OBSERVATION OF QUANTUM EFFECTS IN AN ELECTRON SYNCHROTRON (*)

M. Sands

California Institute of Technology, Pasadena, Cal.

I. INTRODUCTION

When the electron synchrotron at the California Institute of Technology was first put into operation at energies near 1 GeV (in Aug. 1956), the maximum energy which could be reached appeared to be limited by the maximum voltage available at the accelerating gap (120 kV), although this voltage was much larger than the computed radiation loss. It appeared that the limitation was due to the loss of electrons from synchronism owing to the excitation of phase oscillations by the quantum effects in the electron radiation. Measurements were made at that time which gave quantitative agreement with a theoretical analysis of the quantum effects [1]. The recent interest in quantum effects in electron accelerators—particularly with respect to the problems associated with storage rings—has prompted this report.

The Cal. Tech. synchrotron has some properties which make it possible to perform tests on the quantum effects under particularly simple conditions. First, the acceleration time is long ($\simeq$ 200 ms) compared with the damping time constant for radiation effects at high energies ($\simeq$ 10 ms). Second, the magnet excitation equipment can provide a " plateau ", an interval of 20 ms during which the magnet current, and hence, the magnetic field strength, are held constant to within one part in 10^3. The magnet current increases linearly until the plateau is reached. The plateau can be set to occur at magnetic fields which correspond to any electron energy up to the maximum obtainable. Third, the control of the program for the amplitude of the RF voltage on the accelerating cavity permits the selection of an arbitrary constant amplitude at the cavity during the plateau, without influencing the program during the preceding acceleration interval.

We present below the theoretical expectation of the rate of loss from synchronism of electrons in a synchrotron with a stationary guide field and with a constant cavity voltage. We then give the results of some new measurements of the observed loss rates.

II. THEORY

The equation which describes the phase oscillations in an electron synchrotron with a stationary guide field and a constant RF amplitude can be written

$$\frac{d^2\phi}{dt^2}+\rho\frac{d\phi}{dt}+\Omega_0^2(\sin\phi-\sin\phi_s) = g(t)\,, \qquad (1)$$

where

$$\rho = (4-\alpha)\frac{P_{\gamma s}}{E_s}\,, \qquad (2)$$

and

$$\Omega_0^2 = \frac{2\pi k\alpha eV}{\lambda t_s^2 E_s}\,. \qquad (3)$$

Here, θ is the phase angle of the cavity voltage at the instant of passage of the electron, and θ_s is the equilibrium phase angle. The other parameters are

- k : the harmonic number, the ratio of the cavity frequency to the electron rotation frequency.
- λ : the circumference factor, the ratio of the orbit length to 2π times the electron radius in the guide field.

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α: the momentum compaction; $dr/r_s = \alpha\, dp/p_s$.
E_s: the energy of the synchronous electron.
$P_{\gamma s}$: the average power radiated by a synchronous electron.
eV: the energy gain of an electron which traverses the cavity at $\theta = \pi/2$ $\quad (\sin\phi_s = P_{\gamma s}/eV)$.
t_s: the period of circulation of the synchronous electron.

$g(t)$ is the driving function for the phase oscillations and has been discussed in an earlier paper[2].

Equation (1) describes the one dimensional motion of a model particle of unit mass in a potential of the form

$$U = -\Omega_0^2 (\cos\phi + \phi \sin\phi_s) . \tag{4}$$

The model particle will oscillate about ϕ_s if its oscillation energy is less than the difference ΔU between the potential minimum at θ_s and the nearby maximum at $\phi = \pi - \phi_s$. The particles escape from the potential well if their oscillation energy exceeds the barrier height ΔU, which is given by

$$\Delta U = \Omega_0^2 [2\cos\phi_s - (\pi - 2\phi_s)\sin\phi_s] . \tag{5}$$

It has been shown[2-5] that phase oscillations in a synchrotron are excited by quantum emission and damped by the average radiation loss so that there is a " brownian motion " in phase with a mean square fluctuation σ_ϕ^2. This fluctuation corresponds to a mean " excitation energy " of the model particle, described above, of

$$U_{ex} = \sigma_\phi^2 \Omega_0^2 \cos\phi_s . \tag{6}$$

Excitation energies much larger than the mean value—which correspond to abnormal fluctuations in the radiation loss—give rise to amplitudes of phase oscillations sufficiently large that the model particle escapes over the potential barrier. The corresponding electron slips from synchronism.

