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Variable Current Transient Beam Loading Compensation*

Z. D. Farkas
Stanford Linear Accelerator Center, Stanford Univesity, Stanford, CA 94309

Introduction

The energy spread caused by transients during beam turn-on can be reduced by suitable timing of the beam turn-on[1]. If the beam is injected when the no-load voltage reaches the desired loaded voltage, then the energy spread is about 10%. To eliminate this energy spread, one can amplitude or phase modulate the section input power for one fill time, so that when the beam is turned on, the no-load voltage equals the the desired loaded voltage and from then on, the change in no-load voltage tracks the beam induced voltage. It is known that for a constant gradient (CG) structure, and amplitude variation of the form $E(t) = a_o + (1 - a_o)t_p$ will reduce the energy spread to zero for a current that is determined by a_o.

When one uses rf modulation for transient beam loading compensation, the beam is injected a fill time after the rf has been turned on, and one is forced to throw away a section's worth of rf energy. In addition, it requires extra components which use up additional rf energy. This note describes transient beam loading compensation with variable current. It will show that it increases the rf energy to beam energy transfer efficiency.

List of symbols

Structure Parameters

L_s	section length
v_{g0}, v_{gL}	input and output group velocities
h	ratio of output to input group velocity
s_0, s_L	input and output elastance

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$\boldsymbol{h_s}$	ratio of output to input elastance
$\boldsymbol{v_{ga}}$	L_s/T_f, average group velocity
$\boldsymbol{v_{gr}}$	ratio of average to input group velocity
$\boldsymbol{T_a}$,	$Q/\pi f$, TW time constant
$\boldsymbol{T_f}$	section fill time
$\boldsymbol{\tau}$	T_f/T_a, section attenuation in nepers

Operating Parameters - No-load gradient

$\boldsymbol{t'}$	time in units of fill time
$\boldsymbol{P_s}$	rf pulse peak power into section
$\boldsymbol{P(t)}$	power into section as a function of time
$\boldsymbol{E(t)}$	$\sqrt{P(t)/P_s}$, rf field as a function of time
$\boldsymbol{T_s}$	duration of rf pulse into section
$\boldsymbol{G_g}$	no-load gradient as a function of z
$\boldsymbol{E_{gt}}$	no-load average accelerating gradient as a function of time
$\boldsymbol{E_{gs}}$	steady state no-load average gradient

Operating Parameters - Current induced gradient

$\boldsymbol{T_{bi}}$	beam injection time
$\boldsymbol{E_{gi}}$	average gradient at beam injection
$\boldsymbol{E_{gb}}$	no-load gradient minus E_{gi}, starting at T_{bi} as a function of time
$\boldsymbol{E_{gd}}$	maximum no-load gradient minus E_{gi}
$\boldsymbol{E'_{gi}}$	E_{gi}/E_{gs}
$\boldsymbol{B_L}$	beam loading, $1 - E'_{gi}$
$\boldsymbol{i_{bt}}$	current as a function of time
$\boldsymbol{i_b}$	step current as a function of time
$\boldsymbol{E_b}$	step current induced average gradient
$\boldsymbol{E_{bs}}$	steady state current induced gradient
$\boldsymbol{E_{bt}}$	current induced gradient as a funtion of time
$\boldsymbol{T_b}$	duration of beam pulse

Operating Parameters - Loaded gradient

$\boldsymbol{E_{ls}}$	the steady state loaded gradient
$\boldsymbol{E_{lt}}$	loaded gradient as a function of time

No-load effective gradient as a function of time

Consider a linearly varying group velocity (LVG) structure whose input power varies arbitrarily with time. The effective gradient as a function of time is obtained as follows [2]. Let the output to input group velocity ratio $v_{gL}/v_{g0} \equiv h$, and let $g = h-1$. Thus the group velocity varies with normalized distance $z' = z/L$ as $v_g(z') = v_{go}(1+gz')$. The normalized propagation time from input to position z' [3]

$$t'_z = t_z/T_f = \frac{L}{T_f v_{g0}} \int_0^{z'} \frac{dz'}{1+gz'} = \frac{L}{T_f v_{g0}} \frac{\ln(1+gz')}{g} \ .$$

Letting $z' = t'_z = 1$ we obtain

$$v_{gr} \ \equiv \ \frac{L/T_f}{v_{g0}} \ \equiv \ \frac{v_{ga}}{v_{g0}} \ = \frac{g}{\ln h} \ .$$

Substitute for v_{g0} and obtain

$$t'_z = \frac{\ln(1+gz')}{\ln h}, \quad z' = \frac{h^{t'_z}-1}{g}, \quad 1+gz' = h^{t'_z} \ = \ e^{\ln h \ t'_z} \ .$$

If the group velocity varies with z' then the elastance/meter, s $(=\omega r/Q)$ also varies. Let $s_L/s_0 \equiv h_s$, and $g_s = h_s - 1$. Then

$$s_a = (s_0 + s_L)/2, \quad s_0 = 2s_a/(1+h_s), \quad s = s_0(1+g_s z'), \quad s_r \equiv s_a/s_0 \ .$$

The section attenuation is $e^{-2\tau}$, where $\tau = T_f/T_a$ and $T_a = 2Q/\omega$ is the TW time constant. The attenuation to point z, $\tau(z) = t_z/T_a = t'_z\tau$. The accelerating gradient as a function z', the local gradient, is

$$G_g(z') = \sqrt{\frac{P s_0 (1+g_s z') e^{-2\tau(z')}}{v_{go}(1+gz')}} = G_g(0) \times \frac{e^{-\tau t'_z}(1+g_s z')^{0.5}}{(1+gz')^{0.5}}$$

Using $\quad e^{-\tau t'_z} = (1+gz')^{-\tau/\ln h} \quad$ we obtain

$$G_g(z') = G_g(0) \times (1+gz')^{x_1}(1+g_s z')^{0.5}, \quad x_1 = -.5 - \frac{\tau}{\ln h} \ .$$

The ratio of output to input gradient $G_g(1)/G_g(0) = \sqrt{e^{-2\tau}h_s/h}$. If the group velocity varies with position along the section, the elastance will also vary. Thus, for a section that we define as a constant gradient, i.e. $h = e^{-2\tau}$, $x_1 = 0$, the elastance increases with z' and the electric field increases as well. As v_g decreases, s increases. h_s depends on h, if $h = 1$ then $h_s = 1$. A criteria that guaranties that the gradient at the end of the section is the same as at the beginning of the section is $h = e^{-2\tau} h_s$.

The no-load average accelerating gradient, the effective gradient, E_g, is the no-load voltage aquired by a charged particle passing thru the accelerator section divided by the section length L,

$$E_g = \frac{1}{L}\int_0^L G_g(z)dz = \int_0^1 G_g(z')dz' \ .$$

Let $E_{ai} \equiv \sqrt{s_a T_f P/L}$, and let $F_z(z') = \int_0^{z'} (1+gz')^{x_1}(1+g_s z')^{0.5}\, dz' \ .$

If P is constant the we obtain the steady state effective gradient

$$E_{gs} = E_{ai}\sqrt{v_{gr}/s_r} \times \int_0^1 (1+gz')^{x_1}(1+g_s z')^{0.5}\, dz' = E_{ai}\sqrt{v_{gr}/s_r} \times F_z(1)$$

$$E'_{gs} \equiv E_{gs}/E_{ai} = \sqrt{v_{gr}/s_r} \times F_z(1)$$

Define the section efficiency $\eta_s = E'^2_{gs}$, so that $E_{gs} = \sqrt{\eta_s s_a T_f P_s / L}$.

