TEST OF NEWTON'S INVERSE SQUARE LAW OF GRAVITY USING THE 300 m TOWER AT ERIE, COLORADO: NEWTON VINDICATED ON THE PLAINS OF COLORADO

C. C. Speake,[a] T. M. Niebauer,[b] M. P. McHugh
P. T. Keyser, and J. E. Faller[c]
Joint Institute for Laboratory Astrophysics
University of Colorado and National Institute of Standards and Technology
Boulder, CO 80309-0440

J. Y. Cruz and J. C. Harrison
Geodynamics Corporation, 5520 Ekwill Street, Suite A
Santa Barbara, CA 93111

Jaako Mäkinen
Finnish Geodetic Institute, Ilmalankatu 1A, SF-00240
Helsinki, Finland

R. B. Beruff
Defense Mapping Agency
(retired)

ABSTRACT

A 300-m meteorological tower was used to measure gravity at eight different heights above the surface of the earth. The observed gravity values were corrected for tides and gravimeter screw error. Also, tests were performed to look for systematic effects due to tower motion. The resulting values are compared to values predicted by Newton's inverse-square law from upward continuation of surface gravity. The difference between the measured and predicted values at the top of the tower is $21 \pm 27 \times 20^{-8}$ ms^{-2}. This result places new constraints on any non-Newtonian interaction.

[a] Visiting from: Bureau International des Poids et Measures, Pavillion de Breteuil, 92312 Sevres, CEDEX France; now at: School of Physics and Space Science, Univ. of Birmingham, Birmingham B15 2TT, U.K.
[b] Present address: Max Planck Institut für Quanten Optik, Ludwig-Prandtl Str. 10, D-8046 Garching-bei-München, Federal Republic of Germany.
[c] Staff Member, Quantum Physics Division, National Institute of Standards and Technology.

Recent tests of the inverse-square law over geophysical distance scales have been prompted by the reported results of Stacey and his coworkers[1] and Fischbach et al.[2] The first group revived Airy's[3] method of determining G by measuring the change in gravity down a mineshaft and found a significant $\Delta G/G$ 1%; the second group reanalyzed the results of the test of the weak equivalence principle of Eötvös et al.[4] and proposed a new mid-range composition-dependent "fifth force." Although the hypothesized "fifth force" is composition-dependent, for experiments sensitive to its range and strength only, we may write the potential energy of two point test masses due to the new force as:

$$V_5 \equiv -G\, M_1 M_2 \cdot \alpha \frac{e^{-r/\lambda}}{r} \quad , \tag{1}$$

where λ is the range and α is the strength parameter of the interaction.

Assuming this Yukawa parameterization of the putative non-Newtonian interaction the original results of Stacey et al. suggested $\alpha \simeq -0.007 \pm 0.004$ and $\lambda \sim 200$ m. Uncertainty in the topographic corrections is one problem which has been pointed out by Bartlett and Tew.[5]

Ander et al.[6] performed an Airy-type measurement using a borehole in the Greenland ice-cap. However, the advantages of the homogeneity of the ice were somewhat outweighed by the uncertainty in the density of the underlying bedrock. The results of this experiment, which at first appeared to give evidence of non-Newtonian gravity, now seem to be inconclusive.

Eckhardt et al.[7] avoided the uncertainties due to density inhomogeneities and problems associated with downward continuation toward bedrock by measuring the variation of gravity on a tower. In this scheme, data from a comprehensive gravity survey and knowledge of the topography around the tower are used to predict gravity at each level. They initially reported an anomaly which could have been attributed to a Yukawa term with $\alpha = +0.02$ and with a range $\lambda = 300$ m, but also noted that their result is consistent with that of Stacey if a theoretical model with two Yukawa terms is assumed. These results have been questioned[8] and the most recent work by Eckhardt[9] gives results which are consistent with Newtonian gravity. The experiment of Thomas et al.[10] confirms this.

