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Near Horizon Extreme Kerr Geometry

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Abstract. The conjectured Kerr/CFT correspondence states that the quantum theory of gravity in the near horizon of the Near Horizon Extreme Kerr (NHEK) black holes, is holographic dual to a two-dimensional chiral conformal field theory. To obtain NHEK geometry, we take the extreme limit $a = M$ of the Kerr metric and continue by transforming the coordinates. Consequently, we can obtain the new Ernst potentials for that geometry by using the same coordinates transformation. Finally, we can find the central charge as a function of the Papapetrou-Ernst potentials from stretched horizon formalism.

1. Introduction

In mathematical physics, black hole solutions are related to mathematical equations which are equivalent to the Einstein field equation called by Ernst equations in stationary case. The solutions of these equations are named as Ernst potentials [1, 2]. Ernst equations are also related to the general rotating black hole solutions proposed in [3] which correspond to the killing vector $\hat{\xi}_t$ and $\hat{\xi}_\phi$. This metric can be applied to the more general black holes, Kerr-Newman black holes, so by assuming some cases it will reduce to Schwarzschild, Reissner-Nordström, and also Kerr solutions. So in our case, we take the intrinsic charge of the Kerr-Newman spacetimes to zero in order to obtain the Kerr one.

In our universe, we believe that there so many physical phenomena that have been proved indirectly from the observation such as the cosmological model of Dark Energy, that have been modeled in many theories such in [4, 5, 6], and theoretically appear but still less of observational evidence such as holographic duality in black holes theory. Furthermore, physical theory that is recently discussed is about the extreme black holes, especially about the discoveries of the holographic dualities, which tell the relation between the quantum theory of gravity with the quantum field theory or we can call it by AdS/CFT correspondence [7]. AdS/CFT correspondence leads the emergence of the Kerr/CFT correspondence. The conjectured Kerr/CFT correspondence, that states the quantum theory of gravity in the near horizon of extreme black holes is holographic dual to the conformal field theory (CFT), also has been quite successful as the tools to find the microscopic origin of the Bekenstein-Hawking entropy which is proportional to the central charge (that appears from the Cardy formula) times the extremal Frolov-Thorne temperature. To obtain the corresponding central charge, the method proposed by Brown and Henneaux [8] which is associated to AdS₃ spacetime can be adopted. Besides the application for the black holes, AdS/CFT correspondence also has been used on the other gravitational objects such as Neutron stars [9] and Boson stars [10] that have



cosmological constant in the action. In [10], there also a study about the counterpart of the boson stars, i.e. Q-balls with the non-zero cosmological constant. We can also find the others study of the Boson stars and Q-balls in [11, 12, 13].

In the holographic dualities, the central charge of the Virasoro algebra that appears from extremal Kerr which generates the asymptotic symmetries of the near horizon geometry is also found [14, 15]. In addition, the central charge of the extremal magnetized Kerr-Newman metric has been found by Astorino [16]. He also shows the case when the intrinsic charge vanishes and it reduces to magnetized Kerr one such in [17], independently. By taking B to zero, the central charge that appears from extremal Kerr-Newman metric also can be obtained. However, the central charge that is associated with the Reisser-Nordström metric cannot be obtained except we extend to 5 dimensions such in [18] because in 4 dimensions there are no off-diagonal terms of the metric.

The Near Horizon Extreme Kerr (NHEK) geometry of the Kerr black holes has been obtained taking the extreme limit of the event horizon, i.e. $M = a$. Finally, the central charge can also be obtained by using stretched horizon formalism with four-dimensional stationary black hole metric in an ADM form [19]. This method can be applied for all stationary black hole metric in N-dimension.

In this following, we show the Ernst potential of the extremal Kerr by using the coordinate transformation that has never been shown yet. To find that that Ernst potential, we use the same coordinate transformation like we use to find the NHEK metric. We can see that the potential just depends on the angular coordinate because we have taken the radial coordinate on the horizon of the black holes. Note that we know the central charge of extremal geometry, which corresponds to the entropy of the black holes. We also find the central charge as a function of the Papapetrou and Ernst potentials. This result will help us to find the extreme Kerr metric from the Ernst equation and relates to our unpublished work [20].