For a step rf starting at $t' = 0$, using $z' = (h^{t'_z} - 1)/g$, we obtain the no-load average gradient as a function of time,

$$E_{gt} = \int_0^{z'} G(z')dz' = E_{ai}\sqrt{v_{gr}/s_r} \times F_z(z') \ .$$

If s is constant, $s_r = 1$, then

$$\begin{aligned} F_z(z') &= \frac{(1+gz')^{x_{11}} - 1}{x_{11} g} = \frac{h^{x_{11} t'_z} - 1}{x_{11} g}, \qquad x_{11} = x_1 + 1 = .5 - \frac{\tau}{\ln h} \\ F_z(1) &= \frac{h^{x_{11}} - 1}{x_{11} g} \end{aligned}$$

If the section is also constant gradient, i.e. $h = e^{-2\tau}$, then

$$x_{11} = 1, \quad F_z(z') = \frac{h^{t'_z} - 1}{g}, \quad F_z(1) = 1, \quad E'_{gs} = \sqrt{v_{gr}}, \quad \eta_s = \frac{1 - e^{-2\tau}}{2\tau}$$

So far we considered a step rf. But even for constant rf, P varies with time during turn-on due to the rf rise time. The effective gradient when the rf input varies with time is obtained as follows. Let $E(t') = \sqrt{P(t')/P}$, where P is the steady state rf. (In case of SLED it is the section input power with the cavities completely detuned.) Let the normalized effective gradient as a function of time $E'_{gt} = E_{gt}/E_{gs}$. Then at time t'

$$E'_{gt} = \int_{t'}^{t'-t'_z} E(t')(1+gz')^{x_1}(1+g_s z')^{0.5} dz'$$

For some time functions, such as linear or exponential, we can separate t' from z' and obtain a closed form solution for the integral as was done in Ref [3], where expressions for E'_{gt} were obtained for a CG structure. Here, we use numerical integration to obtain E'_{gt}.

Assume an LVG structure having the following NLC like parameters:

L	v_{g0}/c	h	T_f	τ	s_0	h_s
1.8m	0.114	0.24	0.106μ s	0.486	664.7MΩ/μs/m	1.5

The normalized gradient along the section $G'_g(z') = G_g(z')/E_{gs}$, is plotted in Fig. 1 and E'_{gt} is plotted in Fig. 2, for two cases. One, using a constant average elastance and two, using a linear variation in elastance. In both cases $G_g(z') = E_{gs} \times G'_g(z')$, and $E_{gt} = E_{gs} \times E'_{gt}$. Note that $G_g(z')$ changes but η_s and E'_{gt} remain about the same.

Current induced effective gradient as a function of time.

The effective gradient as a function of time, E_{bt} that is induced by a current, i_{bt} that varies with time is obtained as follows. The current $i_{bt}(n)$ is given at points $n = 1, 2, \dots N$, with $i_{bt}(N) = 0$. We expand the current into N step currents, $i_b(n)$, injected at times $t_{bn} = (n-1)\Delta t$.

$$\begin{aligned} i_b(1) &= i_{bt}(1) \\ i_b(n) &= i_{bt}(n+1) - i_{bt}(n), \quad n = 2, 3 \dots N-1 \\ i_b(N) &= -i_{bt}(N-1) \end{aligned}$$

We know the average gradient due to a step current, hence we can obtain the average gradient due to i_{bt}, which is the sum of the gradients induced by the step currents.

A step current, i_b through an accelerator section, induces an effective gradient as a function of time $E_b(t'_b) = (i_b s T_f/4)E'_b(t'_b)$. The function $E'_b(t'_b)$ for an LVG section, derived by P.B. Wilson, is [4], using $\eta \equiv \frac{-\ln h}{2\tau}$,

$$\begin{aligned} E'_b &= \frac{2}{g\tau}\left[\frac{1-e^{-t'_b(1+\eta)\tau}}{1+\eta} - \frac{e^{-2\eta\tau}(1-e^{-t'_b(1-\eta)\tau})}{1-\eta}\right] \quad 0 \le t'_b \le 1 \\ &= \eta_b = \frac{2}{g\tau}\left[\frac{1-e^{-(1+\eta)\tau}}{1+\eta} - \frac{e^{-2\eta\tau}(1-e^{-(1-\eta)\tau})}{1-\eta}\right] \qquad t'_b \ge 1 \end{aligned}$$

For a CG section,

$$\begin{aligned} E'_b &= \frac{1-e^{-2\tau t'_b} - t'_b 2\tau e^{-2\tau}}{\tau(1-e^{-2\tau})} \qquad 0 \le t'_b \le 1 \\ &= \eta_b = \frac{1-e^{-2\tau} - 2\tau e^{-2\tau}}{\tau(1-e^{-2\tau})} \qquad t'_b \ge 1 \end{aligned}$$

Define $r_b = (sT_f/4)E'_b$. Then the gradient due to the nth step current is

$$\begin{aligned} E_b(n) &= i_b(n) r_b(t'_b - (n-1)\Delta t) \qquad 0 < t'_b - (n-1)\Delta t \le 1 \\ &= i_b(n) r_b(1) \qquad t'_b - (n-1)\Delta t \ge 1 \end{aligned}$$

t'_b is time starting at beam injection divided by fill time. The actual gradient is the sum of all the gradients,

$$E_{bt} = \sum_{n=1}^{N} E_b(n)$$

The current starts at time T_{bi} when the no-load gradient is E_{gi}. For zero energy spread, E_{bt} should track E_{gb}, the no-load gradient starting at point T_{bi},E_{gi}. $E_{gb}(t'_b) \equiv E_{gt}(t' - T'_{bi}) - E_{gi}$ is plotted in Fig. 3. We assumed that the no-load gradient reaches E_{gs}, the steady state no-load gradient, hence E_{gb} also reaches steady state. Let $E_{gd} = E_{gs} - E_{gi}$. We equate E_{gi} to the steady state loaded gradient, E_{ls}, so that

$$B_L \equiv (E_{gs} - E_{ls})/E_{gs} = 1 - E_{gi}/E_{gs}, \quad E_{gd} = B_L E_{gs} \quad .$$

The steady state current

$$I_{bs} = \frac{B_L E_{gs}}{\eta_b s T_f/4} \quad .$$

The normalized instantaneous current i_{bt}/I_{bs}, and the step currents that it is expanded into, are plotted in Figure 4. The step current induced gradients divided by E_{gd} and the gradient induced by the actual current, E_{bt} divided by E_{gd}, are plotted in Fig. 5. E_{bt}, divided by E_{gd} is also plotted in Fig. 3. The current i_{bt} was chosen such that its induced gradients at points n equal the no-load gradient minus the no-load gradient when the beam is injected, i.e. $E_{bt}(n) = E_{gb}(n)$. How this current is obtained is given in the next section. The loaded gradient $E_{lt} = E_{gb} - E_{bt}$, in units of E_{gd}, is also plotted in Fig. 3.

