exp(1.77\, i)$. Fig. 2.4 shows the situation when $\Gamma_{12}^{\rm NP}$ is set to vanish.

2.4 Preferred NP models

From the results and discussion in the previous section, it appears that:
(i) The SM by itself is strongly disfavored. Either $M_{12}^{\rm NP}$ or $\Gamma_{12}^{\rm NP}$ should be nonzero.

(ii) $M_{12}^{\rm NP} \neq 0$ but $\Gamma_{12}^{\rm NP} = 0$ is also not allowed at 3σ, but the fit is marginally better than the SM.
(iii) Equally disfavored is the hypothetical case where $\Gamma_{12}^{\rm NP} \neq 0$ but $M_{12}^{\rm NP} = 0$. (This is a rather natural condition, since any interaction that contributes to $\Gamma_{12}^{\rm NP}$ will necessarily contribute to $M_{12}^{\rm NP}$.)

Most of the NP models can contribute significantly to $M_{12}^{\rm NP}$. Leading examples are the MFV models like minimal Supersymmetry, Universal Extra Dimension, Little Higgs with T-parity, etc. Non-MFV models like a fourth chiral generation, Supersymmetry with R-parity violation, two-Higgs doublet models, models with extra Z', etc. can also contribute significantly to $M_{12}^{\rm NP}$.

The NP models that can contribute significantly to $\Gamma_{12}^{\rm NP}$, however, are rather rare. This is because the NP contribution to the absorptive part needs light particles in the final state, and there are strong limits on the decays of B_s to most of the possible light final state particles. One of the few exceptions is the mode $\tau^+\tau^-$, on which there is no available bound at this moment. Thus, the NP that contributes to $\Gamma_{12}^{\rm NP}$ has to do so via the interaction $b \to s\tau^+\tau^-$, but without affecting related decays like $b \to se^+e^-$ or $b \to s\mu^+\mu^-$. This can be achieved only in a limited subset of models, for example those with second and third generation scalar leptoquarks, or those with R-parity violating supersymmetry [21]. It turns out that the former can provide enough contribution to $\Gamma_{12}^{\rm NP}$ to increase $\Delta\Gamma_s$ up to its current experimental upper bound [21, 22]. The amount of NP required for this is consistent with the difference between the decay widths of B_d and B_s mesons ($\Gamma_s/\Gamma_d - 1 = (3.6 \pm 1.8)\%$ [19]), and the recent measurement of the branching ratio of $B^+ \to K^+\tau^+\tau^-$, which is less than 3.3×10^{-3} at 90% C.L. [39].

2.5 Conclusion

Any flavor-dependent new physics model can in general affect both mass and width differences in the $B_s - \overline{B_s}$ system. It can also affect the CP-violating phase, as well as the dimuon asymmetry, which was found by the DØ collaboration to have an anomalously large value. With these four observables, one can constrain the free parameters of the new physics model. We have used the model independent approach where we consider the effective $B_s - \overline{B_s}$ mixing Hamiltonian $\mathcal{H}$ and parameterize the NP through its contribution to $\mathcal{H}$. We quantify the goodness-of-fit for the SM and NP parameter values by performing a combined χ^2-fit to all the four measurements. The tension of the data with the SM is clear by the high value of χ^2 at the SM. Moreover, it is observed that we need NP to contribute to the dispersive as well as absorptive part of the off-diagonal elements of $\mathcal{H}$ in order for the current data to be explained. The absorptive contribution, in particular, can be obtained from a very limited set of models, which will be severely tested in near future.

If the errors and uncertainties shrink keeping the central values more or less intact, this will mean:

- The SM is strongly disfavored. Moreover, the relevant NP should be flavor-dependent, as we do not see much deviation in the $B_d - \overline{B}_d$ sector.

- The NP models that do not contribute to the absorptive amplitude of the $B_s - \overline{B_s}$ mixing are also strongly disfavored if CPT is conserved. The best bets are those NP models that provide both dispersive and absorptive amplitudes in the $B_s - \overline{B_s}$ mixing. This also gives rise to new decay channels for B_s. For example, one might find the branching ratio of $B_s \to \tau^+\tau^-$ enhanced significantly from its SM expectation.

To summarize, the NP models that contribute an absorptive part to $B_s - \overline{B_s}$ mixing seem to be essential if one wants to explain the data on $\beta_s^{J\psi\phi}$ and A_{sl}^b simultaneously. There is only a limited set of such models, and they will be severely tested in near future.

Appendix

Some data that were used in the original work [48] have changed over the last few months. Using the new data, mostly coming frm LHCb as well as the HFAG group, we have redrawn some of the old plots. The following values have been changed to produce new plots:

$$
\begin{aligned}
\Delta m_d &= 0.507 \pm 0.004 \ ps^{-1} \\
\Delta m_s &= 17.69 \pm 0.08 \ ps^{-1} \\
\beta_s^{J/\psi\phi} &= 0.08^{+0.05}_{-0.07} \\
\Delta\Gamma_s &= 0.107^{+0.14}_{-0.11} \ ps^{-1} \\
1/\Gamma_d &= 1.519 \pm 0.007 \ ps \\
1/\Gamma_d &= 1.497 \pm 0.015 \ ps \\
a_{sl}^d &= 0.0038 \pm 0.0036 \\
a_{sl}^s &= -0.0022 \pm 0.0052 \ \text{(Avg. of LHCb and DØ)}
\end{aligned}
$$

Chapter 3

CPT and Possible Violation

"Consistency is the last refuge of the unimaginative."

- Oscar Wilde

CPT theorem is a profound and general result for local relativistic quantum field theories in flat space-time. Pauli[49] used Lorentz invariance to prove the spin-statistics theorem in 1940, whereas Schwinger[50] implicitly used CPT theorem to do that in 1951. Originally proved by direct construction[51], it has since been rigorously derived in the framework of axiomatic field theory [4, 52]. In this chapter, we going to explore the CPT theorem, its origin, some experimental support for it and develop a theoretical premise for possible violation of CPT symmetry in an effective theory.

3.1 Theorem

Reconciling the demands of quantum mechanics with those of special relativity within a *local* description requires the existence of antiparticles. A considerably stronger statement can actually be made concerning the relationship between particles and antiparticles: the combined transformation CPT can always be defined - as an anti-unitary operator - in such a way for a local quantum field theory that it represents a symmetry [5], i.e.

$$CPT\mathcal{L}(t,\vec{x})(CPT)^{-1} = \mathcal{L}(-t,-\vec{x}) \tag{3.1}$$

This theorem can be proven rigorously in axiomatic field theory based on the assumptions of:

- Lorentz invariance;
- the existence of a unique vacuum state;
- weak local commutativity obeying the 'right' statistics.

The transformation properties of a Lagrangian written in terms of bosons is precisely the same as that for a Lagrangian involving fermion bilinears. We shall, therefore, confine our discussion to the transformation properties of a Lagrangian through boson fields. The notation is simpler that way and the essence of the argument becomes more transparent.

Let us consider the simple interaction Lagrangian

$$\begin{aligned}\mathcal{L}_T = & \, aV_\mu^+(t,\vec{x})V^{\mu,-}(t,\vec{x}) + bA_\mu^+(t,\vec{x})A^{\mu,-}(t,\vec{x}) \\ & + cV_\mu^+(t,\vec{x})A^{\mu,-}(t,\vec{x}) + c^*A_\mu^+(t,\vec{x})V^{\mu,-}(t,\vec{x})\end{aligned} \tag{3.2}$$

which under CPT transforms as follows

$$\begin{aligned}& CPT\mathcal{L}_T(CPT)^{-1} \\ & \quad = aV^{\mu,-}(-t,-\vec{x})V_\mu^+(-t,-\vec{x}) + bA^{\mu,-}(-t,-\vec{x})A_\mu^+(-t,-\vec{x}) \\ & \quad\quad + c^*V^{\mu,-}(-t,-\vec{x})A_\mu^+(-t,-\vec{x}) + cA^{\mu,-}(-t,-\vec{x})V_\mu^+(-t,-\vec{x})\end{aligned} \tag{3.3}$$

i.e. CPT is indeed conserved, no matter what the coupling parameters a, b and c are.

The argument is easily repeated for fermions by noting again that each bosonic field can be written in terms of fermionic bilinears which transform in exactly the same way under C, P, and T; likewise for more realistic Lagrangians or Hamiltonians.

3.2 Consequences of CPT conservation

Although the proof of this theorem, at least for the basic cases, appears rather simple, its consequences are far-reaching. The most celebrated ones are presented here:

- Particle and antiparticle must have same mass and same electric charge, e.g.

$$\begin{aligned} M(P) &= \langle P|H|P\rangle \\ &= \langle P|(\boldsymbol{CPT})^{\dagger}\boldsymbol{CPT}H(\boldsymbol{CPT})^{-1}\boldsymbol{CPT}|P\rangle^{*} \\ &= \langle \bar{P}|\boldsymbol{CPT}H\boldsymbol{CPT}^{-1}|\bar{P}\rangle^{*} \\ &= \langle \bar{P}|H|\bar{P}\rangle^{*} = M(\bar{P}) \end{aligned} \tag{3.4}$$

- Particle and antiparticle, if unstable, must have same decay widths or lifetimes. Under time reversal, an incoming spherical wave transforms to an outgoing spherical wave. So, under time reversal, multi-particle states transform as

$$\boldsymbol{T}|p_1, p_2, ...; out\rangle = |-p_1, -p_2, ...; in\rangle.$$

As both 'in' and 'out' states form complete set of states: $\sum_f |f; in\rangle\langle f; in| = \sum_f |f; out\rangle\langle f; out| = 1$, we get,

$$\begin{aligned} \Gamma(P) &= 2\pi \sum_f \delta(M_P - E_f)|\langle f; out|H_{decay}|P\rangle|^2 \\ &= 2\pi \sum_f \delta(M_P - E_f)|\langle f; out|\boldsymbol{CPT}^{\dagger}\boldsymbol{CPT}H_{decay}\boldsymbol{CPT}^{-1}\boldsymbol{CPT}|P\rangle^{*}|^2 \\ &= 2\pi \sum_f \delta(M_P - E_f)|\langle \bar{f}; in|H_{decay}|\bar{P}\rangle|^2 \\ &= 2\pi \sum_f \delta(M_P - E_f)|\langle \bar{f}; out|H_{decay}|\bar{P}\rangle|^2 \\ &= \Gamma(\bar{P}). \end{aligned} \tag{3.5}$$

This is not true if stationary states are particle-antiparticle combinations.

- Particle and antiparticle must have equal and opposite magnetic dipole moments: $\mu_{mag}(P) = -\mu_{mag}(\bar{P})$.

- Hydrogen and anti-hydrogen must have identical spectra.

- T violation necessarily means CP violation.

In case of *local* axiomatic quantum field theories, CPT conservation has another important consequence- if it is violated, it necessarily implies Lorentz violation[53]. This need not be true for nonlocal field theories as well as for theories with noncommutative space-time geometry[54]. Also, the reverse is not true, i.e. Lorentz violation does not necessarily imply CPT violation.

Despite this impeccable pedigree, it makes sense to ask whether limitations exist. There are several motivations for a very precise and multi-pronged test of CPT theorem:

1. precisely because the CPT theorem rests on such essential pillars of our present paradigm, we have to make every reasonable effort to prove its universal validity.

2. the simultaneous existence of a general theoretical proof of CPT invariance in particle physics and accurate experimental tests makes CPT violation an attractive candidate signature for non-particle physics such as string theory[55–58]. These nonlocal and non-renormalizable string-theoretic effects may appear at the Planck scale with a possible ramification at the weak scale through the effective Hamiltonian [62]. CPT through such nonlocal interacting QFT does not necessarily lead to the violation of Lorentz symmetry [54].

   The assumptions needed to prove the CPT theorem are invalid for strings, which are extended objects. Moreover, since the critical string dimensionality is larger than four, it is plausible that higher-dimensional Lorentz breaking would be incorporated in a realistic model. In fact, a mechanism is known in string theory that can cause spontaneous CPT violation[56] with accompanying partial Lorentz-symmetry breaking[59]. The effect can be traced to string interactions that are absent in conventional four-dimensional renormalizable gauge theory. Under suitable circumstances, these interactions can cause instabilities in Lorentz-tensor potentials, thereby inducing spontaneous CPT and Lorentz breaking. If in a realistic theory the spontaneous CPT and partial Lorentz violation extend to the four-dimensional space-time, detectable effects might occur in interferometric experiments with neutral mesons. For example, the quantities parametrizing indirect CPT violation in these systems could be nonzero. There may also be implications for baryogenesis[60].

3. for the bound systems like mesons, asymptotic states, whose existence is a prerequisite for the CPT theorem, are not uniquely defined [61]. Quarks and gluons are bound inside the hadrons and cannot be considered, in a true sense, asymptotic states.

4. An intriguing phenomenon has been suggested by Hawking[63]. Near a black hole *pure* quantum states can evolve into *mixed* ones, since some of the information carried by them gets funnelled into the black hole due to the latter's overpowering gravitational pull, and thus is lost for the 'outside world'. This sequence violates both conventional quantum mechanics and CPT invariance. Hawking used the density formalism to specifically discuss $K^0 - \overline{K}^0$ oscillations in the presence of a black hole. An experimental test of Hawking's idea was proposed in Ref. [64].

3.3 Lorentz violation

The SM, although phenomenologically successful, is believed to be the low-energy limit of a fundamental theory that also provides a quantum description of gravitation. An interesting question is whether any aspect of this underlying theory could be revealed through definite experimental signals accessible with present techniques[65].

The natural scale for a fundamental theory including gravity, governed by the Planck mass M_P, is about 17 orders of magnitude greater than the electroweak scale m_W. This suggests that observable experimental signals from a fundamental theory might be expected to be suppressed by some power of the ratio $r \approx m_W/M_P \simeq 10^{-17}$. Detection of these minuscule effects requires experiments of exceptional sensitivity, preferably ones seeking to observe a signal forbidden in conventional renormalizable gauge theories.

To identify signals of this type, one approach is to examine proposed fundamental theories for effects that are qualitatively different from standard-model physics. It has been shown that spontaneous Lorentz breaking may occur in the context of string theories with Lorentz-covariant dynamics [56, 62, 66]. These theories typically involve interactions that could destabilize the naive vacuum and trigger the generation of nonzero expectation values for Lorentz tensors. We should note that some kind of spontaneous breaking of the higher-dimensional Lorentz symmetry is expected in any realistic Lorentz-covariant fundamental theory involving more than four space-time dimensions. If the breaking extends into the four macroscopic space-time dimensions, apparent Lorentz violation could occur at the level of the standard model. This would represent a possible observable effect from the fundamental theory, originating outside the structure of conventional renormalizable gauge models[55].

An important point is that Lorentz symmetry remains a property of the underlying fundamental theory because the breaking is spontaneous. This implies that various attractive features of conventional theories, including micro-causality and positivity of the energy, are expected to hold in the low-energy effective theory. Also, energy and momentum are conserved as usual, provided the tensor expectation values in the fundamental theory are space-time-position independent. Moreover, standard quantization methods are unaffected, so a relativistic Dirac equation and a non-relativistic Schrödinger equation emerge in the appropriate limits. Also both the fundamental theory and the effective low-energy theory remain invariant under *observer* Lorentz transformations, i.e., rotations or boosts of an observer's inertial frame [55]. The presence of nonzero tensor expectation values in the vacuum affects only invariance properties under *particle* Lorentz transformations, i.e., rotations or boosts of a localized particle or field that leave unchanged the background expectation values.

This framework for treating spontaneous Lorentz violation has been used to obtain a general extension of the minimal SU(3) × SU(2)× U(1) standard model that violates both Lorentz invariance and CPT [55]. In addition to the desirable features of energy-momentum conservation, observer Lorentz invariance, conventional quantization, hermiticity, and the expected micro-causality and positivity of the energy, this standard-model extension maintains gauge invariance and power-counting renormalizability. It would emerge from any fundamental theory (not necessarily string theory) that generates the standard model and contains spontaneous Lorentz and CPT violation. A representative CPT-odd term in the fermion sector in this framework looks like this:

$$\mathcal{L}^{\mathrm{CPT-odd}}_{\mathrm{lepton}} = -(a_L)_{\mu AB}\overline{L}_A\gamma^\mu L_B - (a_R)_{\mu AB}\overline{R}_A\gamma^\mu R_B \tag{3.6}$$

where a_μs are understood to be hermitian in generation space and have dimensions of mass.

The SME predicts some unique signals, such as rotational, sidereal, and annual variations. The effects are likely to be heavily suppressed, perhaps as some power of the ratio of an accessible scale to the underlying scale, but they could be detected using sensitive tools such as interferometry. For example, meson interferometry offers the potential to identify flavor and direction-dependent energy shifts of mesons relative to anti-mesons [67], while exquisite interferometric sensitivity to polarization-dependent effects of photons is attained using cosmological birefringence [68]. Conceivably, SME effects might even be reflected in existing data, such as those for flavor oscillations of neutrinos[69].

3.4 Parametrization(s)

An arbitrary neutral-meson state is a linear combination of the Schrödinger wave functions for the meson P^0 and its anti-meson $\overline{P^0}$. This combination can be represented as a two-component object $\Psi(t)$, with time evolution governed by a 2×2 effective hamiltonian Λ according to the Schrödinger-type equation [70]

$$i\delta_t\Psi = \Lambda\Psi. \tag{3.7}$$

Throughout the rest of this chapter, subscripts P are understood on Ψ, on the components of the effective hamiltonian Λ, and on related quantities such as meson masses and lifetimes.

The physical propagating states are the eigenstates of Λ, analogous to the normal modes of a classical two-dimensional oscillator [71]. These states are generically denoted as $|P_a\rangle$ and $|P_b\rangle$. They evolve in time as

$$\begin{aligned} |P_a(t)\rangle &= \exp(-i\lambda_a t)|P_a\rangle, \\ |P_b(t)\rangle &= \exp(-i\lambda_b t)|P_b\rangle. \end{aligned} \tag{3.8}$$

The complex parameters λ_a, λ_b are the eigenvalues of Λ. They can be decomposed as

$$\lambda_a \equiv m_a - \frac{1}{2}i\gamma_a, \quad \lambda_b \equiv m_b - \frac{1}{2}i\gamma_b, \tag{3.9}$$

where m_a, m_b are the propagating masses and γ_a, γ_b are the associated decay rates.

For calculational purposes, it is useful to introduce a separate notation for the sums and differences of these parameters:

$$\begin{aligned} \lambda &\equiv \lambda_a + \lambda_b = m - \frac{1}{2}i\gamma, \\ \Delta\lambda &\equiv \lambda_a - \lambda_b = -\Delta m - \frac{1}{2}i\Delta\gamma, \end{aligned} \tag{3.10}$$

where $m = m_a + m_b$, $\Delta m = m_b - m_a$, $\gamma = \gamma_a + \gamma_b$, $\Delta\gamma = \gamma_a - \gamma_b$.

The off-diagonal components of Λ control the flavor oscillations between P^0 and $\overline{P^0}$. Indirect CPT violation occurs if and only if the difference of diagonal elements of Λ is nonzero, $\Lambda_{11} - \Lambda_{22} \neq 0$. Indirect T violation occurs if and only if the magnitude of the ratio of off-diagonal components of Λ differs from 1, $|\Lambda_{21}/\Lambda_{12}| \neq 1$.

A priori, the effective hamiltonian Λ can be parametrized by eight independent real quantities. Four of these can be specified in terms of the masses and decay rates, two describe CPT violation, and one describes T violation. The remaining parameter, determined by the relative phase between the off-diagonal components of Λ, is physically irrelevant. It can be dialed at will by rotating the phases of the P^0 and $\overline{P^0}$ wave functions by equal and opposite amounts. The freedom to perform such rotations exists because the wave functions are eigenstates of the strong interactions, which preserve strangeness, charm, and beauty. Under a rotation of this type involving a phase factor of $\exp(i\chi)$ for the P^0 wave function, the off-diagonal elements of Λ are multiplied by equal and opposite phases, becoming $\exp(2i\chi)\Lambda_{12}$ and $\exp(-2i\chi)\Lambda_{21}$.

3.4.1 Different parametrizations

Since relatively little experimental information is available about CPT and T violation in the heavy neutral-meson systems, a general parametrization of Λ is appropriate. It is desirable to have a parametrization that is model independent, valid for arbitrary size CPT and T violation, independent of phase conventions, and expressed in terms of mass and decay rates insofar as possible. A parametrization of this type was originally introduced by Lavoura in the context of the kaon system [72, 73, 108]. For simplicity, it is also attractive to arrange matters so that the quantities controlling T and CPT violation are denoted by single symbols that are distinct from other frequently used notation. In this section, a parametrization convenient to the four meson systems and satisfying all the above criteria is presented and related to formalisms often used in the literature.

The $M\Gamma$ formalism sets

$$\Lambda = M - \frac{1}{2}i\Gamma = \begin{pmatrix} M_{11} - \frac{1}{2}i\Gamma_{11} & M_{12} - \frac{1}{2}i\Gamma_{12} \\ & \\ M_{12}^* - \frac{1}{2}i\Gamma_{12}^* & M_{22} - \frac{1}{2}i\Gamma_{22} \end{pmatrix}. \tag{3.11}$$

The off-diagonal quantities are all phase-convention dependent. The parameter for CPT violation is the combination $(M_{11} - M_{22}) - i(\Gamma_{11} - \Gamma_{22})/2$. The parameter

Formalism	Dependence on phase convention?	λ, $\Delta\lambda$ given as	CPT parameter (complex)	T paran (
$\omega\xi$	No	λ, $\Delta\lambda$	ξ	
$M\Gamma$	Yes(M_{12}, Γ_{12})	See eq.(3.12)	$(M_{11}$ - $M_{12})$ $-\frac{1}{2}i(\Gamma_{11}-\Gamma_{22})$	$\frac{\lvert M_{12}^{*}-i\mathrm{I}}{\lvert M_{12}-i\mathrm{I}}$
D$E_1E_2E_3$	Yes(E_1, E_2)	$2\sqrt{E_1^2+E_2^2+E_3^2}$, $-2iD$	E_3	$i(E_1E_2^{*}-E$
DE$\theta\phi$	Yes(ϕ)	$-2iD$, 2 E	$\cos\theta$	$\lvert\exp$
$pqrs$	Yes(p, q, r, s)	λ, $\Delta\lambda$	$(ps$ - $qr)$	$\lvert pr$
$\epsilon\delta$	Yes(ϵ, δ)	λ, $\Delta\lambda$	δ	Reϵ, if $C\not{P}$ s

Table 3.1: Comparison of formalisms for neutral-meson mixing.(The ω ξ formalism can be found in [119]. All of the others can be traced to early work several decades ago in the context of the K system [70].)

for T violation is $|(M_{12}^{*}-i\Gamma_{12}^{*}/2)/(M_{12}-i\Gamma_{12}/2)|$. The masses and decay rates are given by

$$
\begin{aligned}
\lambda &= (M_{11}+M_{22})-\frac{1}{2}i(\Gamma_{11}+\Gamma_{22}), \\
\Delta\lambda &= 2\big[(M_{12}-\frac{1}{2}i\Gamma_{12})(M_{12}^{*}-\frac{1}{2}i\Gamma_{12}^{*}) \\
&\quad +\frac{1}{4}[(M_{11}-M_{22})-\frac{1}{2}i(\Gamma_{11}-\Gamma_{22})]^2\big]^{1/2},
\end{aligned}
\tag{3.12}
$$

where the definitions in eq.(3.10) are understood to hold.

Our parametrization is very similar to the ω ξ parametrization mentioned here. It will be developed in detail in the next chapter.

Chapter 4

Probing CPT Violation in B Systems

"Because it is there."
- George Leigh Mallory
(On being asked-
"Why did you want to climb Mount Everest?")

As has been explained in the previous chapter, the combined symmetry CPT is supposed to be an exact symmetry of any local axiomatic quantum field theory. This is indeed supported by the experiments: all possible tests so far have yielded negative results, consistent with no CPT violation. Why then should we be interested in the possibility of CPT violation in the B system? There are three main reasons: first, any symmetry which is supposed to be exact ought to be questioned and investigated, and we may get a surprise, just like the discovery of CP violation; second, it is not obvious that CPT will still be an exact symmetry in the bound state of quarks and anti-quarks, where the asymptotic states are not uniquely defined [61]; third, there may be some nonlocal and non-renormalizable string-theoretic effects at the Planck scale which have a ramification at the weak scale through the effective Hamiltonian [62]. Moreover, this effect can very well be flavor-sensitive, and so the constraints obtained from the K system [105] may not be applicable to the B systems. A comprehensive study of CPT violation in the neutral K meson system, with a formulation that is closely analogous to that in the B system, may be found in [81].

There are already some investigations on CPT violation in B systems. Datta *et al.* [106] have shown how CPT violation can lead to a significant lifetime difference

$\Delta\Gamma/\Gamma$ in the generic P^0-$\overline{P}^0$ system, where $P^0 = K^0, B_d$, or B_s. It was discussed in [106] how direct CP asymmetries and semileptonic decays can act as a probe of CPT violation. Signatures of CPT violation in non-CP eigenstate channels was discussed in [107]. The role of dilepton asymmetry as a test of CPT violation and possible discrimination from $\Delta B = -\Delta Q$ processes were investigated in [108]. The BaBar experiment at SLAC has tried to look for CPT violation in the diurnal variations of CP-violating observables and set some limits [82].