In August 1988 we decided that an independent experiment would help clarify the situation, and undertook to perform a tower test of gravity. The Erie tower, although shorter than the WTVD[7] and BREN[9] towers, is conveniently located near our lab. It also has temperature and wind speed instrumentation at all eight levels, is stable, and stands on nearly flat (out to 20 km) and easily-surveyed terrain.

Because of concerns about possible systematics due to the effects of tower vibration on the gravimeters, an investigation of the stability of the tower was undertaken first. The measurements of tilt and horizontal acceleration of the tower at wind speeds of 2.4 ms^{-1} produce a calculated systematic error of at most 2×10^{-8} ms^{-2} in measured gravity at the 295 m level. The precise heights of each tower level (given in Table I) were determined using an electronic distance measurement device (EDM) which we calibrated ourselves. Measurement of the heights at different temperatures yields the expected thermal expansion of the steel tower $(1.35\pm0.05) \times 10^{-5}$°C. This expansion coefficient was used to adjust the gravity measurements made at different temperatures to a standard height.

In all, 27 series of measurements (loops) were taken on the tower during August, September, and October 1988; 13 of these were retained. Several earlier series were rejected because of the high statistical variation due to our initial lack of expertise in making measurements on a tower. Two LaCoste-Romberg G-type gravimeters (G115 and G139) with electrostatic feedback were employed. These meters were calibrated using seven gravity stations in Colorado which include the range of gravity values on the tower. The uncertainty in the calibration factors led to an uncertainty of 9×10^{-8} ms^{-2} over the range of the tower. This was obtained from the standard error of the least-squares fit to a line.

The raw gravity data were corrected for earth tides and values of gravity for each tower level sampled within a measurement loop were calculated using a least-squares fit to a linear drift model. Corrections were made for cyclical screw errors and drift; together these amounted at most to 20×10^{-8} ms^{-2}. We estimate the systematic uncertainty due to wind as 5×10^{-8} ms^{-2} for the 300 m gravity value and scale the wind effect with height so uncertainties can be assigned to the values at the lower levels. This value for the wind effect is in fair agreement with the measurements referred to and calculations. The gravitational attraction of the tower and its foundation were adequately compensated for by the upward continuation except at 295 m where the attraction of the tower required a 1×10^{-8} ms^{-1} correction. The results for gravity values relative to the tower base for each level are listed in Table I.

To take into account the effect of local topography and subsurface density variations we made a local survey comprising 265 stations within 8 km of the base of the tower (191 were within 800 m and 70 within 60 m) supplemented with DMA library gravity data. The relative elevations, latitudes and longitudes of the gravity stations lying within 800 m of the tower were independently surveyed by us. In total we used about 26,000 measurements in a 4° × 5° area, 2640

TABLE I. Comparison between measured and Newtonian predictions.

Observation Height (m)	Measured Δg (10^{-5} ms^{-2})	predicted Δg (10^{-5} ms^{-2})	measured-predicted (10^{-5} ms^{-2})
8.198	-2.556±.009	-2.551±.010	-.005±.013
21.912	-6.789±.010	-6.784±.010	-.005±.014
48.568	-14.986±.010	-14.994±.011	.008±.015
97.323	-30.000±.010	-29.991±.012	-.009±.016
149.136	-45.908±.011	-45.914±.015	.006±.019
197.896	-60.892±.012	-60.890±.018	-.002±.022
249.718	-76.800±.013	-76.802±.021	.002±.025
295.438	-90.816±.014	-90.837±.023	.021±.027

measurements in a 1° × 1° area and 402 measurements in a 10' × 11' area centered on the tower.

The predictions of gravity at the observing platforms were made in two stages: first, the global and regional fields were modeled as the sum of spherical harmonic functions, centrifugal acceleration, the earth's atmosphere, and the attraction of point mass sets. The resolution is of the order of 1-2 km in the immediate vicinity of the tower. This representation is adequate outside a radius of about 20 km around the tower. The second stage treats the residuals (the differences between the measured field and the global/regional model) within the 20 km radius.