Required varying current for zero energy spread

We, now, calculate the current variation that yields a self induced beam voltage identical to a specified change in no-load beam voltage, $E_{gb}(t)$. Let $n = 1, 2, 3 \ldots N_i$, where N_i is the number of step currents. The interval between current injections is $\Delta t = T_b/(N_i - 1)$. Let the time when the nth current is injected $t_n = (n-1) \times \Delta t$. The beam induced voltage at time t_{n+1} is the voltage due to the current injected at time t_n plus the voltage due to the $n-1$ previously step currents. Using $E_b(t_n) = E_{gb}(t_n)$ we obtain the step currents at each point n:

$$
\begin{aligned}
E_{gb}(1) &= 0 \\
E_{gb}(2) &= i_{b1} r_b(\Delta t) \\
i_b(1) &= E_{gb}(2)/r_b(\Delta t) \\
E_{gb}(3) &= i_2 r_b(\Delta t) + i_1 r_b(2\Delta t) \\
i_b(2) &= \frac{E_{gb}(3) - i_1 r_b(2\Delta t)}{r_b(\Delta t)} \\
E_{gb}(4) &= i_3 r_b(\Delta t) + i_1 r_b(3\Delta t) + i_2 r_b(2\Delta t) \\
i_b(3) &= \frac{E_{gb}(4) - [i_1 r_b(3\Delta t) + i_2 r_b(2\Delta t)]}{r_b(\Delta t)}
\end{aligned}
$$

For $n = 2, 3 \ldots N - 1$

$$E_{gb}(n+1) = i_n r_b(\Delta t) + E_p(n+1) \ ,$$

$$E_p(n+1) = \sum_{p=1}^{n-1} i_p \ r_b([n+1-p]\Delta t) \ .$$

$$i_n = \frac{E_{gb}(n+1) - E_p(n+1)}{r_b(\Delta t)} \ .$$

The actual current at point n

$$i_{bt}(n) = \sum_{j=1}^{n} i_j \ .$$

To turn off the beam at point N we have $i_b(N) = -i_{bt}(N-1)$. The step currents are either positive or negative, the the total current is positive as long as E_{gb} does not decrease precipitously.

We can increase the number of step currents and, consequently, decrease Δt and obtain a continuous total current waveform where, during the beam pulse, E_{bt} effectively tracks E_{gb}, resulting in nearly zero energy spread. Knowing the steps currents, we can obtain the current induced gradient as shown in the previous section.

The normalized rf field into the section and the resulting normalized no-load beam voltage, E_{gt}/E_{gm}, are plotted in Fig. 6, top. The normalized no-load, optimum current and required current induced voltages plus E_{gi} and the loaded voltage are plotted in Fig. 6, middle. The energy spread is about 10%. The required current divided by I_{bs} that reduces this energy spread to zero, is plotted in Fig. 6, bottom. The required current oscillates about and approaches I_{bs} and its average value is slightly larger than I_{bs}. The plots are for 15%, left, and for 50%, right, beam loadings. A constant average elastance was assumed. But the curves are nearly the same if a linear variation in elastance is assumed.

For a given E_{gs} the steady state current is proportional to B_L. In terms of beam loading and loaded gradient,

$$E_{gs} = \frac{E_{ls}}{1-B_L}, \qquad I_{bs} = \frac{B_L E_{ls}}{(1-B_L)\eta_b s T_f/4} \ .$$

If the current and loaded gradient are given then the beam loading is determined. The power into the structure

$$P_s = \frac{E_{gs}^2 L_s}{\eta_s s_a T_f} = \frac{E_{ls}^2 L_s}{(1-B_L)^2 \eta_s s_a T_f} \ .$$

$E(t')$ and E'_{gt} for the NLCL are plotted in Fig. 7 for two cases. One, with an rf rise time of $0.1T_f$ and two, with the rf varying linearly for a fill time which results in zero energy spread for a CG section with a step rf input. The time when the currents start is plotted in the figure indicating the increase in beam pulse width due variable current BLC.

Varying current BLC can be used in addition to other forms of BLC. Fig. 8, top, shows that for the NLC like structure with linear rf BLC, there is a small energy spread. This spread can be reduced to zero by varying the current as shown in Fig. 8, bottom.

Fig. 9a shows the gradient as a function of position, 9b shows the normalized input field and average gradient, 9c shows the no-load, current induced and loaded average gradients, and the required current, for a CERN TDS structure with the following parameters:[5]

L	v_{g0}/c	h	T$_f$	τ	s_0	h_s
0.5m	0.108	0.50	0.021μ s	0.511	4204MΩ/μs/m	1.35

The input power is 229 MW. Fig. 10 illustrate CAB for NLCL structure when the rf rise time is about half a fill time. Current amplitude beam loading compensation (CABL) can be used to compensate for the small droop or increase in rf power during the pulse. The same curves as in Fig. 6 are plotted in Fig. 11, except that the left side is for a 10% droop and the right side is for a 10% raise in rf power.

Variable current BLC parameters.

The current $i_{bt}(n)$ is the product of the charge/bunch $q_b(n)$ and the bunch spacing $T_{bs}(t)$, $i_{bt}(n) = q_b(n)T_{bs}(n)$. It can be varied by changing either the charge/bunch or the bunch spacing, or both. The charge per pulse is

$$q_p = \Delta t \sum_{n=1}^{N-1} i_{bt}(n)$$

The number of electrons per pulse in units of 10^{10}, $e_p = q_p/1.6$, if q_p is in nC. The average pulse current is $i_{pa} = q_p/T_b$ and, for fixed bunch spacing, the average charge/bunch is $q_{ba} = i_{pa}T_{bs}$.

The rf and beam pulse energies and the rf to beam energy transfer efficiency, are respectively,

$$U_{rf} = P_s T_s, \qquad U_{pb} = q_p E_{gi} L_s, \qquad \eta_{rb} = U_{pb}/U_{rf} \ .$$

Table 1 lists the parameters as function of beam loading for varying current BLC and for linear rf BLC for comparison, for the NLC like structure, with $E_{ls} = 62.3$ MV/m .

$B_L[\%]$	$T_i[T_f]$	$P_s[MW]$	$I_{bs}[A]$	I_m/I_{bs}	$e_p[10^{10}e]$	$e_b[10^{10}e]$	P_{bp}[MW]	$\eta_{rb}[\%]$
Linear field BLC								
14.9	1	159.16	0.556	1.00	43.80	0.98	62.5	20.9
25.8	1	209.11	1.101	1.00	86.04	1.93	123.7	31.5
42.1	1	343.15	2.301	1.00	179.80	4.03	258.9	40.1
Varying current BLC								
14.9	0.730	156.81	0.550	2.12	58.92	1.09	69.1	28.5
25.6	0.580	204.74	1.081	1.56	121.60	2.04	129.6	45.0
41.1	0.400	330.85	2.209	1.30	263.11	3.98	254.7	60.6
50.2	0.310	460.66	3.179	1.23	389.81	5.63	359.4	64.4

Table 1: Operating Parameters as a function of beam loading. $E_{ls} = 62.3$ MV/m.

We see that η_{rb} and e_p increase as B_L increases and that changing from linear field BLC to varying current BLC increases η_{rb} by about 50%. The increase is greater for larger B_L. At steady state,

$$\eta_{rb} = 4B_L(1 - B_L)\eta_s/\eta_b \ .$$

The rf to beam energy transfer efficiency has a maximum of η_s/η_b at $B_L = 0.5$. For a lossless section $\eta_s = \eta_b = \eta_s/\eta_b = 1$.