Right now, there is no signature of CPT violation, or for that matter any type of new physics, in the width difference of $B_d - \overline{B}_d$ and decay channels of B_d [1]. The width difference for the B_d system, $\Delta\Gamma_d$, is too small yet to be measured experimentally, and the bound is compatible with the Standard Model (SM). On the other hand, it is expected that the width difference $\Delta\Gamma_s$ would be significant for the B_s system, but at the same time we know that the theoretical uncertainties are quite significant [95]. If there is some new physics (NP) that does not contribute to the absorptive part of the $B_s - \overline{B_s}$ box, the width difference can only go down [34], while there are models where this conclusion may not be true [21]. To add to this murky situation, the CP-violating phase β_s, which is expected to be very small from the CKM paradigm, has been measured [75] to be large, compatible with the SM expectations only at the 2.1σ level. The situation is interesting: there is hint of some NP, but we are yet to be certain of its exact nature, not to mention whether it exists at all.

In this situation, let us try to see what we can expect at the LHC, where the B_s meson, along with the B_d, will be copiously produced. We are helped by the fact that the time resolution in ATLAS and CMS are of the order of 40 fs, so one can track the time evolution of even the rapidly oscillating B_s. Thus, we expect excellent tagged and untagged measurements of both B_d and B_s mesons. It is best to focus upon the single-amplitude observables: $B_d \to J/\psi K_S$ and $B_s \to J/\psi\phi$ or $B_s \to D_s^+ D_s^-$ [2]. For the $J/\psi\phi$ mode, one has to perform the angular analysis and untangle the CP-even and CP-odd channels.

In this chapter, we will discuss how one can detect the presence of a CPT violating new physics from the tagged and untagged measurements of the decay. We will confine our discussion to the case where CPT violation is small compared

[1] We use B_d and $\bar{B}_d$ to indicate the flavor eigenstates, B_d as a generic symbol for both of them, and similarly for B_s. The symbol B_q will mean either a B_d or a B_s.

[2] They are not strictly single-channel as there is a penguin process whose dominant part has the same phase as the leading Cabibbo-allowed tree process, but on the other hand these channels are easy to measure, and the penguin pollution is quite small and well under control.

to the SM amplitude, just to make the results more transparent. The conclusions do not change qualitatively if the CPT violation is large, which, we must say, is a far-off possibility based on the data from the other experiments [82]. We will also show how the nature of the CPT violating term can be probed through these measurements.

In Section 2, we mention the relevant expressions, and introduce CPT violation, with relevant expressions, in Section 3. The analysis for both B_d and B_s systems is performed in Section 4, while we summarize and conclude in Section 5.

4.1 Basic formalism

Let us introduce CPT violation in the Hamiltonian matrix through the parameter δ which can potentially be complex:

$$\delta = \frac{H_{22} - H_{11}}{\sqrt{H_{12}H_{21}}}\,, \tag{4.1}$$

so that

$$\mathcal{M} = \left[\begin{pmatrix} M_0 - \delta' & M_{12} \\ M_{12}^* & M_0 + \delta' \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_0 & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_0 \end{pmatrix}\right], \tag{4.2}$$

where δ' is defined by

$$\delta = \frac{2\delta'}{\sqrt{H_{12}H_{21}}}\,. \tag{4.3}$$

Solving the eigenvalue equation of $\mathcal{M}$, we get,

$$\begin{aligned} \lambda &= \left(M_0 - \frac{i}{2}\Gamma_0\right) \pm H_{12}\alpha y \\ \text{or,}\quad \lambda &= \left[H_{11} + H_{12}\alpha\left(y + \frac{\delta}{2}\right)\right],\ \left[H_{22} - H_{12}\alpha\left(y + \frac{\delta}{2}\right)\right], \end{aligned} \tag{4.4}$$

where $y = \sqrt{1 + \frac{\delta^2}{4}}$ and $\alpha = \sqrt{H_{21}/H_{12}}$.

Hence, corresponding eigenvectors or the mass eigenstates are defined as

$$\begin{aligned} |B_H\rangle &= p_1|B_d\rangle + q_1|\bar{B}_d\rangle\,, \\ |B_L\rangle &= p_2|B_d\rangle - q_2|\bar{B}_d\rangle\,. \end{aligned} \tag{4.5}$$

The normalisation satisfies

$$|p_1|^2 + |q_1|^2 = |p_2|^2 + |q_2|^2 = 1\,. \tag{4.6}$$

Let us define,

$$\eta_1 = \frac{q_1}{p_1} = \left(y + \frac{\delta}{2}\right)\alpha\,; \quad \eta_2 = \frac{q_2}{p_2} = \left(y - \frac{\delta}{2}\right)\alpha\,; \quad \omega = \frac{\eta_1}{\eta_2}\,. \tag{4.7}$$

The convention of [82] leads to $z_0 = \delta/2$, where z_0 is a measure of CPT violation as used in [82]. The limits imply that $|z_0| \ll 1$. Even if the origin of CPT violation is something different, it is not unrealistic to assume $|\delta| \ll 1$.

One could even relax the assumption of $H_{21} = H_{12}^*$. However, there are two points that one must note. First, the effect of expressing $H_{12} = h_{12} + \bar{\delta}$, $H_{21} = h_{12}^* - \bar{\delta}$ appears as $\bar{\delta}^2$ in $\sqrt{H_{12}H_{21}}$, the relevant expression in eq. (1), and can be neglected if we assume $\bar{\delta}$ to be small. The second point, which is more important, is that CPT conservation constrains only the diagonal elements and puts no constraint whatsoever on the off-diagonal elements. It has been shown in [81, 106] that $H_{12} \neq H_{21}^*$ leads to T violation, and only $H_{11} \neq H_{22}$ leads to unambiguous CPT violation. Thus, we will focus on the parametrization used in eqs. (1) and (2) to discuss the effects of CPT violation.

The time-dependent flavor eigenstates are given by

$$\begin{aligned} |B_q(t)\rangle &= f_+(t)|B_q\rangle + \eta_1 f_-(t)|\overline{B_q}\rangle \\ |\overline{B_q}(t)\rangle &= \frac{f_-(t)}{\eta_2}|B_q\rangle + \bar{f}_+(t)|\overline{B_q}\rangle\,, \end{aligned} \tag{4.8}$$

where

$$\begin{aligned} f_-(t) &= \frac{1}{(1+\omega)}\left(e^{-i\lambda_1 t} - e^{-i\lambda_2 t}\right)\,, \\ f_+(t) &= \frac{1}{(1+\omega)}\left(e^{-i\lambda_1 t} + \omega e^{-i\lambda_2 t}\right)\,, \\ \bar{f}_+(t) &= \frac{1}{(1+\omega)}\left(\omega e^{-i\lambda_1 t} + e^{-i\lambda_2 t}\right)\,. \end{aligned} \tag{4.9}$$

So, the decay rate of the meson B_q at time t to a CP eigenstate f is given by

$$\begin{aligned}\Gamma(B_q(t) \to f_{CP}) &= \left[|f_+(t)|^2 + |\xi_{f_1}|^2|f_-(t)|^2 + 2\mathrm{Re}\left(\xi_{f_1} f_-(t) f_+^*(t)\right)\right] |A_f|^2 , \\ \Gamma(\overline{B_q}(t) \to f_{CP}) &= \left[|f_-(t)|^2 + |\xi_{f_2}|^2|\bar{f}_+(t)|^2 + 2\mathrm{Re}\left(\xi_{f_2} \bar{f}_+(t) f_-^*(t)\right)\right] \left|\frac{A_f}{\eta_2}\right|^2 \end{aligned} \quad (4.10)$$

where

$$A_f = \langle f|H|B_q\rangle , \quad \bar{A}_f = \langle f|H|\overline{B_q}\rangle . \quad (4.11)$$

Also,

$$\xi_{f_1} = \eta_1 \frac{\bar{A}_f}{A_f} , \quad \xi_{f_2} = \eta_2 \frac{\bar{A}_f}{A_f} . \quad (4.12)$$

In the SM, both are equal and $\xi_{f_1} = \xi_{f_2} = \xi_f$. For single-channel processes, $|\xi_f| = 1$.

Now using eq. (4.7) and eq. (6.27) one gets

$$|f_-(t)|^2 = \frac{2e^{-\Gamma t}}{|1+\omega|^2} \left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) - \cos\left(\Delta m t\right)\right] \text{ ...etc.} \quad (4.13)$$

[Other equations of this form are explicitly written in the appendix of chapter (7)] Here, Δm and $\Delta\Gamma$ are defined through;

$$\lambda_1 - \lambda_2 = \Delta m + \frac{i}{2}\Delta\Gamma , \quad (4.14)$$

with

$$\lambda_{(1,2)} = m_{(1,2)} - \frac{i}{2}\Gamma_{(1,2)} , \quad \Delta m = m_1 - m_2 , \quad \Delta\Gamma = \Gamma_2 - \Gamma_1 . \quad (4.15)$$

4.2 Introducing CPT violation

If we consider a time-independent CPT violation so that δ is a constant, there are only two unknowns in the picture: $\mathrm{Re}(\delta)$ and $\mathrm{Im}(\delta)$, over those in the SM. We will try to see how one can extract information about them. For our analysis, let us take δ to be complex; it will be straightforward to go to the simpler limiting cases where δ is purely real or imaginary. For example, if the width difference $\Delta\Gamma$ is much smaller than Δm, the model of [82] makes δ completely real.

When B_q and $\bar{B}_q$ are produced in equal numbers, the untagged decay rate can be defined as

$$\Gamma_U[f,t] = \Gamma(B_q(t) \to f) + \Gamma(\bar{B}_q(t) \to f)\,, \tag{4.16}$$

using the above expression one could define the branching fraction as

$$Br[f] = \frac{1}{2}\int_0^\infty dt\ \Gamma[f,t]\,. \tag{4.17}$$

The above equation is useful to fix the overall normalization.

We assume, $\delta \ll 1$ and expand any function $f(\delta)$ using Taylor series expansion and drop all the terms $\mathcal{O}(\delta^n)$ for $n > 2$. From eq. (4.16), eq. (4.10) and eq. (4.13) we will get the untagged decay rate for the decay $B_q \to f$,

$$\begin{aligned}\Gamma_U[f,t] = |A_f|^2 e^{-\Gamma_q t}\Bigg[&\left\{(1+|\xi_f|^2)(1+\frac{(\mathrm{Im}(\delta))^2}{4}) - \mathrm{Im}(\delta)\mathrm{Im}(\xi_f)\right\}\cosh\left(\frac{\Delta\Gamma_q t}{2}\right) \\ &- \left\{(1+|\xi_f|^2)\frac{(\mathrm{Im}(\delta))^2}{4} - \mathrm{Im}(\delta)\mathrm{Im}(\xi_f)\right\}\cos(\Delta m_q t) \\ &+ \left\{2\mathrm{Re}(\xi_f) - \frac{1}{2}(1-|\xi_f|^2)\mathrm{Re}(\delta) - \frac{1}{4}\mathrm{Re}(\xi_f)((\mathrm{Re}(\delta))^2 - (\mathrm{Im}(\delta))^2)\right\} \\ &\sinh\left(\frac{\Delta\Gamma_q t}{2}\right) - \frac{1}{2}\mathrm{Im}(\delta)\left\{(1-|\xi_f|^2) + \mathrm{Re}(\delta)\mathrm{Re}(\xi_f)\right\}\sin(\Delta m_q t)\Bigg]\,.\end{aligned} \tag{4.18}$$

Thus, for the B_s system, where the hyperbolic functions are not negligible, we get (keeping up to first order of terms in $\Delta\Gamma_s$):

$$\begin{aligned}Br[f] &= \frac{1}{2}\int_0^\infty dt\ \Gamma[f,t] \\ &= \frac{|A_f|^2}{2}\Bigg[\frac{1}{\Gamma_s}\left\{(1+|\xi_f|^2)(1+\frac{(\mathrm{Im}(\delta))^2}{4}) - \mathrm{Im}(\delta)\mathrm{Im}(\xi_f)\right\} \\ &\quad - \frac{\Gamma_s}{(\Delta m)^2 + (\Gamma_s)^2}\left\{(1+|\xi_f|^2)\frac{(\mathrm{Im}(\delta))^2}{4} - \mathrm{Im}(\delta)\mathrm{Im}(\xi_f)\right\} \\ &\quad + \frac{\Delta\Gamma_s}{2(\Gamma_s)^2}\left\{2\mathrm{Re}(\xi_f) - \frac{1}{2}(1-|\xi_f|^2)\mathrm{Re}(\delta) - \frac{1}{4}\mathrm{Re}(\xi_f)((\mathrm{Re}(\delta))^2 - (\mathrm{Im}(\delta))^2)\right\} \\ &\quad - \frac{1}{2}\mathrm{Im}(\delta)\left\{(1-|\xi_f|^2) + \mathrm{Re}(\delta)\mathrm{Re}(\xi_f)\right\}\frac{\Delta m}{(\Delta m)^2 + (\Gamma_s)^2}\Bigg]\end{aligned} \tag{4.19}$$

Theoretically, one can obtain the coefficients of the trigonometric and the hyperbolic terms by fitting the untagged decay rate. In actual cases this is a difficult task. However, there is one other observable which may help us. Before we go to that, let us note that the above expression is further simplified in the following four cases.

- For the B_d system: We can neglect $\Delta\Gamma_d$ so that the cosh term is unity and the sinh term is zero. Thus, there are only two time-dependent terms, $\cos(\Delta mt)$ and $\sin(\Delta mt)$, and the fitting is easier. Note that $\Delta\Gamma_d$ is measured to be small, so we need not consider the case where it is enhanced to a significant value because of the CPT violation. In fact, if δ is small, $\Delta\Gamma_d$ is bound to be that coming from the SM, as the correction is proportional only to δ^2 and higher.

- For one-amplitude processes: We can put $|\xi_f| = 1$, and only one of $\mathrm{Re}(\xi_f)$ and $\mathrm{Im}(\xi_f)$ remains a free parameter [3].

- For δ being either purely real or purely imaginary: The expressions are straightforward. For example, if δ is purely real, there is no trigonometric dependence on the untagged rate.

- Finally, for $|\delta| \ll 1$: We can neglect terms higher than linear in either $\mathrm{Re}(\delta)$ or $\mathrm{Im}(\delta)$ in eq. (4.19). This is expected to be the case according to [82]. For example, the expression for the branching fraction for a one-amplitude process simplifies to

$$Br[f] = \frac{|A_f|^2}{2}\left[\frac{1}{\Gamma_s}\{2 - \mathrm{Im}(\delta)\mathrm{Im}(\xi_f)\} + \frac{\Gamma_s}{(\Delta m)^2 + (\Gamma_s)^2}\mathrm{Im}(\delta)\mathrm{Im}(\xi_f) + \frac{\Delta\Gamma_s}{(\Gamma_s)^2}\mathrm{Re}(\xi_f)\right. \tag{4.20}$$

[3] ξ_f is a SM quantity, so it is theoretically calculable, but it may also contain other new physics which is CPT conserving, so it is better to obtain both real and imaginary parts of ξ_f and check whether $|\xi_f| = 1$.

One can also tag the B mesons and define a tagged decay rate asymmetry

$$\begin{aligned}\Gamma_T[f,t] &= \Gamma(B_q(t)\to f) - \Gamma(\bar{B}_q(t)\to f)\\ &= |A_f|^2 e^{-\Gamma_q t}\Bigg[\left\{(1-|\xi_f|^2)\frac{(\mathrm{Re}(\delta))^2}{4} - \mathrm{Re}(\delta)\mathrm{Re}(\xi_f)\right\}\cosh\left(\frac{\Delta\Gamma_q t}{2}\right)\\ &\qquad + \left\{(1-|\xi_f|^2)(1-\frac{(\mathrm{Re}(\delta))^2}{4}) + \mathrm{Re}(\delta)\mathrm{Re}(\xi_f)\right\}\cos(\Delta m_q t)\\ &\qquad - \frac{1}{2}\mathrm{Re}(\delta)\left\{(1+|\xi_f|^2) - \mathrm{Im}(\delta)\mathrm{Im}(\xi_f)\right\}\sinh\left(\frac{\Delta\Gamma_q t}{2}\right) + \Big\{2\mathrm{Im}(\xi_f)\\ &\qquad - \frac{1}{2}\mathrm{Im}(\delta)(1+|\xi_f|^2) - \frac{1}{4}\mathrm{Im}(\xi_f)((\mathrm{Re}(\delta))^2 - (\mathrm{Im}(\delta))^2)\Big\}\sin(\Delta m_q t)\Bigg]\end{aligned} \tag{4.21}$$

Note that (i) for Re(δ)=Im(δ)=0, this reverts back to the SM expression, as it should, and (ii) If $|\delta| \ll 1$ and $\Delta\Gamma/\Gamma \ll 1$ as in the B_d system, the tagged rate can measure both Re(δ) and Im(δ).

For one-amplitude processes with $|\delta| \ll 1$, one may write a simplified expression:

$$\begin{aligned}\Gamma_U[f,t] &= |A_f|^2 e^{-\Gamma_q t}\Bigg[(2-\mathrm{Im}(\delta)\mathrm{Im}(\xi_f))\cosh\left(\frac{\Delta\Gamma_q t}{2}\right)\\ &\qquad + \mathrm{Im}(\delta)\mathrm{Im}(\xi_f)\cos(\Delta m_q t) + 2\mathrm{Re}(\xi_f)\sinh\left(\frac{\Delta\Gamma_q t}{2}\right)\Bigg],\\ \Gamma_T[f,t] &= |A_f|^2 e^{-\Gamma_q t}\Bigg[-\mathrm{Re}(\delta)\mathrm{Re}(\xi_f)\cosh\left(\frac{\Delta\Gamma_q t}{2}\right) + \mathrm{Re}(\delta)\mathrm{Re}(\xi_f)\cos(\Delta m_q t)\\ &\qquad - \mathrm{Re}(\delta)\sinh\left(\frac{\Delta\Gamma_q t}{2}\right) + \{2\mathrm{Im}(\xi_f) - \mathrm{Im}(\delta)\}\sin(\Delta m_q t)\Bigg].\end{aligned} \tag{4.22}$$

It is clear from eq. (4.22) how one can extract Re(δ) and Im(δ) by comparing the untagged and tagged analysis. Suppose we consider the B_s system where $\Delta\Gamma_s$ is non-negligible. The coefficient of the sinh term in Γ_T gives Re(δ). However, there is an overall normalization uncertainty given by $|A_f|^2$. To remove this, one can consider a combined study of the coefficients of $\sinh\left(\frac{\Delta\Gamma_s t}{2}\right)$ and $\cos(\Delta m_s t)$ from the untagged and tagged decay rates respectively; their ratio allows for a clean extraction of Re(δ). On the other hand, the ratio of the coefficients of $\cos(\Delta m_s t)$

in Γ_U and $\sin(\Delta m_s t)$ in Γ_T gives a clean measurement of $\mathrm{Im}(\delta)$, as $\mathrm{Im}(\xi_f)$ is known from the SM dynamics. A further check about the one-amplitude nature is provided from $|\mathrm{Re}(\xi_f)|^2 + |\mathrm{Im}(\xi_f)|^2 = 1$. In fact, as long as δ is small, one can extract both $\mathrm{Re}(\delta)$ and $\mathrm{Im}(\delta)$ even if $|\xi_f| \neq 1$, from the coefficients of the sine, cosine, and hyperbolic sine terms in Γ_U and Γ_T.

One may also define the time-dependent CPT asymmetry as

$$A_{CPT}(f,t) = \frac{\Gamma_T[f,t]}{\Gamma_U[f,t]} = \frac{\Gamma(B_q(t) \to f) - \Gamma(\bar{B}_q(t) \to f)}{\Gamma(B_q(t) \to f) + \Gamma(\bar{B}_q(t) \to f)}, \tag{4.23}$$

and the time-independent CPT asymmetry as

$$A_{CPT}(f) = \frac{\int_0^\infty dt\ \Gamma_T[f,t]}{\int_0^\infty dt\ \Gamma_U[f,t]} = \frac{\int_0^\infty dt\ [\Gamma(B_q(t) \to f) - \Gamma(\bar{B}_q(t) \to f)]}{\int_0^\infty dt\ [\Gamma(B_q(t) \to f) + \Gamma(\bar{B}_q(t) \to f)]}. \tag{4.24}$$

This goes to the usual CP asymmetry A_{CP} if $\delta = 0$; thus, any deviation from the expected CP asymmetry calculated from the SM would signal new physics, but one must check all the boxes to pinpoint the nature of the new physics. For example, there would not be any change in the semileptonic CP asymmetry if the new physics is only CPT violating in nature.

4.3 Analysis

There are five *a priori* unknowns: $\mathrm{Re}(\delta)$, $\mathrm{Im}(\delta)$, $\mathrm{Re}(\xi_f)$, $\mathrm{Im}(\xi_f)$, and $|A_f|^2$. For a one-amplitude process $|\xi_f|^2 = 1$ and the number of unknowns reduce to four. The tagged and untagged decay rates, the branching fraction, and the time-independent CPT asymmetry would provide information on all of these unknowns. Assuming the CPT-conserving physics to be purely that of the SM, one may calculate ξ_f following the CKM picture. In the analysis that follows, we take ξ_f to be known from the SM. We would like to point out the following features.

- The overall amplitude $|A_f|^2$ cancels in the CPT asymmetry. This, therefore, is going to be the observable one needs to measure most precisely.

- It is enough to measure the coefficients of the trigonometric terms only. For the B_d system, $\Delta\Gamma_d$ is small anyway, and for the B_s system, $\Delta\Gamma_s$ has a large theoretical uncertainty.

- The analysis holds even if the process under consideration is not a one-amplitude process. In fact, one may check whether there is a second CPT

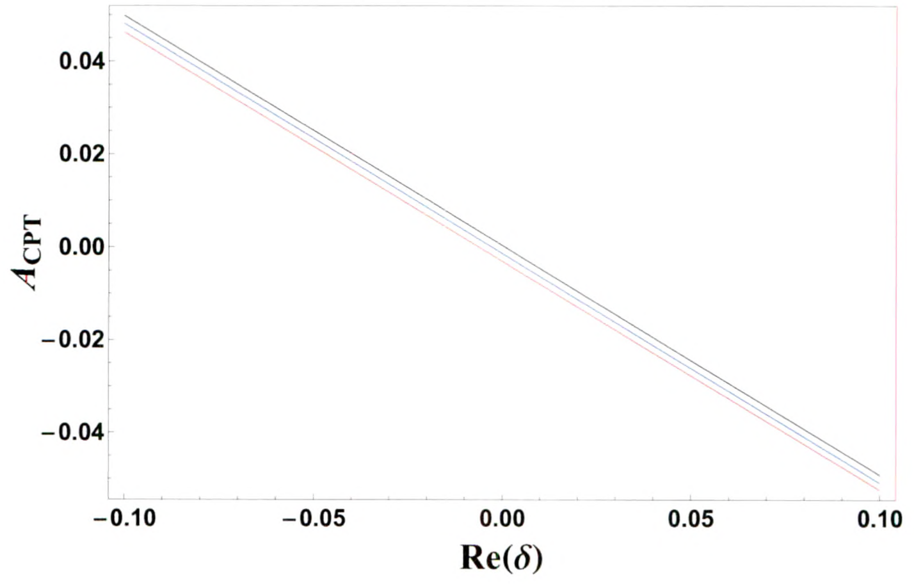

Figure 4.1: Variation of A_{CPT} with $\mathrm{Re}(\delta)$ for the B_s system. The three lines, from top to bottom, are for $\mathrm{Im}(\delta) = -0.1, 0$ and 0.1 respectively.

conserving new physics amplitude just by looking at the extracted values of $\mathrm{Re}(\xi_f)$ and $\mathrm{Im}(\xi_f)$.

- The coefficient of $\sin(\Delta m_q t)$ in the expression for the tagged decay rate Γ_T gives the mixing phase in the box diagram. Thus, $\mathrm{Im}(\delta)$ may be constrained by the CP asymmetry measurements in the B_d system. On the other hand, even those constrained values generate a large mixing phase for the B_s system compatible with the CDF data.

4.3.1 The B_s system

For the B_s system, we take

$$\Delta m_s = 17.77 \pm 0.12 \mathrm{ps}^{-1}\,, \;\; \Delta\Gamma_s = 0.096 \pm 0.039 \mathrm{ps}^{-1}\,, \;\; \frac{\Delta\Gamma_s}{\Gamma_s} = 0.147 \pm 0.060\,,$$
$$\frac{1}{\Gamma_s} = 1.530 \pm 0.009 \mathrm{ps}\,, \;\; \mathrm{Re}(\xi_f) = 0.99\,, \;\; \mathrm{Im}(\xi_f) = -0.04\,. \tag{4.25}$$

In figure 4.1, we show the variation of A_{CPT} with $\mathrm{Re}(\delta)$. For our analysis, we take both $|\mathrm{Re}(\delta)|, |\mathrm{Im}(\delta)| < 0.1$, which is consistent with [82]. The variation of

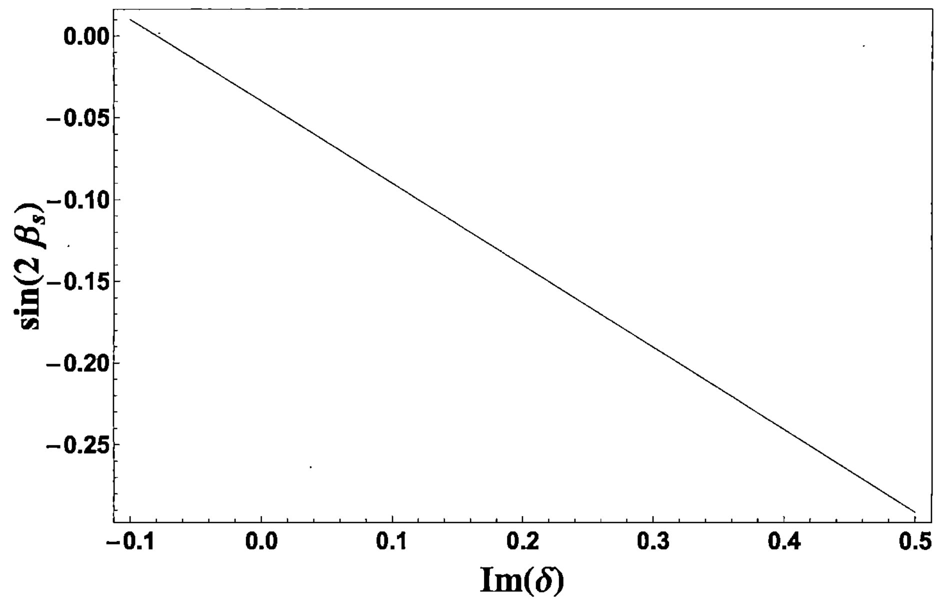

Figure 4.2: Variation of $\sin(2\beta_s)$ with $\mathrm{Im}(\delta)$.

