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Hadron Distributions in the Reactions Hadron + Hadron -> Lepton Pair + Hadrons and Two Virtual Photons -> Hadrons and Tests of the Parton Model*

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Abstract

The parton model description of the hadron distribution can be characterized in terms of a few fragmentation regions and plateaus. We discuss suchdistributions in the reactions hadron + hadron -> lepton pair + hadrons and virtual photon + virtual photon -> hadrons, comparison of different distributions as tests of the parton model, and kinematical conditions for the validity of the asymptotic descriptions of the distributions and the tests.

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The parton model description of the final state hadron distributions has been discussed for various processes, namely, the deep inelastic electro-production[1-3],

lepton + hadron -> lepton + hadrons, (1)

the high energy electron-positron annihilation[1-3],

electron + positron -> hadrons, (2)

and the deep hadron-hadron scattering[4],

hadron + hadron -> large transverse momentum hadron
+ other hadrons (3)

in certain kinematical regions. Such distributions always involve some fragmentation regions and plateaus with the heights of the plateaus, as well as the shapes of the fragmentation regions, for different reactions related to one another. It is interesting to see if such descriptions also can be naturally extended to other reactions in terms of the existing fragmentation regions and plateaus. Furthermore, we indeed need to study some other reactions in order to provide more tests of the parton model. Unless at extremely high energies, there is not enough phase space for the full structure of all these regions and therefore various regions may overlap. Thus it is difficult to use the full structure of the hadron distribution as tests of the model. On the other hand, the overlapping regions can still be related to one another and therefore it is desirable to study more reactions to test such relations. In this note, we study the final state hadron distribution in the reactions

hadron + hadron -> lepton pair + hadrons (4)

and

virtual photon + virtual photon -> hadrons, (5)

tests of the parton model, and estimates of the kinematical conditions for these tests.

We first briefly review the different regions of the hadron distributions. In a purely hadronic collision with limited transverse momentum, namely, the inclusive reaction

a + b -> c + anything (6)

where a,b, and c are all hadrons, there are two hadron fragmentation regions (the target and beam fragmentation regions) and a central hadron plateau[5,6]. Furthermore, according to the Mueller regge analysis and short range correlation models[5,6], the target fragmentation region is independent of the beam particle and vise versa and the central plateau is independent of both the target and the beam, when the distribution of the particle c is devided by the total cross section of the target and beam particles. At very high energies, this distribution scales and a schematic reprsentation is shown in Fig. 1a.

When the Mueller analysis is generalized to the deep inelastic electroproduction, a target fragmentation region, a hadron plateau, and a current fragmentation region are obtained. The hadron fragmentation region and the hadron plateau are the same as those in reaction (6), while, unlike hadrons with fixed mass, the virtual photon is associated with a variable mass and hence the size of its fragmentation region in the rapidity space is proportional to the logrithm of its mass. The structure of this current fragmentation region has been further studied in the parton model and, in this model, it consists of a hole fragmentation region, a current plateau, and a parton fragmentation region. Such regions are formed after a parton in the target hadron is knocked out by the virtual photon, leaving a hole in the initial parton distribution. The hole and the outgoing parton then evolve into hadrons to form these three regions. The final hadron distribution is schematically shown in Fig. 1b. In reaction (3), the lepton pair annihilates into a time-like virtual photon. This virtual photon then creates a parton anti-parton pair which evolves into the final hadrons in a similar way as the hole and the parton in reaction (1) do. The hadron distribution therefore consists of two parton fragmentation regions and a current plateau, as shown in Fig. 1c. Finally, the parton model has also been applied to reaction (3). This reaction takes place via the scattering of two partons at large momentum transfer . The final hadrons are distributed in three jets. One is along the initial collision axis and consists of two hadron fragmentation regions, two hadron plateaus, two hole fragmentation regions, two more plateaus, which shall be called mixed plateaus, and a region overlapping with the two other jets. These two other jets are respectively along the directions of the two scattered partons and each consists of a parton fragmentation region, a current plateau, and an overlapping region. Such a distribution is shown in Fig. 1d. The parton scattering is shown in Figs. 2a and 2b.