For the same power input, increasing beam loading decreases the loaded gradient. To maintain it we have to increase the power and thus increase the no-load surface gradient. But it stops incresing after we turn on the required current.

Beam loading compensation is important when the rf input pulse width, is not much longer than the fill time. If it is much longer then the increase in rf to beam energy transfer efficiency is negligible. But, there are many reasons for having a short rf pulse. Among them

- Klystrons can generate higher peak powers with a shorter pulse.
- A shorter pulse is easier to compress. The delay line length for Binary Pulse Compression and all its variations, is proportional to the accelerator input pulse.

- Many components have pulse energy limits. The same loaded gradient can be achieved with less pulse energy. High peak fields cause breakdown, but high pulse energies do the damage.

- The re-coherence of the wakefields is less of a problem with shorter pulse.

A short rf pulse decreases the efficiency because, then, the modulator rise time is a large fraction of modulator pulse. Work is in progress to make it a smaller fraction.

Variable current BLC can be applied to any variation of no-load gradient as a function of time, (It can be measured.), as long as the function does not decrease by much. The no-load gradient generated at SLAC by the SLEDed rf varies with time because the rf input varies with time, and does reach steady state. Rather than manipulating the rf amplitude and phase in order to reach a flat rf, we can use varying current BLC to obtain zero energy spread for beam pulses as long as half a fill time [6][7], and increase η_{rb}.

CONCLUSION

For systems that tolerate varying currents, such as those that produce x-rays, varying current BLC can be used to increase the rf to beam energy transfer efficiency and the charge per pulse. For a constant P_s, the average gradient decreases as beam loading increases, but it can be kept constant by increasing P_s.

The disadvantage of current BLC is that the current has to vary. But as B_L increases the ratio peak to average current decreases and η_{rb} increases.

References

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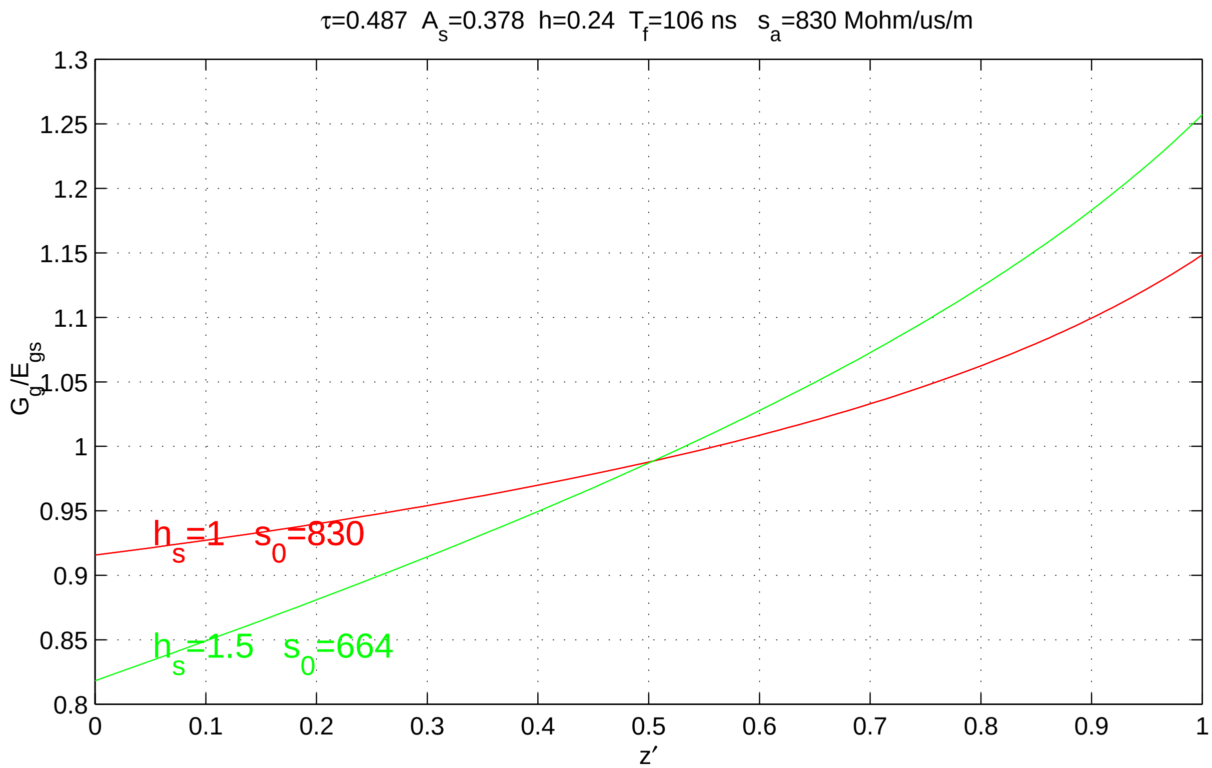

Fig. 1. Gradients G_g/E_{gs} vs position z' for h_s=1 and h_s=1.50

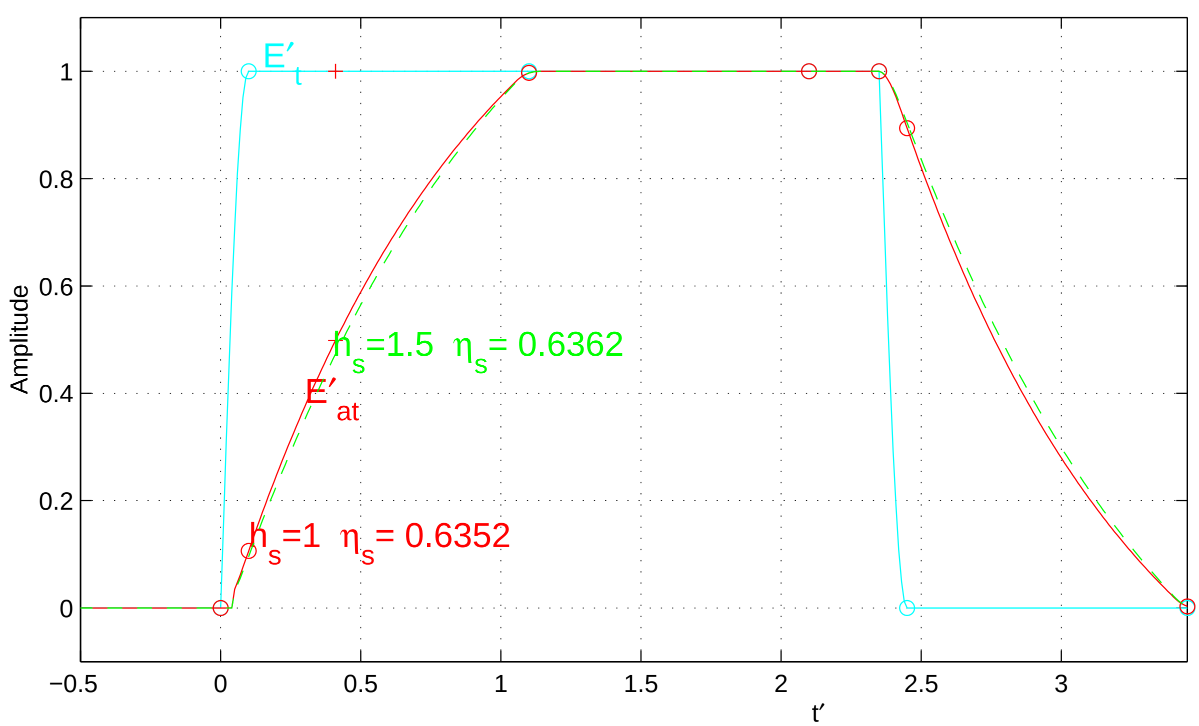

Fig. 2. Normalized rf E'_t and normalized effective gradient E_{gt}/E_{gs} for h_s=1 and h_s=1.50