A_{CPT} with Δm_s and $\Delta\Gamma_s$ is negligible, of the order of 0.2%, so we fix them to their respective central values. Effects of δ in both Δm_s and $\Delta\Gamma_s$ are quadratic in δ, and hence we can use the SM values for them. In fact, A_{CPT} does not depend significantly on the choice of $\mathrm{Im}(\delta)$ either; the variation is less than 1%. This is due to the fact that here, $|\mathrm{Im}(\xi_f)| \ll |\mathrm{Re}(\xi_f)|$ and hence the coefficient of $\mathrm{Re}(\delta)$ is much greater than the coefficient of $\mathrm{Im}(\delta)$ in the expression of A_{CPT}. This feature does not hold for the B_d system. Note that A_{CPT} clearly gives the sign of $\mathrm{Re}(\delta)$. The small nonzero value of A_{CPT} for $\delta = 0$ indicates the small mixing phase in the $B_s - \overline{B_s}$ box diagram. However, the apparent phase, i.e., the coefficient of $\sin(\Delta m_s t)$, can increase with $\mathrm{Im}(\delta)$, as can be seen from figure 4.2.

4.3.2 The B_d system

The inputs that we use for the B_d system are

$$\Delta m_d = 0.507\mathrm{ps}^{-1}\,,\ \Delta\Gamma_d = 0\,,\ \mathrm{Re}(\xi_f) = 0.72\,,\ \mathrm{Im}(\xi_f) = 0.695\,. \qquad (4.26)$$

This follows from the CKM expectation of $\sin(2\beta_d) = 0.695 \pm 0.020$. The constraint on δ comes from the measurement of $\sin(2\beta_d)$ in the $b \to c\bar{c}s$ channel:

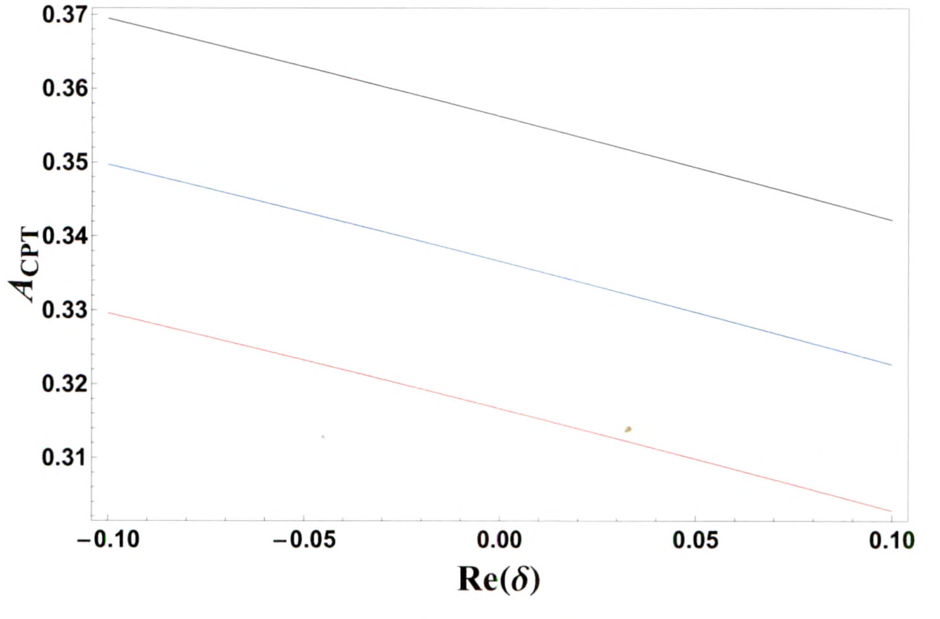

Figure 4.3: Variation of A_{CPT} with $\mathrm{Re}(\delta)$ for the B_d system. The three lines, from top to bottom, are for $\mathrm{Im}(\delta) = -0.1, 0$ and 0.1 respectively.

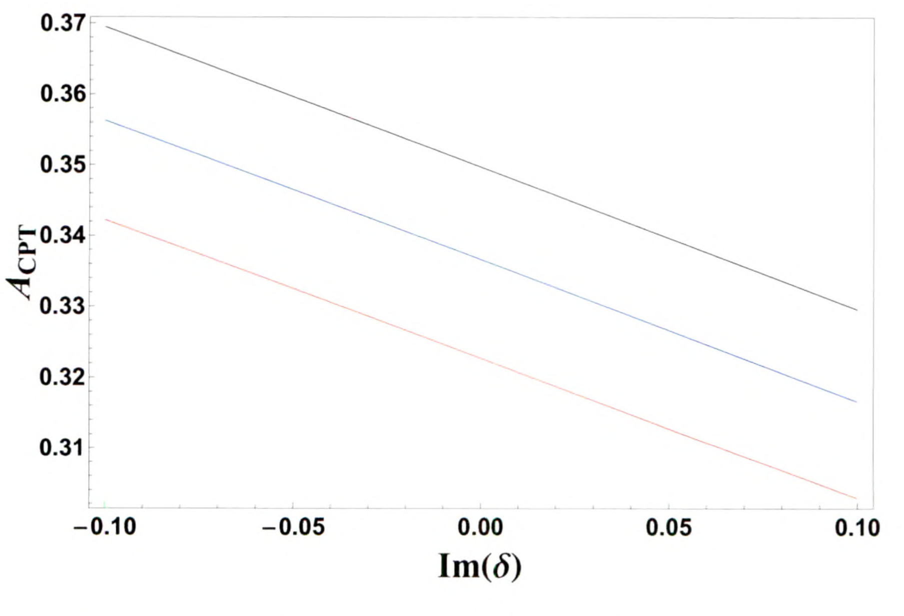

Figure 4.4: Variation of A_{CPT} with $\mathrm{Im}(\delta)$ for the B_d system. The three lines, from top to bottom, are for $\mathrm{Re}(\delta) = -0.1, 0$ and 0.1 respectively.

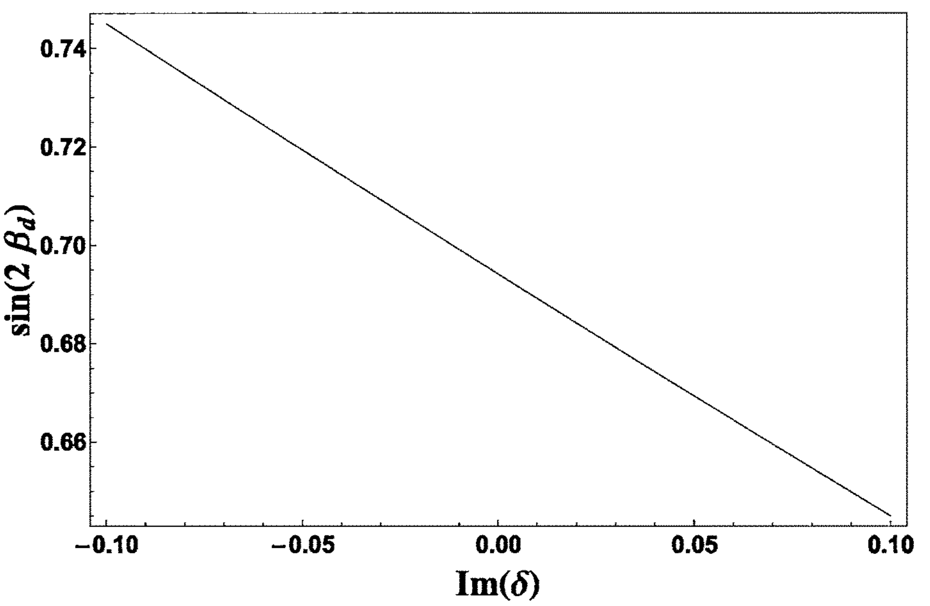

Figure 4.5: Variation of $\sin(2\beta_d)$ with $\mathrm{Im}(\delta)$.

0.668 ± 0.028 [76] [4]. Again, we can fix Δm_d at its central value. This time, due to the comparable values of $\mathrm{Re}(\xi_f)$ and $\mathrm{Im}(\xi_f)$, A_{CPT} is sensitive to both $\mathrm{Re}(\delta)$ and $\mathrm{Im}(\delta)$. The variations are shown in figure 4.3 for three values of $\mathrm{Im}(\delta)$ and figure 4.4 for three values of $\mathrm{Re}(\delta)$. It turns out that A_{CPT} is always positive for $\mathrm{Re}(\delta), \mathrm{Im}(\delta) < 1$; this is a consistency check for the CPT violation. Note that the measured value of $\sin(2\beta_d)$ can go down from its CKM expectation for $\mathrm{Im}(\delta) > 0$, in fact, for $\mathrm{Im}(\delta) \approx 0.07$, $\sin(2\beta_d) \approx 0.66$, as can be seen from figure 4.5. While this value of $\mathrm{Im}(\delta)$ generates a mixing phase for the B_s system that is consistent with the CDF and D0 measurements at 1σ, one must remember that δ need not be a flavor-blind parameter.

[4]We do not take the measurements coming from $b \to s$ penguin channels because of their inherent uncertainties.

4.4 Summary and conclusions

We have investigated the possibility of CPT violation in neutral B systems. CPT is a symmetry that is expected to be exact and the violation, even if it exists, should be quite small. However, it is possible to measure even a small CPT violation from the tagged and untagged decay rates of the neutral B mesons. In particular, for single-amplitude decay channels, the coefficients of the trigonometric terms $\sin(\Delta mt)$ and $\cos(\Delta mt)$ can effectively pinpoint the nature of the CPT violating parameter δ. This is an interesting possibility for the decays $B_s \to D_s^+ D_s^-$ and $B_S \to J/\psi\phi$ (with an angular analysis). Even a small CPT violation, allowed by the mixing constraints for the B_d system, can make the B_s mixing phase more compatible with the Tevatron measurements, at the level of about 1σ. On the other hand CPT violation should not affect the semileptonic CP asymmetries, as the corrections are quadratic in nature, and expected to be negligible for small δ. Thus, a correlated study of the CP asymmetries in $B_s \to J\psi\phi$ and $B_s \to D_s^+ D_s^-$ vis-a-vis $B_s \to D_s \ell\nu$ might be useful to pinpoint the CPT violating effects. This, we feel, is something that the experimentalists should look for in the coming years at the LHC.

Chapter 5

CPT-violating NP and anomaly in $B_S - \overline{B_S}$ mixing:

"The newest is but the oldest made visible to our senses."

- Henry David Thoreau,
A Week on the Concord and Merrimack Rivers
(1849)

Working in the framework of chapter (2), in this chapter we extend it to include possible CPT violation in the $B_s - \overline{B_s}$ mixing, parameterized through the difference in diagonal elements of $\mathcal{H}$. The motivation is to check if this can obviate the need for an absorptive contribution from the NP. Such an analysis to constrain CPT and Lorentz violating parameters was carried out in [33]. However they have used only A^b_{sl} and not $\beta_s^{J/\psi\phi}$ in their analysis, and their parameters are only indirectly connected to the elements of $\mathcal{H}$. We try to account for the two anomalies above with only CPT violation as the source of NP, and with a combination of CPT violation and the NP contribution to the off-diagonal elements of $\mathcal{H}$. As we will show, nothing improves the fit significantly from the SM unless there is a nonzero absorptive part in the $B_s - \overline{B_s}$ mixing amplitude.

5.1 CPT violation: the formalism

The analysis in chapter (2) is valid only if we assume CPT-invariance. However, the CPT symmetry may be violated in theories that break Lorentz invariance [40]. Indeed for local field theories, CPT violation requires Lorentz violation [41].

(This need not be true for nonlocal field theories as well as for theories with noncommutative space-time geometry, see [42].) In general, CPT violation should result in differences in masses and decay widths between particle-antiparticles pairs. However it may be easier to identify even through oscillation experiments, which typically are sensitive to an interference between the CPT-conserving and CPT-violating interactions.

While CPT violation in the K system is severely constrained through the mass difference between the neutral kaons [43], the bounds on the CPT violating parameters in the B_d and B_s systems are rather weak. In fact, the bounds for the B_d sector are about three orders of magnitude weaker than those for the K sector [44]. The bounds on Lorentz-violating parameters using the data on B mesons can be found in [33] and references therein. Here we use a model-independent parameterization, like the one earlier followed in [106] and recently used by two of us [45], and determine the preferred parameter space using the data on $B_s - \overline{B_s}$ oscillations. Unlike [33], we take both $A^b_{\rm sl}$ and $\beta_s^{J/\psi\phi}$ data into account.

The CPT violation manifests itself in the effective Hamiltonian through the difference in the diagonal elements. We write the effective Hamiltonian in eq. (2.1) as

$$\mathcal{H} = \begin{pmatrix} M_0 - \frac{i}{2}\Gamma_0 - \delta' & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M_0 - \frac{i}{2}\Gamma_0 + \delta' \end{pmatrix} , \tag{5.1}$$

and define the dimensionless CPT-violating complex parameter δ as

$$\delta \equiv \frac{H_{22} - H_{11}}{\sqrt{H_{12}H_{21}}} = \frac{2\delta'}{\sqrt{H_{12}H_{21}}} , \tag{5.2}$$

where $H_{ij} \equiv M_{ij} - \frac{i}{2}\Gamma_{ij}$.

The eigenvalues of $\mathcal{H}$ are

$$\lambda = \left(M_0 - \frac{i}{2}\Gamma_0\right) \pm \alpha y H_{12} , \tag{5.3}$$

where $\alpha \equiv \sqrt{H_{21}/H_{12}}$ and $y \equiv \sqrt{1 + \delta^2/4}$. The corresponding mass eigenstates are

$$\begin{aligned} |B_{sH}\rangle &= p_1 |B_s\rangle + q_1 |\overline{B}_s\rangle , \\ |B_{sL}\rangle &= p_2 |B_s\rangle - q_2 |\overline{B}_s\rangle , \end{aligned} \tag{5.4}$$

with $|p_1|^2 + |q_1|^2 = |p_2|^2 + |q_2|^2 = 1$, and

$$
\begin{aligned}
\eta_1 &\equiv \frac{q_1}{p_1} = \sqrt{\frac{H_{21}}{H_{12}}} \left(\sqrt{1 + \frac{\delta^2}{4}} + \frac{\delta}{2} \right) , \\
\eta_2 &\equiv \frac{q_2}{p_2} = \sqrt{\frac{H_{21}}{H_{12}}} \left(\sqrt{1 + \frac{\delta^2}{4}} - \frac{\delta}{2} \right) . \qquad (5.5)
\end{aligned}
$$

Clearly, CPT invariance corresponds to $\eta_1 = \eta_2$.

Let us now determine the dependence of our four observables on the CPT-violating parameters. The differences in masses and decay widths of the eigenstates are related to the difference in eigenvalues as

$$
\lambda_1 - \lambda_2 = \Delta M + \frac{i}{2} \Delta\Gamma , \qquad (5.6)
$$

where λ_1 and λ_2 are ordered such that $\mathrm{Re}(\lambda_1 - \lambda_2) > 0$. From eq. (5.3),

$$
\begin{aligned}
\Delta M &= M_1 - M_2 = 2\mathrm{Re}(\alpha y H_{12}) , \qquad (5.7) \\
\Delta\Gamma &= \Gamma_2 - \Gamma_1 = 4\mathrm{Im}(\alpha y H_{12}) . \qquad (5.8)
\end{aligned}
$$

Since $|\Gamma_{12}| \ll |M_{12}|$, we can write

$$
\begin{aligned}
\alpha H_{12} &= |M_{12}| \left[1 - \frac{1}{4} \frac{|\Gamma_{12}|^2}{|M_{12}|^2} - i\mathrm{Re}\left(\frac{\Gamma_{12}}{M_{12}} \right) \right]^{\frac{1}{2}} \\
&\approx |M_{12}| \left[1 - \frac{i}{2} \mathrm{Re}\left(\frac{\Gamma_{12}}{M_{12}} \right) \right] . \qquad (5.9)
\end{aligned}
$$

Then Eqs. (5.7) and (5.8) yield

$$
\begin{aligned}
\Delta M &\approx |M_{12}| \left[2\mathrm{Re}(y) + \mathrm{Im}(y)\mathrm{Re}\left(\frac{\Gamma_{12}}{M_{12}} \right) \right] , \qquad (5.10) \\
\Delta\Gamma &\approx |M_{12}| \left[4\mathrm{Im}(y) - 2\mathrm{Re}(y)\mathrm{Re}\left(\frac{\Gamma_{12}}{M_{12}} \right) \right] . \qquad (5.11)
\end{aligned}
$$

The dependence on the CPT-violating parameter δ appears entirely through y.

Let us pause here for a moment and find what the above two equations tell us about the allowed parameter space. Let us first focus on the best constraint, ΔM_s, and work in the limit where Γ_{12}/M_{12} is negligible. $|M_{12}|$, and hence M_{12}^{NP}, can be arbitrarily large, as $\mathrm{Re}(y)$ can be made arbitrarily small by an appropriate choice of δ. Similarly, $\mathrm{Re}(y)$ can be quite large (albeit compatible with other constraints) as long as there is a near-perfect cancellation between the SM and NP mixing amplitudes, making $|M_{12}|$ small. However, the smallness of $\Delta\Gamma/\Delta M$ constrains $\mathrm{Im}(y)/\mathrm{Re}(y)$ to be small, thus indicating that y is almost real. Since $y = \sqrt{1+\delta^2/4}$, this implies that δ^2 is almost real and $\mathrm{Re}(\delta^2) \gtrsim -4$. Therefore, one would expect that δ is either almost real, or it is almost imaginary, but with $|\mathrm{Im}(\delta)| < 2$.

Now let us consider the CP-violating observables $\beta_s^{J/\psi\phi}$ and $a_{\rm sl}^s$. The effective value of the former may be obtained in the presence of CPT violation by considering the decay rates of B_s and $\overline{B}_s$ to a final CP eigenstate f_{CP} as [45]:

$$
\begin{aligned}
\Gamma(B_s(t) \to f_{CP}) &= |A_f|^2 \big[|f_+(t)|^2 + |\xi_{f_1}|^2 |f_-(t)|^2 + \\
&\qquad 2\mathrm{Re}(\xi_{f_1} f_-(t) f_+^*(t)) \big] , \qquad (5.12) \\
\Gamma(\overline{B}_s(t) \to f_{CP}) &= |\frac{A_f}{\eta_2}|^2 \big[|f_-(t)|^2 + |\xi_{f_2}|^2 |\overline{f}_+(t)|^2 + \\
&\qquad 2\mathrm{Re}(\xi_{f_2} \overline{f}_+(t) f_-^*(t)) \big] , \qquad (5.13)
\end{aligned}
$$

with

$$
\xi_{f_1} \equiv \eta_1 \frac{\overline{A}_f}{A_f} , \quad \xi_{f_2} \equiv \eta_2 \frac{\overline{A}_f}{A_f} , \quad \omega \equiv \frac{\eta_1}{\eta_2} . \qquad (5.14)
$$

Here A_f and $\overline{A}_f$ are the amplitudes for the processes $B_s \to f_{CP}$ and $\overline{B}_s \to f_{CP}$, respectively. The time evolutions are given by

$$
\begin{aligned}
f_-(t) &= \frac{1}{1+\omega}(e^{-i\lambda_1 t} - e^{-i\lambda_2 t}) , \\
f_+(t) &= \frac{1}{1+\omega}(e^{-i\lambda_1 t} + \omega e^{-i\lambda_2 t}) , \\
\bar{f}_+(t) &= \frac{1}{1+\omega}(\omega e^{-i\lambda_1 t} + e^{-i\lambda_2 t}) . \qquad (5.15)
\end{aligned}
$$

The final state in $B_s \to J/\psi\phi$ is not a CP eigenstate, but a combination of CP-even and CP-odd final states, which may be separated using angular distributions. With the transversity angle distribution [46], the time-dependent decay rate to the CP-even state is given by the coefficient of $(1+\cos^2\theta)$, while the time-dependent decay rate to the CP-odd state is given by the coefficient of $\sin^2\theta$.

The value of effective $\beta_s^{J/\psi\phi}$ in this process is determined by writing the time evolutions (5.12) and (5.13) in the form

$$\begin{aligned}\Gamma(B_s(t) \to f_{CP}) &= c_1 \cosh(\Delta\Gamma_s t/2) + c_2 \sinh(\Delta\Gamma_s t/2) + \\ &\quad c_3 \cos(\Delta M_s t) + c_4 \sin(\Delta M_s t) , \qquad (5.16) \\ \Gamma(\overline{B}_s(t) \to f_{CP}) &= \bar{c}_1 \cosh(\Delta\Gamma_s t/2) + \bar{c}_2 \sinh(\Delta\Gamma_s t/2) + \\ &\quad \bar{c}_3 \cos(\Delta M_s t) + \bar{c}_4 \sin(\Delta M_s t) . \qquad (5.17)\end{aligned}$$

The direct CP violation in $B_s \to J/\psi\phi$ is negligible; i.e. $|\overline{A}_f/A_f| \approx 1$. Also, $|\Gamma_{12}/M_{12}| \ll 1$, so that in the absence of CPT violation, $|\eta_1| = |\eta_2| = 1$. Then in terms of $\xi_f \equiv \xi_{f_1} = \xi_{f_2} = \alpha\overline{A}_f/A_f$, one can write

$$\frac{c_4}{c_1} = -\frac{\bar{c}_4}{\bar{c}_1} = \frac{2\mathrm{Im}(\xi_f)}{1+|\xi_f|^2} \approx -\eta_{CP} \sin(2\beta_s^{J/\psi\phi}) , \qquad (5.18)$$

where η_{CP} is the CP eigenvalue of f_{CP}.

When CPT is violated, the effective phases $\beta_s^{J/\psi\phi}$ and $\bar{\beta}_s^{J/\psi\phi}$ measured through $B_s(t)$ and $\overline{B}_s(t)$ decays, respectively, will turn out to be different. Indeed, the difference between these effective phases will be a clean signal of CPT violation.

$$\begin{aligned}&\sin(2\beta_s^{J/\psi\varphi}) \\ &= -\eta_{CP}\frac{2[-\mathrm{Im}(\omega) - \mathrm{Re}(\xi_{f_1})\mathrm{Im}(\omega) + \mathrm{Im}(\xi_{f_1}) + \mathrm{Im}(\xi_{f_1})\mathrm{Re}(\omega)]}{[1+|\omega|^2 + 2|\xi_{f_1}|^2 + 2\mathrm{Re}(\xi_{f_1}) - 2\mathrm{Re}(\xi_{f_1})\mathrm{Re}(\omega) - 2\mathrm{Im}(\xi_{f_1})\mathrm{Im}(\omega)]} , \qquad (5.19)\end{aligned}$$

$$\begin{aligned}&\sin(2\bar{\beta}_s^{J/\psi\varphi}) \\ &= -\eta_{CP}\frac{2[-|\xi_{f_2}|^2\mathrm{Im}(\omega) + \mathrm{Re}(\xi_{f_2})\mathrm{Im}(\omega) + \mathrm{Im}(\xi_{f_2}) + \mathrm{Im}(\xi_{f_2})\mathrm{Re}(\omega)]}{[2+|\xi_{f_2}|^2(1+|\omega|^2) - 2\mathrm{Re}(\xi_{f_2}) + 2\mathrm{Re}(\xi_{f_2})\mathrm{Re}(\omega) - 2\mathrm{Im}(\xi_{f_2})\mathrm{Im}(\omega)]} , \qquad (5.20)\end{aligned}$$

Though the analysis of the B_s and $\overline{B}_s$ modes needs to be performed separately, here we assume identical detection and tagging efficiencies for both, and use the average of eq. (5.19) and eq. (5.20) for our fit.