We organize the remaining parts of the paper as follows. In section 2, we review the Ernst equations and its solution is related to the Papapetrou metric. In section 3, the central charge of the stretched horizon formalism is obtained in the function of the Papapetrou and Ernst potentials. Here, we investigate the NHEK geometry and find the new Ernst potential. In the last section, we end with the summary and conclude the rest of the paper.

2. Review on the Papapetrou metric and Ernst Equations

We begin with the general stationary metric in Papapetrou form [3] such that

$$ds^2 = f^{-1} \left[e^{2\gamma} (d\rho^2 + dz^2) + \rho^2 d\hat{\phi}^2 \right] - f (d\hat{t} - \omega d\hat{\phi})^2 \quad (1)$$

which is related to the killing vectors $\hat{\xi}_{\hat{t}}$ and $\hat{\xi}_{\hat{\phi}}$. By defining $e^{2\gamma} = -2P^{-2}$, $d\rho^2 + dz^2 = d\zeta \cdot d\zeta^*$, and transforming $\hat{t} \rightarrow \hat{\phi}$ and $\hat{\phi} \rightarrow \hat{t}$, we obtain

$$ds^2 = f^{-1} \left(-2P^{-2} d\zeta \cdot d\zeta^* + \rho^2 d\hat{t}^2 \right) - f \left(d\hat{\phi} - \omega d\hat{t} \right)^2. \quad (2)$$

For the case of the metric which consists of radial and angular coordinates, we can choose

$$d\zeta = \frac{1}{\sqrt{2}} \left(\frac{d\hat{r}}{\sqrt{\Delta}} + i d\theta \right). \quad (3)$$

Hence, equation (1) becomes

$$ds^2 = f^{-1} \left[-P^{-2} \left(\frac{d\hat{r}^2}{\Delta} + d\theta^2 \right) + \rho^2 d\hat{t}^2 \right] - f \left(d\hat{\phi} - \omega d\hat{t} \right)^2, \quad (4)$$

and we have chosen that all potentials in the Papapetrou metric to depend on r, θ coordinates.

These equations, (1) and (4), are the solutions of Ernst equations that are defined by

$$(\text{Re } \varepsilon + |\Phi|^2) \nabla^2 \varepsilon = (\nabla \varepsilon + 2\Phi^* \nabla \Phi) \cdot \nabla \varepsilon, \quad (5)$$

$$(\text{Re } \varepsilon + |\Phi|^2) \nabla^2 \Phi = (\nabla \varepsilon + 2\Phi^* \nabla \Phi) \cdot \nabla \Phi, \quad (6)$$

where $\varepsilon = f - |\Phi|^2 + i\hat{\phi}$ and Φ are the complex function solutions. For example, we show the solution of Kerr-Newman metric with the corresponding Ernst potentials which is found in [2]. The Ernst potentials are

$$\Phi = -iq \cos \theta + \frac{q a \sin^2 \theta}{\hat{r} + i a \cos \theta}, \quad (7)$$

$$\varepsilon = -(\hat{r}^2 + a^2) \sin^2 \theta - q^2 \cos^2 \theta + 2M a i \cos \theta (3 - \cos^2 \theta) - \frac{2a \sin^2 \theta (M a \sin^2 \theta + i q^2 \cos \theta)}{\hat{r} + i a \cos \theta}. \quad (8)$$

By taking these Papapetrou potentials

$$\rho = \sqrt{\Delta} \sin \theta, \quad P = (\sqrt{A} \sin \theta)^{-1}, \quad f = -\frac{A \sin^2 \theta}{\Sigma}, \quad \text{and } \omega = \frac{(2M\hat{r} - q^2)a}{A}, \quad (9)$$

the Kerr-Newman metric is obtained to be ($G = c = \hbar = 1$)

$$ds^2 = \Sigma \left(-\frac{\Delta}{A} dt^2 + \frac{d\hat{r}^2}{\Delta} + d\theta^2 \right) + \frac{A \sin^2 \theta}{\Sigma} \left(d\hat{\phi} - \frac{(2M\hat{r} - q^2)a}{A} dt^2 \right)^2, \quad (10)$$

where

$$\Delta \equiv \hat{r}^2 - 2M\hat{r} + a^2 + q^2, \quad \Sigma \equiv \hat{r}^2 + a^2 \cos^2 \theta, \quad A \equiv (\hat{r}^2 + a^2)^2 - \Delta a^2 \sin^2 \theta.$$

By setting $q = 0$, Kerr-Newman metric reduces to Kerr or $a = 0$ reduces to Reissner-Nordström, or setting both of $a = q = 0$ reduces to Schwarzschild.