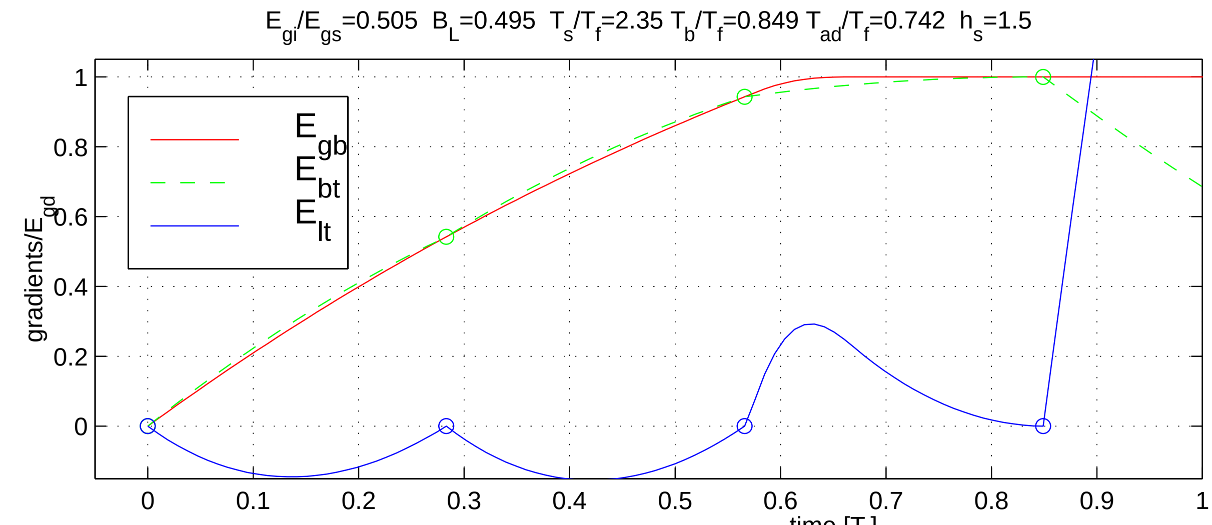

Fig. 3. No−load E_{gt}, current induced, E_{bt} and loaded, 10×E_{lt} average gradients in units of E_{gd} vs time in units of fill time

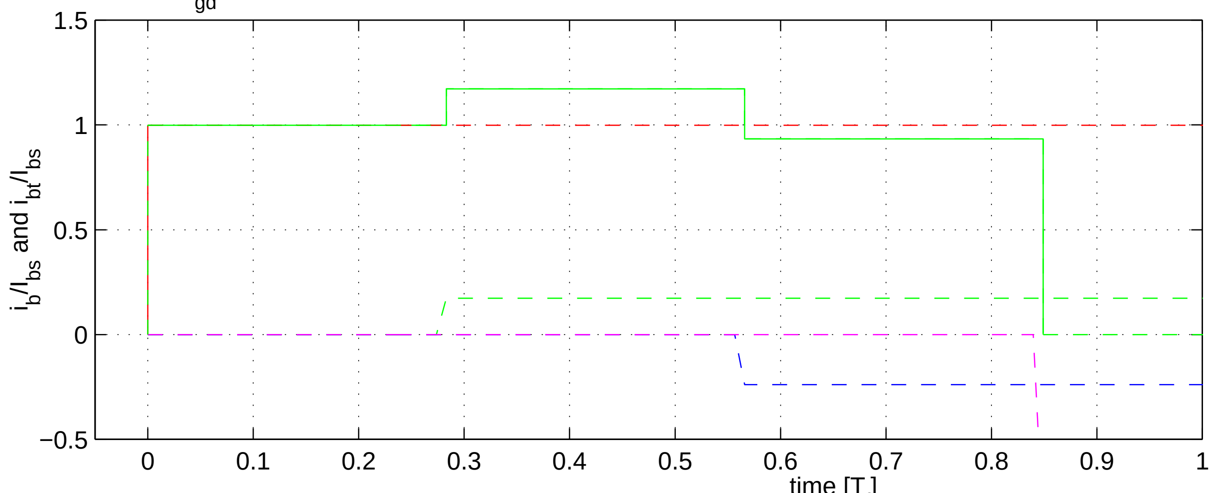

Fig. 4. Normalized step currents (dash), i_b/I_{bs} and total current, i_{bt}/I_{bs}, vs time

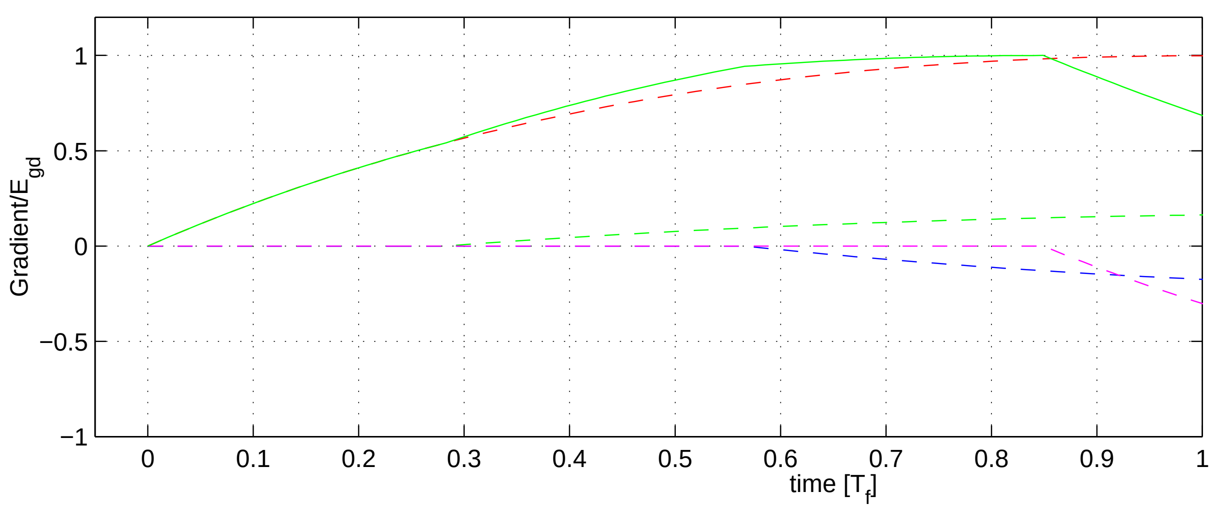

Fig. 5. Gradients induced by step currents (dash) and total gradient E_{bt} vs time