The semileptonic CP asymmetry $a_{\rm sl}^s$ is measured through the "wrong-sign" lepton signal:

$$a_{\rm sl}^s = \frac{\Gamma(\overline{B}_s(t) \to \mu^+ X) - \Gamma(B_s(t) \to \mu^- X)}{\Gamma(\overline{B}_s(t) \to \mu^+ X) + \Gamma(B_s(t) \to \mu^- X)} . \qquad (5.21)$$

Here,

$$\begin{aligned} \Gamma(B_s(t) \to \mu^- X) &= |\eta_1 f_- A(B_s \to \mu^+ X)|^2 , && (5.22) \\ \Gamma(\overline{B}_s(t) \to \mu^+ X) &= |(f_-/\eta_2) A(\overline{B}_s \to \mu^+ X)|^2 , && (5.23) \end{aligned}$$

and since $|A(\overline{B}_s \to \mu^+ X)| = |A(B_s \to \mu^+ X)|$,

$$a_{\rm sl}^s = \frac{\frac{1}{|\eta_2|^2} - |\eta_1|^2}{\frac{1}{|\eta_2|^2} + |\eta_1|^2} = \frac{1-|\alpha|^4}{1+|\alpha|^4} , \qquad (5.24)$$

which is independent of the CPT-violating parameter δ. That the semileptonic asymmetry does not contain a CPT violating term in the leading order was also noted earlier [47].

5.2 CPTV: statistical analysis

In this Section, we perform a χ^2-fit to the observables ΔM_s, $\Delta\Gamma_s$, the effective phase $\beta_s^{J/\psi\phi}$, and $a_{\rm sl}^s$. Let us first assume that there is no CPT-conserving NP contribution coming from $M_{12}^{\rm NP}$ and $\Gamma_{12}^{\rm NP}$, so that the only relevant NP contribution is CPT violating, and is parameterized by Re(δ) and Im(δ). The allowed parameter space is shown in Fig. 5.1. It turns out that in this case, the value of $\chi^2_{\rm min}$ is $\approx$ 11.5 (at $\delta = 0.008 + 0.958\ i$ and $\delta = -0.024 + 0.958\ i$), marginally better than the one obtained in the ($\Gamma_{12}^{\rm NP} = 0, M_{12}^{\rm NP} \neq 0$) case discussed above in Fig. 2.4. There are some, albeit small, regions in the parameter space that are allowed to 3σ. However a fit good to 2σ or better is still not possible.

We therefore need to add the CPT-conserving NP to the CPT-violating contribution. However we have already seen in the preceding section that $M_{12}^{\rm NP}$ and $\Gamma_{12}^{\rm NP}$ together are capable of explaining the data by themselves. Therefore the fit using δ, $M_{12}^{\rm NP}$ as well as $\Gamma_{12}^{\rm NP}$ is redundant. With six independent parameters and only four observables, not only is $\chi^2_{min} = 0$ guaranteed, but no effective limits on CPT-conserving and CPT-violating parameters are generated.

We, therefore, go directly to the possibility where there is CPT-conserving NP, but without an absorptive part: $\Gamma_{12}^{\rm NP} = 0$. We have already observed (Fig. 2.4) that the entire region in the $|M_{12}^{\rm NP}| - {\rm Arg}(M_{12}^{\rm NP})$ is outside the 3σ region in such a scenario. We would now ask what happens if we enhance the two-parameter NP with two more CPT violating parameters, *viz.*, Re(δ) and Im(δ). This scenario is interesting because, as we have seen before, only very specific kind of NP can

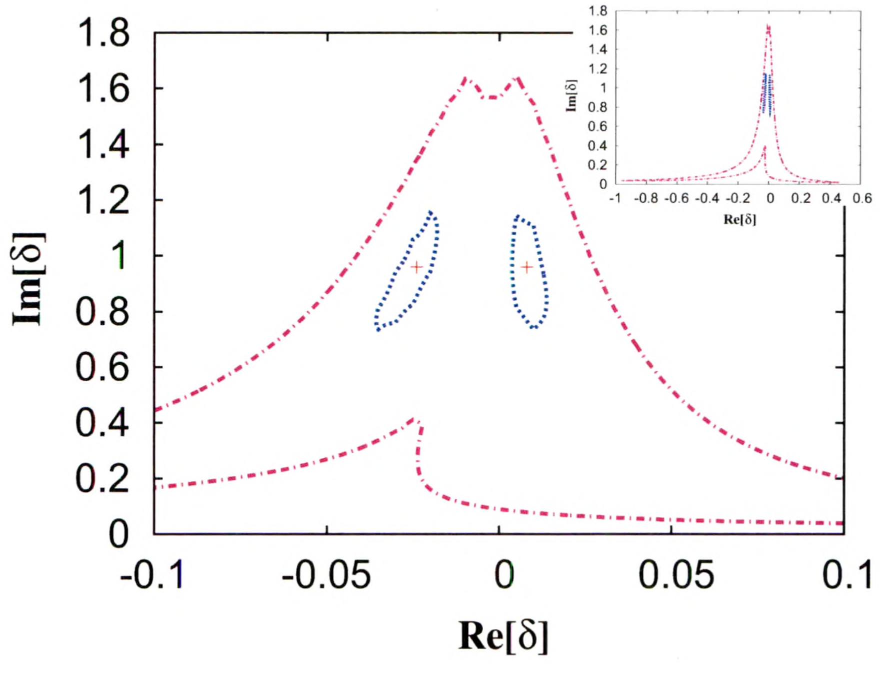

Figure 5.1: The 3σ (blue/dotted) and 4σ (pink/dash-dotted) goodness-of-fit contours in the $\text{Re}(\delta) - \text{Im}(\delta)$ plane, when the only relevant NP contribution is CPT violating, parameterized entirely by δ. There are no points that are allowed to within 2σ. The inset shows the complete allowed region, while the main figure shows the expanded form of the region of interest, where the favored parameter space is clearly visible. The crosses show the best fit points, with $\chi^2 = 11.5$.

contribute to $\Gamma_{12}^{\rm NP}$, which would be tested severely in near future. In case no evidence for the relevant NP is found (e.g. the branching ratio of $B_s \to \tau^+\tau^-$ is observed to be the same as its SM prediction), the next step would be to check if CPT violation, along with the NP contribution through $M_{12}^{\rm NP}$, would be able to account for the anomalies. For example, one may want to determine β_s and $\bar{\beta}_s$ of eqs. (5.19) and (5.20) separately and see whether they are different.

Fig. 5.2 shows the situation in the $|M_{12}^{\rm NP}| - {\rm Arg}(M_{12}^{\rm NP})$ plane. As compared to Fig. 2.4, one can see that once we marginalize over δ, we now have some regions allowed to within 3σ. Indeed, $\chi^2_{min} = 9.6$ at $M_{12}^{\rm NP} = 3.40 \exp(0\ i)$. This clearly does not improve the goodness-of-fit substantially, indicating that there is no good alternative for $\Gamma_{12}^{\rm NP}$.

Fig. 5.3 shows the situation in the complex δ plane, when $M_{12}^{\rm NP}$ has been marginalized over. One observes that the current data allows rather large (~ 1) positive values of ${\rm Im}(\delta)$ even at 3σ. The best-fit point corresponds to $\delta = 0.0037 + 1.40\ i$, which gives $\chi^2_{min} = 9.6$ as mentioned earlier. The CPT conserving point ($\delta = 0$) lies outside the 3σ region. As expected from the discussion in Sec. 5.1, the allowed values of δ are close to the ${\rm Re}(\delta)$ or ${\rm Im}(\delta)$ axis, with $|{\rm Im}(\delta)|$ restricted to 2.

5.3 Conclusion

We introduce the possibility of CPT violation by adding unequal NP contributions to the diagonal elements of $\mathcal{H}$. We explicitly show how CPT violation might affect the observables, especially dwelling on the effect on $\beta_s^{J/\psi\phi}$. Taken alone, the CPT violation cannot affect the dimuon asymmetry, and it can make the fit to the $B_s - \overline{B_s}$ mixing data only marginally better. In combination with a CPT conserving NP, it can enhance the allowed parameter space for that NP, however it does not seem to be able to obviate the need of an absorptive contribution from NP.

If the errors and uncertainties shrink keeping the central values more or less intact, this will mean:

Without any CPT-conserving NP, only CPT violation is only of marginal help, as it cannot enhance the semileptonic asymmetry. In combination with the CPT-conserving dispersive NP, however, it allows regions in the parameter space to better than 3σ.

To summarize, in the scenario that an absorptive NP contribution is ruled out,

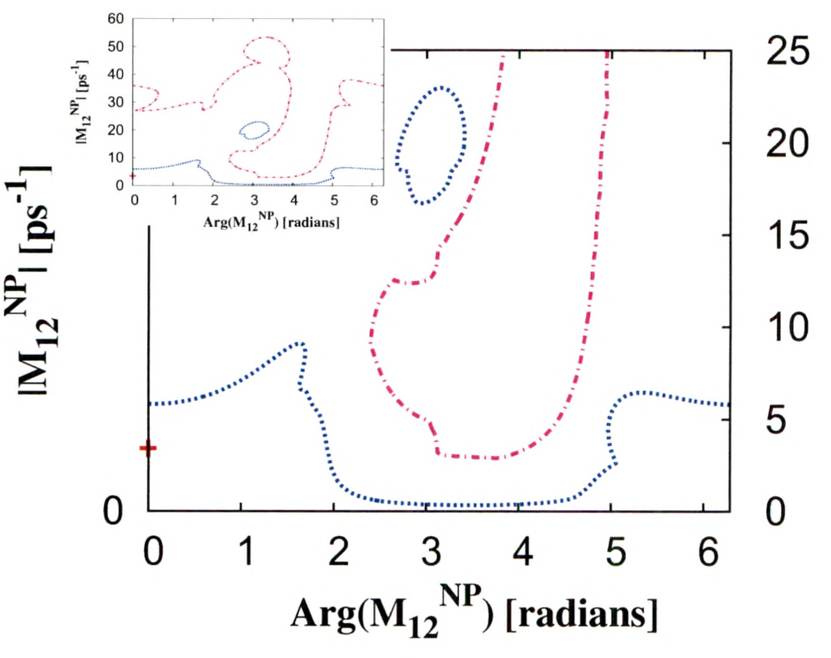

Figure 5.2: The 3σ (blue/dotted) and 4σ (pink/dot-dashed) goodness-of-fit contours in the $|M_{12}^{\rm NP}| - {\rm Arg}(M_{12}^{\rm NP})$ plane, when $\Gamma_{12}^{\rm NP} = 0$, i.e. NP does not contribute to the absorptive part of the effective hamiltonian. The CPT-violating complex parameter δ has been marginalized over. There are no points that are allowed to within 2σ. The inset shows the complete allowed region, while the main figure shows the expanded form of the region of interest, where the favored parameter space is clearly visible. The cross shows the best fit point, with $\chi^2 = 9.6$.

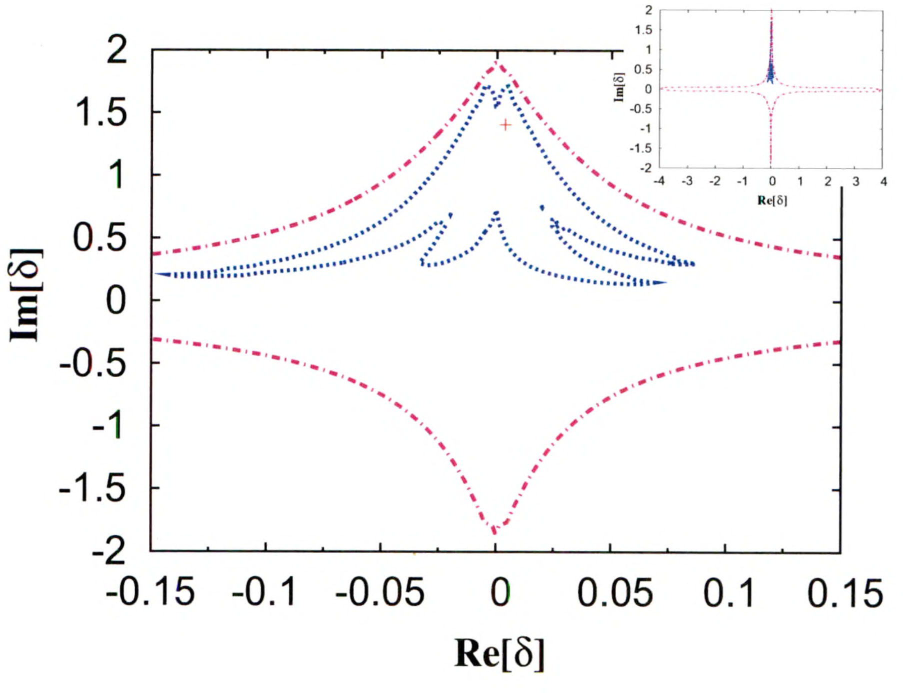

Figure 5.3: The 3σ (blue/dotted) and 4σ (pink/dot-dashed) goodness-of-fit contours in the $\mathrm{Re}(\delta) - \mathrm{Im}(\delta)$ plane, when the complex NP parameter M_{12}^{NP} is marginalized over, while $\Gamma_{12}^{\mathrm{NP}}$ has been constrained to vanish. There are no points that are allowed to within 2σ. The inset shows the complete allowed region, while the main figure shows the expanded form of the region of interest, where the favored parameter space is clearly visible. The cross shows the best fit point, with $\chi^2 = 9.6$.

one may have to resort to CPT violation in order to explain the data. A prominent signature of such a CPT violation would be a difference in $\beta_s^{J\psi/\phi}$ obtained from the tagged decays of B_s and $\overline{B}_s$.

Chapter 6

$B_S \to D_S K$ as a Probe of CPT Violation

"Extraordinary claims require extraordinary evidence."

- Carl Sagan,
Cosmos

In this chapter, we would like to investigate the signatures of CPT violation in the B_s system, both in $B_s - \overline{B_s}$ mixing and in B_s decays. We would like to emphasize that this is a model-independent approach in the sense that we do not specify any definite model that might lead to CPT violation; in fact, as far as we know, all studies on CPT violation are based on some phenomenological Lagrangian to start with.

As an illustrative example, we consider the non-leptonic $B_s(\bar{B}_s) \to D_s^+ K^-$ and $B_s(\bar{B}_s) \to D_s^- K^+$ decays. The $\bar{B}_s$ decays are mediated by color-allowed tree-level transitions $b \to u\bar{c}s$ and $b \to c\bar{u}s$. These are single-amplitude processes in the SM, so that any non-trivial contribution beyond the SM expectations, like direct CP asymmetry, is a clear signal of NP. This set of channels is also of interest as in the SM, both the amplitudes are of same order, $\mathcal{O}(\lambda^3)$ in the standard Wolfenstein parametrization of the CKM matrix (so that the event rates are comparable), and same final states can be reached both from B_s and $\bar{B}_s$. The importance of such modes to unveil any NP has already been emphasized; *e.g.*, see [84–87]. The decay was first observed by the CDF and the Belle collaborations [88, 89], and recently the LHCb collaboration has measured the branching ratio to be [90]

$$\mathrm{Br}(B_s \to D_s^{\mp} K^{\pm}) = (1.90 \pm 0.23) \times 10^{-4}$$

where the errors have been added in quadrature. We also note that flavor-specific NP in these channels is relatively unconstrained [29]. LHCb has also measured several time-dependent CP violating observables in $B_s \to D_s^{\mp} K^{\pm}$ using flavor-tagged and flavor-untagged observables [91].

Here we do a more general analysis considering both the CPT violating and CPT conserving NP contributions to $B_s - \overline{B_s}$ mixing. We show how one can construct combinations of observables coming from tagged and untagged decay rates that can unambiguously differentiate between CPT violating and CPT conserving NP models. On the other hand, if there is some CPTV contribution only to B_s decays, it might be difficult to differentiate it from CPT conserving NP in this approach. We define an observable which is useful to extract the CPT violating parameter in decay.

We will consider both these cases separately: first, when CPTV (or CPT conserving NP) is present only in the operators responsible for decay but not in those responsible for the mixing; and second, when the same is present also in the $B_s - \overline{B_s}$ mixing amplitude. As we will show explicitly, the extraction of CPTV in mixing is independent of the CPTV in decay and any other CPT conserving NP either in decay or mixing.

The first possibility of NP (including CPT violation) only in decay can arise if the NP operators are strongly flavor-dependent, like those in R-parity violating supersymmetry, or leptoquark models. As we are considering final states that can be accessed both from B_s and $\bar{B}_s$, any such NP will necessarily contribute in $B_s - \overline{B_s}$ mixing, in particular to its absorptive part, and will change the decay width difference $\Delta\Gamma_s$. Apart from the short-distance contributions to the absorptive part, there can be non-negligible long-distance effects too, coming from mesonic intermediate states [93]. However, the accuracy of the present data on $\Delta\Gamma_s$, the lifetime difference of two B_s mass eigenstates, is relatively weak. The most accurate result comes from the LHCb collaboration [94]: $\Delta\Gamma_s/\Gamma_s = 0.176 \pm 0.028$. Even the SM prediction [95] has a large uncertainty. Thus, as a first approximation, one can consider such NP effects only in decay and not in mixing, where it is in all probability sub-leading.

For the second case, one can construct several observables from the time-dependent tagged and untagged decay rates, and some of them are identically zero if there is no CPTV in mixing, irrespective of whether there is any CPTV in decay, or some CPT conserving NP.

The Belle Collaboration [83] places limits on the CPTV parameters in mixing, but no such limits exist for CPTV in decay. Also, the Belle limits are valid for the B_d system, but one can expect similar numbers for the B_s system too, even if CPTV is flavor-dependent. Like the experimental tests on CP-violation, various independent cross-checks on CPTV are also essential. Needless to say, one can play the same game with decays like $B_s \to D^0\phi$ and $B_s \to \bar{D}^0\phi$, and can form more observables (although not independent of the original ones) out of the CP-eigenstates of D^0 and $\bar{D}^0$ in the final state.

6.1 CPT violation in decay

6.1.1 $B_s - \overline{B_s}$ mixing and $B_s \to D_s^{\pm} K^{\mp}$ in the SM

The $B_s - \overline{B_s}$ mixing is controlled by the off-diagonal term $H_{12} = M_{12} - (i/2)\Gamma_{12}$ of the 2×2 Hamiltonian matrix, with the mass difference between two mass eigenstates B_H and B_L given by (in the limit $|\Gamma_{12}| \ll |M_{12}|$)

$$\Delta M_s \equiv M_{sH} - M_{sL} \approx 2|M_{12}|\,, \tag{6.1}$$

and the width difference by

$$\Delta\Gamma_s \equiv \Gamma_{sL} - \Gamma_{sH} \approx 2|\Gamma_{12}|\cos\phi_s\,, \tag{6.2}$$

where $\phi_s \equiv \arg(-M_{12}/\Gamma_{12})$. CPT conservation ensures $H_{11} = H_{22}$.

The eigenstates are defined as

$$|B_{H(L)}\rangle = p|B_s\rangle + (-)q|\bar{B}_s\rangle\,, \tag{6.3}$$

where $|p|^2 + |q|^2 = 1$ is the normalization, and one defines

$$\alpha \equiv q/p = \exp(-2\beta_s) \tag{6.4}$$

where $2\beta_s$ is the mixing phase of the $B_s - \overline{B_s}$ box diagram.

For the single-amplitude decays $B_s \to D_s^{\pm} K^{\mp}$, the amplitudes are of the form

$$\begin{aligned} A(B_s \to D_s^+ K^-) &= T_1 e^{i\gamma}, \qquad & A(B_s \to D_s^- K^+) &= T_2, \\ A(\bar{B}_s \to D_s^+ K^-) &= T_2, \qquad & A(\bar{B}_s \to D_s^- K^+) &= T_1 e^{-i\gamma}, \end{aligned} \tag{6.5}$$

where T_1 and T_2 are real amplitudes times the strong phase, which we parametrize as

$$\arg\left(\frac{T_1}{T_2}\right) = \Delta, \tag{6.6}$$

and $\gamma = \arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$, so that to a very good approximation, $V_{ub} \approx |V_{ub}|\exp(-i\gamma)$. The quantity $\xi_f \equiv \alpha \bar{A}_f / A_f$, where $A_f \equiv A(B_s \to D_s^+ K^-)$ and $\bar{A}_f \equiv A(\bar{B}_s \to D_s^+ K^-)$, carries a weak phase of $-(2\beta_s + \gamma)$.

Let us define, following [84],

$$\begin{aligned} \langle \mathrm{Br}(B_s \to D_s^+ K^-) \rangle &= \mathrm{Br}(B_s \to D_s^+ K^-) + \mathrm{Br}(\bar{B}_s \to D_s^+ K^-), \\ \langle \mathrm{Br}(B_s \to D_s^- K^+) \rangle &= \mathrm{Br}(B_s \to D_s^- K^+) + \mathrm{Br}(\bar{B}_s \to D_s^- K^+), \end{aligned} \tag{6.7}$$

so that these untagged rates are the same in the SM, even though a future measurement of the time-dependent branching fractions at the LHCb may show nonzero CP violation.

6.1.2 CPT violation in B_s decay

In order to take into account CPTV in decay, we parametrize various transition amplitudes for the decay $B_s \to D_s^{\pm} K^{\mp}$ as [96, 97]

$$\begin{aligned} A(B_s \to D_s^+ K^-) &= T_1 e^{i\gamma}(1 - y_f), \qquad & A(B_s \to D_s^- K^+) &= T_2\left(1 + y_f^*\right), \\ A(\bar{B}_s \to D_s^+ K^-) &= T_2(1 - y_f), \qquad & A(\bar{B}_s \to D_s^- K^+) &= T_1 e^{-i\gamma}\left(1 + y_f^*\right) \end{aligned} \tag{6.8}$$

where CPT violation (in decay) is parametrized by the complex parameter y_f, and y_f is real if T is conserved. The CPT violation is proportional to the difference $A(B_s \to D_s^+ K^-)^* - A(\bar{B}_s \to D_s^- K^+)$ or $A(\bar{B}_s \to D_s^+ K^-)^* - A(B_s \to D_s^- K^+)$.

We define the complete set of four relevant amplitudes, with $|f\rangle \equiv |D_s^+ K^-\rangle$ and $|\bar{f}\rangle \equiv |D_s^- K^+\rangle$,

$$\begin{aligned} A_f &= \langle f|H|B_s\rangle, \quad & A_{\bar{f}} &= \langle \bar{f}|H|B_s\rangle, \\ \bar{A}_f &= \langle f|H|\bar{B}_s\rangle, \quad & \bar{A}_{\bar{f}} &= \langle \bar{f}|H|\bar{B}_s\rangle, \end{aligned} \tag{6.9}$$

so that the ratios

$$\xi_f = \alpha \bar{A}_f / A_f\,, \quad \xi_{\bar{f}} = \alpha \bar{A}_{\bar{f}} / A_{\bar{f}}\,, \tag{6.10}$$

are independent of y_f; the CPTV effect in the decays cancels in the ratio. We also have $|\xi_f| = 1/|\xi_{\bar{f}}|$ and $\arg(\xi_{f(\bar{f})}) = -(2\beta_s + \gamma + (-)\Delta)$ where Δ is defined in eq. (6.6).

From eq. (6.8) we get

$$\begin{aligned} |A(B_s \to D_s^+ K^-)|^2 + |A(\bar{B}_s \to D_s^+ K^-)|^2 &= (|T_1|^2 + |T_2|^2)\, |1 - y_f|^2\,, \\ |A(B_s \to D_s^- K^+)|^2 + |A(\bar{B}_s \to D_s^- K^+)|^2 &= (|T_1|^2 + |T_2|^2)\, \left|1 + y_f^*\right|^2 \end{aligned} \tag{6.11}$$

Thus we can define an asymmetry

$$A_{br}^{CPT} = \frac{\langle \mathrm{Br}(B_s \to D_s^+ K^-)\rangle - \langle \mathrm{Br}(B_s \to D_s^- K^+)\rangle}{\langle \mathrm{Br}(B_s \to D_s^+ K^-)\rangle + \langle \mathrm{Br}(B_s \to D_s^- K^+)\rangle} \tag{6.12}$$

$$= -2\,\frac{\mathrm{Re}(y_f)}{1 + |y_f|^2} \approx -2\,\mathrm{Re}(y_f)\,, \text{ for } |y_f|^2 \ll 1 \tag{6.13}$$

We have already seen that this asymmetry is zero in the SM. Using eq. (6.13), the real part of the CPTV parameter y_f can be directly probed from the difference of the untagged rates (as the initial state B_s flavor is summed over) $\mathrm{Br}(B_s \to D_s^+ K^-)$ and $\mathrm{Br}(B_s \to D_s^- K^+)$.

One can have a rough idea of the LHCb reach in measuring $\mathrm{Re}(y_f)$. With 1 fb^{-1} of integrated luminosity, LHCb has obtained 1390 ± 98 events [91]. With full LHCb upgrade to an integrated luminosity of 50 fb^{-1}, total number of events should go up by a factor of about 200, as a twofold gain in the yield is expected when the LHC reaches $\sqrt{s} = 13$-14 TeV (as the cross section of $pp \to b\bar{b}X$ scales almost linearly with $\sqrt{s}$), and another twofold gain is expected in the trigger efficiency when the detector is upgraded. These 0.28 million events should be roughly equally divided between $D_s^+ K^-$ and $D_s^- K^+$. The advantage is that there is no need to tag the flavor of the initial B_s. The statistical fluctuation for each channel is about 375, and detection of CPT violation over such fluctuations results in a sensitivity of $375/140000 \approx 0.0027$ for $\mathrm{Re}(y_f)$. Note that LHCb already has a plan to measure CPT violation in the decay $B_d \to J/\psi[\to \pi^\mp \mu^\pm \nu(\bar{\nu})]K^0$ [92]. However, in this estimate we have only concerned ourselves with the statistical reach; we leave it to the experimentalists to address the systematic errors.