The problem of the diffusion of thermally excited particles over a barrier near a harmonic well has been considered by Kramers[6] who finds that the relative loss rate of particles from the well is

$$-\frac{1}{N}\frac{dN}{dt} = \rho \frac{\Delta U}{U_{ex}} \exp\left[\frac{\Delta U}{U_{ex}}\right], \tag{7}$$

where ρ is the damping coefficient of Eq. (1).

R.F. Christy has shown[7] that the method of Kramers is applicable to the problem of electron loss in a synchrotron, and that the approximations made in obtaining Eq. (7) should probably introduce errors of less than 20 per cent.

Robinson has considered[8] the problem of the diffusion of electrons in the phase space of energy oscillations and arrived independently at the formula of Eq. (7). No estimate was given of the errors introduced by the approximations made. Matveev[9] and Moroz[10] have also considered the problem of loss from phase stability, but do not give expressions for the loss rate.

We define the time constant τ for electron loss as the inverse of the relative loss rate of Eq. (7). Substituting the quantities of Eqs. (4), (5) we obtain

$$\tau = \left[\frac{1}{N}\frac{dN}{dt}\right]^{-1} = \theta \exp\delta \tag{8}$$

with

$$\theta = \frac{k\alpha t_s}{(4-\alpha)^2\lambda} \frac{E_1}{\Delta E_\gamma} \frac{1}{H(s)} \tag{9}$$

and

$$\delta = \frac{(4-\alpha)\lambda}{k\alpha} \frac{E_s}{E_1} H(s) \tag{10}$$

We have written $\Delta E_\gamma = t_s P_{\gamma s}$ for the radiation loss in one revolution. E_1 is a universal constant with the dimensions of an energy:

$$E_1 = \frac{55\sqrt{3}}{64} \frac{hc}{e^2} mc^2 = 1.04 \times 10^8 \text{ eV}. \tag{11}$$

The function $H(s)$ depends only on the " overvoltage "

$$s = eV/\Delta E_\gamma = [\sin\phi_s]^{-1}.$$

$$H(s) = 2\cot\phi_s - \pi + 2\phi_s$$
$$= 2\sqrt{s^2 - 1} - \pi + 2\sin^{-1}\frac{1}{s} \tag{12}$$

In the equations above we have used for σ_ϕ^2 the relation given in Sands[2] (*) (there denoted by $\langle \psi^2 \rangle_{AV}$).

The time constant for electron loss has been computed for the Cal. Tech. synchrotron for various

(*) Kolomenskij and Lebedev[5] state that the result of Sands[2] is in error by a factor of λ. The result of Sands[2] agrees with that of Robinson[4] and I believe it to be correct.

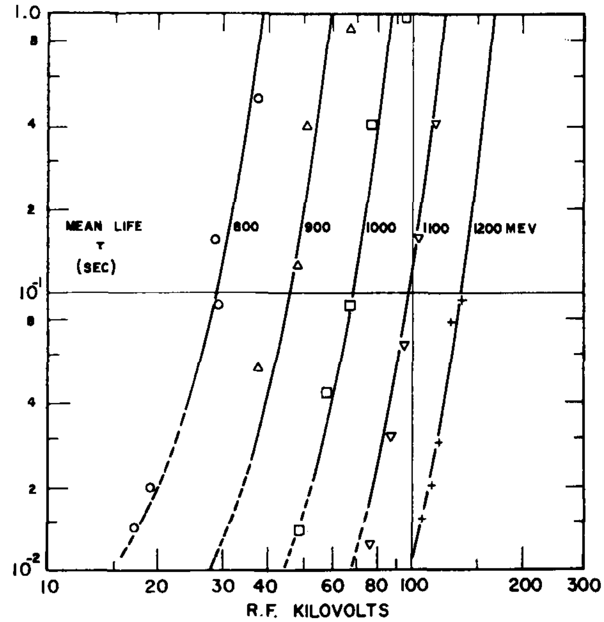

Fig. 1 The mean life of the circulating electrons as a function of the amplitude of the RF voltage on the accelerating gap for various synchronous electron energies. The curves were computed from Eq. (8).

energies and RF voltages, and is plotted in Fig. 1. The parameters used for the computations are

$k = 4 \qquad t_s = 0.96\times10^{-7}$ s

$\lambda = 1.26 \qquad \Delta E_\gamma = 50$ keV $(E_s/1\ \mathrm{GeV})^4$

$\alpha = 2.5$

The broken line part of the curves corresponds to loss time constants less than 5 times the damping constant $1/\rho$. The theory should not be expected to apply for such high loss rates.