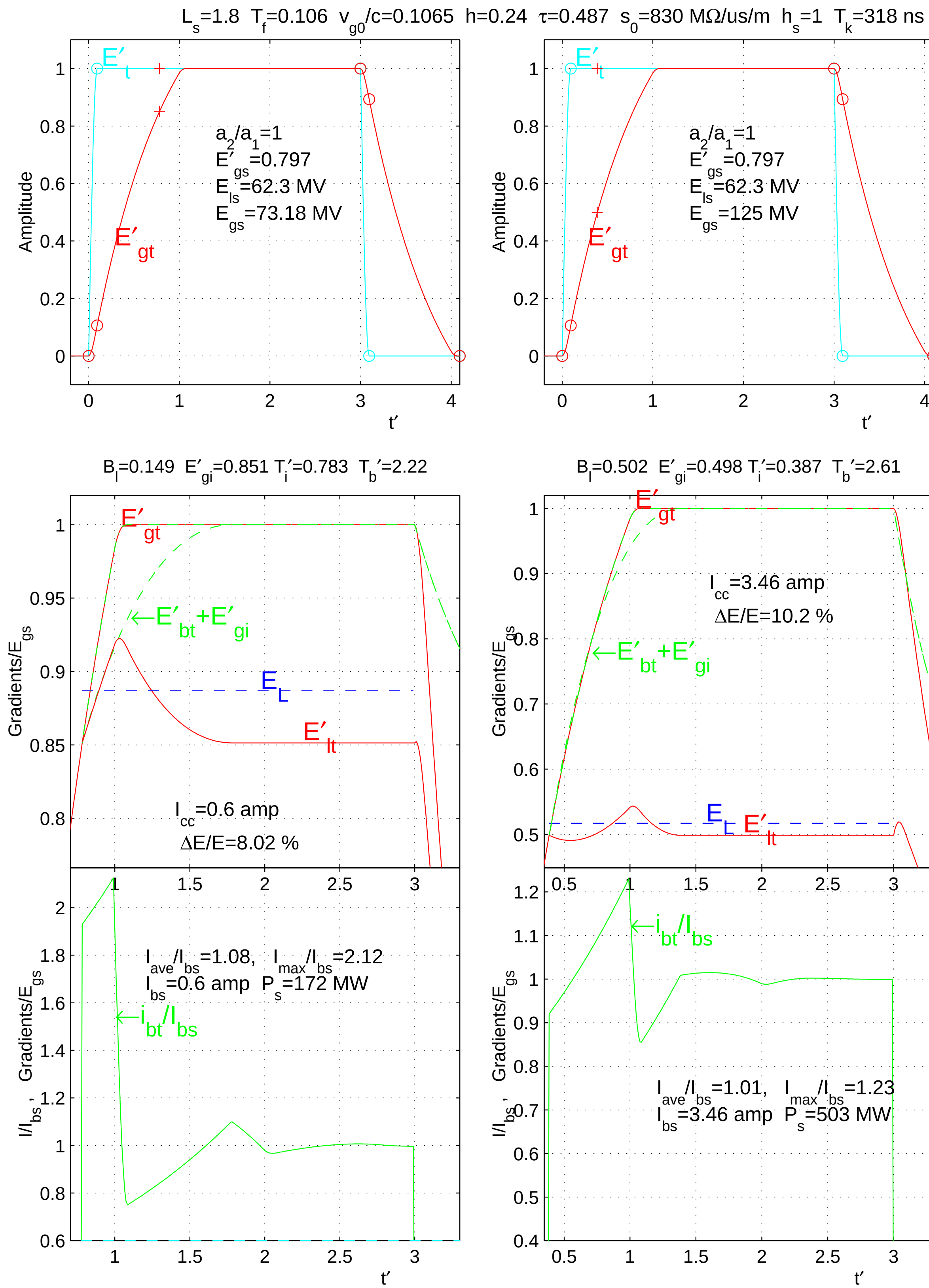

Fig. 6. Top: Normalized rf E_t and normalized avarage gradient E_{gt}/E_{gs} as a function of time.
Middle: No−load, current induced and loaded gradients vs time
Bottom: The current required to reduce the energy to zero.

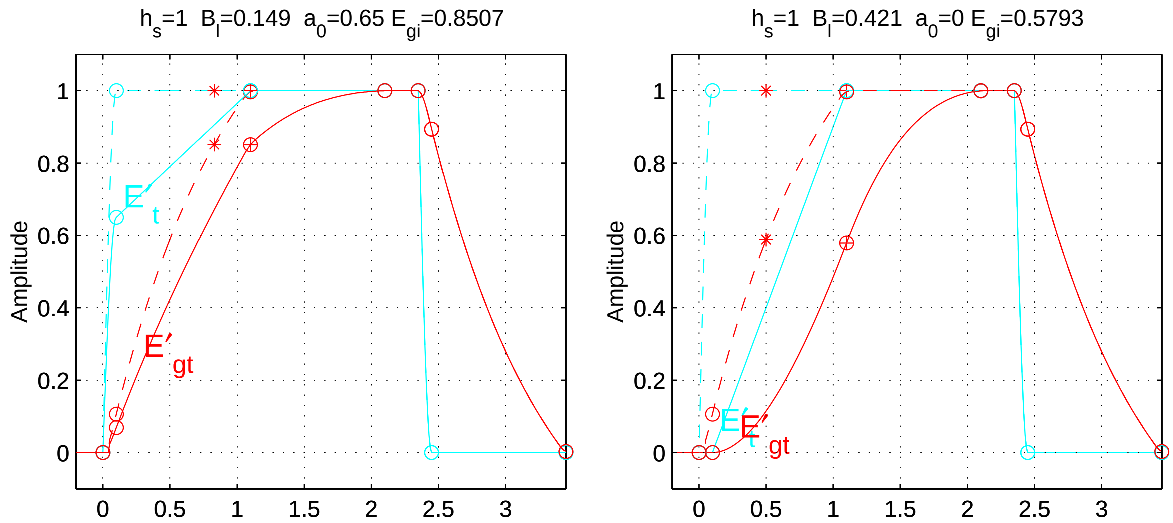

Fig. 7. Normalized rf E'_t and normalized avarege gradient E_{gt}/E_{gs} as a function of time. With ramp (solid) and without ramp (dash).