Let us compare this to a case where there is no CPT violation, but some CPT conserving NP is present which contributes to either $b \to u\bar{c}s$ or $b \to c\bar{u}s$

transitions, or maybe both. If this NP leads to observable CP violating effects, we can write the various amplitudes for the $B_s \to D_s^{\pm} K^{\mp}$ decays as

$$\begin{aligned}
A(B_s \to D_s^+ K^-) &= T_1 e^{i\gamma}\left(1 + a\; e^{i(\theta-\gamma+\sigma)}\right), \\
A(B_s \to D_s^- K^+) &= T_2\left(1 + a'\; e^{i(\theta'+\sigma')}\right), \\
A(\bar{B}_s \to D_s^+ K^-) &= T_2\left(1 + a'\; e^{-i(\theta'-\sigma')}\right), \\
A(\bar{B}_s \to D_s^- K^+) &= T_1 e^{-i\gamma}\left(1 + a\; e^{-i(\theta-\gamma-\sigma)}\right). \qquad (6.14)
\end{aligned}$$

The amplitudes, obviously, are related by CP conjugation. The NP is parametrized by the (relative) amplitudes a, a', the new weak phases θ, θ', and the new strong phase differences σ, σ'. Therefore, the asymmetry defined in eq. 6.13 is given by

$$A_{br}^{NP} = -2\frac{a\,|T_1|^2\sin(\theta-\gamma)\sin\sigma + a'\,|T_2|^2\sin\theta'\sin\sigma'}{|T_1|^2\left(1+a^2+2\,a\cos(\theta-\gamma)\cos\sigma\right) + |T_2|^2\left(1+a'^2+2\,a'\cos\theta'\cos\sigma'\right)}. \qquad (6.15)$$

Hence, a nonzero value of A_{br} could be due to either CPTV or CPT conserving NP (which, perhaps, is flavor-dependent, and definitely not of the minimal flavor violation type). As both the decays are color-allowed, one can even invoke the color-transparency argument [98] to claim that all strong phases are small; but CPTV effects are not expected to be large either.

eq. (6.13) is in general true for all decays which are either (i) single-amplitude in the SM, be it tree or penguin, or (ii) multi-amplitude in the SM but with one amplitude highly dominant over the others. Single-amplitude decays are preferred simply because any nonzero asymmetry as in eqs.~(6.13) or (6.15) can be unambiguously correlated with NP. The same observable A_{br}^{CPT} can be defined for charged B decays, or even D and K decays. However, in all cases, CPT conserving (but necessarily CP violating) NP can always mimic the asymmetry, unless there are strong motivations for the corresponding amplitudes to be highly subdominant, or the strong phase difference between the two amplitudes to be zero or vanishingly small.

On the other hand, if there is CPT violation in mixing too, this formalism does not hold, because the definition of the mass eigenstates also contains CPT violating parameters (see later). In that case, we suggest using single-amplitude charged B meson decay modes, like $B^+ \to D^0 K^+$ and $B^+ \to \overline{D}^0 K^+$.

If there is no other CPT conserving NP, but the $B_s - \overline{B_s}$ mixing matrix has CPTV built in, the asymmetry is still nonzero, as the individual branching fractions are functions of the CPTV parameter δ (see below) in the mixing matrix [45].

6.2 CPT violation in mixing

This section closely follows the formulation developed in [45], but let us quote some relevant expressions for completeness. CPT violation in the Hamiltonian matrix is introduced through the complex parameter δ:

$$\delta = \frac{H_{22} - H_{11}}{\sqrt{H_{12}H_{21}}}, \tag{6.16}$$

so that the Hamiltonian matrix looks like

$$\mathcal{H} = \left[\begin{pmatrix} M_0 - \mathrm{Re}(\delta') & M_{12} \\ M_{12}^* & M_0 + \mathrm{Re}(\delta') \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_0 + 2\mathrm{Im}(\delta') & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_0 - 2\mathrm{Im}(\delta') \end{pmatrix} \right], \tag{6.17}$$

where δ' is defined by

$$\delta = \frac{2\delta'}{\sqrt{H_{12}H_{21}}}. \tag{6.18}$$

One could even relax the assumption of $H_{21} = H_{12}^*$. However, there are two points that one must note. First, the effect of expressing $H_{12} = h_{12} + \bar{\delta}$, $H_{21} = h_{12}^* - \bar{\delta}$ appears as $\bar{\delta}^2$ in $\sqrt{H_{12}H_{21}}$, the relevant expression in eq. (6.16), and can be neglected if we assume $\bar{\delta}$ to be small. The second point, which is more important, is that CPT conservation constrains only the diagonal elements and puts no constraint whatsoever on the off-diagonal elements. It has been shown in [81] that $H_{12} \neq H_{21}^*$ leads to T violation, and only $H_{11} \neq H_{22}$ leads to unambiguous CPT violation. Thus, we will focus on the parametrization used in eqs. (6.16) and (6.17) to discuss the effects of CPT violation.

In the review on CPT violation in [9], the authors have used a formalism which is close to ours. While their treatment is for the K_S-K_L pair, this can be generalized to any neutral meson system. The mass eigenstates are defined as

$$|K_S(K_L)\rangle = \frac{1}{\sqrt{2(1 + |\epsilon_{s(L)}|^2)}} \left[(1 + \epsilon_{S(L)})|K^0\rangle + (1 - \epsilon_{S(L)})|\overline{K^0}\rangle\right] \tag{6.19}$$

where

$$\begin{aligned} \epsilon_{S(L)} &= \frac{-i\text{Im}(M_{12}) - \frac{1}{2}\text{Im}(\Gamma_{12}) \mp \frac{1}{2}\left[M_{11} - M_{22} - \frac{i}{2}(\Gamma_{11} - \Gamma_{22})\right]}{M_L - M_S + i(\Gamma_S - \Gamma_L)/2} \\ &\equiv \epsilon \pm \tilde{\delta}\,. \end{aligned} \tag{6.20}$$

Note that $\tilde{\delta}$ and δ are not the same, but related; both parametrize CPT violation. On the other hand, $\epsilon_{S(L)}$ is not truly a CPT conserving quantity, as the expression contains the mass and width differences of the two eigenstates, and both depend on the CPT violating parameter δ that we have used here.

The Belle collaboration [83] recently put stringent limits on the real and imaginary parts of δ,

$$\text{Re}(\delta_d) = (-3.8 \pm 9.9) \times 10^{-2}\,, \quad \text{Im}(\delta_d) = (1.14 \pm 0.93) \times 10^{-2}\,, \tag{6.21}$$

where we have added the errors in quadrature, and used the straightforward translation valid for small δ, *viz.*, $\delta = -2z$ (the subscript emphasizes that these results are for the B_d system). The CPT violating parameter z is defined as

$$|B_{L(H)}\rangle = p\sqrt{1-(+)z}|B_d\rangle + (-)q\sqrt{1+(-)z}|\bar{B}_d\rangle\,. \tag{6.22}$$

We can see that within the error bars data are consistent with no CPTV case i.e $\text{Re}(\delta_d) = \text{Im}(\delta_d) = 0$. However, more precise measurements are important and essential. In any case it is safe to assume $|\delta| \ll 1$, even for the B_s system. In ΔM_s and $\Delta\Gamma_s$ the CPT-violating effects are quadratic in δ and hence negligible.

We can write

$$|B_H\rangle = p_1|B_s\rangle + q_1|\bar{B}_s\rangle\,, \quad |B_L\rangle = p_2|B_s\rangle - q_2|\bar{B}_s\rangle\,. \tag{6.23}$$

with the normalization conditions $|p_1|^2 + |q_1|^2 = |p_2|^2 + |q_2|^2 = 1$, so that with CPT violation, $p_1 \neq p_2$ and $q_1 \neq q_2$. The time evolutions of B_H and B_L are controlled by $\lambda_1 \equiv m_1 - i\Gamma_1/2$ and $\lambda_2 \equiv m_2 - i\Gamma_2/2$ respectively. We also use

$$\Delta M_s = m_1 - m_2\,, \quad \Delta\Gamma_s = \Gamma_2 - \Gamma_1\,. \tag{6.24}$$

Let us define,

$$y = \sqrt{1+\frac{\delta^2}{4}}\,; \quad \eta_1 \equiv \frac{q_1}{p_1} = \left(y+\frac{\delta}{2}\right)\alpha\,; \quad \eta_2 \equiv \frac{q_2}{p_2} = \left(y-\frac{\delta}{2}\right)\alpha\,; \quad \omega = \frac{\eta_1}{\eta_2}\,, \tag{6.25}$$

where $\alpha = \sqrt{H_{21}/H_{12}}$. For $|\delta| \ll 1$, we can approximate y with unity.

The time-dependent flavor eigenstates are given by

$$\begin{aligned} |B_s(t)\rangle &= h_+(t)|B_s\rangle + \eta_1 h_-(t)|\bar{B}_s\rangle \\ |\bar{B}_s(t)\rangle &= \frac{h_-(t)}{\eta_2}|B_s\rangle + \bar{h}_+(t)|\bar{B}_s\rangle\,, \end{aligned} \tag{6.26}$$

where

$$\begin{aligned} h_-(t) &= \frac{1}{(1+\omega)}\left(e^{-i\lambda_1 t} - e^{-i\lambda_2 t}\right)\,, \\ h_+(t) &= \frac{1}{(1+\omega)}\left(e^{-i\lambda_1 t} + \omega e^{-i\lambda_2 t}\right)\,, \\ \bar{h}_+(t) &= \frac{1}{(1+\omega)}\left(\omega e^{-i\lambda_1 t} + e^{-i\lambda_2 t}\right)\,. \end{aligned} \tag{6.27}$$

and we refer the reader to [45] for detailed expressions. Note that in the absence of CPTV, $\eta_1 = \eta_2$, $\omega = 1$, and hence $h_+(t) = \bar{h}_+(t)$. In the limit $|\delta| \ll 1$, $\omega \approx 1+\delta$.

With our convention of $|f\rangle \equiv |D_s^+ K^-\rangle$ and $|\bar{f}\rangle \equiv |D_s^- K^+\rangle$, where both the states are directly accessible to B_s and $\bar{B}_s$, the time dependent decay rates are [45]

$$\begin{aligned} \Gamma(B_s(t)\to f) &= \left[|h_+(t)|^2 + |\xi_{f_1}|^2|h_-(t)|^2 + 2\mathrm{Re}\left(\xi_{f_1} h_-(t) h_+^*(t)\right)\right]|A_f|^2\,, \\ \Gamma(\bar{B}_s(t)\to f) &= \left[|h_-(t)|^2 + |\xi_{f_2}|^2|\bar{h}_+(t)|^2 + 2\mathrm{Re}\left(\xi_{f_2} \bar{h}_+(t) h_-^*(t)\right)\right]\left|\frac{A_f}{\eta_2}\right|^2\,, \\ \Gamma(B_s(t)\to \bar{f}) &= \left[|h_+(t)|^2 + |\xi'_{f_1}|^2|h_-(t)|^2 + 2\mathrm{Re}\left(\xi'_{f_1} h_-(t) h_+^*(t)\right)\right]|A_{\bar{f}}|^2\,, \\ \Gamma(\bar{B}_s(t)\to \bar{f}) &= \left[|h_-(t)|^2 + |\xi'_{f_2}|^2|\bar{h}_+(t)|^2 + 2\mathrm{Re}\left(\xi'_{f_2} \bar{h}_+(t) h_-^*(t)\right)\right]\left|\frac{A_{\bar{f}}}{\eta_2}\right|^2 \end{aligned} \tag{6.28}$$

where,

$$\begin{aligned} \xi_{f_1} &= \eta_1\frac{\bar{A}_f}{A_f} = \left(1+\frac{\delta}{2}\right)\xi_f\,, \quad \xi_{f_2} = \eta_2\frac{\bar{A}_f}{A_f} = \left(1-\frac{\delta}{2}\right)\xi_f\,, \\ \xi'_{f_1} &= \eta_1\frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}} = \left(1+\frac{\delta}{2}\right)\xi_{\bar{f}}\,, \quad \xi'_{f_2} = \eta_2\frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}} = \left(1-\frac{\delta}{2}\right)\xi_{\bar{f}}\,. \end{aligned} \tag{6.29}$$

Dropping terms $\mathcal{O}(\delta^2)$ or higher, we get the following expressions for the tagged and untagged time-dependent decay rates:

$$\begin{aligned}\Gamma(B_s(t) \to f) - \Gamma(\bar{B}_s(t) \to f) &= [P_1 \sinh(\Delta\Gamma_s t/2) + Q_1 \cosh(\Delta\Gamma_s t/2) \\ &\quad + R_1 \cos(\Delta M_s t) + S_1 \sin(\Delta M_s t)]\, e^{-\Gamma_s t} |A_f|^2\,, \\ \Gamma(B_s(t) \to f) + \Gamma(\bar{B}_s(t) \to f) &= [P_2 \sinh(\Delta\Gamma_s t/2) + Q_2 \cosh(\Delta\Gamma_s t/2) \\ &\quad + R_2 \cos(\Delta M_s t) + S_2 \sin(\Delta M_s t)]\, e^{-\Gamma_s t} |A_f|^2\,,\end{aligned} \tag{6.30}$$

with

$$\begin{aligned}P_1 &= -\frac{1}{2} Re(\delta) \left(1 + |\xi_f|^2\right)\,, \\ Q_1 &= -|\xi_f| \cos(\gamma + 2\beta_s + \Delta)\, \mathrm{Re}(\delta)\,, \\ R_1 &= 1 - |\xi_f|^2 + |\xi_f| \cos(\gamma + 2\beta_s + \Delta) \mathrm{Re}(\delta)\,, \\ S_1 &= 2\,|\xi_f| \sin(\gamma + 2\beta_s + \Delta) - \frac{1}{2}\,\mathrm{Im}(\delta) \left(1 + |\xi_f|^2\right)\,, \\ P_2 &= 2\,|\xi_f| \cos(\gamma + 2\beta_s + \Delta) - \frac{1}{2}\,\mathrm{Re}(\delta) \left(1 - |\xi_f|^2\right)\,, \\ Q_2 &= 1 + |\xi_f|^2 - |\xi_f| \sin(\gamma + 2\beta_s + \Delta) \mathrm{Im}(\delta)\,, \\ R_2 &= |\xi_f| \sin(\gamma + 2\beta_s + \Delta) Im(\delta)\,, \\ S_2 &= -\frac{1}{2}\,\mathrm{Im}(\delta) \left(1 - |\xi_f|^2\right)\,.\end{aligned} \tag{6.31}$$

It is clear from eq. (6.31) that CPT violating effects in decay will not affect the determination of these 8 coefficients. Whatever the effects are, they will be lumped in the overall normalization $|A_f|^2$ and will not appear in the coefficients of the trigonometric and hyperbolic functions.

All the 8 coefficients can theoretically be extracted from a fit to the time-dependent decay rates, but admittedly the coefficients of the hyperbolic functions are harder to extract and need more statistics. The coefficients P_1 - S_1 are to be extracted from the tagged measurements, and P_2 - S_2 from untagged measurements. Note that whether or not any CPT-conserving NP is present, absence of CPT violation definitely means $\delta = 0$, so $P_1 = Q_1 = R_2 = S_2 = 0$. If any of these four observables are found to be nonzero, that is a sure signal of CPT violation. (While P_1 and S_2 depend only on δ, Q_1 and R_2 also have an implicit dependence on the $B_s - \overline{B_s}$ mixing phase $2\beta_s$, which might depend on CPT conserving NP effects.) Therefore, if CPT is conserved, the tagged measurements are sensitive only to the trigonometric functions, and the untagged measurements only to the hyperbolic functions, but we urge our experimental colleagues to perform a complete fit.

If at least P_1 or S_2 be nonzero (maybe with nonzero Q_1 and R_2), one gets

$$\mathrm{Im}(\delta) = -\frac{2S_2}{R_1 + Q_1}, \qquad \mathrm{Re}(\delta) = -\frac{2P_1}{R_2 + Q_2}, \tag{6.32}$$

which is theoretically clean, *i.e.* free from hadronic uncertainties. The overall normalization can be extracted from the CP averaged branching fractions.

Even if the experiment is not sensitive enough to extract unambiguously nonzero values of P_1, Q_1, R_2, or S_2, one can still find signals of CPTV, from the fact that P_2, Q_2, R_1, and S_1 contain CPTV terms over and above CPT conserving but CP violating terms. For example, one can extract the following analogous quantities from the tagged and untagged $B_s \to \bar{f}$ decays:

$$\begin{aligned}
\bar{P}_2 &= 2\,|\xi_f|\,\cos(\gamma + 2\beta_s - \Delta) + \frac{1}{2}\,\mathrm{Re}(\delta)\left(1 - |\xi_f|^2\right), \\
\bar{Q}_2 &= 1 + |\xi_f|^2 - |\xi_f|\,\sin(\gamma + 2\beta_s - \Delta)\mathrm{Im}(\delta), \\
\bar{R}_1 &= -1 + |\xi_f|^2 + |\xi_f|\,\cos(\gamma + 2\beta_s - \Delta)\mathrm{Re}(\delta), \\
\bar{S}_1 &= 2\,|\xi_f|\,\sin(\gamma + 2\beta_s - \Delta) - \frac{1}{2}\,\mathrm{Im}(\delta)\left(1 + |\xi_f|^2\right).
\end{aligned} \tag{6.33}$$

It is easy to derive eq. (6.33) from eq. (6.31). First, note that the relevant expressions contain $|A_{\bar{f}}|$ and $\xi_{\bar{f}}$. eq. (6.33) follows when one substitutes $|\xi_{\bar{f}}| = 1/|\xi_f|$ and $|A_{\bar{f}}|^2/|\xi_f|^2 = |A_f|^2$. However, the strong phase changes sign because of the definitions of ξ_f and $\xi_{\bar{f}}$.

Therefore, from eqs. (6.31) and (6.33) we can define observables which are only sensitive to the CPTV effect independent of any other NP effects in mixing,

$$\frac{R_1 + \bar{R}_1}{P_2 + \bar{P}_2} = \frac{\mathrm{Re}(\delta)}{2}, \qquad \frac{Q_2 - \bar{Q}_2}{S_1 - \bar{S}_1} = \frac{\mathrm{Im}(\delta)}{2}. \tag{6.34}$$

From eq. (6.34) we note that it is possible to probe the CPTV parameter δ even in the presence of any other generic NP in mixing or decays (which modifies $2\beta_s$); the NP effects in mixing are canceled in the ratio. In addition we note that the strong phase is also exactly canceled in the ratio, hence the measurement of δ is free from hadronic uncertainties.

LHCb performs the decay profile fit assuming CPT invariance [91], so it is not easy to predict the reach for the new CPT violating parameters, or even the CPT conserving ones. For this we need a full fit, assuming the possibility of CPT violation. Still, one can try to have an estimate of the reach. As there exists no measurement on the CPT violating parameters, let us use the first relation of eq. (6.34). The parameter R_1 (called C in [91]) has an error of about 56% right now; if the data sample increases by a factor of 200, this might come down to 4%. The same is true for $\bar{R}_1$, which should be measured independently (the central value, in the absence of CPT violation, should be equal and opposite to that of R_1). Thus the total uncertainty, added in quadrature, should be about 6%. Similarly, the uncertainty in the denominator should be about 6%, and is to be added in quadrature with the numerator. Thus, $\mathrm{Re}(\delta) \geq 0.16$ should be measurable using this relationship. Of course, we expect a much better reach with a full 4-parameter fit to each decay profile.

We reiterate that even if CPTV is present in decay, the conclusion that a nonzero value of any one of the four observables P_1, Q_1, R_2, or S_2 indicates CPTV in mixing remains valid. Consider the expressions for the tagged and untagged decay rates, eq. (6.30). With enough statistics, one gets the coefficients of the trigonometric and the hyperbolic functions, as well as the overall normalization $|A_f|^2$. If CPTV is present in decay, the expression for $|A_f|^2$ will change and be a function of y_f, but the eight coefficients of eq. (6.30) will remain the same.

The same method is applicable to decays like $B_s \to D_0\phi$, with $\bar{b} \to \bar{c}u\bar{s}$ and $\bar{b} \to \bar{u}c\bar{s}$ transitions.

6.3 Summary and conclusions

While the effects of CPT violation are severely constrained for systems with first and/or second generation fermions, the B systems, in particular B_s, are relatively less constrained. This opens up the possibility of a CPT violating action that is flavor-dependent. As a typical example of the effects of CPT violation, we consider the decays $B_s, \bar{B}_s \to D_s^{\pm} K^{\mp}$. These decays are excellent probes of any NP; in the SM, they are single-amplitude processes, and both $B_s \to D_s^+ K^-$ and $B_s \to D_s^- K^+$ amplitudes are of the same order in Wolfenstein parametrization. Thus, the number of events for both $D_s^+ K^-$ and $D_s^- K^+$, summing over parents B_s and $\bar{B}_s$, should be the same in the SM. We show how this asymmetry becomes nonzero if there is CPT violation in the decay.

At the same time, we see that if there is some NP that conserves CPT but comes with different strong and weak phases from the corresponding SM amplitude, the asymmetry is again nonzero. So, while this asymmetry serves as an excellent indicator of any NP, it might be either CPT conserving (but necessarily CP violating) or CPT violating, and further checks are necessary.

The situation is far better if there is CPT violation in mixing. The best way to put CPTV in mixing is to make the diagonal terms of the 2×2 mixing Hamiltonian unequal. With this, the CPTV parameter enters the definition of the mass eigenstates, and through that, to various time-dependent decay rates. With sufficient statistics, one can extract the coefficients of the trigonometric and hyperbolic terms of both tagged and untagged time-dependent rates. We find that there are four coefficients which are zero not only in the SM but also any extension with CPT conservation, so any nonzero value for any of them is a definite indication for CPT violation. There are several ways to extract these coefficients, and LHCb should have enough statistics to be able to measure them with sufficient precision. The argument goes through even if CPTV is present in both decay and mixing; this is because different sets of observables are extracted for the two different cases.

Chapter 7

CPT violation and triple-product correlations in B decays

"All animals are equal,
But some animals are
More equal than others!"
- G. Orwell, Animal Farm

Triple product (TP) correlations are known to be a good probe of CP violation in B decays [100–104]. Consider a B meson decaying into two vector mesons V_1 and V_2:

$$B(p) \to V_1(k_1, \varepsilon_1) + V_2(k_2, \varepsilon_2)\,,$$

where k and ε are respectively the four-momentum and polarization of the vector mesons. Suppose one constructs an observable $\alpha \equiv \vec{k}_1.(\vec{\varepsilon}_1 \times \vec{\varepsilon}_2)$, where we have taken out the spatial components of the respective four-vectors. The asymmetry

$$\frac{\Gamma(\alpha > 0) - \Gamma(\alpha < 0)}{\Gamma(\alpha > 0) + \Gamma(\alpha < 0)}$$

is odd under the time-reversal operator T as α itself is T-odd. As CPT is supposed to be a good symmetry of the Hamiltonian, the asymmetry is CP-odd too, and can be taken as a probe and measure of CP violation.

TP asymmetries are also an excellent probe of new physics (NP) beyond the Standard Model (SM). There are many TP asymmetries which are either zero or tiny in the SM but can go up to observable range under some new physics (NP) dynamics. Also, true TP asymmetries, unlike direct CP asymmetries, are nonzero even if the strong phase difference between two competing amplitudes is small or even zero. Of course, TP asymmetries can be faked by a sizable strong phase difference. The authors of Ref. [103] have discussed in detail the conditions for observation of TP asymmetries, and also the feasibility of measuring such asymmetries for different decay channels. The analysis has been extended by the authors of Ref. [104] for 4-body final states.

A crucial ingredient of extracting CP-violating signals from TP asymmetries is the CPT theorem: the combined discrete symmetry CPT, taken in any order, is an exact symmetry of any local axiomatic quantum field theory (QFT) [4]. Experiments have put stringent limits on CPT violation (CPTV), as all tests performed so far to probe CPTV yielded null results. Still, one should try to measure CPTV in B systems in as many ways as possible, irrespective of the theoretical dogma, as CPTV can be a flavor-dependent phenomenon, and the constraints obtained from the K system [105] may not be applicable to the B systems. One might also want to know whether any tension between data and the SM expectation is due to CPT conserving canonical NP, or or just due to CPTV.

The issue of CPTV has started to receive significant attention due to the growing phenomenological importance of CPTV scenarios in neutrino physics and cosmology [80]. A comprehensive study of CPTV in the neutral K meson system, with a formulation that is closely analogous to that in the B system, may be found in Ref. [81]. CPTV in the B systems, and its possible signatures, including differentiation from CPT conserving NP models, have been already investigated by several authors [61, 106? –109]. It was shown that the lifetime difference of the two mass eigenstates, or the direct CP asymmetries and semileptonic observables, may be affected by such new physics. The experimental limits are set by both BaBar, who looked for diurnal variations of CP-violating observables [82], and Belle, who looked for lifetime difference of B_d mass eigenstates [83]. This makes it worthwhile to look for possible CPTV effects in the B_s system (by B_s we generically mean both B_s and $\bar{B}_s$ mesons).

In this chapter, we would like to develop the formalism of TP asymmetries with possible CPTV terms in the Lagrangian. Thus, T violation and CP violation are no longer correlated. We will show, in detail, how and where deviations occur from the standard CPT conserving cases. In particular, it will be shown that some decay channels where TP asymmetries are not expected might throw up new surprises[99]. We will also relate the TP violating observables with the transversity amplitudes [103], and discuss the implications of the LHCb results [110] on $B_s \to$

$\phi\phi$.