The global/regional model starts with a spherical harmonic expansion of the gravitational potential of degree and order 8, plus centrifugal acceleration due to the Earth's rotation. This model is refined by the addition of the attractions of five mass sets centered on the tower with spacings of 5°, 1°, 15', 5' and 1' (30") for model 1 (model 2) in latitude and longitude, at depths equal to their horizontal spacing and of horizontal extents which get smaller as the grid spacing decreases. These mass sets are determined from deviations of the gravity field from the standard ellipsoidal model which are estimated primarily from terrestrial gravity measurements on the North American continent and from satellite altimetry over the oceans. The 5° × 5° mass set can be considered to add detail down to 10° wavelength, the 1° × 1° set to refine this detail to 2° wavelengths, and so on. The residuals in the two cases (model 1 and model 2) are quite different but the platform values, computed as the sum of the model plus upward continued surface residuals, are in agreement to within their uncertainties. This provides a check as to whether the granularity used was fine enough. The average of the results of these two models is shown in Table I.

In view of the proximity of the Erie tower to the Rocky Mountains, an additional complication to modeling gravity in mountainous terrain must be considered. Anomalies are estimated at fine (about 1 km) granularity on the topographic surface, then continued analytically down to sea level, and averaged into blocks of dimension appropriate to the granularity of the point mass sets. The point masses of the global/regional models are fitted to these sea level averages and give an unbiased representation of the average field on level surfaces above the topography. Mass sets of granularity finer than 5' are, however, fitted to residuals on the topographic surface. In fact, the Erie tower is far enough from the Rocky Mountains that these problems, which are associated with the modeling of gravity in mountainous terrain, give rise to a correction only of the order of 13×10^{-8} ms^{-2} to the predicted gravity difference over the height of the tower.

When finding the residual field inside 20 km, the surface field is first estimated as a grid of values representing averages over 30" × 30" or finer blocks. As the topographically-corrected Bouguer anomalies vary more smoothly than the field itself, the irregularly spaced gravity observations are reduced to Bouguer anomalies. These anomalies are interpolated onto the desired grid and gravity values recovered by reversing the Bouguer corrections. This is accomplished by using the appropriate mean topographic elevations. These elevations are usually obtained from DMA's digital terrain elevation data base but within about 2.5 km we hand-digitized 3" × 3" mean elevations from large scale topographic maps contoured at 10 foot intervals. Even this refined topographic model contains significant errors and the density of gravity measurements within 800 m of the tower is high enough that these topographically-derived estimates were corrected using our own elevation observations.

Values computed from the spherical harmonic/point mass model were subtracted from the gridded estimates of surface gravity. These residuals were continued analytically to a plane through the base of the tower using Fourier series techniques,[11] and then to the platform levels using the Poisson integral. Gravity values were then computed as the sum of the spherical harmonic/point mass model and the continued residual field. Uncertainties in the upward continuation are based on collocation techniques.

Differences between the measured and predicted gravity intervals are given in Table I. Agreement of the measured values with the Newtonian predictions is clearly excellent and the validity of the inverse-square law under the conditions of the experiment is confirmed. This agreement is, in fact, rather better than expected from the estimated standard errors. These errors are very difficult to estimate and our values are evidently somewhat conservative. The residuals of the present experiment are shown together with those of other tower experiments in Fig. 1.

A Yukawa potential of the type in Eq. (1) would predict a difference in acceleration between a point at height z and a point on the ground equal to $2\pi G\rho\alpha\lambda(e^{-z/\lambda}-1)$, for $\lambda \ll$ radius of the earth, where ρ is the density of the earth's surface. Figure 2 illustrates the constraints placed by this experiment on such an interaction. Also shown are the constraints placed by other recent experiments.