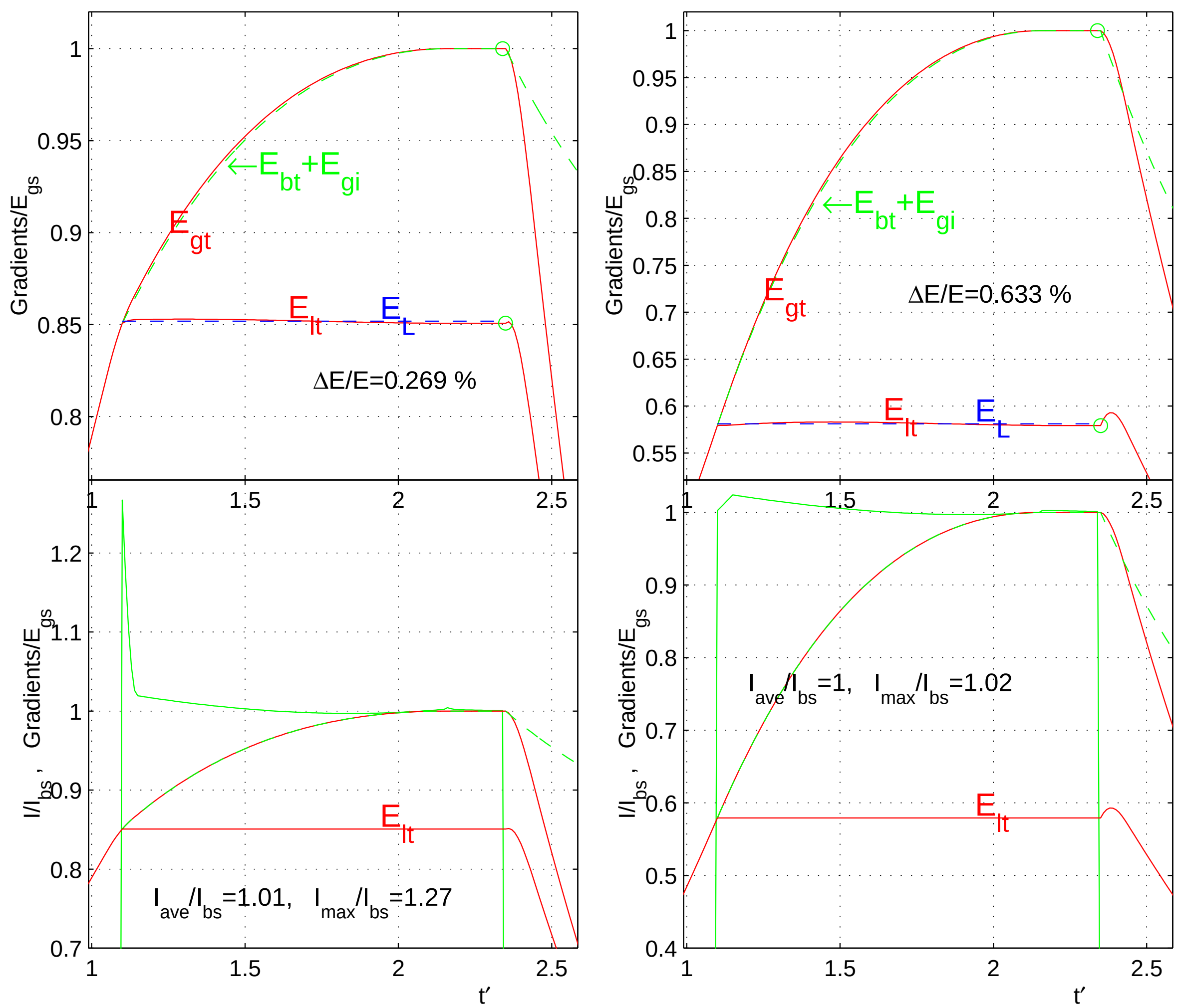

Fig. 8. Top: No−load, current induced and loaded gradients with ramp, vs time
Bottom: Same as top, but with variable current BLC.
Also plotted the current required to reduce the energy to zero.

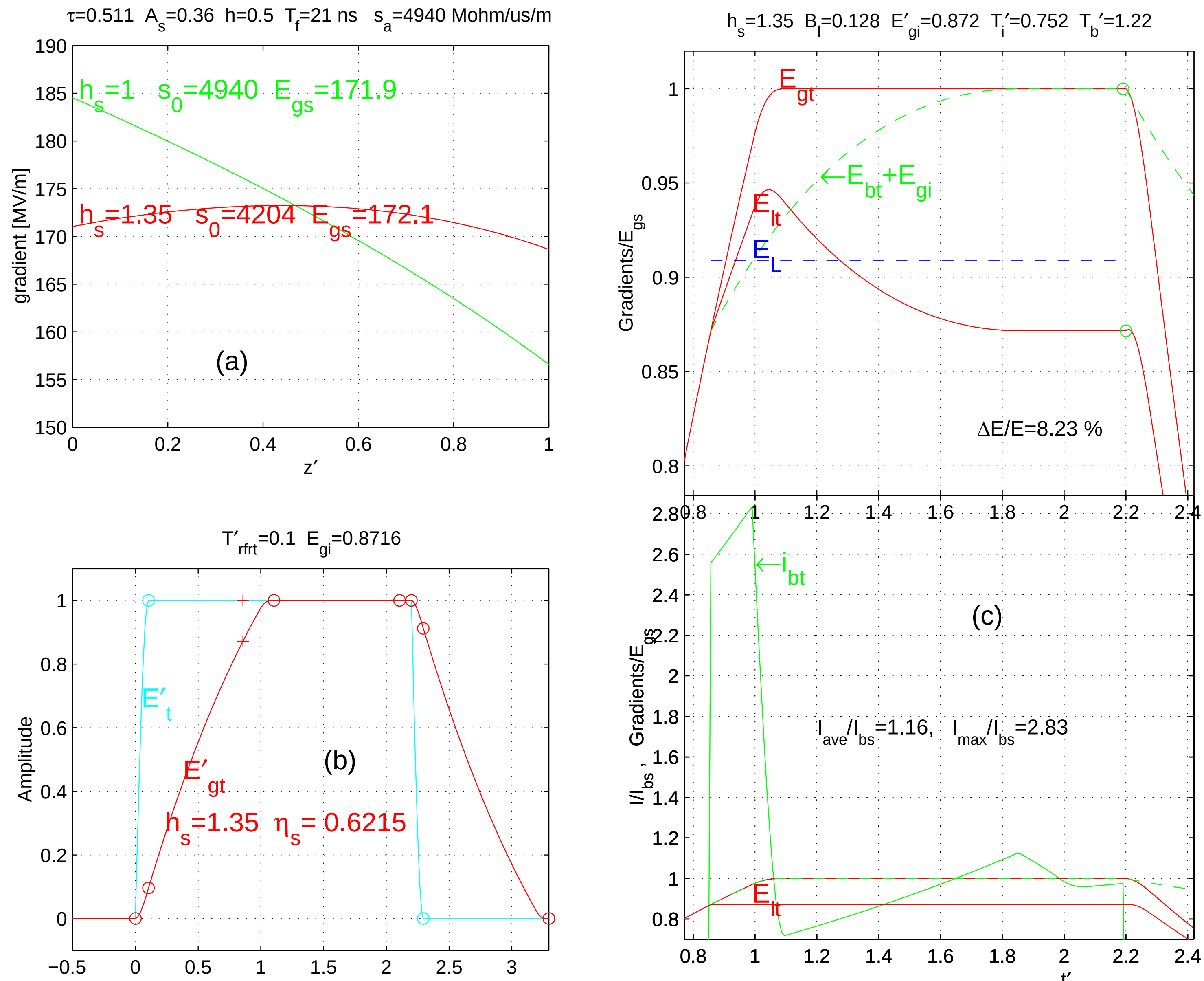

Fig. 9. a) Gradient as a function of z′, P_s=230 MW. b) Input field and average gradient vs time

c) No−load, current induced and loaded gradients. Top: constant current; bottom: required current