III. EXPERIMENT

Measurements were made of the time dependence of the number of electrons circulating in the synchrotron while holding constant the strength of the magnetic guide field and the amplitude of excitation of the RF cavity. Electrons were accelerated to the plateau of the guide field by the normal RF program with which few electrons are lost. At the start of the 20 ms plateau of the guide field, the RF amplitude is reduced to a predetermined value and held at this value for the duration of the plateau.

Oscilloscope signals proportional to the guide field strength at the orbit and to the RF amplitude are shown in the photograph of Fig. 2(a) for which the plateau has been set at 1000 MeV. The electron energy at the plateau is determined to 0.1 per cent by a separate measurement of the magnetic field strength at the orbit.

The signal which indicates the RF amplitude is obtained from a diode rectifier driven by a loop in the RF cavity. The signal is proportional to the amplitude of excitation of the cavity for excitations encountered in these measurements and has been calibrated in terms of the cavity voltage (*).

The number of circulating electrons is determined by means of an induction electrode which encircles the electron trajectories. The current signal to the electrode is fed to an amplifier which is tuned (broadly)

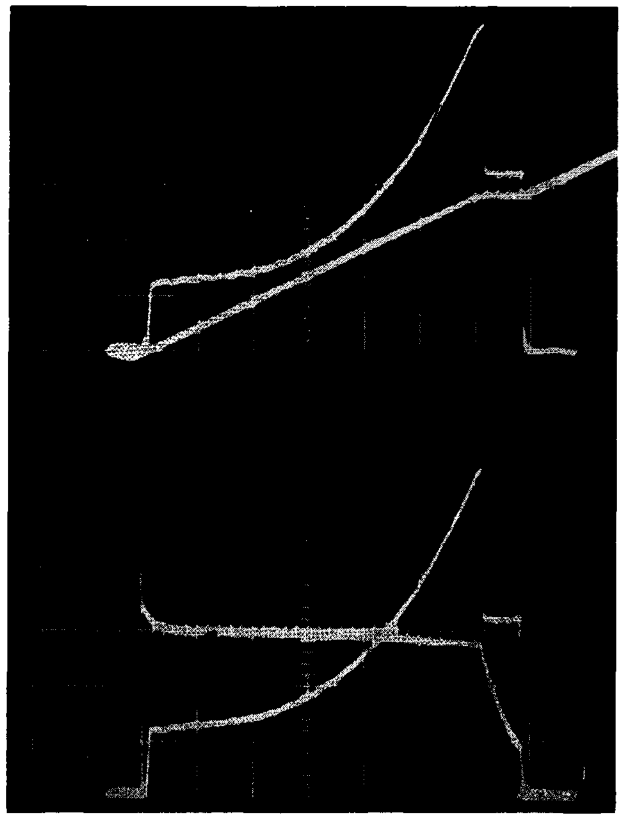

Fig. 2 (a) Oscillograms of magnetic field strength and RF voltage for the measurements at 1 GeV. (b) Oscillograms of the RF voltage and the electron monitor.

(*) Determined by R. M. Worlock and R. V. Langmuir who were also responsible for the construction of the RF cavity and drive system.

to the frequency of the accelerating cavity. The tuned amplifier operates a diode detector whose output is proportional to the number of electrons circulating in the synchrotron—averaged over about 100 μs (*).

The rate of electron loss was measured by photographing an oscilloscope on which were displayed, simultaneously, the RF amplitude signal and the electron monitor signal. A typical photograph (for 1000 MeV) is shown in Fig. 2(b). From measurements on such photographs, the RF amplitude and the relative decrease in the number of circulating electrons during the 20 ms plateau interval were determined. From this latter number the loss time constant was computed.

The experimental results are shown by the indicated points in Fig. 1. The measurements were not extended to include loss time constants greater than one second, because electron losses of less than 5 per cent could not be determined with significant accuracy.