Let us now construct the parton model for the hadron distribution in reaction (4). The lepton pair mass distribution has already been discussed[7] and we only need to study how the final state hadrons are distributed. In this process, a parton in one hadron annihilates with an anti-parton in the other to form the lepton pair via a massive virtual photon, leaving two holes in the initial parton distribution, as graphically illustrated in Figs. 2a and 2c. In the center of mass frame of the initial hadrons, if the parton has a momentum $x_1 p$ and the anti-parton has momentum $-x_2 p$, then the lepton pair momentum and invariant mass respectively are given by

$$q = (x_1 - x_2) p \qquad (7)$$

$$Q^2 = x_1 x_2 s, \qquad (8)$$

where x_1 and x_2 are positive, s is the total invariant energy squared, and $p = s^{1/2}/2$ is the incident hadron momentum. In the parton model, the parton and anti-parton distributions are only functions of x_1 and x_2, then the lepton pair distribution is given by $d\sigma/dQ^2 = f(x_1 x_2)/Q^4$, which is the Drell-Yan scaling[7]. The remaining parton distribution is shown in Fig. 2c. Notice that this remaining system is the same as that along the collision axis in the reaction (3), as in Fig. 2b. Thus we naturally expect that the final hadron distribution is the same as the first jet in reaction (3), except that the overlapping region is absent and the two mixed plateaus join together. We thus have two hadron fragmentation regions, two hadron plateaus, two hole fragmentation regions, and a single hole plateau, as shown in Fig. 1e.

Invert Eqs. (7) and (8), we obtain

$$x_1 = [\, q + (\, q^2 + Q^2\,)^{1/2}]/2s^{1/2} \tag{9}$$

$$x_2 = -x_1 + q/s^{1/2} \tag{10}$$

Thus for an observe lepton pair with momentum q and mass Q, we can tell where were the origional parton anti-parton pair. The energy remaining in the origional hadrons are, respectively, $(1 - x_1)p$ and $(1 - x_2)p$. If $x_1 \ll 1 - x_1$ and $x_2 \ll 1 - x_2$, as shown in Fig. 1e, then there is a hadron fragmentation region and a hadron plateau between $(1-x_1)p$ and $x_1 p$, Near $x_1 p$, where there was a hole, the hadron distribution should be that of the hole fragmentation. Between $x_1 p$ and $-x_2 p$, there is a mixed plateau, which shall be discussed later, then there is another hole fragmentation region, another hadron plateau, and so on. The average multiplicity is given by

$$\langle n\rangle = \theta(1-2x_1)c_h \ln (1/x_1 -1) + \theta(1-2x_2)c_h \ln (1/x_2 -1) + c_x \ln Q^2 + \text{constant}, \qquad (10)$$

where c_x and c_h are respectively the heights of the mixed and hadron plateaus. We want to stress that in the view of the hole fragmentation in the parton model, the hadron distribution in reaction (4) "remenbers" where a parton and an anti-parton has annihilated into the lepton pair and may show some difference from the hadron distribution in reaction (6). We also want to remark that in a multiperipheral model[8] and in the lepton pair production by two photon process[9], there should be almost no hadron present between x_1 and x_2 and therefore there should be a dramatic difference from the parton model.

Next we study the hadron distribution from reaction (5). First let us consider the situation when the total energy, s, is much greater than the magnitudes of the phton masses, q_1^2 and q_2^2. In this energy range, each photon creates a virtual parton anti-parton pair[10], which then interact strongly to form the final hadrons. This is illustrated in Fig. 3a. The hadron distribution depends on the masses of the virtual photons. There are three different cases in this situation.

The most obvious one is when both q_1^2 and q_2^2 are small. In this case reaction (5) is just like $\rho^c + \rho^c \to$ hadrons in reaction (6), where the hadron distribution consists of two photon fragmentation regions and a hadron plateau in between. The second case is when one of the photon masses becomes large. This case is like the deep inelastic scattering off a photon target[11] and therefore the hadron distribution should be similar to that in reaction (1), that is, one of the photon fragmentation region reveals its full structure of a parton fragmentation region, a current plateau, and a hole fragmentation region. The last case is when both q_1^2 and q_2^2 are large but still much less than s. This is the scaling regge limit . In this case, both of the photon fragmentation region reveal their full parton model structures and the hadron distribution contains two parton fragmentation regions, two current plateaus, two hole fragmentation regions, and a hadron plateau, as shown in Fig. 3b. The average multiplicity in these cases is

$$\langle n \rangle = c_h \ln (s/q_1^2 q_2^2) + c_e \ln (q_1^2 q_2^2) + \text{constant}, \qquad (11)$$

where c_e is the height of the current plateau.