At this point, we note that violations of different conservation rules lead to different signals. For example, violation of $\Delta B = \Delta Q$ keeping CPT invariant would lead to some interesting time-integrated dilepton asymmetries [111]. While a systematic study of the inverse problem, (i.e. going from the signal to the underlying model) in the B sector is worthwhile, it is outside the ambit of this chapter. We would like to refer the reader to [109] for ways to differentiate between CPT conserving and CPT violating NP under certain conditions; such a differentiation is not always possible.

The chapter is arranged as follows. In Section II, we discuss the essential formalism of TP asymmetries when CPTV terms are present in the decay amplitudes. In Section III, we show how the transversity amplitudes are modified by the CPTV terms. Section IV is devoted to the case where CPTV terms are present in the neutral B meson mixing Hamiltonian but not in the subsequent decay processes. In Section V, we correlate the expressions with the data from LHCb. In Section VI, we summarize and conclude. Some calculational details and a compendium of relevant expressions, not strictly necessary to catch the main flow of the chapter, have been relegated to the two appendices.

7.1 Formalism

Following Ref. [103], we can write the decay amplitude for $B(p) \to V_1(k_1, \varepsilon_1) + V_2(k_2, \varepsilon_2)$ as

$$M = \mathbf{a}\mathcal{S} + \mathbf{b}\mathcal{D} + i\mathbf{c}\mathcal{P} = \mathbf{a}\,\varepsilon_1^* \cdot \varepsilon_2^* + \frac{\mathbf{b}}{m_B^2}(p \cdot \varepsilon_1^*)(p \cdot \varepsilon_2^*) + i\frac{\mathbf{c}}{m_B^2}\epsilon_{\mu\nu\rho\sigma}p^\mu q^\nu \varepsilon_1^{*\rho}\varepsilon_2^{*\sigma} \ , \quad (7.1)$$

where $q \equiv k_1 - k_2$. Terms are normalized with a factor m_B^2, so that each of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ is expected to be of the same order of magnitude. The $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ terms correspond to combinations of s, d and p-wave amplitudes for the final state, denoted by $\mathcal{S}$, $\mathcal{D}$, and $\mathcal{P}$ respectively. The quantities $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are complex and will in general contain both CP-conserving strong phases and CP-violating weak phases.

Similarly, the amplitude for the CP-conjugate process $\bar{B}(p) \to \bar{V}_1(k_1, \varepsilon_1) + \bar{V}_2(k_2, \varepsilon_2)$ can be expressed as:

$$\overline{M} = \bar{\mathbf{a}}\,\varepsilon_1^* \cdot \varepsilon_2^* + \frac{\bar{\mathbf{b}}}{m_B^2}(p \cdot \varepsilon_1^*)(p \cdot \varepsilon_2^*) - i\frac{\bar{\mathbf{c}}}{m_B^2}\epsilon_{\mu\nu\rho\sigma}p^\mu q^\nu \varepsilon_1^{*\rho}\varepsilon_2^{*\sigma} \ , \quad (7.2)$$

where, considering CPT conservation, $\bar{\mathbf{a}}$, $\bar{\mathbf{b}}$ and $\bar{\mathbf{c}}$ can be obtained from $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ by changing the sign of the weak phases.

In that case, one can write

$$\begin{aligned} \mathbf{a} &= \sum_i a_i e^{i\phi_i^a} e^{i\zeta_i^a} \; , & \bar{\mathbf{a}} &= \sum_i a_i e^{-i\phi_i^a} e^{i\zeta_i^a} \; , \\ \mathbf{b} &= \sum_i b_i e^{i\phi_i^b} e^{i\zeta_i^b} \; , & \bar{\mathbf{b}} &= \sum_i b_i e^{-i\phi_i^b} e^{i\zeta_i^b} \; , \\ \mathbf{c} &= \sum_i c_i e^{i\phi_i^c} e^{i\zeta_i^c} \; , & \bar{\mathbf{c}} &= \sum_i c_i e^{-i\phi_i^c} e^{i\zeta_i^c} \end{aligned} \tag{7.3}$$

where $\phi_i^{a,b,c}$ ($\zeta_i^{a,b,c}$) are weak (strong) phases of the respective amplitudes. The relevant quantities for true T-violating TP asymmetries are $[\mathrm{Im}(\mathbf{a}\mathbf{c}^*) - \mathrm{Im}(\bar{\mathbf{a}}\bar{\mathbf{c}}^*)]$ and $\left[\mathrm{Im}(\mathbf{b}\mathbf{c}^*) - \mathrm{Im}(\bar{\mathbf{b}}\bar{\mathbf{c}}^*)\right]$, which we get by adding T-odd asymmetries in $|M|^2$ and $|\overline{M}|^2$. One can show [103] that TPs would be non-zero in $B \to V_1 V_2$ decays as long as $\mathrm{Im}(\mathbf{a}\mathbf{c}^*)$ or $\mathrm{Im}(\mathbf{b}\mathbf{c}^*)$ is non-zero. For that, both $B \to V_1$ and $B \to V_2$ channels must be present with different weak phases, following a naive factorization argument, detailed in Appendix A following Ref. [103].

There are two ways to introduce CPT violation in the formalism, namely,

1. CPTV in the decay amplitude, and
2. CPTV in the mixing amplitude.

We will discuss the former here and postpone the latter for Section IV. However, note that even if CPTV is present in the decay amplitudes, one can still have a mixing-induced CPT violation, characterized by time-dependent TP asymmetries, as discussed below.

7.1.1 CPTV in decay

Let us start with the first option, which can be subdivided into two categories:

Type I: CPTV present only in the p-wave amplitude

We introduce the CPTV parameter $f \equiv \mathrm{Re}(f) + i\mathrm{Im}(f)$ in the following way:

$$\mathbf{c} = \sum_i c_i e^{i\phi_i^c} e^{i\zeta_i^c} (1 - f) \, , \quad \bar{\mathbf{c}} = \sum_i c_i e^{-i\phi_i^c} e^{i\zeta_i^c} (1 + f^*) \, , \tag{7.4}$$

and other amplitudes remaining the same. This is the simplest way to introduce CPTV; a channel-dependent CPTV parameter f_i would only complicate the calculation without giving any extra insight.

The relevant quantity for TP is

$$\begin{aligned}\frac{1}{2}\left[\mathrm{Im}(\mathbf{ac}^*) - \mathrm{Im}(\bar{\mathbf{a}}\bar{\mathbf{c}}^*)\right] = \sum_{i,j} a_i c_j [&\sin\left(\phi_i^a - \phi_j^c\right)\cos\left(\zeta_i^a - \zeta_j^c\right) \\ &- \mathrm{Re}(f)\cos\left(\phi_i^a - \phi_j^c\right)\sin\left(\zeta_i^a - \zeta_j^c\right) \\ &- \mathrm{Im}(f)\sin\left(\phi_i^a - \phi_j^c\right)\sin\left(\zeta_i^a - \zeta_j^c\right)] \,. \end{aligned} \quad (7.5)$$

A similar expression is obtained for $\frac{1}{2}\left[\mathrm{Im}(\mathbf{bc}^*) - \mathrm{Im}(\bar{\mathbf{b}}\bar{\mathbf{c}}^*)\right]$. Even if the weak phase difference vanishes, these are still nonzero because of the second term, so the TP asymmetry will essentially probe $\mathrm{Re}(f)$.

Type II: Universal CPTV present in all amplitudes

In this case, the coefficients from eqs. (7.1) and (7.2) are modified as

$$(\mathbf{a}, \mathbf{b}, \mathbf{c}) \to (\mathbf{a}, \mathbf{b}, \mathbf{c})(1-f) \,, \quad (\bar{\mathbf{a}}, \bar{\mathbf{b}}, \bar{\mathbf{c}}) \to (\bar{\mathbf{a}}, \bar{\mathbf{b}}, \bar{\mathbf{c}})(1+f^*) \,. \quad (7.6)$$

Thus, the relevant expression for TP becomes,

$$\begin{aligned}\frac{1}{2}\left[\mathrm{Im}(\mathbf{ac}^*) - \mathrm{Im}(\bar{\mathbf{a}}\bar{\mathbf{c}}^*)\right] = \sum_{i,j} a_i c_j [&\sin\left(\phi_i^a - \phi_j^c\right)\cos\left(\zeta_i^a - \zeta_j^c\right) \\ &- 2\mathrm{Re}(f)\cos\left(\phi_i^a - \phi_j^c\right)\sin\left(\zeta_i^a - \zeta_j^c\right)] \end{aligned} \quad (7.7)$$

Here too, only the second term remains in absence of weak phase.

Following eq. (7.49) taken from [103], one finds the cases where no TP asymmetry is expected in the SM. On the other hand, introduction of CPTV may induce nonzero TP asymmetries for some of the cases as follows:

1. In order to have a TP correlation in a given decay, both of the amplitudes in eq. (7.46) must be present, otherwise either X or Y becomes zero. This remains true for CPTV of type II, but for type I, even in the absence of either X or Y, TPs can be generated.

2. For the same reason as above, CPTV of type I can produce nonzero TPs even if V_1 and V_2 have identical flavor wavefunctions (same meson, or an

excited state). Such nonzero TPs are not allowed in the SM as then **a**, **b**, and **c** are all proportional to the same factor and there is no relative phase.

3. In the SM (or in any NP model with CPT conservation), two kinematical amplitudes must have different weak phases for a nonzero TP asymmetry. Thus, if the quark-level decay is dominated by a single decay amplitude, a nonzero TP can never be generated. This is again not necessarily true for CPTV of either type I or type II, as we have seen from eqs. (7.5) and (7.7) that even in the absence of weak phase difference, one of the terms in the relevant expressions can have a nonzero value.

Effects of Type I and Type II CPTV in mixing

There could be another way to induce CPTV. Let us suppose CPTV to be present only for $B \to V_1$ and not for $B \to V_2$. As can be seen from eq. (7.47), this changes only the terms with the same phase in the expressions for **a**, **b**, and **c**. Thus, $|f|$ is absorbed in the form factors and $\arg(f)$ in the phase. Obviously, this scenario does not produce any TP even if CPTV is present.

Now let us consider the special case where V_1 can be accessed from B but not from $\bar{B}$, and vice versa. Let us also take, for simplicity, $B \to V_1$ and $\bar{B} \to V_2$ to be single-amplitude processes. For $B = B_{d,s}$, there will be a mixing-induced TP because the B meson can oscillate into $\bar{B}$ and hence decay to V_2, thus providing the second amplitude. The relevant T-violating terms, as shown in Ref. [103], are proportional to the **a**-**c** (and **b**-**c**) interference contributions, and are given by

$$
\begin{aligned}
|M|^2_{\mathbf{ac}} + |\overline{M}|^2_{\mathbf{ac}} &\sim \mathrm{Im}(\mathbf{ac}^*) - \mathrm{Im}(\bar{\mathbf{a}}\bar{\mathbf{c}}^*) \\
&= \cos^2\left(\frac{\Delta M t}{2}\right) \mathrm{Im}(\mathbf{a}_1\mathbf{c}_1^* - \bar{\mathbf{a}}_1\bar{\mathbf{c}}_1^*) + \sin^2\left(\frac{\Delta M t}{2}\right) \mathrm{Im}(\mathbf{a}_2\mathbf{c}_2^* - \bar{\mathbf{a}}_2\bar{\mathbf{c}}_2^*) \\
&+ \sin\left(\frac{\Delta M t}{2}\right) \cos\left(\frac{\Delta M t}{2}\right) \mathrm{Re}\left[e^{-2i\phi_M}\mathbf{a}_2\,\mathbf{c}_1^* - e^{2i\phi_M}\bar{\mathbf{a}}_2\,\bar{\mathbf{c}}_1^* - e^{2i\phi_M}\mathbf{a}_1\,\mathbf{c}_2^* \right. \\
&\left. + e^{-2i\phi_M}\bar{\mathbf{a}}_1\,\bar{\mathbf{c}}_2^*\right] . \qquad (7.8)
\end{aligned}
$$

where ΔM is the mass difference of the two eigenstates, and following eq. (7.1),

$$
\begin{aligned}
A(B \to V_1V_2) &= \mathbf{a}_1\mathcal{S} + \mathbf{b}_1\mathcal{D} + i\mathbf{c}_1\mathcal{P}, \qquad & A(\bar{B} \to V_1V_2) &= \mathbf{a}_2\mathcal{S} + \mathbf{b}_2\mathcal{D} + i\mathbf{c}_2\mathcal{P}, \\
A(B \to \bar{V}_1\bar{V}_2) &= \bar{\mathbf{a}}_2\mathcal{S} + \bar{\mathbf{b}}_2\mathcal{D} - i\bar{\mathbf{c}}_2\mathcal{P}, \qquad & A(\bar{B} \to \bar{V}_1\bar{V}_2) &= \bar{\mathbf{a}}_1\mathcal{S} + \bar{\mathbf{b}}_1\mathcal{D} - i\bar{\mathbf{c}}_1\mathcal{P},
\end{aligned}
\qquad (7.9)
$$

so that,

$$M \equiv A(B(t) \to V_1V_2) = e^{-i\left(M-\frac{i}{2}\Gamma\right)t}\left[\mathbf{a}\mathcal{S} + \mathbf{b}\mathcal{D} + i\mathbf{c}\mathcal{P}\right],$$
$$\bar{M} \equiv A(\bar{B}(t) \to \bar{V}_1\bar{V}_2) = e^{-i\left(M-\frac{i}{2}\Gamma\right)t}\left[\bar{\mathbf{a}}\mathcal{S} + \bar{\mathbf{b}}\mathcal{D} - i\bar{\mathbf{c}}\mathcal{P}\right], \quad (7.10)$$

with

$$\mathbf{a} = \mathbf{a}_1 \cos\left(\frac{\Delta Mt}{2}\right) - i\,e^{-2i\phi_M}\sin\left(\frac{\Delta Mt}{2}\right)\mathbf{a}_2\,, \quad (7.11)$$
$$\bar{\mathbf{a}} = \bar{\mathbf{a}}_1 \cos\left(\frac{\Delta Mt}{2}\right) - i\,e^{2i\phi_M}\sin\left(\frac{\Delta Mt}{2}\right)\bar{\mathbf{a}}_2\,,$$
$$\mathbf{b} = \mathbf{b}_1 \cos\left(\frac{\Delta Mt}{2}\right) - i\,e^{-2i\phi_M}\sin\left(\frac{\Delta Mt}{2}\right)\mathbf{b}_2\,,$$
$$\bar{\mathbf{b}} = \bar{\mathbf{b}}_1 \cos\left(\frac{\Delta Mt}{2}\right) - i\,e^{2i\phi_M}\sin\left(\frac{\Delta Mt}{2}\right)\bar{\mathbf{b}}_2\,,$$
$$\mathbf{c} = \mathbf{c}_1 \cos\left(\frac{\Delta Mt}{2}\right) - i\,e^{-2i\phi_M}\sin\left(\frac{\Delta Mt}{2}\right)\mathbf{c}_2\,,$$
$$\bar{\mathbf{c}} = \bar{\mathbf{c}}_1 \cos\left(\frac{\Delta Mt}{2}\right) - i\,e^{2i\phi_M}\sin\left(\frac{\Delta Mt}{2}\right)\bar{\mathbf{c}}_2\,. \quad (7.12)$$

Note that amplitudes like $\mathbf{a}_1$ are complex, with relevant weak and strong phases:

$$\mathbf{a}_1 = a_1 e^{i\phi_1^a} e^{i\zeta_1^a}\,, \quad (7.13)$$

The first term in eq.(7.8) describes the time evolution of the TP in $B \to V_1V_2$ and the second term, generated due to B–$\bar{B}$ mixing, describes the time evolution of the TP in $\bar{B} \to V_1V_2$. The third term can potentially generate a TP due to B–$\bar{B}$ mixing even in the absence of TP in $B \to V_1V_2$. This term can be rewritten after explicitly writing down $\mathbf{a}_1$, $\mathbf{a}_2$ etc. following Eq. (7.3):

$$-(\sin\Delta Mt)[a_2c_1\sin(\phi_2^a - \phi_1^c - 2\phi_M)\sin(\zeta_2^a - \zeta_1^c) - a_1c_2\sin(\phi_1^a - \phi_2^c + 2\phi_M)\sin(\zeta_1^a - \zeta_2^c \quad (7.14)$$

This expression goes to zero in the absence of strong phase differences, which is intuitively obvious as strong phase differences are related in part to kinematics, and the TP vanishes if kinematics of $\bar{B} \to V_2$ is identical to $B \to V_1$.

However, in the presence of CPTV of Type I, the expression in (7.14) is modified to

$$
\begin{aligned}
&-(\sin\Delta Mt)[a_2c_1\sin(\phi_2^a-\phi_1^c-2\phi_M)\sin(\zeta_2^a-\zeta_1^c)-a_1c_2\sin(\phi_1^a-\phi_2^c+2\phi_M)\sin(\zeta_1^a-\zeta\\
&\quad-2\mathrm{Re}(f)[a_2c_1\cos(\phi_2^a-\phi_1^c-2\phi_M)\cos(\zeta_2^a-\zeta_1^c)-a_1c_2\cos(\phi_1^a-\phi_2^c+2\phi_M)\cos(\zeta_1^a-\zeta\\
&\quad-\mathrm{Im}(f)[a_2c_1\sin(\phi_2^a-\phi_1^c-2\phi_M)\cos(\zeta_2^a-\zeta_1^c)-a_1c_2\sin(\phi_1^a-\phi_2^c+2\phi_M)\cos(\zeta_1^a-\zeta_2^c)
\end{aligned}
\tag{7.15}
$$

while for CPTV of Type II, the same expression takes the form

$$
\begin{aligned}
&-(\sin\Delta Mt)[a_2c_1\sin(\phi_2^a-\phi_1^c-2\phi_M)\sin(\zeta_2^a-\zeta_1^c)-a_1c_2\sin(\phi_1^a-\phi_2^c+2\phi_M)\sin(\zeta_1^a-\zeta\\
&\quad-2\mathrm{Re}(f)[a_2c_1\cos(\phi_2^a-\phi_1^c-2\phi_M)\cos(\zeta_2^a-\zeta_1^c)-a_1c_2\cos(\phi_1^a-\phi_2^c+2\phi_M)\cos(\zeta_1^a-\zeta
\end{aligned}
\tag{7.16}
$$

The last two equations show that in the presence of CPTV, we can get a non-zero TP from mixing, even if the strong phase differences vanish. Only if the final state is self-conjugate, the third term in eq.(7.8) is zero and the first two terms add up, so the TP in $B\to V_1V_2$ is time-independent and this remains true even in the presence of CPTV.

7.2 Relation to transversity amplitudes

The angular momentum amplitudes are related to the transversity amplitudes by the following relations [103]:

$$
A_\| = \sqrt{2}\mathbf{a}\,,\quad A_0 = -\mathbf{a}x - \frac{m_1m_2}{m_B^2}\mathbf{b}(x^2-1)\,,\quad A_\perp = 2\sqrt{2}\,\frac{m_1m_2}{m_B^2}\mathbf{c}\sqrt{x^2-1}\,. \tag{7.17}
$$

Let us consider, following Ref. [104], the channels in which each of the two vector mesons in $B\to V_1V_2$ further decays into two pseudoscalar mesons. The decay angular distribution in three dimensions is given in terms of the three transversity amplitudes. We take $\theta_1(\theta_2)$ to be the angle between the direction of motion of P_1 (P_2) in the V_1 (V_2) rest frame and that of V_1 (V_2) in the B rest frame. The angle between the planes defined by P_1P_1' and P_2P_2' in the B rest frame is denoted by

φ. One obtains [104]

$$\begin{aligned}
\frac{d\Gamma}{d\cos\theta_1 d\cos\theta_2 d\varphi} &= N\Bigg[|A_0|^2\cos^2\theta_1\cos^2\theta_2 + \frac{|A_\parallel|^2}{2}\sin^2\theta_1\sin^2\theta_2\cos^2\varphi \\
&+\frac{|A_\perp|^2}{2}\sin^2\theta_1\sin^2\theta_2\sin^2\varphi + \frac{\mathrm{Re}(A_0A_\parallel^*)}{2\sqrt{2}}\sin 2\theta_1\sin 2\theta_2\cos\varphi \\
&-\frac{\mathrm{Im}(A_\perp A_0^*)}{2\sqrt{2}}\sin 2\theta_1\sin 2\theta_2\sin\varphi - \frac{\mathrm{Im}(A_\perp A_\parallel^*)}{2}\sin^2\theta_1\sin^2\theta_2\sin 2\varphi\Bigg] , \\
\frac{d\bar{\Gamma}}{d\cos\bar{\theta}_1 d\cos\bar{\theta}_2 d\bar{\varphi}} &= N\Bigg[|\bar{A}_0|^2\cos^2\bar{\theta}_1\cos^2\bar{\theta}_2 + \frac{|\bar{A}_\perp|^2}{2}\sin^2\bar{\theta}_1\sin^2\bar{\theta}_2\sin^2\bar{\varphi} \\
&+\frac{|\bar{A}_\parallel|^2}{2}\sin^2\bar{\theta}_1\sin^2\bar{\theta}_2\cos^2\bar{\varphi} + \frac{\mathrm{Re}(\bar{A}_0\bar{A}_\parallel^*)}{2\sqrt{2}}\sin 2\bar{\theta}_1\sin 2\bar{\theta}_2\cos\bar{\varphi} \\
&+\frac{\mathrm{Im}(\bar{A}_\perp \bar{A}_0^*)}{2\sqrt{2}}\sin 2\bar{\theta}_1\sin 2\bar{\theta}_2\sin\bar{\varphi} + \frac{\mathrm{Im}(\bar{A}_\perp \bar{A}_\parallel^*)}{2}\sin^2\bar{\theta}_1\sin^2\bar{\theta}_2\sin 2\bar{\varphi}\Bigg] .
\end{aligned} \tag{7.18}$$

Integrating these over θ_1 and θ_2 gives a T-odd asymmetry involving $\sin 2\varphi$ [103]

$$A_T^{(2)} \equiv \frac{\Gamma(\sin 2\varphi > 0) - \Gamma(\sin 2\varphi < 0)}{\Gamma(\sin 2\varphi > 0) + \Gamma(\sin 2\varphi < 0)} = -\frac{4}{\pi}\frac{\mathrm{Im}(A_\perp A_\parallel^*)}{|A_0|^2 + |A_\perp|^2 + |A_\parallel|^2} . \tag{7.19}$$

Similarly, we may define an asymmetry with respect to the values of $\sin\varphi$, assigning it the sign of $\cos\theta_1\cos\theta_2$ and integrating over all angles,

$$\begin{aligned}
A_T^{(1)} &\equiv \frac{\Gamma[\mathrm{sign}(\cos\theta_1\cos\theta_2)\sin\varphi > 0] - \Gamma[\mathrm{sign}(\cos\theta_1\cos\theta_2)\sin\varphi < 0]}{\Gamma[\mathrm{sign}(\cos\theta_1\cos\theta_2)\sin\varphi > 0] + \Gamma[\mathrm{sign}(\cos\theta_1\cos\theta_2)\sin\varphi < 0]} \\
&= -\frac{2\sqrt{2}}{\pi}\frac{\mathrm{Im}(A_\perp A_0^*)}{|A_0|^2 + |A_\perp|^2 + |A_\parallel|^2} .
\end{aligned} \tag{7.20}$$

One can define similar asymmetries $\bar{A}_T^{(1)}$ and $\bar{A}_T^{(2)}$ by integrating the second part of eq. (7.18) and proceeding in a similar manner. As the p-wave amplitude in $\overline{M}$ changes sign relative to that of M (eqs. (7.1) and (7.2)), the sign of the T-odd asymmetry in $|\overline{M}|^2$ is opposite that in $|M|^2$. The true T-violating asymmetry is therefore found by *adding* the T-odd asymmetries in $|M|^2$ and $|\overline{M}|^2$ [101]:

$$\mathcal{A}_T \equiv \frac{1}{2}\left(A_T + \bar{A}_T\right) . \tag{7.21}$$

This essentially means that instead of $\mathrm{Im}(A_\perp A_i^*)$, we should look for expressions involving $\mathrm{Im}(A_\perp A_i^* + \bar{A}_\perp \bar{A}_i^*)$ in search of true TP-violating asymmetries. If we consider specifically the decay $B_s \to \phi\phi$, following Ref. [104], we notice that final states are flavorless and accessible to both B_s and $\bar{B}_s$. As a result of B_s–$\bar{B}_s$ oscillation, the angular decay distributions become time-dependent. Using standard notations for B_s-$\bar{B}_s$ mixing, and assuming no CP violation in mixing ($|q/p| = 1$) and decay ($|\bar{A}_k| = |A_k|$), one has [112]

$$\frac{q}{p}\frac{\bar{A}_k}{A_k} = \eta_k e^{-2i\phi_k} . \tag{7.22}$$

Here η_k is the CP parity for a state of transversity k ($\eta_0 = \eta_\| = -\eta_\perp = +1$), while ϕ_k is the weak phase involved in an interference between mixing and decay amplitudes. Denoting the CP conserving strong phase of A_k by ζ_k, one can write $A_k = |A_k| e^{i\zeta_k} e^{i\phi_k}$, so that $\bar{A}_k = (p/q)\eta_k e^{i\zeta_k} e^{-i\phi_k}$. One thus has for $i = 0, \|$:

$$\mathrm{Im}(A_\perp A_i^* + \bar{A}_\perp \bar{A}_i^*) = |A_\perp||A_i| \mathrm{Im}\left[e^{i\zeta^-}(e^{i\phi^-} - e^{-i\phi^-})\right] = 2|A_\perp||A_i| \cos(\zeta^-)\sin(\phi^-) , \tag{7.23}$$

where we define the notations for our future references:

$$\zeta^- = \zeta_\perp - \zeta_i , \quad \phi^- = \phi_\perp - \phi_i , \quad \phi^+ = \phi_\perp + \phi_i . \tag{7.24}$$

One finds from eq. (7.17) that expressions such as $\mathrm{Im}(A_\perp A_0^*)$ are proportional to linear combinations of terms like $\mathrm{Im}(\mathbf{a}^*\mathbf{c})$ and $\mathrm{Im}(\mathbf{b}^*\mathbf{c})$. Now, as per eq. (7.49), they are all zero for decays like $B_s \to \phi\phi$; thus, $A_T^{(1)}$, $A_T^{(2)}$, and consequently all of their combinations are zero. This can also be seen from eq. (7.23) if the weak phases for all the transversity amplitudes are the same. So, any nonzero values to any of these observables unambiguously point to new physics.