3. NHEK Black Holes and The Corresponding Ernst Potentials

Here, we will study the extremal Kerr geometry by using new coordinates [14, 15, 21] on Kerr metric in Boyer-Lindquist coordinates defined by

$$ds^2 = \Sigma \left(-\frac{\Delta}{A} dt^2 + \frac{d\hat{r}^2}{\Delta} + d\theta^2 \right) + \frac{A \sin^2 \theta}{\Sigma} \left(d\hat{\phi} - \frac{2M\hat{r}a}{A} dt^2 \right)^2, \quad (11)$$

Potential ω is the angular velocity of the Kerr black hole which has an important role in finding the central charge in the extremal case. There is an event horizon at

$$r_{\pm} = M \pm \sqrt{M^2 - a^2}. \quad (12)$$

The surface gravity, Hawking temperature, and the angular velocity of the event horizon are

$$\kappa = \frac{r_+ - M}{2Mr_+}, \quad T_H = \frac{\kappa}{2\pi}, \quad \Omega_H = \frac{a}{2Mr_+}. \quad (13)$$

For the extremal case $a = M$, the event horizon is at $r_+ = M$ and the near horizon is at $\hat{r} = r_+$. We see that the Hawking temperature vanishes on the extremal case but later we will see that the temperature on the stretched horizon has a non-zero value.

Before showing the Ernst potential of the extreme Kerr, we introduce this coordinates transformation

$$\hat{t} = \frac{2Mt}{\lambda}, \quad \hat{\phi} = \phi + \frac{t}{\lambda}, \quad \hat{r} = \frac{\lambda M}{y} + M, \quad (14)$$

on the extremal Kerr metric and take the limit $\lambda \rightarrow 0$. Finally, by using (14), we can obtain

$$ds^2 = 2J\Omega^2 \left[\frac{-dt^2 + dy^2}{y^2} + d\theta^2 + \Lambda^2 \left(\frac{dt}{y} + d\phi \right)^2 \right], \quad (15)$$

which is in the Poincaré-type coordinates and where

$$J = M^2, \quad \Omega^2 = \frac{1 + \cos^2\theta}{2}, \quad \Lambda = \frac{2\sin\theta}{1 + \cos^2\theta}.$$

By using $d\hat{t} \rightarrow y^2 d\hat{t}$, we have

$$ds^2 = 2J\Omega^2 \left[-y^2 d\hat{t}^2 + \frac{dy^2}{y^2} + d\theta^2 + \Lambda^2 \left(d\phi + y d\hat{t} \right)^2 \right], \quad (16)$$

which is NHEK in the form of general near horizon geometry that is still a vacuum solution. We also find the new Ernst potential along with using this transformation (14) that only depends on the angular coordinate because we have take $\hat{r} = M$. The corresponding Ernst potential is

$$\varepsilon = -2M^2 \sin^2\theta - \frac{2M^2 \sin^4\theta}{1 + \cos^2\theta} + 2iM^2 \cos\theta \left[(3 - \cos^2\theta) + \frac{\sin^4\theta}{1 + \cos^2\theta} \right]. \quad (17)$$

Then global NHEK geometry [22] also can be obtained from coordinates transformation (14) on the metric (15), which are defined by

$$y = \frac{1}{r + \sqrt{1 + r^2} \cos\tau}, \quad t = y \sin\tau \sqrt{1 + r^2}, \quad \text{and} \quad \phi = \varphi + \ln \left(\frac{\cos\tau + r \sin\tau}{1 + \sin\tau \sqrt{1 + r^2}} \right). \quad (18)$$

Then we find

$$ds^2 = 2J\Omega^2 \left[-(1 + r^2) d\tau^2 + \frac{dr^2}{1 + r^2} + d\theta^2 + \Lambda^2 (d\varphi + r d\tau)^2 \right]. \quad (19)$$

Now, the central charge that is associated with NHEK geometry can be calculated in the stretched horizon formalism. In this formalism, the metric which is used is four-dimensional stationary black hole in ADM form [19]

$$ds^2 = -N^2 d\hat{t}^2 + h_{\hat{r}\hat{r}} d\hat{r}^2 + h_{\hat{\theta}\hat{\theta}} d\hat{\theta}^2 + h_{\hat{\phi}\hat{\phi}} (d\hat{\phi} + N^{\hat{\phi}} d\hat{t})^2, \quad (20)$$