Since both photons can be space-like, we can also consider the situation where s is large but much smaller than one or both of the magnitudes of the photon masses, namely, the limit $|q_1^2|, |q_2^2| \to \infty$ and $s/(q_1^2 + q_2^2) \to 0$. This is the light cone limit[12], in which the two photons together create a parton anti-parton pair which then evolve into hadrons, as schematically shown in Fig. 3c. In this limit, the parton anti-parton pair should evolve in a way similar to that in reaction (2). The hadron distribution then have two parton fragmentation regions and a current plateau in between, similar to the one shown in Fig. 1c. Since the maximum rapidity length can not exceed lns, the average multiplicity in this limit is given by

$$\langle n\rangle = c_e \ln s + \text{constant} \tag{12}$$

We proceed to discuss some tests of the parton model by comparing the different regions of the hadron distributions. There are three plateaus, namely, the hadron plateau with height c_h , the current plateau with height c_e , and the mixed plateau with height c_x . The first one occures in reactions (1) and (3) - (6), the second one in reactions (1) - (3) and (5), while the last one only in (3) and (4). Therefore, reaction (4) is needed for a comparison of the hole plateaus in different reactions. Furthermore, this plateau is not well understood even in the parton model. In Ref.4 , there are four arguements leading to three different possibilities for the height of this plateau. The first possibility is that $c_x = c_e$, as obtained from the arguements in Ref.4 and also the triality arguement.[3] The second possibility is that the holes do not significantly affect the hadron distribution and therefore $c_x = c_h$. The third one is obtained from the arguement that the holes form their own plateau which is the same as that formed by the partons and the rest of the partons in the central region creates another plateau which is the same as the hadron plateau, hence we obtain $c_x = c_e + c_h$. Such an arguement immediately leads to yet another possibility that the holes and the rest of the partons creates two plateaus but not independently. The last possibility gives a value for c_x which is not related to either c_e or c_h . Although the value of c can de determined from reaction (3) alone, a study of reaction (4) offers a consistency check for the arguements.

As learned from reaction (6), only the fragmentation regions may be present and the plateaus may not develope until at extremely high energies. Thus it is difficult to test the model by trying to observe the full structure of the hadron distribution and comparing the heights of the plateaus. Thus we want to compare the fragmentation regions and even some overlapping fragmentation regions. There are three types of fragmentation regions, namely, the hadron fragmentation region, the parton fragmentation region, and the hole fragmentation region, which is always smoothly joined to one plateau at each side. The first one occurs in reactions (1), (3), (4), and (6), the second one in (1) - (3) and (5), and the last one in (1) and (3) - (5). Thus reactions (1), (2), (3), and (6) are enough to form a necessary set of consistency checks and the addition of reactions (4) and (5) only offers more constraints to the model.

When the energy is not very high, even the fragmentation regions may overlap. The overlapping of the fragmentation regions can be an asset instead of a liability. For example, we can form an overlapping fragmentation region of the hadron and the hole. This region is interesting by itself, since it characterizes the hadron distribution of an excited target with one hard parton at x knocked out. Such a region can only occur in reactions (1), (3), and (4), as schematically shown in Fig. 4. For this region, the importance of reaction (4) is at least two fold. (a) It is almost impossible to do electron scattering on a meson target, hence an overlapping meson and hole fragmentation region can only be explored by studying reactions (3) and (4) and comparing the results. (b) For reaction (3), it may be difficult to separate the three jets in an unambigious way, hence a comparison of the overlapping nucleon and hole fragmentation region is easier to carry out for reactions (1) and (4) than (1) and (3).

The hadron distrobution from reaction (6) is interesting by itself. As discussed before, the full structure shown in Fig. 3b can be revealed only if $s \gg |q_1^2|, |q_2^2|$. However, when the energy is not high enough, the different regions may not overlap in the sense discussed above. In the light cone limit, the hadron plateau will disappear and the two hole fragmentation regions will also disappear, instead of overlapping with the remaining regions. In principle, it is thus interesting to see how such a transition occure. However, as we shall discuss, it probably requires too high an energy too study this transition in the near future.