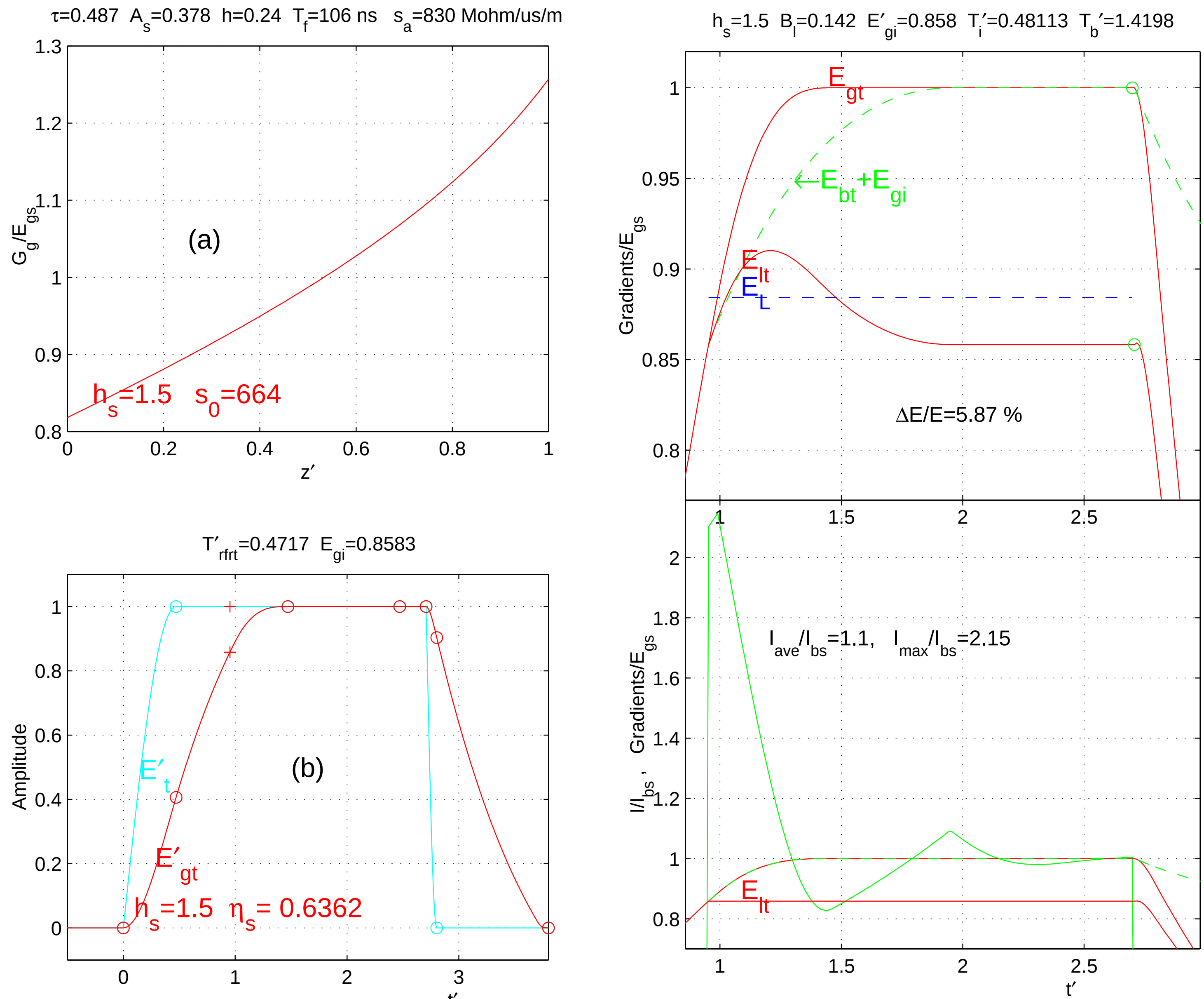

Fig. 10. a) Gradient as a function of z′ b) Input field and average gradient vs time
c) No−load, current induced and loaded gradients. Top: constant current; bottom: required current

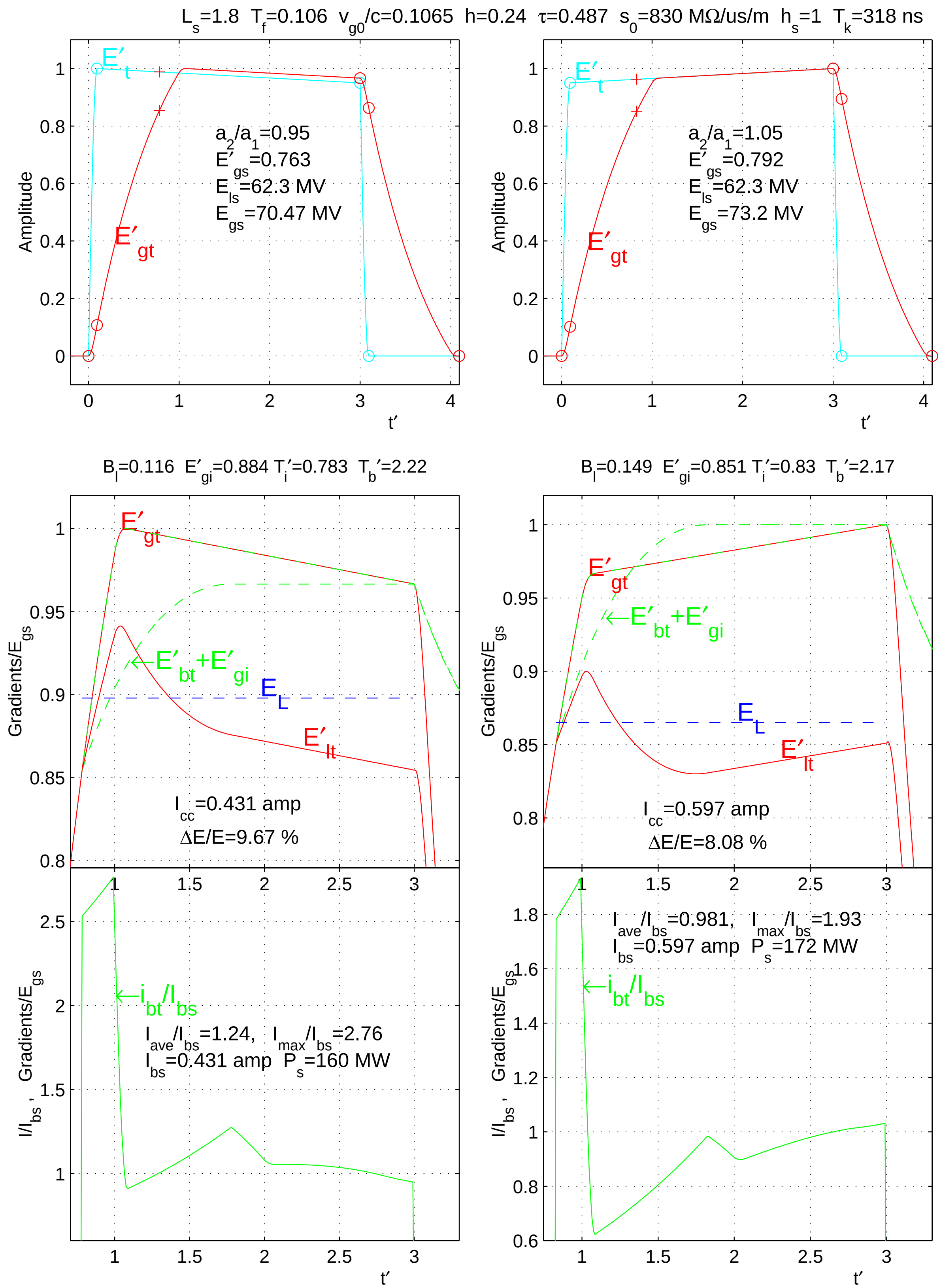

Fig. 11. Top: Normalized rf E_t and normalized avarage gradient E_{gt}/E_{gs} as a function of time.
Middle: No−load, current induced and loaded gradients vs time
Bottom: The current required to reduce the energy to zero.