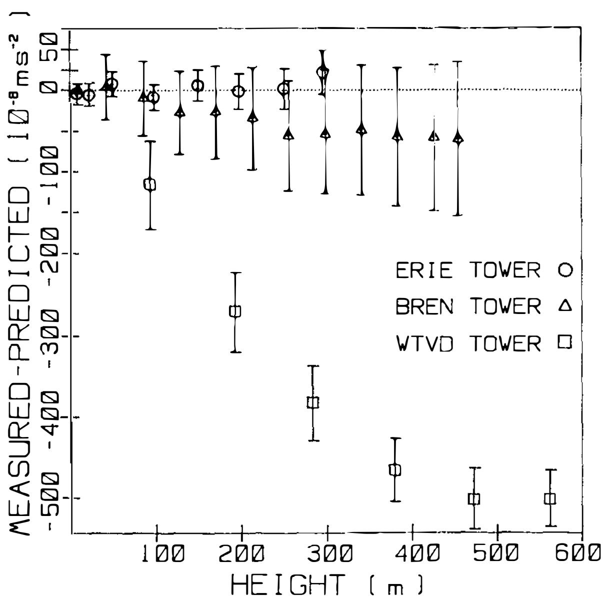

Fig. 1. The dotted line corresponds to the Newtonian prediction. The BREN tower result is from data in Ref. 9. The WTVD tower points are from the result in Ref. 7.

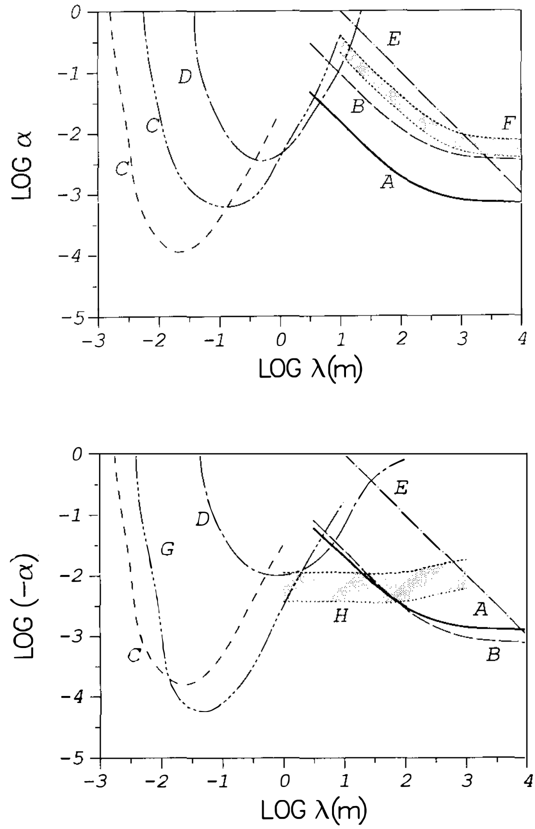

Fig. 2. Excluded strengths (α) and ranges (λ) for a single Yukawa model are in the area above the curves at the 1σ level. Shaded regions represent positive results. The log(α) graph (a) is for an attractive force, the log(-α) graph (b) is for a repulsive force. Solid curves labeled A represent this work; curves labeled B are from data in Ref. 9 (note that Thomas et al. use the opposite sign convention for α); curves C-G are from Refs. 12-16, respectively; curve H is from data in Ref. 1.

The Quantum Physics Division of NIST provided the $1000 tower rental fee. We are indebted to NOAA for allowing us flexibility in the use of the tower, to NGS and DMA for helping to carry out the local surveys and for providing personnel and instruments. We thank P. Bender, R. Sands, D. Eckhardt, and J. Thomas for helpful discussions. Salary for personnel involved came from BIPM, NIST, AFGL, Fort Belvoir RD&E Center, NGS, USAF Ballistics Systems Division and DMA. We are also grateful to Mr. H. Malek for loaning us the EDM.

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