Finally we should estimate some kinematical conditions for the validity of the asymptotic descriptions of these distributions and the tests mentioned above. In rapidity space, the distance between the hadron and the hole is $\ln(1/x-1)$, where x is either x_1 or x_2 for reaction (4) and $x=Q^2/2m\nu=1/\omega$, $-Q^2$ and ν are respectively the virtual photon mass and energy, and m is the target hadron mass for reaction (1). Since the hadron fragmentation region has a typical length of two to three, the hole and hadron fragmentation regions will definitely overlap if $\ln(1/x-1)<2$, which corresponds to $x>0.1$. Most optimistically, we can avoid the overlap only if $x<0.1$. However, the hole fragmentation region also spans a certain length on both sides of the hole. If the hole fragmentation region also have a typical length of two, the two regions may still overlap for $\ln(1/x-x)$ as large as three. Thus, very likely, these two region can be separated only for $x<0.03$. We must also study the conditions on Q^2. If Q^2 is too small, not only the current or mixed plateau will completely disappear but the hole and parton fragmentation region will also overlap. Assuming that the parton and hole fragmentation regions all have length two, we can avoid a complete overlap only if $\ln Q^2>2$ and therefore $Q^2>10$ GeV^2. For these two regions to be completely separated and a current or mixed plateau to develope, we probably need $\ln Q^2>4$ and $Q^2>100$ GeV^2. Therefore, we encounter the following cases.

(a) For $Q^2 > 100$ GeV^2 and $x < 0.1$, we can probe into the full structure of the different regions. This requires $2m\nu > 1000$ GeV^2 for reaction (1), momentum of at least 5 GeV/c per beam for reaction (2), and $s > 10^4$ GeV^2 for reaction (4).

(b) For $Q^2 > 10$ GeV^2 and $1-x > x > 0.1$, we can probe into an overlapping hadron and hole fragmentation region for reactions (1) and (4) and also a small portion of the parton fragmentation for reactions (1) and (2). Obviously, the hadron distribution in this overlapping region depends on where is the overlap. Most reasonably, we should compare the distributions as functions of the rapidity for two different reactions with the same value of x. This energy range requires 20 GeV^2 $< 2m\nu <$ 100 GeV^2 for reaction (1), $s > 10$ GeV^2 for reaction (2), and 50 GeV^2 $< s <$ 100 GeV^2 for reaction (4).

(c) For $Q^2 \cong 1$ GeV^2 and $x < 0.1$, the parton and/ or hole fragmentation regions and the current or mixed plateau all overlap into a finite current fragmentation region but it can be separated from the other regions. In this case, we have a hadron fragmentation region, a current fragmentation region, and even a hadron plateau at high energies. Such a distribution is very similar to that from reaction (6) and is straightforwardly expected from generalized Mueller analysis. This energy range is $2m\nu > 100$ GeV^2 for reaction (1) and $s > 1000$ GeV^2 for reaction (4).

(d) For $Q^2 \cong 1$ GeV^2 and $x > 0.1$, all the regions are probably still under developement and overlap with one another. Even if the parton model itself can still apply to these reactions at this energy range, we do not expect its asymptotic predictions on the hadron distributions to be relevant. Such an energy range is $2m\nu < 10\ GeV^2$ for reaction (1) and $s < 50\ GeV^2$ for reaction (4).

As far as reaction (5) is concerned, the parton model description of the hadron distribution may seem to be purely academic in the foreseable future. Since observing the full structure would probably require $-q_1^2, -q_2^2 > 100 GeV^2$ and $s > 10^5\ GeV^2$, which is impossible to achieve with the electron-electron colliding beams. Even to observe the hadron plateau for large s and small photon masses would require $s > 100\ GeV^2$, which is also unlikely to achieve. The only hope is at reasonbly high value of the photon masses, we may see some part of the photon fragmentation region.

To conclude, we remark that the parton model description of the hadron distributions in different reactions in terms of only a few regions is a very interesting one but probably difficult to test. Even the above estimation of the energy required for the tests may be too optimistic. As learned from reaction (6) at NAL and ISR energies, the height of the hadron plateau is still energy dependent and no asymptopia has been reached yet. We may well expect similar situations to happen in the other reactions. Only experiments at high energies may tell us whether this is the case. Furthermore, we do have some knowledge about the corrections to the asymptotic behavior for reaction (6) but much less knowledge about the corrections to the other reactions. At present energies, we probably need a good nonasymptotic model to describe the observed data.