For the experimental points of Fig. 1 the scale of RF voltage was adjusted to give the best fit with the theoretical curves. The scale of RF amplitude obtained in this way was 0.9 times that given by the calibration referred to above. Since the accuracy of that determination is estimated to be about 10 per cent, the scale adjustment is probably justified.

It appears from Fig. 1 that, over the region of energies and loss rates covered in this experiment, the agreement with the theory is satisfactory.

It may be remarked that the sudden decrease of the RF voltage at the start of the plateau occurs in a time of a few hundred microseconds. It is, thus, not strictly true that the electron loss occurs under stationary conditions, particularly since the time to establish these conditions ($1/\rho$) is comparable with the plateau duration. No attempt has been made to correct for such effects.

The experimental results given here were obtained in July 1959 after the installation of a new RF cavity and other equipment. The measurements reported earlier [1] (**) were made with a different electron monitor and at energies up to only 1100 MeV. For those measurements an adjustment of 15 per cent in the RF calibration was required in order to obtain agreement with the theory.

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(*) The author is indebted to H. Seidel and R. Hundley for assistance in the design and construction of this electron monitoring system.

(**) The earlier measurements were made in collaboration with A. B. Clegg.

(***) See note on reports, p. 696.

DISCUSSION

LITTAUER: I would like to ask Sands to comment on the effect of a sudden decrease in the RF amplitude preceding his measurements.

SANDS: That will be mentioned in the Proceedings. No attempt has been made to take into account the fact that measurements are not really made in a stationary condition. When the RF voltage drops suddenly one does see a sudden decrease in the number of electrons and then an exponential fall afterwards. Brief consideration of this does not indicate to me that this is a serious effect.

QUERCIA: In the experimental procedure for measuring the decay of a number of electrons circulating in the machine, do you take into account the fact that due to the phase oscillation of electrons there is some debunching of electrons and the induction probe can pick up less amplitude than before?

SANDS: An evaluation of the effect of the change in shape of the bunch and the sensitivity of the induction probe to the shape of the bunch in this region indicates that the effect is only about 10% and would not have a significant effect on the points shown in the graph.

KOLOMENSKIJ: Have you made observations on the dimensions of the cross-section of the beam, particularly of the vertical dimensions? I think these dimensions must be very small; it is important to have some experimental data on this because their interpretation is simple, simpler than for radial dimensions.

SANDS: The question was about the radial and vertical dimensions of the beam, which were related to the damping and excitation of betatron oscillations. We have only some very crude measurements of this, made by photographing the radiation from the electrons through a window in the wall of the synchrotron. The photographs were not made for this purpose, but for other purposes, so they are not very satisfactory. It appears that the vertical dimension is much larger than one would calculate theoretically, but this may be due to carelessness in focusing of the camera and things of this sort. So we are not sure. The radial size seems to be comparable with what one would expect from theory and has an increase with increasing energy with about the slope one expects from theory.

VOROB'EV: I would like to ask Sands to give more details on the calculation of betatron oscillations in this case. Sands has convincingly shown the agreement between his theory with the experiment, but his theory considers only phase oscillations. There are reasons to believe that the electron does not know this and may actually perform betatron oscillations at the same time. As the losses are actually due to both betatron and phase oscillations, this should be taken into account in the summation.

SANDS: The question, as I understand it, is that the theory includes only the synchrotron oscillation equation and the radiation effects which appear therein. There are, of course, betatron oscillations and I believe he is suggesting that the betatron oscillations will modify the radiation effects which also appear in the synchrotron equations. Is that right?

VOROB'EV: Yes.

SANDS: There, of course, exist betatron oscillations, but I believe that for the small amplitudes and the magnitude of the radiation effects that we have now, the betatron and synchrotron equations can be considered to be completely uncoupled, and one may consider independently the betatron and synchrotron oscillations, including the radiation damping terms, as Robinson talked of just a little while ago. The losses which occur in the measurements reported here are all due exclusively to phase oscillations.

O'NEILL: I might add to this that, in some calculations we did last year, we made a rough, simple theory of radiation damping based on decoupled betatron and synchrotron oscillations and then later checked it against a computer calculation which made no such approximation; the agreement was very good.