Let us assume the NP to be CPT violating in nature, and parametrize the amplitudes following eqs. (7.3) and (7.17):

$$\begin{aligned}
A_\perp &= \sum_l |A_\perp^l|\, e^{i\phi_\perp^l} e^{i\zeta_\perp^l}(1-f) , \qquad A_i = \sum_m |A_i^m| e^{i\zeta_i^m} e^{i\phi_i^m} , \\
\bar{A}_\perp &= \eta_\perp \sum_l |A_\perp^l|\, e^{-i\phi_\perp^l} e^{i\zeta_\perp^l}(1+f^*) , \qquad \bar{A}_i = \sum_m |A_i^m| e^{i\zeta_i^m} e^{-i\phi_i^m} , \\
&(i = 0, \|) .
\end{aligned} \tag{7.25}$$

Using the notation $\zeta^-_{l,m} = \left(\zeta^l_\perp - \zeta^m_i\right)$ and $\phi^-_{l,m} = \left(\phi^l_\perp - \phi^m_i\right)$, we obtain,

$$\begin{aligned}\mathrm{Im}(A_\perp A_i^* + \bar{A}_\perp \bar{A}_i^*) = 2 \sum_{l,m} |A^l_\perp||A^m_i| \big[\sin(\phi^-_{l,m}) \cos(\zeta^-_{l,m}) - \mathrm{Re}(f) \sin(\zeta^-_{l,m}) \cos(\phi^-_{l,m}) \\ + \mathrm{Im}(f) \sin(\phi^-_{l,m}) \sin(\zeta^-_{l,m})\big] \,. \end{aligned} \tag{7.26}$$

For $f = 0$ this reduces to eq. (7.23). On the other hand, even if $\phi^-_{l,m} = 0$, we still get a nonzero result:

$$\mathrm{Im}(A_\perp A_i^* + \bar{A}_\perp \bar{A}_i^*) = -2 \sum_{l,m} |A^l_\perp||A^m_i| \mathrm{Re}(f) \sin(\zeta^-_{l,m}) \,. \tag{7.27}$$

7.2.1 Time dependence of the transversity amplitudes

Next, let us consider the time dependence of transversity amplitudes; we will use a formalism closely following Ref. [104]. The states B and $\bar{B}$ evolve in time as

$$B(t) = f_+(t) B + (q/p) f_-(t) \bar{B} \ , \qquad \bar{B}(t) = (p/q) f_-(t) B + f_+(t) \bar{B} \,, \tag{7.28}$$

where

$$\begin{aligned} f_+(t) &= \frac{1}{2}\left(e^{-i\lambda_1^{(q)}t} + e^{-i\lambda_2^{(q)}t}\right) = \frac{1}{2}\left(e^{-im_1 t - (\Gamma_1 t/2)} + e^{-im_2 t - (\Gamma_2 t/2)}\right) \\ f_-(t) &= \frac{1}{2}\left(e^{-i\lambda_1^{(q)}t} - e^{-i\lambda_2^{(q)}t}\right) = \frac{1}{2}\left(e^{-im_1 t - (\Gamma_1 t/2)} - e^{-im_2 t - (\Gamma_2 t/2)}\right), \\ |f_\pm(t)|^2 &= (e^{-\Gamma t}/2)[\cosh(\Delta\Gamma t/2) \pm \cos(\Delta M t)], \\ f_+^*(t) f_-(t) &= (e^{-\Gamma t}/2)[\sinh(\Delta\Gamma t/2) - i \sin(\Delta M t)] \,, \end{aligned} \tag{7.29}$$

ΔM and $\Delta\Gamma$ being the mass and width differences of the stationary states respectively.

Time dependence of transversity amplitudes, $A_k \equiv \langle k|B\rangle, \bar{A}_k \equiv \langle k|\bar{B}\rangle$ ($k = 0, \|, \perp$), is given by:

$$\begin{aligned} A_k(t) &\equiv \langle k|B(t)\rangle = f_+(t) A_k + (q/p) f_-(t) \bar{A}_k \,, \\ \bar{A}_k(t) &\equiv \langle k|\bar{B}(t)\rangle = (p/q) f_-(t) A_k + f_+(t) \bar{A}_k \,. \end{aligned} \tag{7.30}$$

Let us calculate the interference terms $A_i^*(t) A_k(t)$ and $\bar{A}_i^*(t) \bar{A}_k(t)$, where $i = 0, \|, k = \perp$. Inserting $A_i^* A_k = |A_i||A_k|(1-f) \exp[i(\zeta_k - \zeta_i)] \exp[i(\phi_k - \phi_i)]$, and

$\bar{A}_i^* \bar{A}_k = \eta_i \eta_k |A_i||A_k|(1+f^*) \exp[i(\zeta_k - \zeta_i)] \exp[-i(\phi_k - \phi_i)]$, one gets, using eq. (7.24),

$$\begin{aligned}\mathrm{Im}[A_\perp(t)A_i^*(t) + \bar{A}_\perp(t)\bar{A}_i^*(t)] &= 2|A_\perp||A_i|e^{-\Gamma t}\times \\ &\big[\left\{\cos(\zeta^-)\sin(\phi^-) - \sin(\zeta^-)\left(\mathrm{Re}(f)\cos(\phi^-) - \mathrm{Im}(f)\sin(\phi^-)\right)\right\}\cosh(\Delta\Gamma t/2) \\ &+ \left\{\cos(\zeta^-)\sin(\phi^+) + \sin(\zeta^-)\left(\mathrm{Re}(f)\cos(\phi^+) + \mathrm{Im}(f)\sin(\phi^+)\right)\right\}\sinh(\Delta\Gamma t/2)\big]\end{aligned} \tag{7.31}$$

This, again, agrees with eq. (7.23) at $t = 0, f = 0$. When CPT is conserved, it shows the variation of a genuine CP violating quantity with time which requires no strong phase differences. The CPTV contribution is nonzero even if the weak phase difference vanishes but the strong phase difference ζ^- must be nonzero.

If there are more than one decay channel contributing to the transversity amplitudes, eq. (7.31) can be generalized to

$$\begin{aligned}\mathrm{Im}[A_\perp(t)A_i^*(t) + \bar{A}_\perp(t)\bar{A}_i^*(t)] &= \sum_{l,m} 2|A_\perp^l||A_i^m|e^{-\Gamma t}\times \\ \big[\left\{\cos(\zeta_{l,m}^-)\sin(\phi_{l,m}^-) - \sin(\zeta_{l,m}^-)\left[\mathrm{Re}(f)\cos(\phi_{l,m}^-) - \mathrm{Im}(f)\sin(\phi_{l,m}^-)\right]\right\}&\cosh(\Delta\Gamma t/2) \\ + \left\{\cos(\zeta_{l,m}^-)\sin(\phi_{l,m}^+) + \sin(\zeta_{l,m}^-)\left[\mathrm{Re}(f)\cos(\phi_{l,m}^+) + \mathrm{Im}(f)\sin(\phi_{l,m}^+)\right]\right\}&\sinh(\Delta\Gamma t/2)\big].\end{aligned} \tag{7.32}$$

The two "true" CP violating time-integrated triple product asymmetries ($i = 0, \|$) for untagged decays are proportional to

$$\begin{aligned}\Gamma\int_0^\infty \mathrm{Im}[A_\perp(t)A_i^*(t) + \bar{A}_\perp(t)\bar{A}_i^*(t)]dt &= \sum_{l,m} 2|A_\perp^l||A_i^m|\times \\ &\big[\left\{\cos(\zeta_{l,m}^-)\sin(\phi_{l,m}^-) - \sin(\zeta_{l,m}^-)\left(\mathrm{Re}(f)\cos(\phi_{l,m}^-) - \mathrm{Im}(f)\sin(\phi_{l,m}^-)\right)\right\} \\ &+ \left\{\cos(\zeta_{l,m}^-)\sin(\phi_{l,m}^+) + \sin(\zeta_{l,m}^-)\left(\mathrm{Re}(f)\cos(\phi_{l,m}^+) + \mathrm{Im}(f)\sin(\phi_{l,m}^+)\right)\right\}(\Delta\Gamma/2\Gamma) \\ &+ \mathcal{O}[(\Delta\Gamma/2\Gamma)^2]\big].\end{aligned} \tag{7.33}$$

In the limit $\Delta\Gamma \ll \Gamma$, one can neglect everything apart from the first term in eq. (7.33) and finds

$$\mathcal{A}_T^{(1)\text{untagged}}$$
$$= -\frac{4\sqrt{2}}{\pi}\sum_{l,m}\frac{|A_\perp^l||A_0^m|\,[\cos(\zeta_{l,m}^{0-})\sin(\phi_{l,m}^{0-}) - \sin(\zeta_{l,m}^{0-})\,(\mathrm{Re}(f)\cos(\phi_{l,m}^{0-}) - \mathrm{Im}(f)\sin(\phi_{l,m}^{0-}}{\left(|A_0|^2+|A_\perp|^2+|A_\parallel|^2\right)+\left(|\bar{A}_0|^2+|\bar{A}_\perp|^2+|\bar{A}_\parallel|^2\right)}$$
$$+\,\mathcal{O}[(\Delta\Gamma/2\Gamma)]$$
$$\mathcal{A}_T^{(2)\text{untagged}}$$
$$= -\frac{8}{\pi}\sum_{l,m}\frac{|A_\perp^l||A_\parallel^m|\left[\cos(\zeta_{l,m}^{\parallel-})\sin(\phi_{l,m}^{\parallel-}) - \sin(\zeta_{l,m}^{\parallel-})\left(\mathrm{Re}(f)\cos(\phi_{l,m}^{\parallel-}) - \mathrm{Im}(f)\sin(\phi_{l,m}^{\parallel-})\right)\right]}{\left(|A_0|^2+|A_\perp|^2+|A_\parallel|^2\right)+\left(|\bar{A}_0|^2+|\bar{A}_\perp|^2+|\bar{A}_\parallel|^2\right)}$$
$$+\,\mathcal{O}[(\Delta\Gamma/2\Gamma)]\,, \tag{7.34}$$

where $\zeta_{l,m}^{i-} = (\zeta_\perp^l - \zeta_i^m)$ and $\phi_{l,m}^{i-} = (\phi_\perp^l - \phi_i^m)$ for $i = 0, \parallel$, and the coefficients of the $\Delta\Gamma/2\Gamma$ terms can be easily found out from eq. (7.33).

In the absence of weak phase difference, $\phi_\perp = \phi_0 = \phi_\parallel$, *i.e.* $\phi_{l,m}^{i-} = 0$, the asymmetries vanish in the leading order if CPT is conserved [104] but is nonzero if CPT is violated. Again, a nonzero strong phase difference $\zeta_{l,m}^{i-}$ is obligatory for this.

In the SM, all the three transversity amplitudes have approximately equal and very small weak phases. Thus, one expects the asymmetries to be quite small. On the other hand, if CPTV is present, these asymmetries, measured in self-tagged decays to final CP eigenstates, need not be nonzero; thus, measurements of such asymmetries may either put stringent limits on the CPT violating parameter f, or indicate physics beyond SM.

7.3 CPT violation in mixing

One can also consider the case where CPTV is present not in decay but in $B_d - \overline{B}_d$ mixing, and parametrize the 2×2 Hamiltonian matrix with the introduction of an extra complex parameter δ which incorporates CPT violation [?]:

$$\delta = \frac{H_{22} - H_{11}}{\sqrt{H_{12}H_{21}}}\,, \tag{7.35}$$

so that

$$\mathcal{M} = \left[\begin{pmatrix} M_0 - \delta' & M_{12} \\ M_{12}^* & M_0 + \delta' \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_0 & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_0 \end{pmatrix}\right]\,, \tag{7.36}$$

where δ' is defined by

$$\delta = \frac{2\delta'}{\sqrt{H_{12}H_{21}}}\,. \tag{7.37}$$

We work within the Wigner-Weisskopf approximation which is a reliable one after a time scale of $\sim 1/M_B$. Violation of this approximation, which has nevertheless been considered in the literature [113], would change all the subsequent expressions, and we refrain from considering such a possibility. This will give, akin to the Bell-Steinberger analysis [114], a way to measure the CPT violating parameter δ in terms of the interference amplitudes which are supposed to be good probes of CP violation.

Eq. (7.30) can be written as

$$\begin{aligned} A_i(t) &\equiv \langle k|B(t)\rangle = f_+(t)A_i + \eta_1 f_-(t)\bar{A}_i\,, \\ \bar{A}_i(t) &\equiv \langle k|\bar{B}(t)\rangle = \frac{f_-(t)}{\eta_2}A_i + \bar{f}_+(t)\bar{A}_i\,, \end{aligned} \tag{7.38}$$

where $f_\pm(t)$, $\bar{f}_+(t)$ and $\eta_{(1,2)}$ are defined in Appendix 7.5. Using Eq. (7.24), one gets,

$$\begin{aligned} \mathrm{Im}[A_\perp(t)A_i^*(t) + \bar{A}_\perp(t)\bar{A}_i^*(t)] &= 2e^{-\Gamma t}|A_i||A_\perp|\times \\ &\Bigg[\cosh(\Delta\Gamma t/2)\left\{\cos\zeta^-\sin\phi^- - \frac{1}{4}\mathrm{Im}\delta\cos\phi^+\left(1+\sin\zeta^-\right)\right\} \\ &+\sinh(\Delta\Gamma t/2)\left\{\cos\zeta^-\left(\sin\phi^+ - \frac{1}{2}\mathrm{Re}\delta\sin\phi^-\right) - \frac{1}{2}\mathrm{Re}\delta\sin\zeta^-\cos\phi^-\right\} \\ &+\frac{1}{2}\cos(\Delta M t)\mathrm{Im}\delta\cos\zeta^-\cos\phi^+ - \frac{1}{2}\sin(\Delta M t)\mathrm{Im}\delta\sin\zeta^-\cos\phi^-\Bigg]\,. \end{aligned} \tag{7.39}$$

If there are multiple decay channels, one can generalize the above expression, by replacing ζ^-, ϕ^-, ϕ^+ with $\zeta^-_{l,m}$ etc., $|A_i||A_\perp|$ with $|A_i^m||A_\perp^l|$ and then taking a summation over l and m.

Then the two "true" CP violating time-integrated triple product asymmetries

$(i = 0, \|)$ for untagged decays are proportional to

$$\Gamma \int_0^\infty \mathrm{Im}[A_\perp(t)A_i^*(t) + \bar{A}_\perp(t)\bar{A}_i^*(t)] = \sum_{l,m} 2|A_i^m||A_\perp^l| \times$$
$$\left[\left\{\cos\zeta_{l,m}^- \sin\phi_{l,m}^- - \frac{1}{4}\mathrm{Im}\delta\cos\phi_{l,m}^+\left(1+\sin\zeta_{l,m}^-\right)\right\}\right.$$
$$+\left(\frac{\Delta\Gamma}{2\Gamma}\right)\left\{\cos\zeta_{l,m}^-\left(\sin\phi_{l,m}^+ - \frac{1}{2}\mathrm{Re}\delta\sin\phi_{l,m}^-\right) - \frac{1}{2}\mathrm{Re}\delta\sin\zeta_{l,m}^-\cos\right.$$
$$+\frac{1}{2}\left(\frac{1}{1+\left(\frac{\Delta M}{\Gamma}\right)^2}\right)\mathrm{Im}\delta\cos\zeta_{l,m}^-\cos\phi_{l,m}^+$$
$$\left.-\frac{1}{2}\left(\frac{\frac{\Delta M}{\Gamma}}{1+\left(\frac{\Delta M}{\Gamma}\right)^2}\right)\mathrm{Im}\delta\sin\zeta_{l,m}^-\cos\phi_{l,m}^-\right] . \qquad (7.40)$$

In the limit $\Delta M/\Gamma \ll 1$, one can neglect the last term and simplify the expression considerably.

We also note that even in the case $\zeta_{l,m}^- = \phi_{l,m}^- = 0$, *i.e.* when all strong and weak phase differences cancel out individually, there is a nonzero TP asymmetry that gives a clean measurement of Imδ:

$$\Gamma \int_0^\infty \mathrm{Im}[A_\perp(t)A_i^*(t) + \bar{A}_\perp(t)\bar{A}_i^*(t)] \approx \sum_{l,m} \frac{1}{2}|A_i^m||A_\perp^l|\mathrm{Im}\delta\cos\phi_{l,m}^+ , \qquad (7.41)$$

where we have used $\Delta M/\Gamma \approx 0$ and neglected the sub-leading $\Delta\Gamma/\Gamma$ terms.

7.4 $B_s \to \phi\phi$ at LHCb

The LHCb collaboration has recently measured the transversity amplitudes for the decay $B_s \to \phi\phi$ [110], which is a pure penguin process and hence dominated by a single amplitude in the SM. Thus, for all l, m, $A_i^l = A_i^m$ (for $i = 0, \|, \perp$) The analysis also assumes that the weak phases of the three polarization amplitudes are all equal; thus, all $\phi_{l,m}^{i-}$ (for $i = 0, \|$) in our notation become zero. The correspondence between our notation and that of Ref. [110] is as follows:

$$\mathcal{A}_T^{(2)\mathrm{untagged}} \to A_U , \quad \mathcal{A}_T^{(1)\mathrm{untagged}} \to A_V$$
$$(\zeta_\perp - \zeta_\|) \to \delta_1 , \quad (\zeta_\perp - \zeta_0) \to \delta_2 , \quad (\zeta_\| - \zeta_0) \to \delta_\| \equiv (\delta_2 - \delta_1) . \qquad (7.42)$$

With the standard normalization of the transversity amplitudes, *viz.* $|A_0|^2 +$

$|A_\perp|^2 + |A_\parallel|^2 = |\bar{A}_0|^2 + |\bar{A}_\perp|^2 + |\bar{A}_\parallel|^2 = 1$, Eq. (7.34) becomes

$$\begin{aligned}
A_V &= -\frac{2\sqrt{2}}{\pi}|A_\perp||A_0|\,[-\sin(\delta_2)\,(\mathrm{Re}(f)) \\
&\quad + \{\cos(\delta_2)\sin(2\phi_s) + \sin(\delta_2)\,(\mathrm{Re}(f)\cos(2\phi_s) + \mathrm{Im}(f)\sin(2\phi_s))\}\,(\Delta\Gamma/2\Gamma)] \\
&\quad + \mathcal{O}[(\Delta\Gamma/2\Gamma)^2] \\
A_U &= -\frac{4}{\pi}|A_\perp||A_\parallel|\,[-\sin(\delta_1)\mathrm{Re}(f) \\
&\quad + \{\cos(\delta_1)\sin(2\phi_s) + \sin(\delta_1)\,(\mathrm{Re}(f)\cos(2\phi_s) + \mathrm{Im}(f)\sin(2\phi_s))\}\,(\Delta\Gamma/2\Gamma)] \\
&\quad + \mathcal{O}[(\Delta\Gamma/2\Gamma)^2]\,. \qquad (7.43)
\end{aligned}$$

We will use the following numbers from Ref. [110]:

$$\begin{aligned}
|A_0|^2 &= 0.365 \pm 0.022(\mathrm{stat}) \pm 0.012(\mathrm{syst}), \\
|A_\perp|^2 &= 0.291 \pm 0.024(\mathrm{stat}) \pm 0.010(\mathrm{syst}), \\
|A_\parallel|^2 &= 0.344 \pm 0.024(\mathrm{stat}) \pm 0.014(\mathrm{syst}), \\
\cos(\delta_\parallel) &= -0.844 \pm 0.068(\mathrm{stat}) \pm 0.029(\mathrm{syst}), \\
A_U &= -0.055 \pm 0.036(\mathrm{stat}) \pm 0.018(\mathrm{syst}) \\
A_V &= 0.010 \pm 0.036(\mathrm{stat}) \pm 0.018(\mathrm{syst})\,. \qquad (7.44)
\end{aligned}$$

For our analysis, we use Eqs. (7.42), (7.43) and (7.44), and keep terms only up to the first order in $\Delta\Gamma/\Gamma$. Even for the B_s system, this is a good approximation. All $\phi^{i+}_{l,m}$s in Eq. (7.33) (for $i = 0, \|$) are now equal to $2\phi_s$, where ϕ_s is the weak CP violating phase which is the same for the three polarization amplitudes, and very small in the SM ($\phi_s \sim 0.02$ [116, 117] based on QCD factorization) [1]. Even if there is some new physics making ϕ_s large, the effects will be suppressed by $\Delta\Gamma/\Gamma$, so we do not expect much sensitivity on the precise value of ϕ_s. One may note that this phase has recently been measured by the LHCb collaboration [118] to be between -2.46 and -0.76 rad with 68% confidence level, which is not exactly in total conformity with the SM prediction.

As is evident from Eq. (7.43), if we neglect higher order terms in $\Delta\Gamma/\Gamma$, both A_U and A_V are zero in the SM; thus, any definite nonzero value for these observables would point to the presence of some NP. Considering CPT violation as the source of NP, one sees that there is a definite deviation from zero even at the

[1]This should not be confused with the phase ϕ_s relevant for $B_s - \overline{B_s}$ mixing and defined as $\phi_s = \arg(-M_{12}/\Gamma_{12})$.

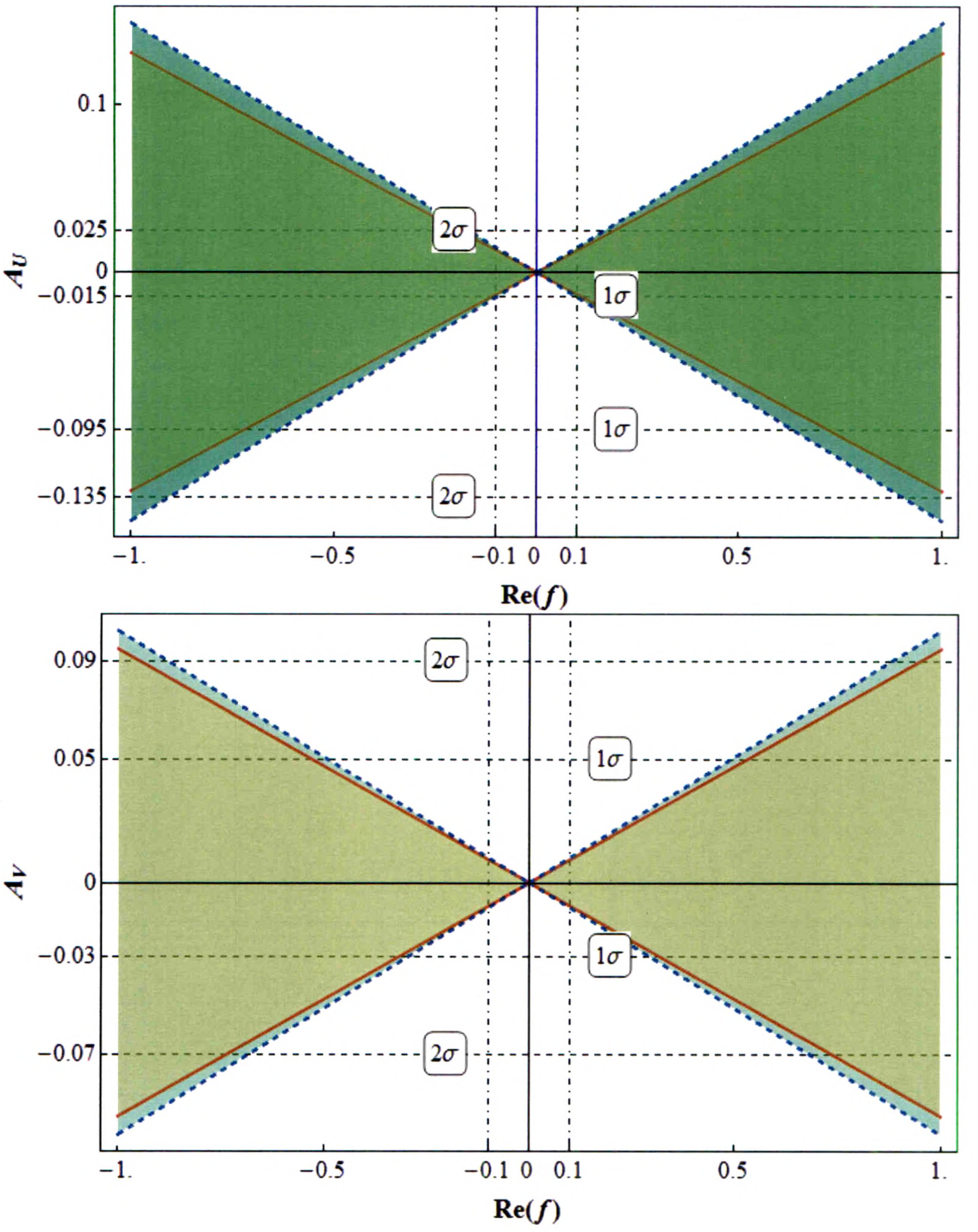

Figure 7.1: Upper panel: Allowed values of A_U for $-1 \leq \text{Re}(f) \leq 1$. The inner wedge is for the input parameters varied in their 1σ ranges, the outer wedge is for 2σ variation. Also shown are the 1σ and 2σ experimental bands for A_U, and the allowed region for a smaller range of $\text{Re}(f)$, namely, $|\text{Re}(f)| \leq 0.1$. Lower panel: Same plot for A_V.