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Figure Captions

Fig. 1 (a) Schematic illustration of a hadron distribution, as a function of the rapidity in an arbitrary frame along the collision axis, in reaction (6).

(b) - (e) Similar illustrations for reactions (1) - (4) respectively, except that Fig 1d only shows the phase space distribution in transverse momentum and rapidity.

In all of the above figures, the various regions are represented by the capital letters, where A-C respectively represent the hadron, the parton, and the hole fragmentation regions, D-F respectively represent the hadron, the current, and the mixed plateaus, and G represents the overlapping region of the three jets in reaction (3); $m_{\perp}$ is the average transverse mass, typically of the order of 1 GeV; a, b, and c are respectively the typical lengths of the regions A, B, and C; c_h, c_e, and c_x are the heights of the plateaus D, E, and F.

Fig. 2 (a) Parton distribution before a collision.

(b) Parton scattering in reaction (3).

(c) Parton annihilation into lepton pair in reaction (4), where the dotted lines represent the lepton pair.

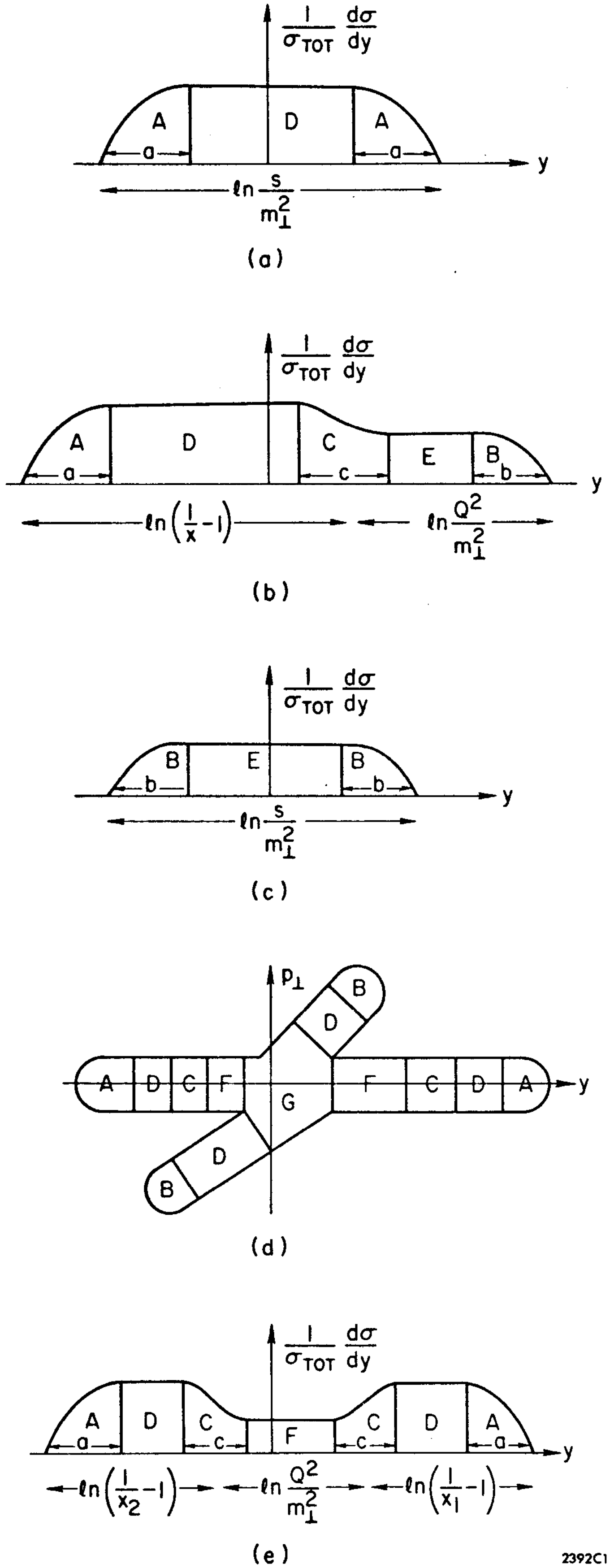
Fig. 3 (a) Two photon go into hadrons via the process in which each photon first creates a parton anti-parton pair. The shaded area represents strong interactions.