The central charge which corresponds to the (20) has general form as

$$c = \frac{3f A_{BH}}{2\pi}, \quad (21)$$

where A_{BH} is the horizon area of the black hole and \hat{f} is a function which is defined by

$$\hat{f} = \left. \frac{f_2 f_3}{f_1} \right|_{\hat{r}=\hat{r}_+} \quad (22)$$

where

$$f_1 = \frac{N}{\hat{r} - \hat{r}_+}, f_2 = (\hat{r} - \hat{r}_+)\sqrt{h_{\hat{r}\hat{r}}}, \text{ and } f_3 = \frac{N\hat{\phi} + \Omega_H}{\hat{r} - \hat{r}_+}. \quad (23)$$

The Hawking temperature is defined by

$$T_H = \frac{\partial_r N}{2\pi\sqrt{h_{\hat{r}\hat{r}}}} = \frac{f_1(r - r_+)}{2\pi f_2}, \quad (24)$$

and the extremal Frolov-Thorne temperature in the stretched horizon formalism [19] is

$$T = \frac{T_H}{\epsilon}, \quad (25)$$

where $\epsilon = (r - r_+)f_3|_{r=r_+}$. Then we have

$$T = \frac{1}{2\pi\hat{f}}. \quad (26)$$

We will see the central charge of the NHEK geometry. The event horizon of the (16) is in $r_+ = 0$ [19, 23, 24]. So, from the metric (19), we see that

$$f_1 = f_2 = \sqrt{2M^2\Omega^2}, \quad f_3 = 1, \quad (27)$$

and it causes $\hat{f} = 1$. In addition, the horizon area is

$$A_{BH} = \int \sqrt{|\gamma|} d\theta d\phi, \quad (28)$$

where γ is the determinant of the Kerr metric only on the components g_{22} and g_{33} . Then we find A_{BH} is $8\pi M^2$ for extreme Kerr. Then we end with the central charge (21) and the temperature (26) to be

$$c = 12M^2, \quad (29)$$

$$T = \frac{1}{2\pi}, \quad (30)$$

where it is equal to the Kerr metric by taking $r_+ = M$. Actually, we can find the central charge directly from the Papapetrou and Ernst potentials in the definition of function \hat{f} . So, from (23) we can find

$$f_1 = \frac{\rho}{(\hat{r} - \hat{r}_+)\sqrt{-\text{Re } \varepsilon}}, f_2 = (\hat{r} - \hat{r}_+)(\text{Re } \varepsilon \Delta P^2)^{-1/2}, f_3 = \frac{\Omega_H - \omega}{\hat{r} - \hat{r}_+}. \quad (31)$$

4. Summary and Conclusion

Black holes solutions are related to mathematical equations which are equivalent to the Einstein field equation in stationary case called by Ernst equations. Ernst equations are also related to the general rotating black hole metric. This metric can be applied to the more general black holes, Kerr-Newman black holes. In our case, we take the intrinsic charge of the Kerr-Newman spacetimes to be zero in order to obtain Kerr spacetimes. One of the great discoveries recently in theoretical physics is holographic dualities which happen in extremal black holes such as Kerr one and are called by Kerr/CFT correspondence. The conjectured Kerr/CFT correspondence

has been quite successful as the tools to find the microscopic origin of the Bekenstein-Hawking entropy. For calculating the entropy of the extremal black holes, one also need to find the central charge that has appeared from the Cardy formula.

Here, we reconstruct the Near Horizon Extreme Kerr geometry. Then by using the same coordinate transformation on the Kerr metric, we obtain the Ernst potential for this geometry. We find that the Ernst potential only depends on θ

$$\varepsilon = -2M^2\sin^2\theta - \frac{2M^2\sin^4\theta}{1 + \cos^2\theta} + 2iM^2\cos\theta \left[(3 - \cos^2\theta) + \frac{\sin^4\theta}{1 + \cos^2\theta} \right],$$

because of the near horizon condition. This Ernst potential is the solution of the Ernst equation represents the vacuum Einstein field equation. We show that central charge of the metric can be defined in the function of Papapetrou and Ernst potentials. So, we can find directly the central charge and the extremal Frolov-Thorne temperature from those potentials. This result will help us to find the extreme Kerr metric from the Ernst equation.

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