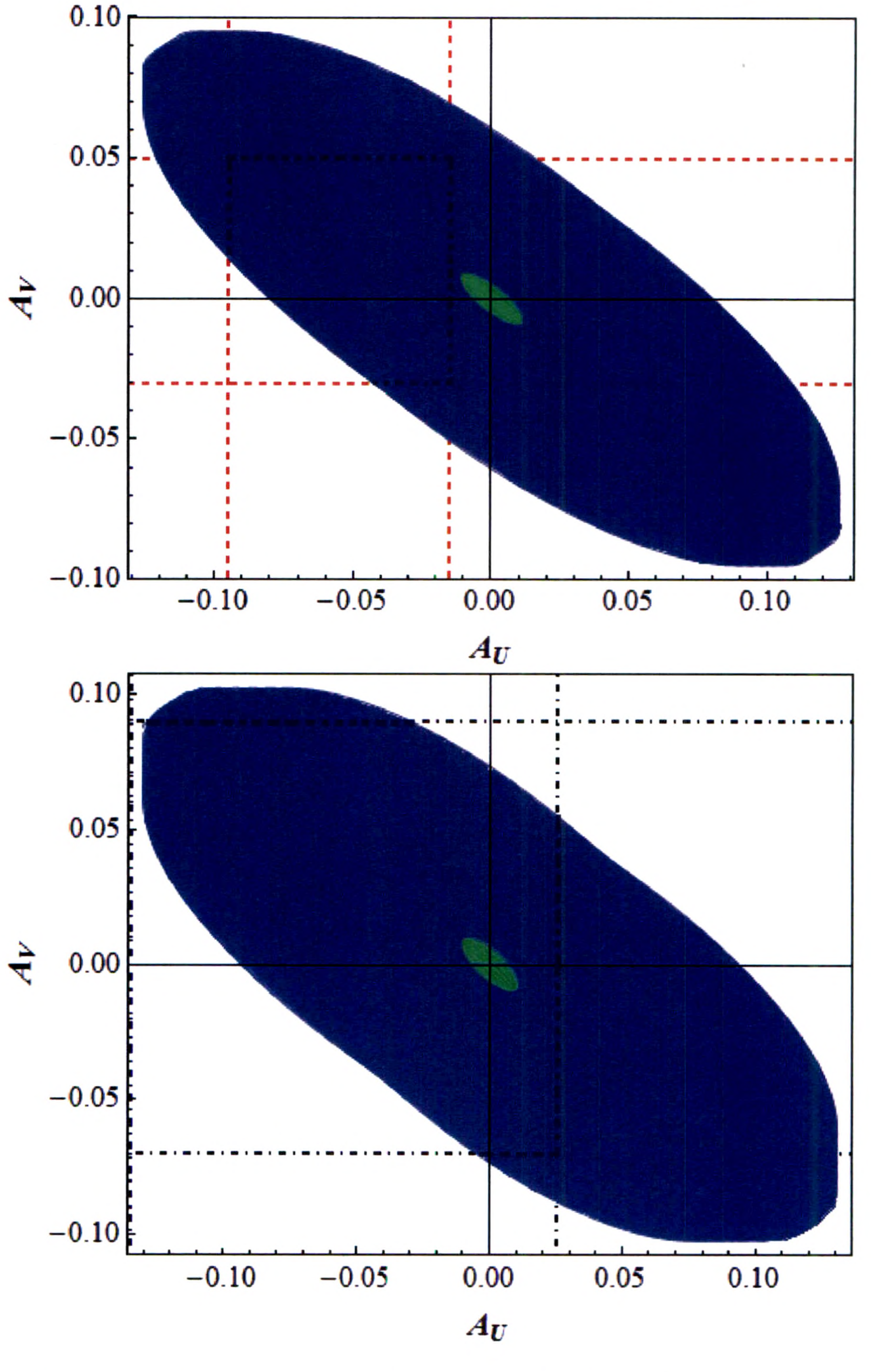

Figure 7.2: Upper panel: Allowed region in the A_U-A_V plane when all the input parameters are varied over their 1σ ranges. The outer ellipse is for $-1 \leq \mathrm{Re}(f) \leq 1$ and the inner green ellipse is for $-0.1 \leq \mathrm{Re}(f) \leq 0.1$. The 1σ bands for A_U and A_V are shown as dashed lines. Lower panel: The same plot when the input parameters are varied over 2σ; also, the 2σ bands are shown. The left edge of the A_U band coincides with the left edge of the plot.

zero-th order of $\Delta\Gamma/\Gamma$; unfortunately, the shift depends only on $\mathrm{Re}(f)$, as $\mathrm{Im}(f)$ comes as a coefficient of $\sin(2\phi_s)$ in the sub-leading order. Fig. 7.1 shows the allowed ranges for A_U and A_V when the input parameters are varied over their experimental ranges. We have varied the three transversity amplitudes over their allowed ranges keeping the normalization to unity fixed, and also varied the strong phase differences δ_1 and δ_2 over the entire range of $[0 : 2\pi]$ keeping the constraint on $\cos(\delta_\|)$. This gives a bound on A_U and A_V, although this is quite weak at present (however, note that if we take the 1σ region on A_U seriously, small values of $\mathrm{Re}(f)$ are ruled out, as is the SM). The allowed region will shrink considerably with more data.

In Fig. 7.2 we show the allowed region in the A_U-A_V plane for large and small values of $\mathrm{Re}(f)$, varying all other input parameters as above. Again, with more data, the elliptic figures are bound to shrink, as well as the horizontal and vertical bands, constraining CPT violation. If finally the intersection of the bands settle outside the ellipses, that will rule out CPT violation in this channel at least, but that will also rule out the pure-SM explanation and call for some other NP.

7.5 Conclusions

The role of TP asymmetries as a probe of CP violation crucially hinges on the CPT theorem which relates a possible T violating observable to a CP violating one. If CPT is not conserved, there is no such relationship, and observables that are not supposed to show any TP asymmetries in the SM might do so. For example, if CPT violation is present in one or more decay amplitudes, there will be a nonzero TP asymmetry even if the weak phases of all the amplitudes are equal. The same trend persists in the time-dependence of the TP asymmetries.

One might trade the s, p, and d-wave amplitudes with the transversity amplitudes, which are directly accessible to the experiments. Some of the interference terms between these amplitudes are CP violating only if the corresponding weak phases are different; in the presence of CPT violation, we again observe that a nonzero signal can be observed even if all the weak phases are equal. The observables A_U and A_V, as measured by LHCb, are supposed to be zero in the SM for channels like $B_s \to \phi\phi$. We show how one gets nonzero and possibly large values for these observables with CPT violation; a more canonical NP that contributes only to the $B_s - \overline{B_s}$ mixing and hence modifies the weak CP violating phase ϕ_s in the decay can hardly generate such large values as all ϕ_s-dependent terms are suppressed by $\Delta\Gamma/\Gamma$. The other side of the coin is that with more data, one can successfully constrain the parameter space for the CPT violating parameters.

Appendix

A: Factorization

Following Ref. [103], we briefly describe the main results of naive factorization. The prediction of naive factorization, that most TP asymmetries with ground state vector mesons are expected to be small in the SM, will necessarily hold in PQCD or QCD factorization too.

The starting point for factorization is the SM effective hamiltonian for B decays [115]:

$$\begin{aligned} H_{eff}^q = \frac{G_F}{\sqrt{2}}&[V_{fb}V_{fq}^*(c_1O_{1f}^q + c_2O_{2f}^q) \\ &- \sum_{i=3}^{10}(V_{ub}V_{uq}^*c_i^u + V_{cb}V_{cq}^*c_i^c + V_{tb}V_{tq}^*c_i^t)O_i^q] + h.c., \end{aligned} \tag{7.45}$$

where the superscript u, c, t indicates the internal quark, f can be the u or c quark, and q can be either a d or s quark.

Within factorization, the amplitude for $B \to V_1V_2$ can be written as

$$\mathcal{A}(B \to V_1V_2) = \sum_{\mathcal{O},\mathcal{O}'}\{\langle V_1\mathcal{O}\rangle 0\langle V_2\mathcal{O}'\rangle B + \langle V_2\mathcal{O}\rangle 0\langle V_1\mathcal{O}'\rangle B\}\ , \tag{7.46}$$

where $\mathcal{O}$ and $\mathcal{O}'$ are some relevant four-fermion operators. The first amplitude, $\langle V_1\mathcal{O}\rangle 0$, is proportional to the polarization vector of V_1, named, ε_1^*. The second amplitude, $\langle V_2\mathcal{O}'\rangle B$, can be written in terms of the usual vector and axial-vector form factors. Thus, the first term of Eq. (7.46) is given by

$$\begin{aligned} &\sum_{\mathcal{O},\mathcal{O}'}\langle V_1\mathcal{O}\rangle 0\langle V_2\mathcal{O}'\rangle B \\ &= -(m_B+m_2)m_1 g_{V_1} X A_1^{(2)}(m_1^2)\varepsilon_1^*\cdot\varepsilon_2^* + 2\frac{m_1}{m_B+m_2}g_{V_1}XA_2^{(2)}(m_1^2)\varepsilon_2^*\cdot p\varepsilon_1^*\cdot p \\ &- i\frac{m_1}{(m_B+m_2)}g_{V_1}XV^{(2)}(m_1^2)\epsilon_{\mu\nu\rho\sigma}p^\mu q^\nu \varepsilon_1^{*\rho}\varepsilon_2^{*\sigma}\ , \end{aligned} \tag{7.47}$$

All phase information is contained within the factor X, which is common to all the three independent amplitudes. Thus, these quantities must have the same phase.

A similar treatment for the second term in Eq. (7.46) gives

$$
\begin{aligned}
&\sum_{O,O'} \langle V_2 O \rangle 0 \langle V_1 O' \rangle B \\
&= -(m_B+m_1)m_2 g_{V_2} Y A_1^{(1)}(m_2^2)\varepsilon_1^*\cdot\varepsilon_2^* + 2\frac{m_2}{m_B+m_1} g_{V_2} Y A_2^{(1)}(m_2^2)\varepsilon_2^*\cdot p\varepsilon_1^*\cdot p \\
&- i\frac{m_2}{(m_B+m_1)} g_{V_2} Y V^{(1)}(m_2^2)\epsilon_{\mu\nu\rho\sigma}p^\mu q^\nu \varepsilon_1^{*\rho}\varepsilon_2^{*\sigma}\,, \qquad (7.48)
\end{aligned}
$$

where the phase information are contained in the common factor Y, which need not be the same as X.

We can now express the quantities a, b and c of Eq. (7.1) as follows:

$$
\begin{aligned}
a &= -m_1 g_{V_1}(m_B+m_2)A_1^{(2)}(m_1^2)X - m_2 g_{V_2}(m_B+m_1)A_1^{(1)}(m_2^2)Y \\
b &= 2m_1 g_{V_1}\frac{m_B}{(m_B+m_2)}m_B A_2^{(2)}(m_1^2)X + 2m_2 g_{V_2}\frac{m_B}{(m_B+m_1)}m_B A_2^{(1)}(m_2^2)Y \\
c &= -m_1 g_{V_1}\frac{m_B}{(m_B+m_2)}m_B V^{(2)}(m_1^2)X - m_2 g_{V_2}\frac{m_B}{(m_B+m_1)}m_B V^{(1)}(m_2^2)Y \qquad (7.49)
\end{aligned}
$$

Thus, nonzero TP asymmetries are generated from $\mathrm{Im}(ac^*)$ or $\mathrm{Im}(bc^*)$ if and only if both X and Y are present with different phase. Thus, if $V_1 = V_2$, there cannot be any TP asymmetry in the SM.

B: CPT Violation in mixing

This closely follows Ref. [?] with a coupe of typographical errors corrected. Consider the 2×2 Hamiltonian matrix with an explicit CPT violating term δ. Let us define,

$$
\eta_1 = \frac{q_1}{p_1} = \left(y+\frac{\delta}{2}\right)\alpha\,; \quad \eta_2 = \frac{q_2}{p_2} = \left(y-\frac{\delta}{2}\right)\alpha\,; \quad \omega = \frac{\eta_1}{\eta_2}\,, \qquad (7.50)
$$

and

$$
\begin{aligned}
f_-(t) &= \frac{1}{(1+\omega)}\left(e^{-i\lambda_1 t} - e^{-i\lambda_2 t}\right)\,, \\
f_+(t) &= \frac{1}{(1+\omega)}\left(e^{-i\lambda_1 t} + \omega e^{-i\lambda_2 t}\right)\,, \\
\bar{f}_+(t) &= \frac{1}{(1+\omega)}\left(\omega e^{-i\lambda_1 t} + e^{-i\lambda_2 t}\right)\,. \qquad (7.51)
\end{aligned}
$$

Thus,

$$
\begin{aligned}
|f_-(t)|^2 &= \frac{2e^{-\Gamma t}}{|1+\omega|^2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) - \cos\left(\Delta M t\right)\right] \\
&\approx \frac{e^{-\Gamma t}\left(1-\mathrm{Re}\delta\right)}{2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) - \cos\left(\Delta M t\right)\right], \\
|f_+(t)|^2 &= \frac{e^{-\Gamma t}}{|1+\omega|^2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right)(1+|\omega|^2) + \sinh\left(\frac{\Delta\Gamma t}{2}\right)(1-|\omega|^2)\right. \\
&\left. +2\mathrm{Re}(\omega)\cos\left(\Delta M t\right) - 2\mathrm{Im}(\omega)\sin\left(\Delta M t\right)\right], \\
&\approx \frac{e^{-\Gamma t}}{2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) - \sinh\left(\frac{\Delta\Gamma t}{2}\right)\mathrm{Re}\delta + \cos\left(\Delta M t\right) - \mathrm{Im}\delta\sin\left(\Delta M t\right)\right], \\
|\bar{f}_+(t)|^2 &= \frac{e^{-\Gamma t}}{|1+\omega|^2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right)(1+|\omega|^2) - \sinh\left(\frac{\Delta\Gamma t}{2}\right)(1-|\omega|^2)\right. \\
&\left. +2\mathrm{Re}(\omega)\cos\left(\Delta M t\right) + 2\mathrm{Im}(\omega)\sin\left(\Delta M t\right)\right], \\
&\approx \frac{e^{-\Gamma t}}{2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) + \sinh\left(\frac{\Delta\Gamma t}{2}\right)\mathrm{Re}\delta + \cos\left(\Delta M t\right) + \mathrm{Im}\delta\sin\left(\Delta M t\right)\right], \\
f_+^*(t)f_-(t) &= \frac{e^{-\Gamma t}}{|1+\omega|^2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right)(1-\omega^*) + \sinh\left(\frac{\Delta\Gamma t}{2}\right)(1+\omega^*)\right. \\
&\left. +\cos\left(\Delta M t\right)(-1+\omega^*) - i\sin\left(\Delta M t\right)(1+\omega^*)\right], \\
&\approx \frac{e^{-\Gamma t}}{4}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right)(-\mathrm{Re}\delta + i\mathrm{Im}\delta) + \sinh\left(\frac{\Delta\Gamma t}{2}\right)(2-\mathrm{Re}\delta - i\mathrm{Im}\delta)\right. \\
&\left. +\cos\left(\Delta M t\right)(\mathrm{Re}\delta - i\mathrm{Im}\delta) - \sin\left(\Delta M t\right)(\mathrm{Im}\delta + i(2-\mathrm{Re}\delta))\right], \\
\bar{f}_+(t)f_-^*(t) &= \frac{e^{-\Gamma t}}{|1+\omega|^2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right)(\omega-1) + \sinh\left(\frac{\Delta\Gamma t}{2}\right)(1+\omega)\right. \\
&\left. +\cos\left(\Delta M t\right)(1-\omega) + i\sin\left(\Delta M t\right)(1+\omega)\right] \\
&\approx \frac{e^{-\Gamma t}}{4}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right)(\mathrm{Re}\delta + i\mathrm{Im}\delta) + \sinh\left(\frac{\Delta\Gamma t}{2}\right)(2-\mathrm{Re}\delta + i\mathrm{Im}\delta)\right. \\
&\left. -\cos\left(\Delta M t\right)(\mathrm{Re}\delta + i\mathrm{Im}\delta) + i\sin\left(\Delta M t\right)(-\mathrm{Im}\delta + i(2-\mathrm{Re}\delta))\right],
\end{aligned}
\tag{7.52}
$$

Where we take, $y \approx 1\,, \eta_{1(2)} \approx \left(1+(-)\frac{\delta}{2}\right)\,, \omega \approx (1+\delta)\,, |\omega|^2 \approx (1+2\mathrm{Re}\delta)\,, |1+\omega|^{-2} \approx \frac{1}{4}(1-\mathrm{Re}\delta)\,, |\eta_{1(2)}|^2 \approx (1+(-)\mathrm{Re}\delta)$.

This gives,

$$
\begin{aligned}
A_i^*(t)A_k(t) &= \left[f_+^* A_i^* + \eta_1^* f_-^* \bar{A}_i^*\right]\left[f_+ A_k + \eta_1 f_- \bar{A}_k\right] \\
&= A_i^* A_k\left[|f_+|^2 + \eta_1(\bar{A}_k/A_k) f_+^* f_-\right] + \bar{A}_i^*\bar{A}_k\left[|\eta_1|^2|f_-|^2 + \eta_1^*(A_k/\bar{A}_k) f_+ f_-^*\right] \\
&= \frac{e^{-\Gamma t}}{2}\left[A_i^* A_k\left\{\cosh(\Delta\Gamma t/2) + \cos(\Delta m t) - \mathrm{Re}\delta\sinh(\Delta\Gamma t/2) - \mathrm{Im}\delta\sin(\Delta m t\right.\right. \\
&+ \frac{\eta_k e^{-2i\phi_k}}{2} A_i^* A_k\left\{2\sinh(\Delta\Gamma t/2) - 2i\sin(\Delta M t) + (-\mathrm{Re}\delta + i\mathrm{Im}\delta)\cosh(\Delta\Gamma t/\right. \\
&+ (\mathrm{Re}\delta - i\mathrm{Im}\delta)\cos(\Delta M t)\} + \bar{A}_i^*\bar{A}_k\left\{\cosh(\Delta\Gamma t/2) - \cos(\Delta M t)\right\} \\
&+ \frac{\eta_k e^{2i\phi_k}}{2}\bar{A}_i^*\bar{A}_k\left\{2\sinh(\Delta\Gamma t/2) + 2i\sin(\Delta M t) + \cosh(\Delta\Gamma t/2)(-\mathrm{Re}\delta - i\mathrm{Im}\delta\right. \\
&+ (\mathrm{Re}\delta + i\mathrm{Im}\delta)\cos(\Delta m t)\}]\ ,
\end{aligned}
\tag{7.53}
$$

$$
\begin{aligned}
\bar{A}_i^*(t)\bar{A}_k(t) &= \left[\frac{f_-^*}{\eta_2^*}A_i^* + \bar{f}_+^*\bar{A}_i^*\right]\left[\bar{f}_+\bar{A}_k + \frac{f_-}{\eta_2}A_k\right] \\
&= A_i^* A_k\left[\frac{|f_-|^2}{|\eta_2|^2} + (\bar{A}_k/A_k)\frac{\bar{f}_+ f_-^*}{\eta_2^*}\right] + \bar{A}_i^*\bar{A}_k\left[|\bar{f}_+|^2 + (A_k/\bar{A}_k)\frac{f_-\bar{f}_+^*}{\eta_2}\right] \\
&= \frac{e^{-\Gamma t}}{2}\left[A_i^* A_k\left\{\cosh(\Delta\Gamma t/2) - \cos(\Delta M t)\right\}\right. \\
&+ \frac{\eta_k e^{-2i\phi_k}}{2}A_i^* A_k\{2\sinh(\Delta\Gamma t/2) + 2i\sin(\Delta M t) \\
&+ (\mathrm{Re}\delta + i\mathrm{Im}\delta)\cosh(\Delta\Gamma t/2) - (\mathrm{Re}\delta + i\mathrm{Im}\delta)\cos(\Delta M t)\} \\
&+ \bar{A}_i^*\bar{A}_k\left\{\cosh(\Delta\Gamma t/2) + \cos(\Delta M t) + \mathrm{Re}\delta\sinh(\Delta\Gamma t/2) + \mathrm{Im}\delta\sin(\Delta M t)\right\} \\
&+ \frac{\eta_k e^{2i\phi_k}}{2}\bar{A}_i^*\bar{A}_k\{2\sinh(\Delta\Gamma t/2) - 2i\sin(\Delta M t) \\
&+ (\mathrm{Re}\delta - i\mathrm{Im}\delta)\cosh(\Delta\Gamma t/2) - (\mathrm{Re}\delta - i\mathrm{Im}\delta)\cos(\Delta M t)\}]\ .
\end{aligned}
\tag{7.54}
$$

Chapter 8

Conclusion

For the most part of this thesis, we have tried to investigate some possible manifestations of CPT violation in B systems, in particular B_s. CPT, or any combination thereof, is a discrete symmetry that is respected by all local axiomatic quantum field theories. However, even this statement is motivation enough to look for signatures of CPT violation, which might be a glimpse to the underlying nonlocal aspects coming from some ultraviolet completed theory, or something even more fundamental. Indeed, searches for CPT violation in different systems have been going on for quite some time; while all results are consistent with CPT conservation, one must look for signals in all possible systems, because CPT violation might not be a universal phenomenon.

CPT violation is intricately related with Lorentz symmetry violation. Only a subset of Lorentz violating operators are CPT violating too, and tight constraints have been placed upon most of them from various observables. For example, there are stringent results for muons or K mesons. The third generation systems, like the B mesons, have not been investigated to that detail, so we take that up in this thesis.

In Chapter 1, we have outlined the basic ingredients of the Standard Model that are necessary for our studies. Before going into CPT violation proper, we have tried to investigate how far the SM is consistent with the B physics data. Concentrating on the $B_s - \overline{B_s}$ system, we find that there is a serious tension between the data and the theory, which stems mostly from the measurement of the dimuon asymmetry. We have also made some comments on the nature of possible new physics that may ameliorate this tension: such a new physics must contribute to the absorptive part of the $B_s - \overline{B_s}$ mixing, and a possible option is some effective operator of the form $\bar{b}\Gamma^A s \bar{\tau}\Gamma^B \tau$ where Γ^A, Γ^B are some Dirac matrices.

Chapter 3 deals with the formalism and different parametrizations of CPT violation. In Chapter 4, we discuss how CPT violation can manifest itself in $B_d - \overline{B_d}$ and $B_s - \overline{B_s}$ mixing. To be precise, we show how CPT violation present in the mixing amplitudes can affect tagged and untagged decay rates. We have constructed time-dependent and time-independent CPT asymmetries and shown how they depend on the CPT violating parameters, commenting on their possible observability at the LHC.

In the next chapter, we have taken up the analysis in Chapter 2, that of analyzing the possible nature of new physics from B_s data, but this time with the possibility of a nonzero CPT violation in the $B_s - \overline{B_s}$ mixing amplitude. While this reduces the existing tension a bit, the improvement still leaves much room for CPT conserving new physics.

In Chapter 6, we consider the possibility of CPT violation either in mixing or in the subsequent decay amplitude. As an example, we discuss the decay $B_s \to D_s^{\pm} K^{\mp}$. Observables that can unambiguously extract CPT violation have been defined. We show that the extraction of CPT violation in mixing is independent of any possible CPT violation in decay. We find that it is possible for LHCb to disentangle such CPT violating signals, and this method has been included in the LHCb program.

Chapter 7 is about the triple product asymmetries in B decays in the presence of CPT violation. As is well known, triple product asymmetries are T violating and hence CP violating by the CPT theorem, but such a correlation is lost if the possibility of CPT violation is taken into account. After working out the formalism in detail in terms of the transversity amplitudes, we show how CPT violation can create nonzero asymmetries where no such asymmetries are expected. As a practical example, we consider the decay $B_s \to \phi\phi$ recently measured by the LHCb collaboration, and constrain the CPT violating parameters.

Thus, even though CPT violation seems to be in conflict with the common wisdom of axiomatic field theories, one should look for such signals and be rewarded with some pleasant surprise, just as Christenson, Cronin, Fitch, and Turlay were almost fifty years ago. The B_s system provides an interesting laboratory, and there are several ways to look for CPT violation, some of them evidently within reach of the complete run of LHCb. It will be nice if the experimentalists can surprise the theoreticians and force them to go back to the drawing board.

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