(b) Illustration of the full structure of the hadron distribution from reaction (5).

(c) Parton model description of the reaction (5) in the light cone limit.

Fig. 4 (a) Illustration of the overlapping regions in reaction (1), where the dotted line represent the position of the hole and the shaded area is the mixed hadron and hole fragmentation region to be compared with that from reactions (3) and (4).

(b) Similar illustration for reaction (4).



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ln(1/x - 1)
ln Q²/m⊥²
(b)
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Fig. 1

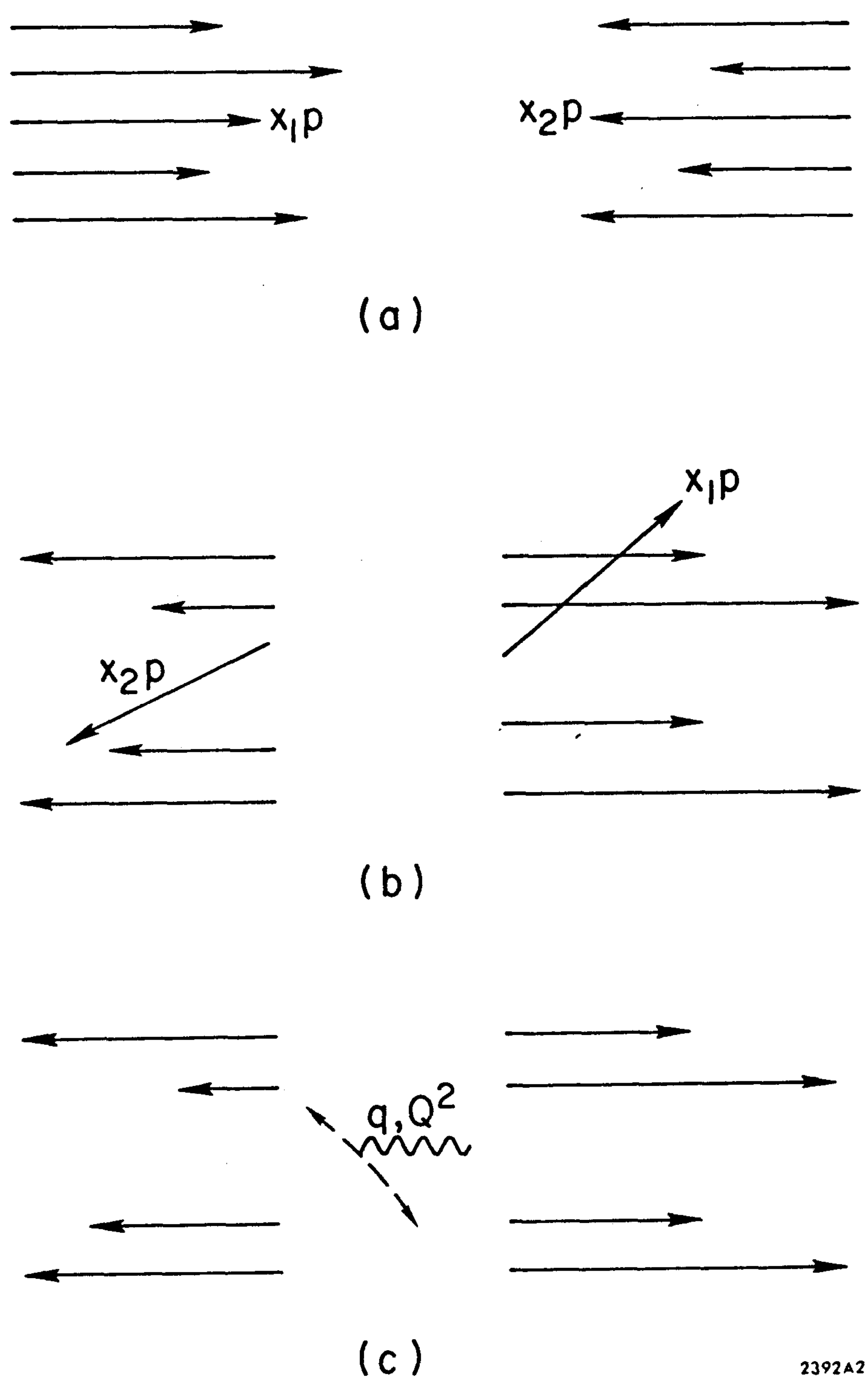


Fig. 2

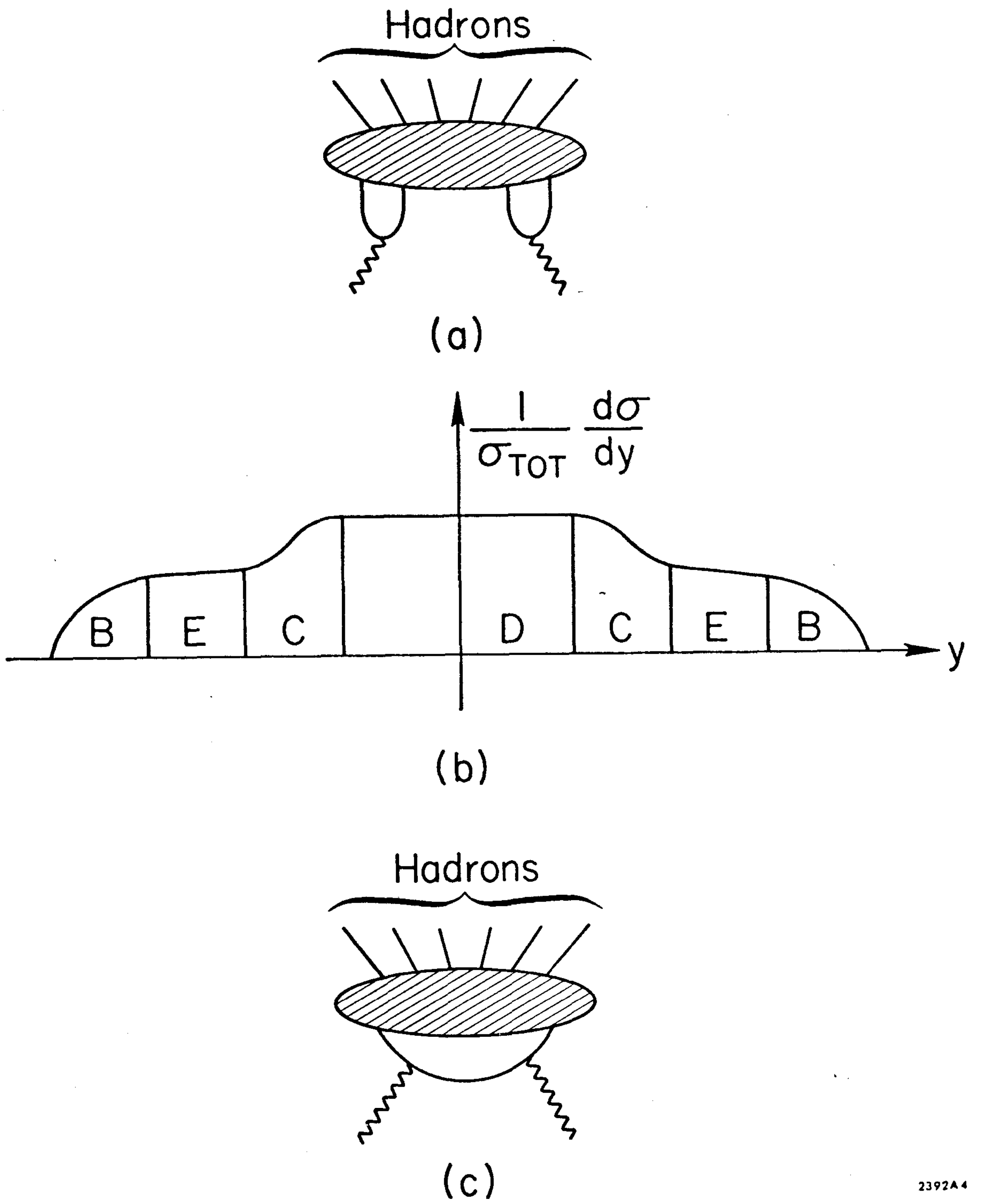
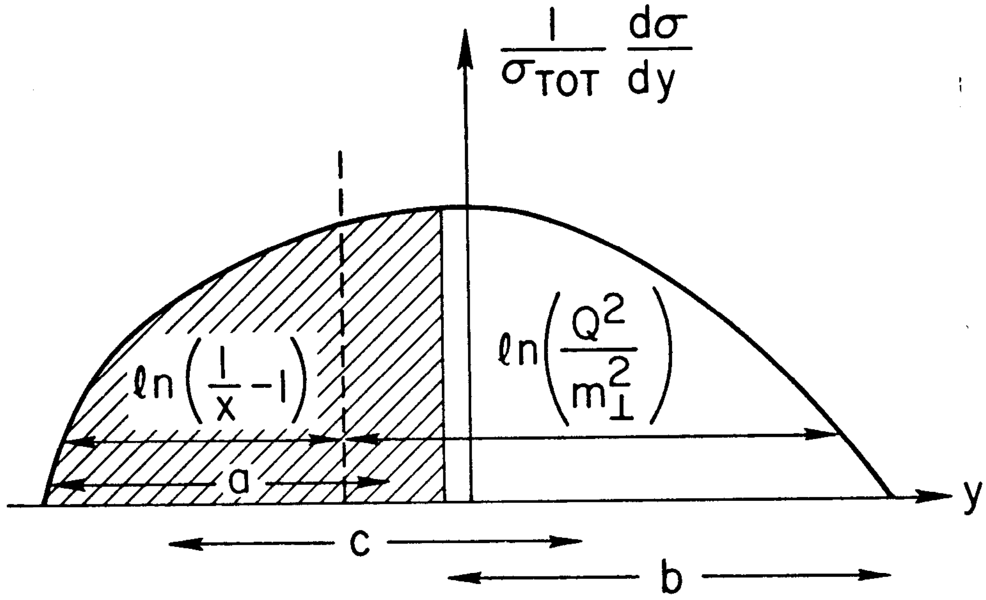
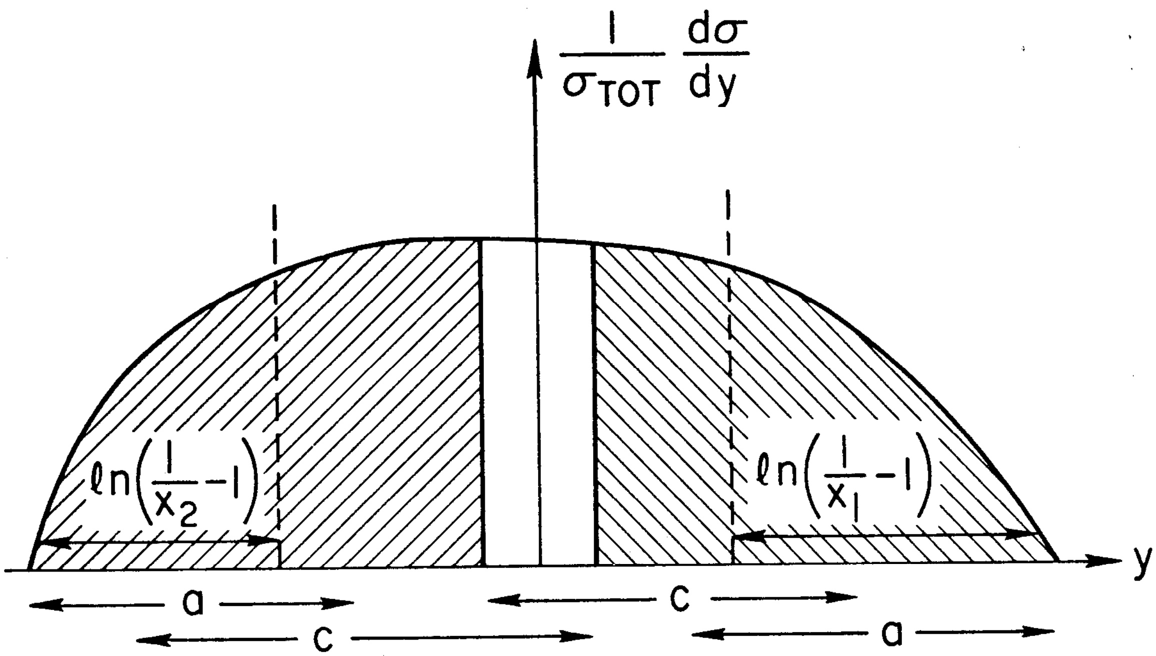


Fig. 3



(a)



(b)

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Fig. 4